Langmuir_probe_wsolutions

October 28, 2022

1 Langmuir probes measurement techniques

2 Plasma

Plasma is created when a gas is heated to such a degree that at least one of the electrons in an atom has been stripped free (ionized), leaving behind a positively charge nucleus, called an ion. Plasma is often referred to as the fourth state of matter; solid, liquid, and gas being the three others. Since plasma are made up free charge carriers (ions and electrons), it is subject to influence from electromagnetic fields and forces. Emerging electric fields become rampant everywhere meaning that the particles do not simply collide mechanically, but are affected by the electromagnetic fields between each other.

Plasma usually exist only in vacuum conditions. In air, plasma will cool and electrons and ions will easily recombine into neutral atoms. Plasma is the most abundant form of matter in the visible universe. E.g., plasmas are present in stars (Sun), the solar wind, the Earth's ionosphere, interstella space, nebulas.

On Earth, natural plasmas are less frequent but still present. They appear in lightning and auroras (ionosphere), or can be man made, by e.g., using electromagnetic fields to partly ionize a gas within a confined volume (Neon or fluorecent light, pixels of a plasma TV, nuclear fusion plasma reactors).

Because so much of the universe is made of plasma, its behaviour and properties are naturally of interest to scientists in many disicplines and even for industrial applications. Plasmas are e.g., essential for producing computer chips. Plasma diagnostic techniques are therefore an important field of study.

Within the 4DSpace research initiative at the University of Oslo, we use Langmuir probes to measure the plasma properties of the ionosphere.

2.1 Sheath

When you introduce a charged object into a plasma it will start to attracted particles of opposite charge. This builds a sheath of opposite charge around the object that balances out the introduced charge. The field of the negativly charged object will not extend beyond the edge of the sheath, and the negatively charge object is effectively **shielded** from the surrounding plasma.

In a plasma the Debye length (λ_D) is considered a fundamental plasma parameter and describes change in the local potential due to the presence of some charged object, and is defined as:

$\lambda_D = \sqrt{\frac{\epsilon_0 k T}{n q^2}}$

where ϵ_0 is the permittivity of free space, k is the Boltzmann constant, T is the particle temperature, n is the plasma density and q is the particle charge.

The Debye length is a measure for how far the electrostatic effect created by the introduced charge persist in the plasma. With each Debye length the introduced charge is increasingly electrically screened and the electric potential decreases in magnitude by $1/e \approx 0.368$.

2.2 Criteria to be called a plasma

To be defined as a plasma, an ionized gas must be quasineutral ($n_e \simeq n_i$) and exhibit a collective behaviour.

While a plasma can contain local concentrations of positive and negative charge which give rise to electric fields, quasineutral means that as a whole the plasma must contain an equal amount of negative and positive charge (on average); the net charge of the plasma is neutral.

Motion of charges also generates currents, and hence magnetic fields. These electric and magnetic fields are long range and affect particles even at large distances. Plasma is therefore a collection of charged particles that behave in a collective way because of attractive and repelling electric and magnetic forces.

A plasma must also have enough charged particles so that it can act as shield to an external potential. Its volume must be large enough to cover many Debye lengths in each direction.

3 Langmuir probe

A Langmuir probe is a device that can be used to determine the - **electron temperature (T_e)**, and the - **electron and ion density (N_e and N_i)** of a plasma.

It is the oldest and most often used technique to for low-temperature plasmas and was introduced by Irving Langmuir in the 1920's. In fact, Langmuir coined the term plasma (MOTT-SMITH, H. History of "Plasmas". Nature 233, 219 (1971). https://doi.org/10.1038/233219a0).

In its simplest form, it works by inserting an electrode into a plasma, applying a bias voltage to it relative to some reference potential, and then measure the current collected by the electrode.

The Langmuir probe has been extensively used for many years on rockets and satellites to perform in situ measurement of the plasma density and temperature. The University of Oslo as developed the multi-Needle Langmuir Probe (mNLP) system to achieved high resolution measurements of the electron density in the ionosphere [Jacobsen 2010], [Bekkeng 2010]. Before we introduce this multi-needle technqiue, we should however first study the basic measurement principle of a single Langmuir probe.

`Example of a cylindrical Langmuir probe manufactured by the University of Oslo.`

Langmuir probes usually take the shape of a spherical, cylindrical or planer object. We will focus on the cylindrical probe in this module.

3.1 Langmuir probe measurement technique

The principle measurement technique is to vary the bias voltage of the electrode and to measure the current collected by the electrode as a function of the applied voltage. In other words, to measure the current-voltage (I-V) characterisitc of the electrode, as seen in the figure.

The analysis of the I-V curve will then provide information about the plasma density and temperature. The theory was developed by Mott-Smith and Langmuir in the 1920's [Mott-Smith and Langmuir, The theory of collectors in gaseous discharges, Phys. Rev. 28, 727, 1926, https://doi.org/10.1103/PhysRev.28.727]. There is no general theory that applies to all probe geometries and orientations, plasma conditions, and spacecraft designs. Probes are therefore designed for special limiting cases. In this module we consider the orbital-motion-limited (OML) described in Mott-Smith and Langmuir 1926. For this condition, the probe radius must be smaller than the thickness of the sheath that surrounds it ($r_{probe} < \lambda_D$).

The material introduced in this module is for a cylindrical probe satisfying the OML theory - with probe radius smaller than the Debye length, and assumes an unmagnetized and collisionless Maxwellian plasma.

3.2 Thermal current

When there is no potential difference between the probe and the plasma the current to the probe is determined by the particle mean speed (according to a Maxwell-Boltzmann distributed population).

$$< v > = \sqrt{\frac{8kT}{\pi m}}$$

where m is the particle mass. Since the mass of an electron is much smaller than the mass of an ion, the thermal velocity of electrons is $\sqrt{(m_i/m_e)}$ larger than the ion, e.g., ~43 in the case of hydrogen. The thermal current to the probe is therefore dominated by the electron thermal current, which is given by the following expression:

$$I_{e,th} = \frac{1}{4} n_e \cdot q \cdot A \cdot < v >$$

$$I_{e,th} = n_e \cdot q \cdot A \cdot \sqrt{\frac{kT_e}{2\pi m_e}}$$

where A is the collecting surface area of the probe.

3.3 Floating potential (V_f)

Consider a neutral object immersed in a plasma. Since electrons are more mobile than ions (higher velocity due to small mass), more electrons than ions will collide with and be collected by the object. The object will therefore aquire a negative charge with respect to the ambient plasma. This will in turn start to attract positive ions and repel electrons, until eventually an equilibrium is reach where the electron current balances the ion current ($I_e = I_i$).

The floating potential V_f refers to the potential aquired by an unbiased object relative to the plasma at the equilibrium condition where the electron current is equal to the ion current ($I_e = I_i$). In space, the plasma potential is usually defined to be 0 and thus a spacecraft will aquire a negative floating potential with respect to the plasma.

3.4 Electron retardation region (T_e)

In the region where the probe potential is between the floating potential and the plasma potential, the probe potential is slightly negative relative to the plasma potential. In this region, only electrons with a sufficiently high thermal speed will be collected by the probe. Electrons with a lower thermal speed will be repelled. This is called the electron retardation region, and can be used to characterize the thermal distribution of the electrons, and thus also to determine the electron temperature T_e.

Since a probe immersed in a plasma will aquire a negative floating potential, a positive bias voltage must be applied to the probe to measure the electron current in this region.

As the potential difference between the probe and the plasma is reduced (increasing probe bias relative to V_f, the electron current will increase exponentially. As mentioned above, when reaching the plasma potential, all the thermal electrons with a direction towards the probe will be collected since no potential barrier exists between the probe and the plasma.

The electron current to the probe in retardation region is:

$I_{e,ret} = I_{e,th} \cdot e^{(qV/kT_e)}$

where V is the probe bias relative to the plasma potential.

At V=0, the potential difference between the probe and the plasma is 0 and there exist not sheth around the probe. The current to the probe is then equal to the thermal current $I_{e,th}$ as discussed above.

3.4.1 Finding T_e

The electron temperature can be determined from the slope of the $\ln(I_e)$

$ln(I_{e,ret}) = ln(I_{e,th}) + \frac{q}{KT_e}V$

where the slope S is:

$S = \frac{q}{KT_e}$

and

$T_e = \frac{q}{kS}$

3.5 Electron saturation region

Increasing the probe voltage beyond the plasma, electrons will be attracted to the probe and form a sheath around the probe to shield the positively charged probe from the plasma. While the electric field set up by the probe does not affect the plasma outside the of the sheat, electrons that enter the sheath will be accelerated towards the probe and be collected as an electron current. Ions on the other hand will be repelled by the sheath. As the bias voltage is increased further, the collection area of the sheat will increase. The electron current to the probe in this region is given by:

$I_{e,sat} = CI_{e,th} \left(1 + \frac{qV}{k_B T_e}\right)^{\beta}$

where where V is the probe bias relative to the plasma potential:

C : $\frac{2}{\sqrt{\pi}}$ (cylindrical), 1 (spherical)

β: 0 (planer), 0.5 (cylindrical), 1 (spherical)

3.6 Ion saturation region (N_i)

If negative voltage is applied to the object or probe, the ion current to the probe will increase, and eventually only ions wil be collected. This region of the I-V curve is referred to as the ion saturation region and can be used to determine the ion density. For ionospheric plasmas we are usually conserned about the electron density. This region is therefore of less interest for this module.

4 Practical exercise

This exercise we will demonstrate how you can determine the electron temperature and electron density by sweeping the bias voltage of a Langmuir probe immersed into a plasma.

The method is adopted from in J.-P. Lebreton et al., "The ISL Langmuir probe experiment processing onboard DEMETER: Scientific objectives, description and first results", Planetary and Space Science, Vol. 54, Issue 5, April 2006, Pages 472-486. https://doi.org/10.1016/j.pss.2005.10.017 Section 4. Data analysis method (4.1 - 4.3)

Step by step the following analysis is performed:

- You will start by producing and plotting a syntetic dataset generated using the an available Langmuir probe Python library.
- The sweep includes both the electron and ion current to the probe. In order to determine the electron temperature from the slope of $\ln(I_{e,ret})$, you first need to subtract the ion current. This can be achieved by deriving an approximate curve for the ion saturation.
- Then you will have to identify the electron retardation region by determining the floating potential and the plasma potential.
- The electron temperature can now be estimated from the slope of straight line that is fitted to natural logarithm of the electron current in the electron retardation region.
- Finally you will determine the electron density at the point where the probe is at the plasma potential and no sheet exist between the probe and the plasma.

```python
[1]: # Load relevant Python libraries
     import numpy as np
     import matplotlib.pyplot as plt
     import langmuir as lm
     import scipy.constants as sc
     from scipy.optimize import curve_fit
```

5 Lets generate the data

To validate the method we will use syntetic data. For this you can use the Langmuir probe python library by Sigvald Marholm and Diako Darian

Documentation: https://langmuir.readthedocs.io/en/latest/index.html

Github: https://github.com/langmuirproject/langmuir

The Langmuir library contains a collection of functions that compute the current collected by a conductor immersed in a plasma according to various models (characteristics). These functions take as arguments the probe geometry, for instance Sphere, and a plasma, where a plasma may consist of one or more Species.

```python
[2]: # Example

     # The OML current from a cylindrical probe biased at 5 V with radius 0.255 mm
     ↪and length 2.5 mm immersed in a plasma with a density of
```

```
# n_e = n_i = 1e11 m^3, and  Temperature of T = 1000 K, can be calculated as␣
  ↪shown below.



# Probe dimensions
probe_radius = 0.255e-3
probe_length = 2.5e-3
probe_area = 2 * np.pi * probe_radius * probe_length

# For simplicity lets assume Te = Ti = T and Ne = Ni = N
N = 1e11
T = 1200

# Define the geometry of the cylinder
probe_geometry = lm.Cylinder(r=probe_radius, l=probe_length)

# Define the plasma as list of Electrons and Argon ions
plasma = []
plasma.append(lm.Electron(n=N, T=T))
plasma.append(lm.Argon(n=N,T=800))

# Choose a probe bias of 5 V
probe_bias = 5

# Calculate the electron current to the probe, assuming the V=5 is well in to␣
  ↪the electron saturation region
Ie = lm.OML_current(probe_geometry, plasma, V=probe_bias)


print("Electron current: {:.2e} A at probe bias of {:n} V.".
  ↪format(Ie,probe_bias))
```

```
Electron current: -2.74e-08 A at probe bias of 5 V.
```

5.1   EX1 - Does the probe geometry satisfy the OML theory?

Determine if the above specifed probe geometry satisfies the OML theory.

```
[3]: # Solution EX1

# Calculate the debye length
def debye_length(T,n):
    return np.sqrt((sc.epsilon_0*T*sc.k)/(n*sc.e**2))

# Typical values for the electron density in the ionosphere is 1e8 - 1e12 m^3.
# Since the OML theory requires  the probe radius < debye length,
```

6

```
# we consider only the high end of the electron density. We use typical low end␣
 ↪value of T=1000K.



dl = debye_length(1000,1e12)
ratio = dl / 0.255e-3

print("Debye length: {:.2f} mm".format(dl*1e3))
print("Probe radius: {:.2f} mm".format(probe_radius*1e3))
print("Probe radius / Debye length: {:.1f} mm".format(ratio))



# We see that the probe radius is smaller than the Debye length. The probe␣
 ↪geometry therefore
# satisfies the OML theory.
```

```
Debye length: 2.18 mm
Probe radius: 0.26 mm
Probe radius / Debye length: 8.6 mm
```

5.2 EX2 - Generate sweep data

Generate and plot data for a bias sweep from -10 to +10 V with the probe geometry and plasma conditions as specified above

```
[4]:  # Solution EX2

      # Create a numpy array from -10 to 10 in steps of 5 mV
      probe_bias = np.arange(-10,10,0.005)

      # Use the OML_current function to calculate the corresponding currents. It
      # returns a negative current for electrons. For space plasmas the electron␣
       ↪current
      # is usually defined as positive.
      probe_current = -lm.OML_current(probe_geometry,
                                       plasma,
                                       V=probe_bias)

      # Plot the sweep data
      fig = plt.figure(figsize=(8,4))
      ax = fig.add_subplot(121)
      ax_log = fig.add_subplot(122)


      ax.set_xlabel("Probe bias voltage [V]")
      ax.set_ylabel("Probe current [nA]")
      ax.grid()
```
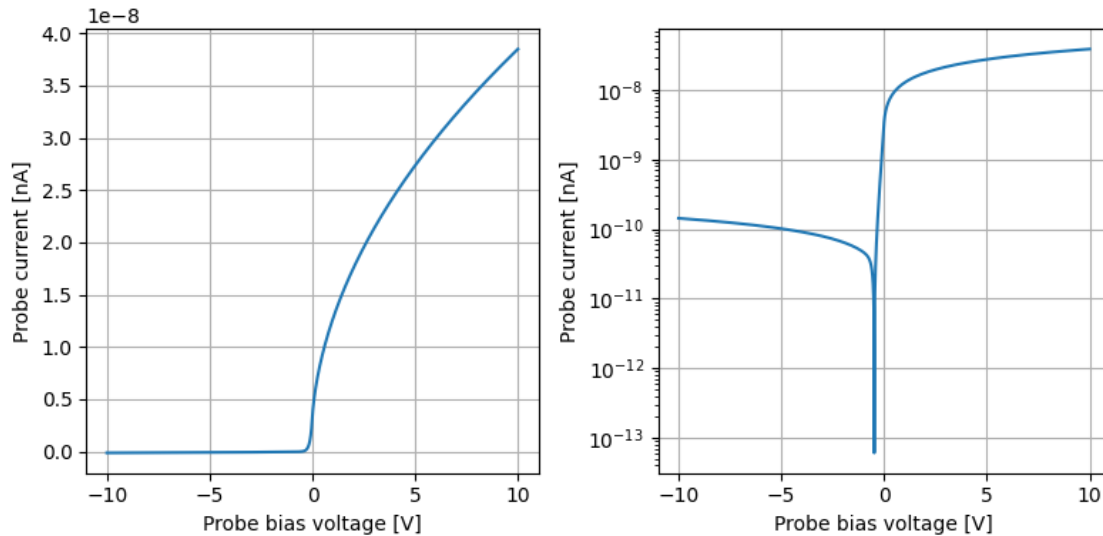
```
# For the log-plot we use absolute values
ax_log.set_xlabel("Probe bias voltage [V]")
ax_log.set_ylabel("Probe current [nA]")
ax_log.grid()
ax.plot(probe_bias,probe_current);
ax_log.plot(probe_bias, np.abs(probe_current))
ax_log.set_yscale("log")
fig.tight_layout()
```


6 EX3: Estimate fit for ion saturation current

To determine the electron temperature we need remove the contribution from the ion current to the total current. Only then we can extract the electron temperature by fitting a straight line to the natural log of the electron current in the electron retardation region.

Fit a straight line to the ion saturation region and subract the result from the total probe current.

```
[5]: # Solution EX3

# The ion saturation region can be approximated to a straight line
def straight_line(x, slope, b):
    return slope * x + b


# Limit to ion sat. I.e., get indecies for probe_biases lower than a given␣
 ↪voltage
isat_idx = probe_bias < -6
```

```python
# Fit a straight line to the probe current below the specified voltage
[slope, b], pcov = curve_fit(straight_line, probe_bias[isat_idx],
 ↪probe_current[isat_idx] )

# Calculate the probe ion current for all probe biases
probe_current_i = straight_line(slope,probe_bias,b)


# Visualize the fit to determine how well it works
fig = plt.figure(figsize=(8,4))
ax = fig.add_subplot(111)

# Plot the result, iterate to find the best fit
ax.set_xlabel("Probe bias voltage [V]")
ax.set_ylabel("Probe current [A]")
ax.grid()

ax.plot(probe_bias, probe_current,label="$I_{tot}$")
ax.plot(probe_bias, probe_current_i,label="$I_i$")
# Zoom into region of interest
ax.set_ylim(-3e-10,3e-10)
ax.set_xlim(-10,1)


# Calculate the electron current by subtracting the ion contribution from the
 ↪total current
probe_current_e = probe_current - probe_current_i
# Plot the electron current
ax.plot(probe_bias, probe_current_e,label="$I_e$")
ax.legend()
```
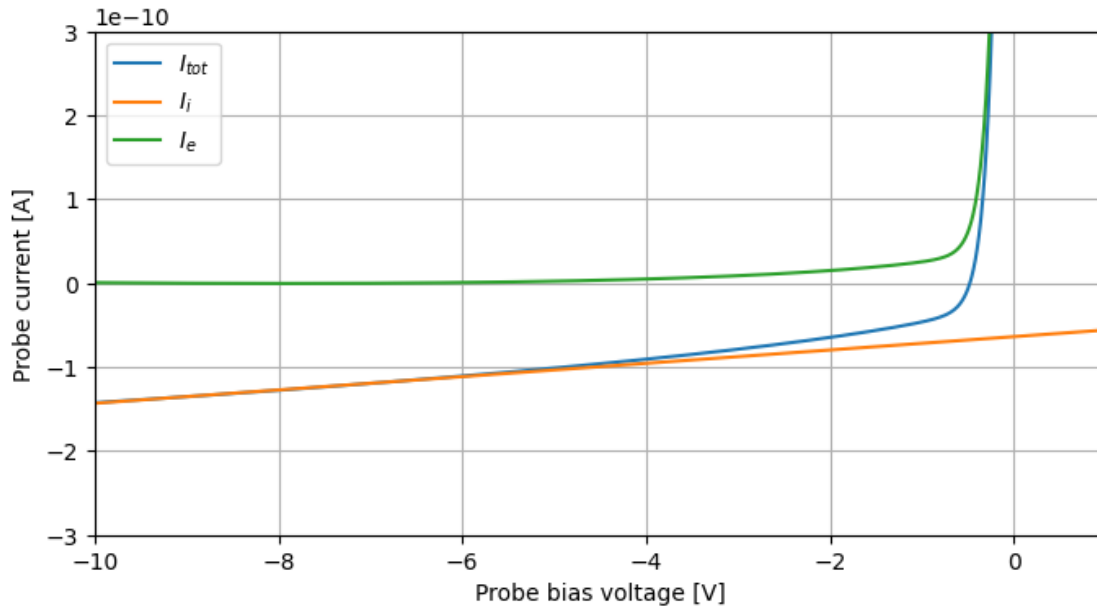
[5]: <matplotlib.legend.Legend at 0x12bcf5360>

6.1 EX4: Estimate the floating potential

To identify the electron retardation region we need to first identify the floating potential and the plasma potential.

An estimate for the floating potential can be found by finding voltage at which the total current is 0, or if we take the absolute value of the current, the voltage at which the current is the lowest.

```
[6]: # Solution EX4

floating_potential_idx = np.abs(probe_current).argmin()
floating_potential = probe_bias[floating_potential_idx]
print("The floating potential is: {:.2f} V".format(floating_potential))
```

The floating potential is: -0.48 V

6.2 EX5: Estimate the plasma potential

The floating potential can be found at the transition point between the electron retardation and saturation region. At this point the current changes rapidly, and we can find this point by identify the point at which the derivativ of the electron current is at its maximum: max (dIe / dV)

Tips: use np.gradient to find the delta of a data series

```
[7]: # Solution EX5
# In space we usually define the plasma potential to be zero.

probe_current_e_gradient = np.gradient(probe_current_e)/np.gradient(probe_bias)
```

```
plasma_potential_idx = probe_current_e_gradient.argmax()
plasma_potential = probe_bias[plasma_potential_idx]
print("Plasma potential is: {:.6} V".format(plasma_potential))


# visualize the gradient
fig = plt.figure(figsize=(8,4))
ax = fig.add_subplot(111)
ax2 = ax.twinx()

# Plot the result, iterate to find the best fit
ax.set_xlabel("Probe bias voltage [V]")
ax.set_ylabel("Probe current [A]")
ax.grid()

le = ax.plot(probe_bias, probe_current,"b-",label="$I_e$")
lg = ax2.plot(probe_bias, probe_current_e_gradient,"r:", label="$dI_e/dV$")
lns = le + lg
labs = [l.get_label() for l in lns ]
ax2.legend(lns, labs, loc=0)

#ax.set_ylim(-1e-10,5e-11)
ax.set_xlim(-1.6,1.6)
```
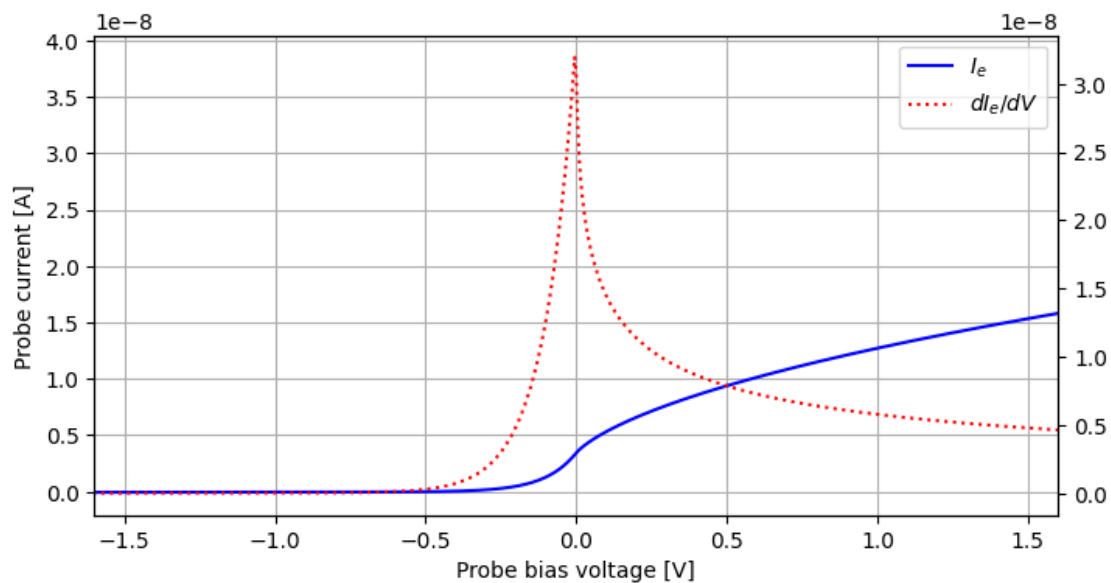
Plasma potential is: -0.005 V

[7]: (-1.6, 1.6)

6.3 EX6: Estimate the electron temperature

The electron temperature can be estimated by fitting a straight line to the electron retardation region between the floating potential and the plasma potential.

Plot the line fit to get a feel for how well it fits. Adjust Vf and Vplasma manually if needed.

```
[8]: # Solution EX6

     # Extract only the data from the retardation region
     manual_adjustment_vf = 0.2
     manual_adjustment_vp = 0

     # Finde the np array indicies for the probe biases that lies in the electron⌴
      ↪retardation region, i.e.
     retardation_idx = (probe_bias > floating_potential + manual_adjustment_vf)  &⌴
      ↪(probe_bias < plasma_potential+manual_adjustment_vp)


     # Extract the ln of the probe electron current in the retardation region
     probe_current_ret = np.log(probe_current_e[retardation_idx])
     probe_bias_ret = probe_bias[retardation_idx]



     # Fit a straight line to the ln(Ie)
     [slope_ret, b], pcov = curve_fit(straight_line,probe_bias_ret,⌴
      ↪probe_current_ret)

     # Calculate the fit line
     fit_line = slope_ret * probe_bias + b

     # Visually check the fit and iterate to find the best fit
     fig = plt.figure(figsize=(8,4))
     ax = fig.add_subplot(111)


     # Plot the result, iterate to find the best fit
     ax.set_xlabel("Probe bias voltage [V]")
     ax.set_ylabel("Probe current [ln(nA)]")
     ax.grid()

     ax.plot(probe_bias, np.log(np.abs(probe_current_e)),"b-",label="ln($I_e$)")
     ax.plot(probe_bias, fit_line,"r:", label="y=ax+b")
     ax.set_xlim(-1,0.5)
     ax.set_ylim(-30,-18)

     ax.axvline(x = floating_potential, color="grey",linestyle="--",label="$V_f$")
```

```
ax.axvline(x = plasma_potential, color="grey",linestyle="-.
 ↪",label="$V_{plasma}$")

ax.legend()


Te_calc = sc.e / (sc.k * slope_ret)

print("Calculated electron temperature is: {:.2f} K".format(Te_calc))
print("True electron temperature is: {:.2f} K".format(T))
```
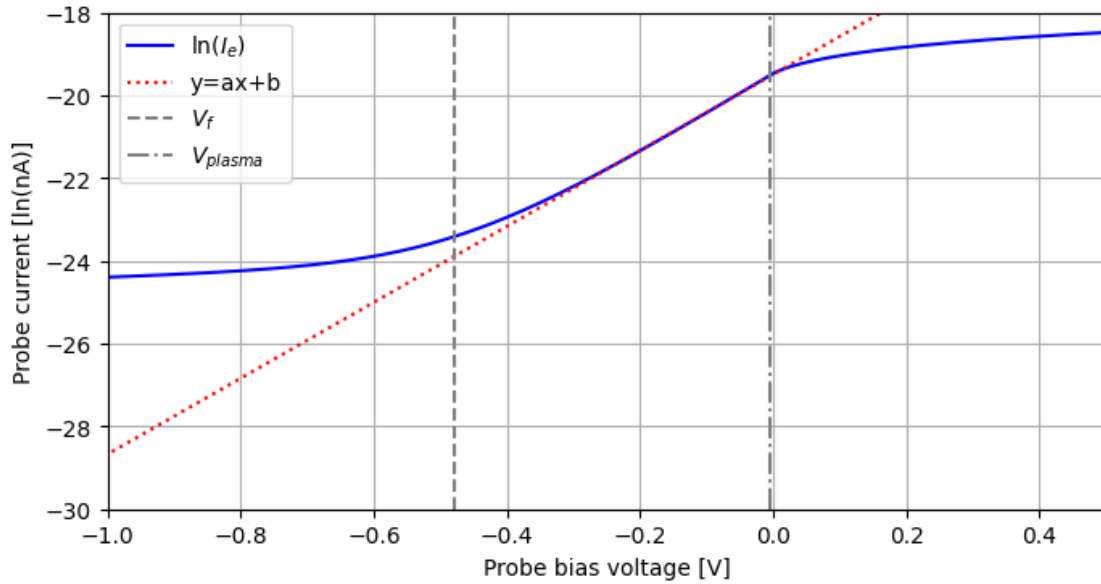
```
Calculated electron temperature is: 1264.05 K
True electron temperature is: 1200.00 K
```


6.4 EX7: Estimate the electron density

Estimate the electron density in the point where no sheet exist between the probe and the plasma. This is the point when only the thermal electrons contribute. (V = Vbias - Vplasma = 0)

6.5 Electron density

The last step is to calculat the electron density.

$$I_e = N_e A_e q \sqrt{\frac{k_B T_e}{2\pi m_e}} e^{qV/k_B T_e}$$

In the equation above, when the plasma potential, $V = 0$, the second term is equal to zero, and the electron density can be obtained by the following formula:

$$I_e(V = 0) = N_e A_e q \sqrt{\frac{k_B T_e}{2\pi m_e}}$$

$$N_e = \frac{I_e(V=0)}{qA_e} \frac{1}{\sqrt{\frac{k_B T_e}{2\pi m_e}}}$$

$$N_e = \frac{I_e(V=0)}{qA_e} \sqrt{\frac{2\pi m_e}{k_B T_e}}$$

```python
[9]: # The last step is to calculate the electron density

     # Find index where V = 0, Vbias-Vplasma = 0)
     # Convert np array to absolute values and use .argmin() to find the index of␣
      ↪the lowest value
     i_vp_idx = np.abs(probe_bias - plasma_potential).argmin()
     # Get the lowest value
     i_vp = probe_current_e[i_vp_idx]

     # Calculate Ne
     C = (np.sqrt( (2*np.pi*sc.m_e) / (sc.k*Te_calc) ))/( sc.e * probe_area)
     Ne = i_vp * C


     print("Calucalated Ne: {0:.4e}".format(Ne))
     print("True Ne: {0:.4e}".format(N))
```

```
Calucalated Ne: 9.4321e+10
True Ne: 1.0000e+11
```

6.6 Fixed bias approach (m-NLP)

This method is requires that the transition from the electron retardation to electron saturation region is nicely defined with this sharp knee. This is however not always the case with real data, and the knee may be hard to locate.

The fixed bias approach is an alternative method that can be used to calculate the electron density from the electron saturation region without the need to know neither the electron temperature nor the plasma potential.

It is based on measuring the current at two or more voltages in the electron retardation region. The electron density can then be calculated by dividing the difference between the two squared currents by the difference between the two bias voltages, multiply by a constant, and then taking the square root of the results.

$$N_e = \sqrt{K \frac{\Delta(I_e^2)}{\Delta V}}$$

where

$$K = \frac{m_e}{2q(2qrl)^2}$$

The method is described in [Jacobsen 2010] and [Bekkeng 2010], and is named the multi-Needle Langmuir Probe approach (m-NLP). The derivation is shown below:

$$I_e = I_{e,th} \frac{2}{\sqrt{\pi}} \sqrt{1 + \frac{qV}{k_B T_e}}$$

$$I_e^2 = I_{e,th}^2 \frac{4}{\pi} \left(1 + \frac{qV}{k_B T_e}\right)$$

$$I_e^2 = I_{e,th}^2 \frac{4}{\pi} + I_{e,th}^2 \frac{4}{\pi} \frac{qV}{k_B T_e}$$

$$I_{e2}^2 - I_{e1}^2 = \left(I_{e,th}^2 \frac{4}{\pi} + I_{e,th}^2 \frac{4}{\pi} \frac{qV_2}{k_B T_e}\right) - \left(I_{e,th}^2 \frac{4}{\pi} + I_{eo}^2 \frac{4}{\pi} \frac{qV_1}{k_B T_e}\right)$$

$$I_{e2}^2 - I_{e1}^2 = I_{e,th}^2 \frac{4}{\pi} \frac{qV_2}{k_B T_e} - I_{e,th}^2 \frac{4}{\pi} \frac{qV_1}{k_B T_e}$$

$$I_{e2}^2 - I_{e1}^2 = I_{e,th}^2 \frac{4}{\pi} \frac{q}{k_B T_e} (V_2 - V_1)$$

$$\Delta(I_e^2) = I_{e,th}^2 \frac{4}{\pi} \frac{q}{k_B T_e} \Delta V$$

$$\Delta(I_e^2) = \left(N_e q 2\pi r l \sqrt{\frac{k_B T_e}{2\pi m_e}}\right)^2 \frac{4}{\pi} \frac{q}{k_B T_e} \Delta V$$

$$\Delta(I_e^2) = (N_e q 2\pi r l)^2 \frac{k_B T_e}{2\pi m_e} \frac{4}{\pi} \frac{q}{k_B T_e} \Delta V$$

$$\Delta(I_e^2) = (N_e q 2 r l)^2 \frac{2q}{m_e} \Delta V$$

$$N_e = \sqrt{\frac{m_e}{2q(2qrl)^2} \frac{\Delta(I_e^2)}{\Delta V}}$$

$$N_e = \sqrt{K \frac{\Delta(I_e^2)}{\Delta V}}$$

6.7 EX8: Estimate the electron density using the mNLP approach

```
[10]: # Solution EX8


      # Find to value pairs of I^2 and V for the electron current in the electron
      # saturation region

      V1 = probe_bias[2000]
      V2 = probe_bias[2500]

      I1_squared = probe_current_e[2000]**2
      I2_squared = probe_current_e[2500]**2

      dI_squared = I2_squared - I1_squared
      dV = V2 - V1

      K = sc.m_e / ( (2*sc.e*probe_radius*probe_length)**2 * 2 * sc.e)

      Ne_mnlp = np.sqrt(K * dI_squared/dV)

      print("Calucalated Ne (mNLP): {0:.4e}".format(Ne_mnlp))
      print("True Ne: {0:.4e}".format(N))
```

```
Calucalated Ne (mNLP): 1.0061e+11
True Ne: 1.0000e+11
```

This technique can also be realised by taking a linear fit to a current collected by a few probes biased at different potentials in the electron saturation regime.

$$I_e^2 = I_{e,th}^2 \frac{4}{\pi} + I_{e,th}^2 \frac{4}{\pi} \frac{qV}{k_B T_e}$$

$$y = b + ax$$

$$a = I_{e,th}^2 \frac{4}{\pi} \frac{qV}{k_B T_e} = (N_e q A \sqrt{\frac{k_B T_e}{2\pi m_e}})^2 \frac{4}{\pi} \frac{qV}{k_B T_e} = \frac{2(N_e q A)^2 q}{\pi^2 m_e}$$

$$N_e = \sqrt{\frac{a\pi^2 \cdot m_e}{2q^3 A^2}}$$

```
[11]:   # Adding two additional potentials and squared currents
        V3 = probe_bias[3000]
        V4 = probe_bias[3000]

        I3_squared = probe_current_e[3500]**2
        I4_squared = probe_current_e[3500]**2


        V = [V1, V2, V3, V4]
        I = [I1_squared,I4_squared,I4_squared,I4_squared]

        # Fit a straight line and extract the slope a
        [a,b],pcov = curve_fit(straight_line, V, I)


        # Calculate density
        Ne_mnlp_fit = np.sqrt(a*sc.m_e *np.pi**2 / (2*sc.e**3 * probe_area**2))

        print("Calucalated Ne (mNLP-fit): {0:.4e}".format(Ne_mnlp_fit))
        print("True Ne: {0:.4e}".format(N))
```

```
Calucalated Ne (mNLP-fit): 1.1694e+11
True Ne: 1.0000e+11
```