

This is a thesis manuscript of Mariia Legenkaia, cited in *Uncertainties in signal recovery from heterogeneous and convoluted time series with principal component analysis* by Mariia Legenkaia, Laurent Bourdieu, and Rémi Monasson.

There are some differences in the notation between the article and the manuscript. The model defined in the equation (6) of the article,

$$s_{i,t} = \sum_{\tau} (F_{\tau} + \delta F_{i,\tau}) \sum_k \left(x_{t-\tau}^{(k)} + \delta x_{t-\tau}^{(k)} \right) e_i^{(k)} + z_{i,t}$$

and convoluted later with the smoothing kernel in equation (7),

$$\bar{s}_{i,t} = \sum_{\tau'} G_{\tau'} s_{i,t-\tau'} , \quad (1)$$

corresponds to the model defined in the eq.(2.37) of the manuscript:

$$s_{i,t} = \sum_{\tau_2} G_{t-\tau_2} \left[\sum_k \sum_{\tau_1} (F_{\tau_2-\tau_1} + \delta F_{i,\tau_2-\tau_1}) \left(x_{\tau_1}^{(k)} + \delta x_{\tau_1}^{(k)} \right) e_i^{(k)} + z_{i,\tau_2} \right] . \quad (2)$$

The expression for the ground-state energy in the equation (18) of the article corresponds to the equation with the following changes of notation:

Order parameters appearing in the ground-state energy E_{GS} .

Notation in the article	Definition	Notation in the manuscript
$R^{(k,k')}$	$\frac{1}{N} \sum_i [\langle v_i^{(k)} \rangle] e_i^{(k')}$	$R^{(k,k')}$
$W^{(k,k')}$	$\frac{\beta}{N} \sum_i \bar{\sigma}_i^2 [\langle v_i^{(k)} v_i^{(k')} \rangle - \langle v_i^{(k)} \rangle \langle v_i^{(k')} \rangle]$	$W^{(k,k')}$
$M^{(k,k')}$	$\frac{1}{N} \sum_i \bar{\sigma}_i^2 [\langle v_i^{(k)} \rangle \langle v_i^{(k')} \rangle]$	$L^{(k,k')}$
$v^{(k,k',\ell,\ell')}$	$\frac{\beta}{N} \sum_i e_i^{(k)} e_i^{(k')} [\langle v_i^{(\ell)} v_i^{(\ell')} \rangle - \langle v_i^{(\ell)} \rangle \langle v_i^{(\ell')} \rangle]$	$v^{(k,k',\ell,\ell')}$
$q^{(k,k',\ell,\ell')}$	$\frac{1}{N} \sum_i e_i^{(k)} e_i^{(k')} [\langle v_i^{(\ell)} \rangle \langle v_i^{(\ell')} \rangle]$	$r^{(k,k',\ell,\ell')}$

Conjugated parameters appearing in the ground-state energy.

Notation in the article	Conjugated to	Notation in the manuscript
$\hat{R}^{(k,k')}$	$R^{(k,k')}$	$\hat{R}^{(k,k')}$
$\hat{W}^{(k,k')}$	$W^{(k,k')}$	$\tilde{W}^{(k,k')}$
$\hat{M}^{(k,k')}$	$M^{(k,k')}$	$S^{(k,k')}$
$\hat{v}^{(k,k',\ell,\ell')}$	$v^{(k,k',\ell,\ell')}$	$\tilde{v}^{(k,k',\ell,\ell')}$
$\hat{q}^{(k,k',\ell,\ell')}$	$q^{(k,k',\ell,\ell')}$	$p^{(k,k',\ell,\ell')}$
$\hat{U}^{(k,k')}$	normalization and orthogonality of $\{\mathbf{v}^{(k)}, k = 1, \dots, K\}$	$U^{(k,k')}$