Avaliação 03 de Álgebra Linear

30. [**M**] Let $A = \begin{bmatrix} -6 & 28 & 21 \\ 4 & -15 & -12 \\ -8 & a & 25 \end{bmatrix}$. For each value of a in the set $\{32, 31.9, 31.8, 32.1, 32.2\}$, compute the characteristic polynomial of A and the eigenvalues. In each case, create a graph of the characteristic polynomial $p(t) = \det(A - tI)$ for $0 \leq t \leq 3$. If possible, construct all graphs on one coordinate system. Describe how the graphs reveal the changes in the eigenvalues as a changes.

33. [M] Generate random vectors $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{v}$ in $\mathbb{R}^4$ with integer entries (and $\mathbf{v} \neq \mathbf{0}$), and compute the quantities

$$\left(\frac{\mathbf{x}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v},\ \left(\frac{\mathbf{y}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v},\ \frac{(\mathbf{x}+\mathbf{y})\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\mathbf{v},\ \frac{(10\mathbf{x})\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\mathbf{v}$$

Repeat the computations with new random vectors $\mathbf{x}$ and $\mathbf{y}$. What do you conjecture about the mapping $\mathbf{x} \mapsto T(\mathbf{x}) = \left(\frac{\mathbf{x}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v}$ (for $\mathbf{v} \neq \mathbf{0}$)? Verify your conjecture algebraically.

7. A certain experiment produces the data $(1, 1.8)$, $(2, 2.7)$, $(3, 3.4)$, $(4, 3.8)$, $(5, 3.9)$. Describe the model that produces a least-squares fit of these points by a function of the form

$$y = \beta_1 x + \beta_2 x^2$$

Such a function might arise, for example, as the revenue from the sale of x units of a product, when the amount offered for sale affects the price to be set for the product.

a. Give the design matrix, the observation vector, and the unknown parameter vector.

b. [**M**] Find the associated least-squares curve for the data.

25. [**M**] Use a matrix program to diagonalize

$$A = \begin{bmatrix} -3 & -2 & 0 \\ 14 & 7 & -1 \\ -6 & -3 & 1 \end{bmatrix}$$

if possible. Use the eigenvalue command to create the diagonal matrix D. If the program has a command that produces eigenvectors, use it to create an invertible matrix P. Then compute $AP - PD$ and PDP^{-1}. Discuss your results.

26. [**M**] Repeat Exercise 25 for $A = \begin{bmatrix} -8 & 5 & -2 & 0 \\ -5 & 2 & 1 & -2 \\ 10 & -8 & 6 & -3 \\ 3 & -2 & 1 & 0 \end{bmatrix}$.

33. [**M**] According to Theorem 11, a linearly independent set $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$ in $\mathbb{R}^n$ can be expanded to a basis for $\mathbb{R}^n$. One way to do this is to create $A = [\,\mathbf{v}_1 \;\; \cdots \;\; \mathbf{v}_k \;\; \mathbf{e}_1 \;\; \cdots \;\; \mathbf{e}_n\,]$, with $\mathbf{e}_1, \dots, \mathbf{e}_n$ the columns of the identity matrix; the pivot columns of A form a basis for $\mathbb{R}^n$.

a. Use the method described to extend the following vectors to a basis for $\mathbb{R}^5$:

$$\mathbf{v}_1 = \begin{bmatrix} -9 \\ -7 \\ 8 \\ -5 \\ 7 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 9 \\ 4 \\ 1 \\ 6 \\ -7 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 6 \\ 7 \\ -8 \\ 5 \\ -7 \end{bmatrix}$$

b. Explain why the method works in general: Why are the original vectors $\mathbf{v}_1, \dots, \mathbf{v}_k$ included in the basis found for Col A? Why is Col $A = \mathbb{R}^n$?

8. The length $|\mathbf{u}|$ (magnitude) of a vector $\mathbf{u} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is given by $|\mathbf{u}| = \sqrt{x^2 + y^2 + z^2}$. Given the vector $\mathbf{u} = 23.5\mathbf{i} - 17\mathbf{j} + 6\mathbf{k}$, determine its length two ways:
   (*a*) Define the vector in MATLAB, and then write a mathematical expression that uses the components of the vector.
   (*b*) Define the vector in MATLAB, then use element-by element operations to create a new vector with elements that are the squares of the elements of the original vector. Then use MATLAB built-in functions `sum` and `sqrt` to calculate the length. All of these steps can be written in one command.

11. Two vectors are given:

$$\mathbf{u} = -3\mathbf{i} + 8\mathbf{j} - 2\mathbf{k} \quad \text{and} \quad \mathbf{v} = 6.5\mathbf{i} - 5\mathbf{j} - 4\mathbf{k}$$

Use MATLAB to calculate the dot product $\mathbf{u} \cdot \mathbf{v}$ of the vectors in three ways:

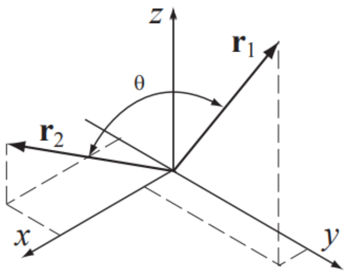
(*a*) Write an expression using element-by-element calculation and the MATLAB built-in function `sum`.

(*b*) Define **u** as a row vector and **v** as a column vector, and then use matrix multiplication.

(*c*) Use the MATLAB built-in function `dot`.

18. The dot product can be used for determining the angle between two vectors:

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{|\mathbf{r}_1||\mathbf{r}_2|}\right)$$



Use MATLAB's built-in functions `cosd`, `sqrt`, and `dot` to find the angle (in degrees) between $\mathbf{r}_1 = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{r}_2 = 1\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$.
Recall that $|\mathbf{r}| = \sqrt{\mathbf{r} \cdot \mathbf{r}}$.

31. Solve the following system of three linear equations:

$$
\begin{aligned}
3x - 2y + 5z &= 7.5 \\
-4.5 + 2y + 3z &= 5.5 \\
5x + y - 2.5z &= 4.5
\end{aligned}
$$

32. Solve the following system of five linear equations:

$$
\begin{aligned}
3u + 1.5v + w + 0.5x + 4y &= -11.75 \\
-2u + v + 4w - 3.5x + 2y &= 19 \\
6u - 3v + 2w + 2.5x + y &= -23 \\
u + 4v - 3w + 0.5x - 2y &= -1.5 \\
3u + 2v - w + 1.5x - 3y &= -3.5
\end{aligned}
$$