Problem 1:

1.) $T(N) = 2T(N-1) + 1$

$$T(N) = \begin{cases} 1 & N = 0 \\ 2T(N-1)+1 & N > 0 \end{cases}$$

$T(N) = 2T(N-1)+1$

$T(N-1) = 2T(N-2)+1$

$T(N) = 2[2T(N-2)+1]+1$

$T(N-2) = 2T(N-3)+1$

$T(N) = 2(2[2T(N-3)+1])+1$

$T(N) = 2^3T(N-3) + 2^2 + 1$

$\downarrow$

$T(N) = 2^kT(N-k) + 2^{k-1} + 2^{k-2} + 2^{k-3} \ldots + 2^2 + 1$

Assume $N-k=0$, $N=k$

$T(N) = 2^N(0) + 1 + 2 + 2^2 + 2^3 + \ldots + 2^{k-1}$

$T(N) = 2^NT(0) + 2^{k-1} - 1$, $T(0) = 1$

$T(N) = 2^N + 2^N - 1 = 2^{N+1} - 1$

$$\boxed{T(N) = \Theta(2^N)}$$

Master Theorem for Decreasing Functions:

$T(N) = 2T(N-1)+1$

$a = 2$

$a > 1$ $\Theta(a^n * f(n))$

$\Theta(2^N * 1) = \Theta(2^N)$

2.) $T(N) = 3T(N-1) + n$

$$T(N) = \begin{cases} 1 & N = 0 \\ 3T(N-1) + n & N > 0 \end{cases}$$

$T(N) = 3T(N-1) + n$

$T(N-1) = 3T(N-2) + N-1$

$T(N) = 3[3T(N-2) + N-1] + N$

$T(N-2) = 3T(N-3) + N-2$

$T(N) = 3^2[3T(N-3) + (N-2)] + 3(N-1) + N$

$= 3^3T(N-3) + 3^2(N-2) + 3(N-1) + N$

$\downarrow$

$T(N) = 3^k T(N-k) + 3^{k-1}(N-k+1) + \ldots + 3(N-1) + N$

Assume $N-k=0$, $N=k$

$T(N) = 3^N T(0) + 3^{N-1}(1) \ldots$

$\boxed{T(N) = O(n3^N)}$

Master theorem for decreasing function:

$T(N) = 3T(N-1) + n$

$a = 3$

$a > 1$, $O(a^n * f(n))$

$O(3^N * n) = O(n3^N)$

$T(n) = aT(n/b) + f(n)$, $f(n) = \Theta(n^k \log^p n)$

3.) $T(N) = 9T(N/2) + n^2$

Master Theorem for Dividing Functions:

$a = 9$, $b = 2$, $f(n) = n^2$, $k = 2$

Case 1: if $\log_b a > k$, then $\Theta(n^{\log_b a})$

$\log_2 9 > 2$, $\Theta(n^{\log_2 9})$

$\boxed{T(N) = \Theta(n^{\log_2 9}) = \Theta(n^{3.17})}$

4.) $T(N) = 100T(N/2) + n^{\log_2 cn + 1}$

Master Theorem:

$a = 100$, $b = 2$, $f(n) = n^{\log_2 cn+1}$, $k = \log_2 cn + 1$

Case 3: if $\log_b a < k$, if $p \geq 0$, $\Theta(n^k \log^p n)$
if $p < 0$, $\Theta(n^k)$

$\log_2 100 < \log_2 cn + 1$

$p = 0$, $\Theta(n^{\log_2 cn+1})$

$\boxed{T(N) = \Theta(n^{\log_2 cn+1})}$

5.) $T(N) = 4T(N/2) + n^2 \log n$

Master Theorem:

$a = 4,\ b = 2,\ f(n) = n^2 \log n,\ k = 2,\ p = 1$

Case 2: if $\log_b a = k$, if $p > -1$, $\Theta(n^k \log^{p+1} n)$

if $p = -1$, $\Theta(n^k \log \log n)$

if $p < -1$, $\Theta(n^k)$

$\log_2 4 = 2$

$p = 1$, $\Theta(n^2 \log^2 n)$

$\boxed{T(N) = \Theta(n^2 \log^2 n)}$

6.) $T(N) = 5T(N/2) + n^2 / \log n$

$a = 5,\ b = 2,\ f(n) = n^2 / \log n,\ k = 2,\ p = -1$

Case 1: if $\log_b a > k$, then $\Theta(n^{\log_b a})$

$\log_2 5 > 2$

$\boxed{T(N) = \Theta(n^{\log_2 5}) = \Theta(n^{2.32})}$

Problem 2:

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yetAnotherFunc(n):
  if n>1:                        -> 1
    for(i=0; i<10n; i++)         -> n
      doSomething;               -> 1
    yetAnotherFunc(n/2);         -> T(n/2)
    yetAnotherFunc(n/2);         -> T(n/2)
```

Recurrence Relation:

$T(N) = 2T(n/2) + n$

$T(N/2) = 2T(n/4) + n/2$

$T(N) = 2(2T(n/4) + n/2) + n = 4T(n/4) + 2n$

$T(n/4) = 2T(n/8) + n/4$

$T(N) = 4(2T(n/8) + n/4) + 2n = 8T(n/8) + 3n$

$T(N) = 2^k T(n/2^k)\ k \cdot n$

Assume $n/2^k = 1$, $n = 2^k$, $k = \log(n)$

$T(N) = 2^{\log n} T(n/n = 1) \log n \cdot n$

$T(N) = nT(1) + n\log n$

$$T(N) = \begin{cases} 1 & n = 1 \\ 2T(n/2) + n & n > 1 \end{cases}$$

$T(1) = 1$,

$T(N) = n + n\log n$

$$\boxed{T(N) = O(n\log n)}$$