

Real-Time End-to-End Scheduling

做下一件事要等其他事一起完成

Embedded System Software Design

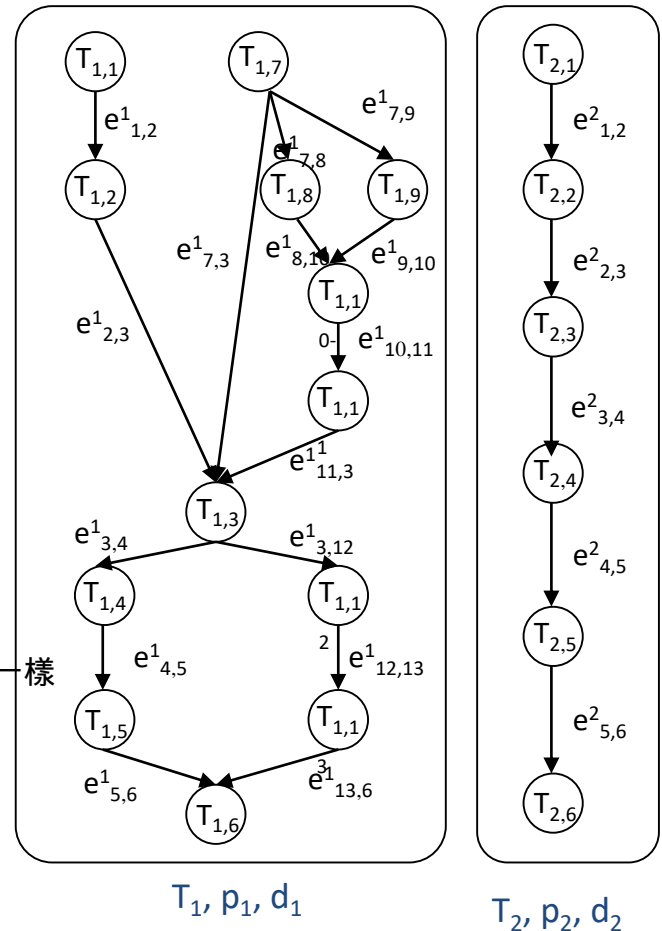
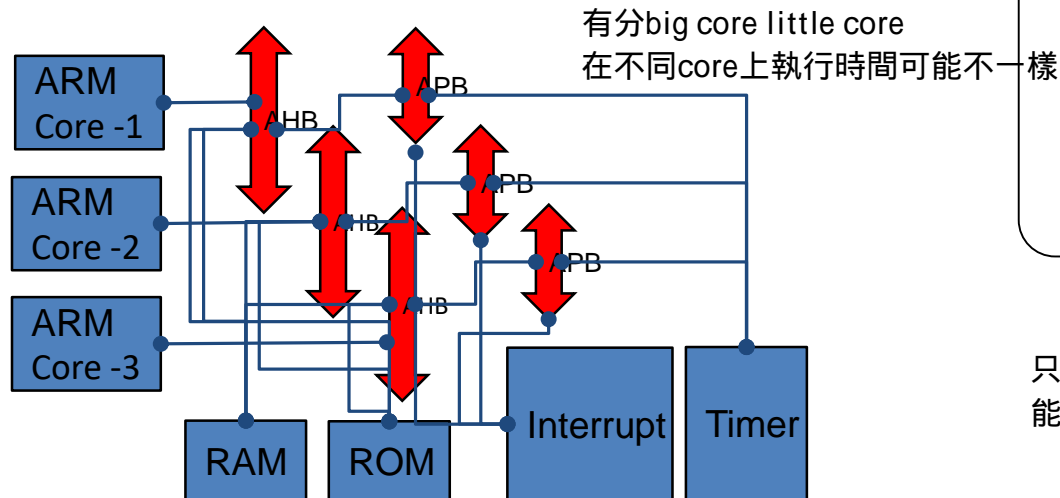
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End-to-End Scheduling

- Task model
 - Each task needs to execute on a set of processors in a certain order
 - Each task may require a different order
- Problems in End-to-End scheduling
 - Priority assignment 優先分配
 - Assign fixed priorities to tasks so that the system is schedulable
 - Synchronization of tasks 任務同步
 - Control the releases of subtask instances (non-first subtasks)
 - Schedulability analysis 可調度性分析
 - For a given priority assignment and a given synchronization protocol, whether every instance of each task meets its deadline

System Model

- Platform: A set of processors
- Task graph, $G = \{T_1, T_2, \dots, T_n\}$
 - Sink node
 - Deadline: d_i
 - Precedence edge: $e_{j,k}^i$
 - Predecessors and Successors
 - Period or Minimum separation time: p_i
- Characteristics of $T_{i,j}$:
 - Execution time (on processor m): $c_{i,j}^m$



只要找到最長的路徑
能排就能排

Priority Assignment

- To find feasible priority assignments off-line if all tasks executed are known in prior 同一條路優先權怎麼設
不一定 這有很多作法
- NP-hard problem 沒有定論 哪一個表現特別好
- Algorithms
 - Branch and bound
 - Search algorithm
 - Simulated annealing
 - Generic algorithm
 - Heuristic
 - Deadline assignment

Deadline Assignment

- Ultimate deadline
 - $UD_{i,k} = D_i$ 期限
- Effective deadline
 - $ED_{i,k} = D_i - \sum_{l=k+1}^{n(i)} e_{i,l}$ 期限 - 同T比他低的執行時間
- Proportional deadline
 - $PD_{i,k} = D_i e_{i,k} / e_i$ 期限 * 執行時間 / 總執行時間
- Normalized Proportional deadline
 - $NPD_{i,k} = D_i \frac{e_{i,k} U(V_{i,k})}{\sum_{l=1}^{n(i)} e_{i,l} U(V_{i,l})}$ (執行時間 * U) / (總執行時間 * U)
 整個T的執行時間*U 總和
 - $U(V_{i,l})$ is the total utilization of all the subtasks that execute on the processor $V_{i,l}$

$$U = e / d$$

Example

| $T_{i,k}$ | $V_{i,k}$ | p_i | $e_{i,k}$ | $UD_{i,k}$ | $ED_{i,k}$ | $PD_{i,k}$ | $NPD_{i,k}$ |
|-----------|-----------|-------|-----------|------------|------------|------------|-------------|
| $T_{1,1}$ | P_1 | 15 | 1 | 15 | 11 | 3 | 2.0 |
| $T_{1,3}$ | P_1 | 15 | 2 | 15 | 15 | 6 | 4.1 |
| $T_{2,1}$ | P_1 | 20 | 4 | 20 | 20 | 20 | 20 |
| $T_{3,1}$ | P_2 | 2 | 1 | 2 | 2 | 2 | 2 |
| $T_{1,2}$ | P_2 | 15 | 2 | 15 | 13 | 6 | 8.9 |
| $T_{4,1}$ | P_2 | 20 | 5 | 20 | 20 | 20 | 20 |

UD = D
UD11=15
UD13=15
UD21=20

ED11=15-(2+2)=11
ED12=15-(2)=13
ED13=15
ED21=20
ED31=2
ED41=20

PD11=(15*1)/(1+2+2)=3
PD13=(15*2)/(1+2+2)=6
PD12=(15*2)/(1+2+2)=6
PD21=(20*4)/(4)=20
PD31=(2*1)/(1)=2
PD41=(20*5)/(5)=20

U=E/D

Up1=1/15+2/15+4/20=0.4
Up2=1/2+2/15+5/20=0.88

NPD11=15*(1*0.4)/(1*0.4+2*0.88+2*0.4)=2
NPD13=15*(2*0.4)/(1*0.4+2*0.88+2*0.4)=4.1
NPD21=20*(4*0.4)/(4*0.4)=20
NPD31=2*(1*0.88)/(1*0.88)=2
NPD12=15*(2*0.88)/(1*0.4+2*0.88+2*0.4)=8.9
NPD41=20*(5*0.88)/(5*0.88)=20

The Synchronization Problem

同步問題

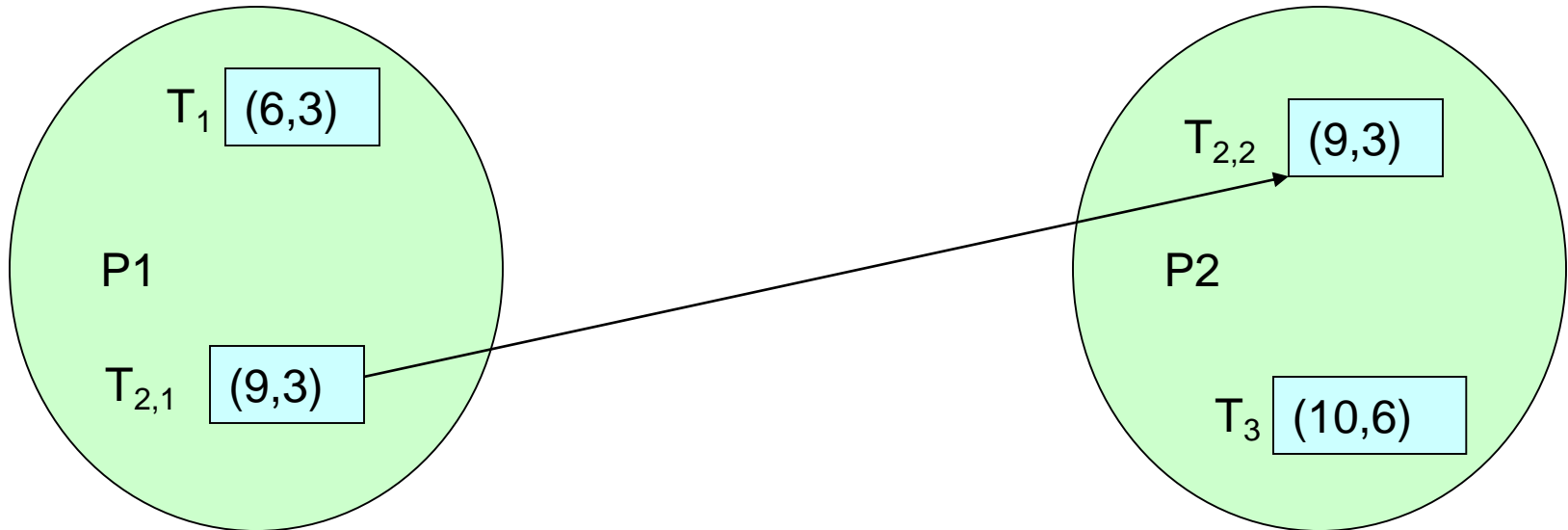
- Given that
 - Priorities are assigned to subtasks in a task chain using some fixed priority assignment algorithm
- How do we coordinate the release of subtasks in a task chain so that
 - Precedence constraints among subtasks are satisfied
 - Subtask deadlines are met
 - End-to-end deadlines are met

Synchronization Protocols

同步協議

- Direct Synchronization (DS) Protocol 做完呼叫 繼續做
 - Simple and straightforward 簡單直覺
- Phase Modification (PM) Protocol 相位修改
 - Used by flow-shop tasks
 - Extension called Modified Phase Modification (MPM) Protocol
- Release Guard Protocol 釋放
 - Reclaim the idle time 回收 idle 時

Example



$T_{i,j}$ – j^{th} subtask of task T_i

(period, execution time)

Period = relative deadline of parent task

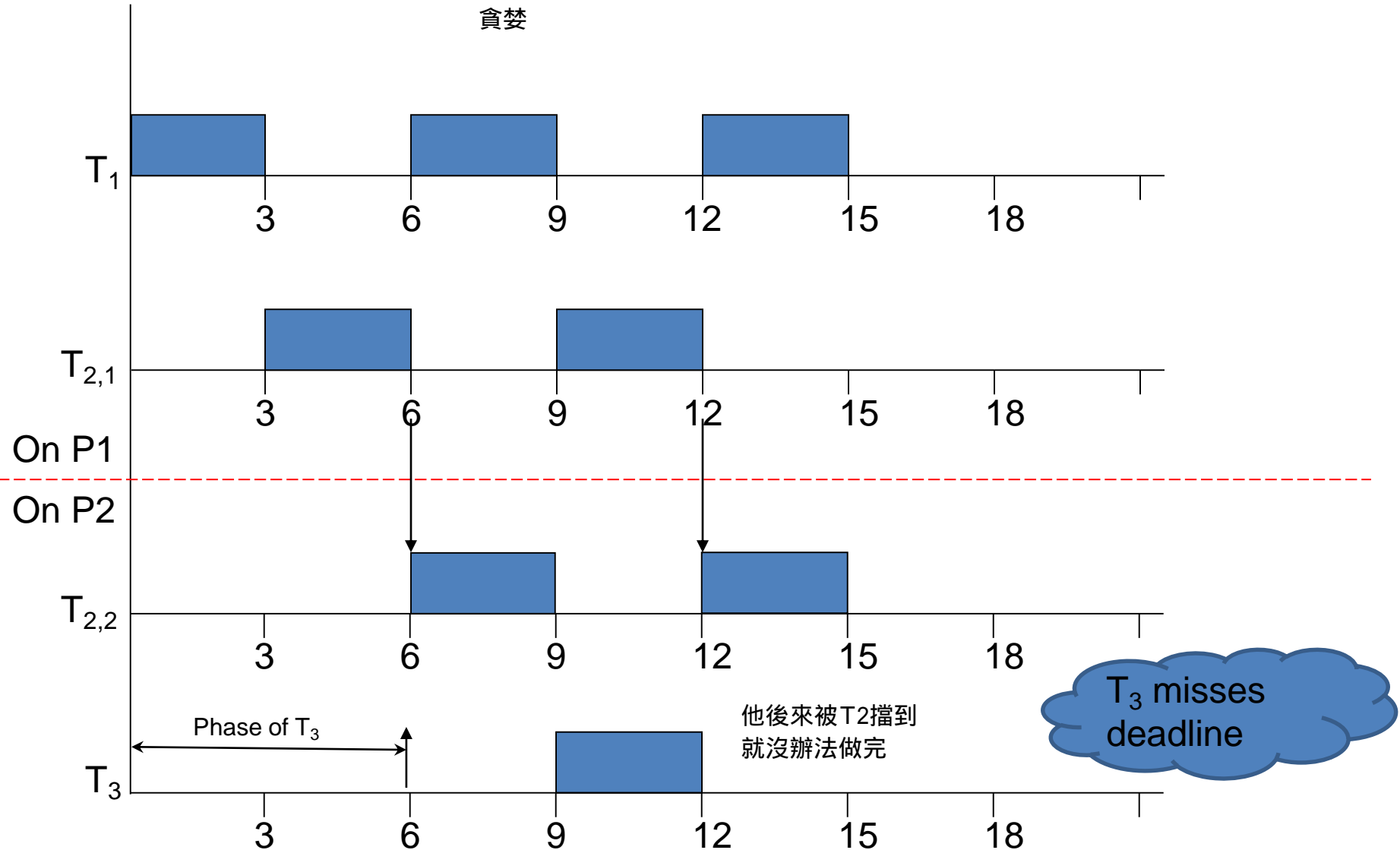
Task T_3 releases at 6

Direct Synchronization Protocol

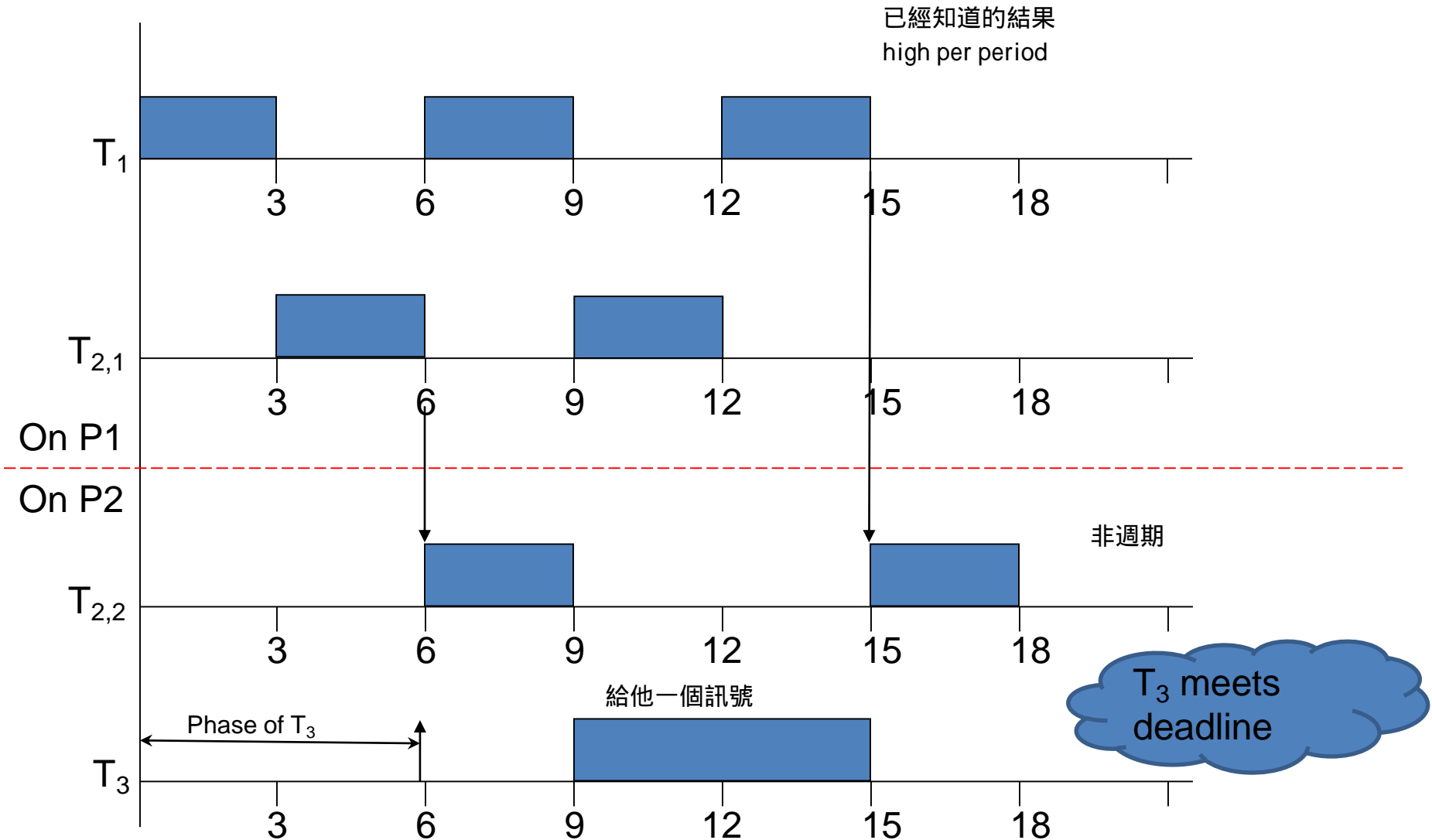
- Greedy strategy
- On completion of subtask
 - A synchronization signal sent to the next processor
 - Successor subtask competes with other tasks/subtasks on the next processor

Greedy Example

貪婪



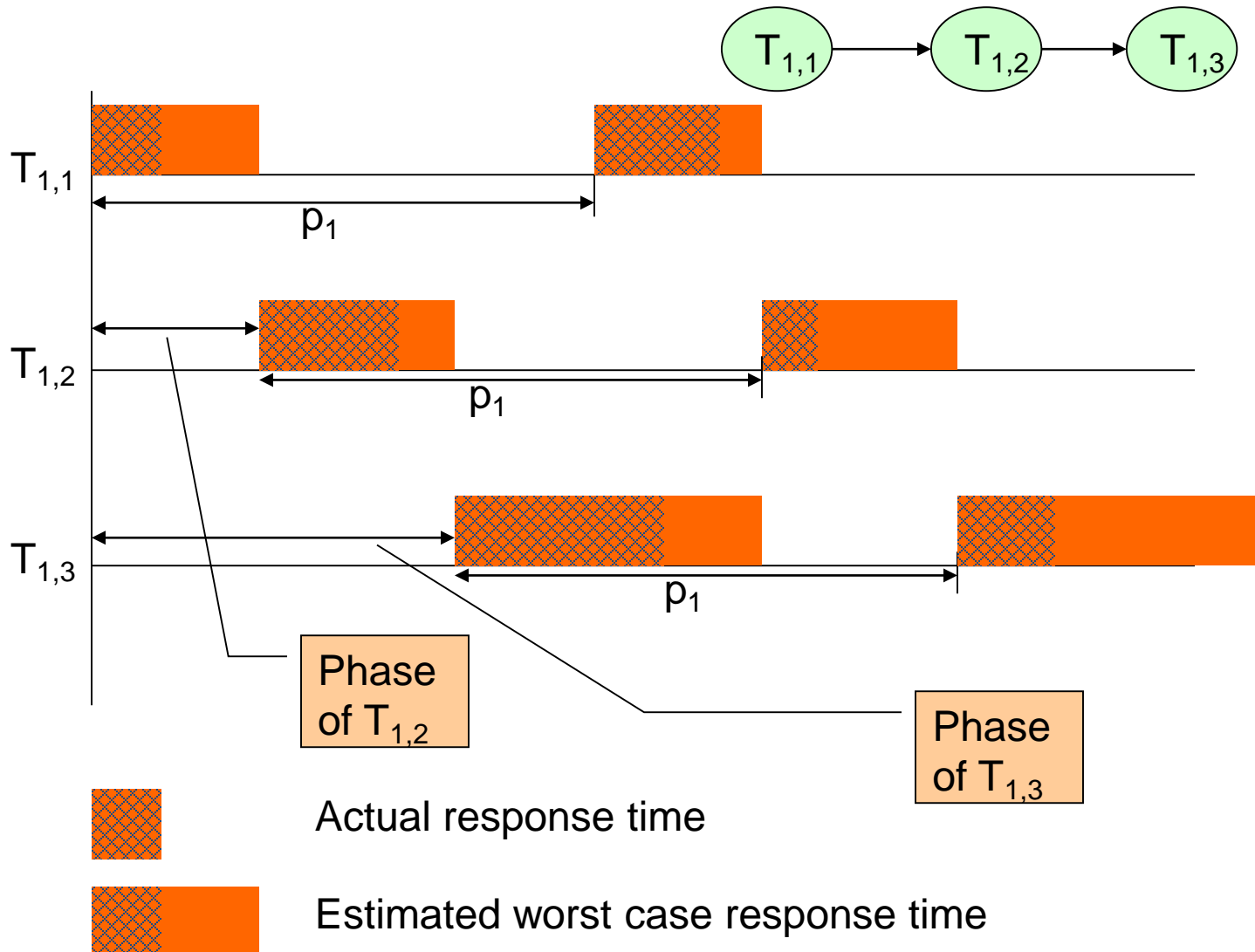
Non-greedy Example



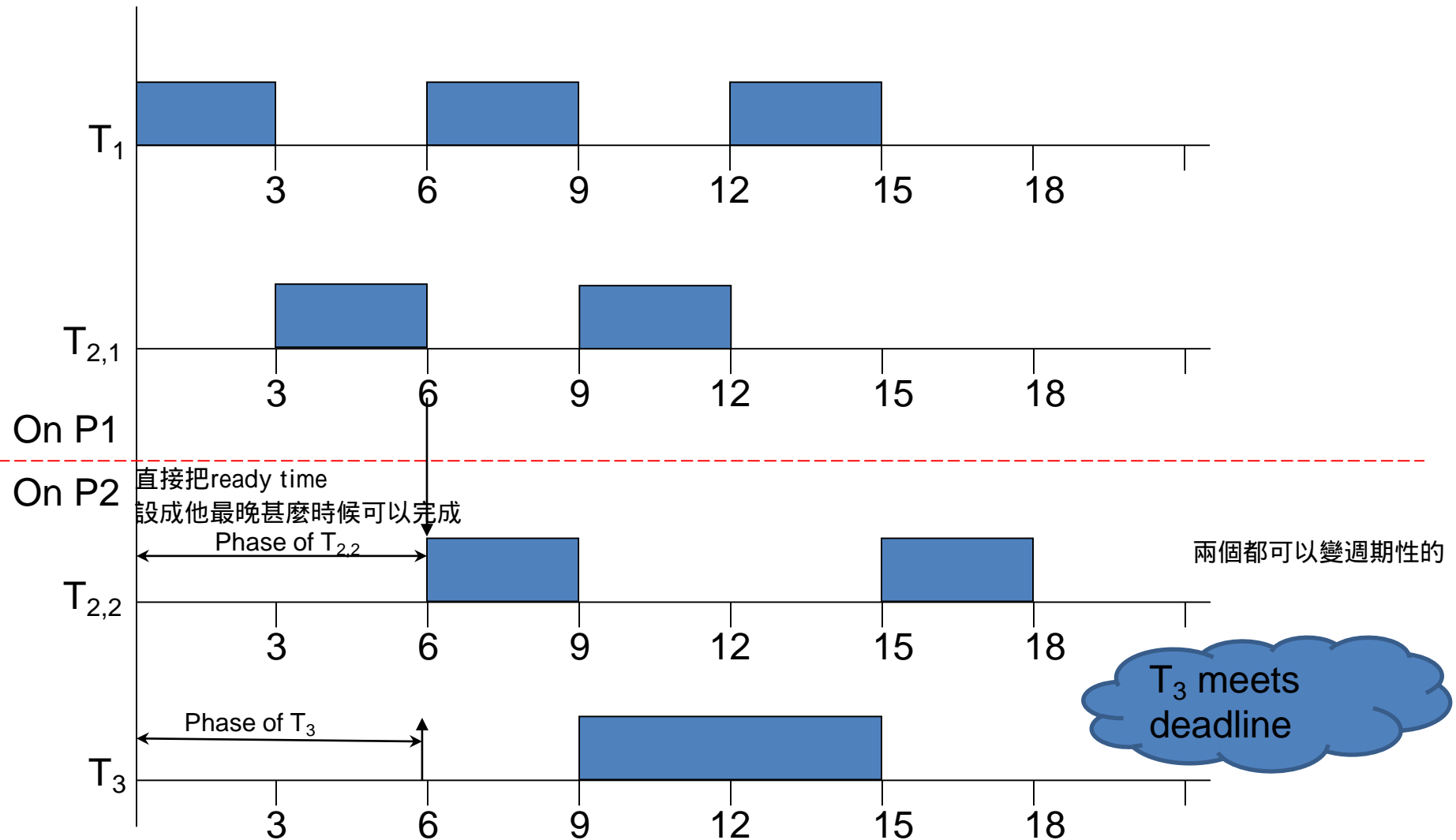
Phase Modification Protocol

- Release subtasks periodically 定期釋放子任務
 - According to the periods of their parent tasks 根據其上級任務的期限
- Each subtask given its own phase 每個子任務都有自己的階段
- Phase determined by subtask precedence constraints 階段由子任務優先級確定
約束條件

Phase Modification Protocol (1/2)



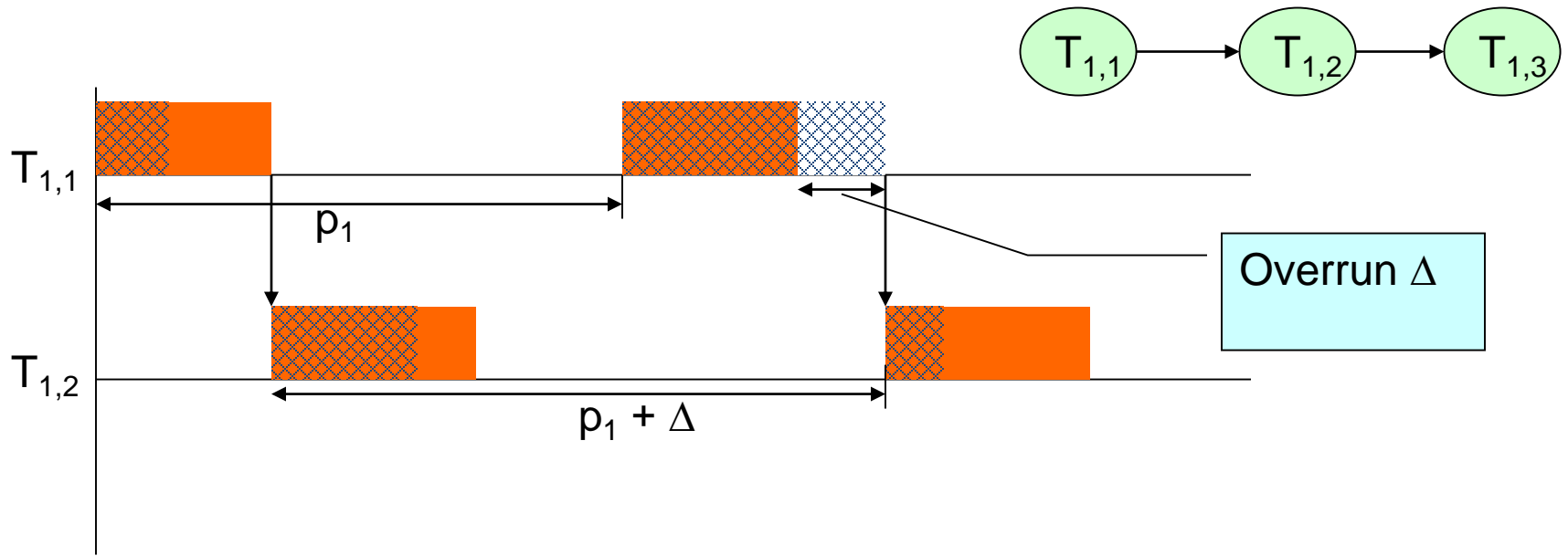
Phase Modification Protocol (2/2)



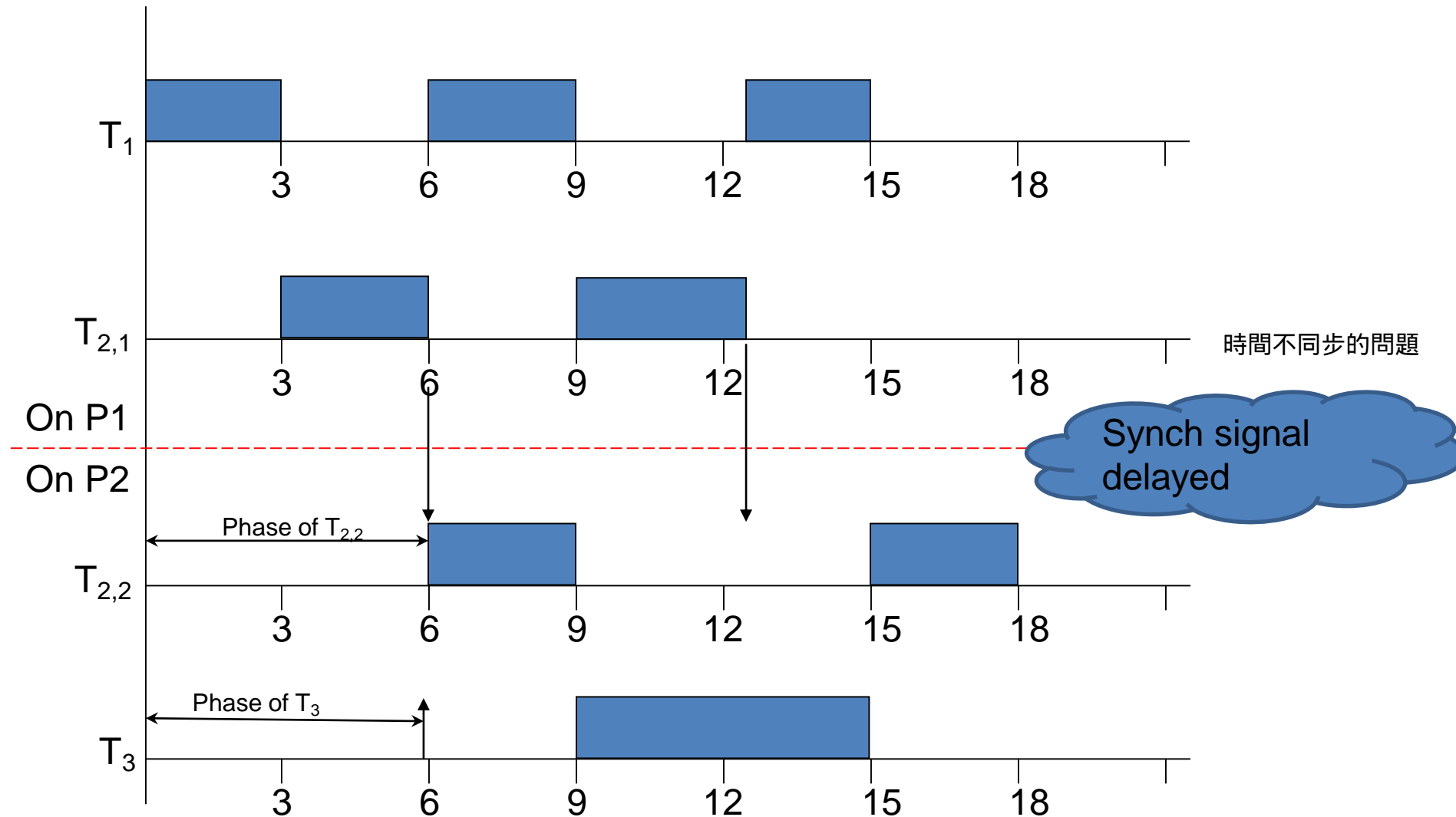
Phase Modification Protocol - Analysis

- Periodic timer interrupt to release subtasks
- Centralized clock or strict clock synchronization
- Task overruns could cause precedence constraint violations

Modified PM Protocol (1/2)



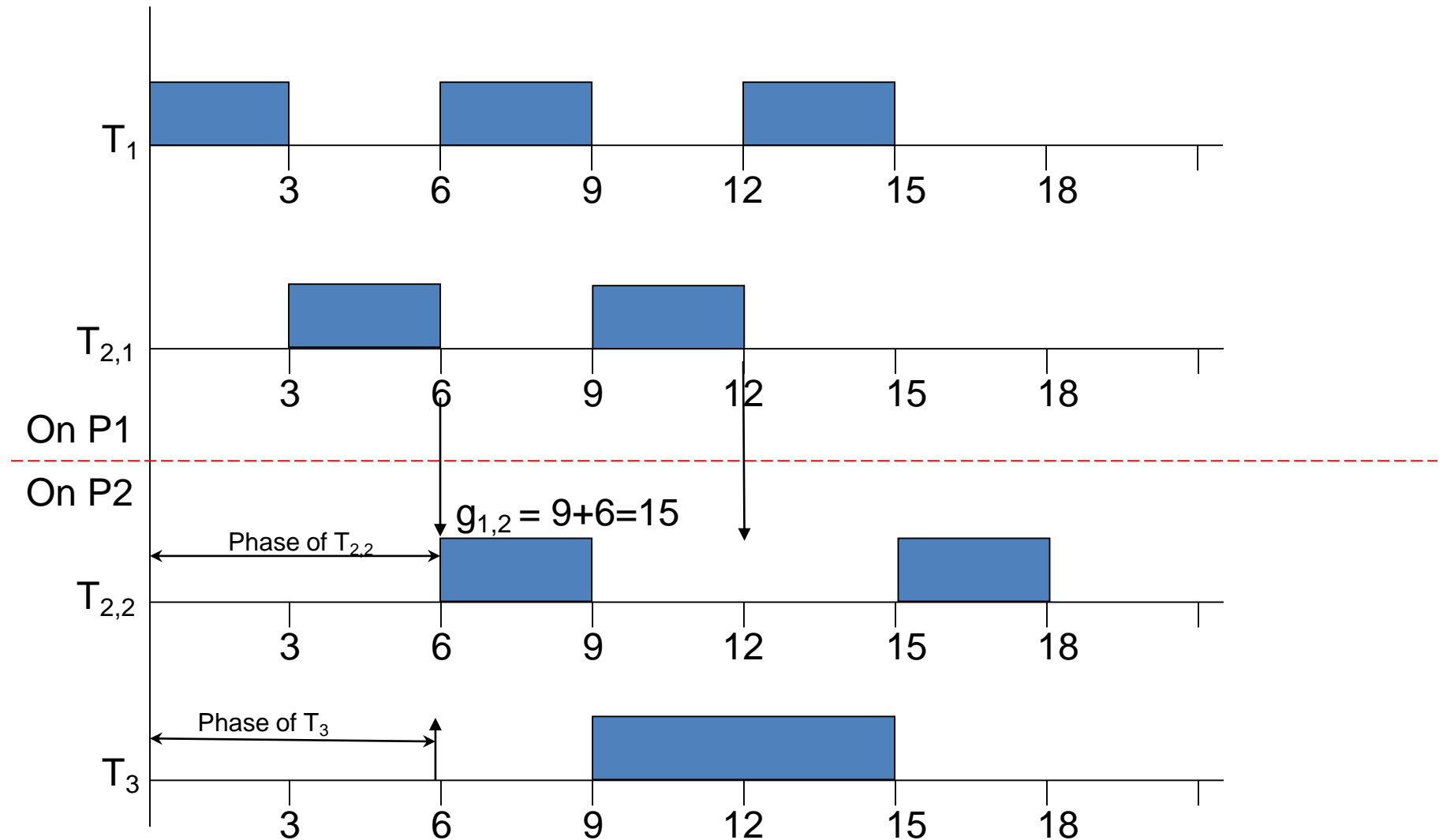
Modified PM Protocol (2/2)



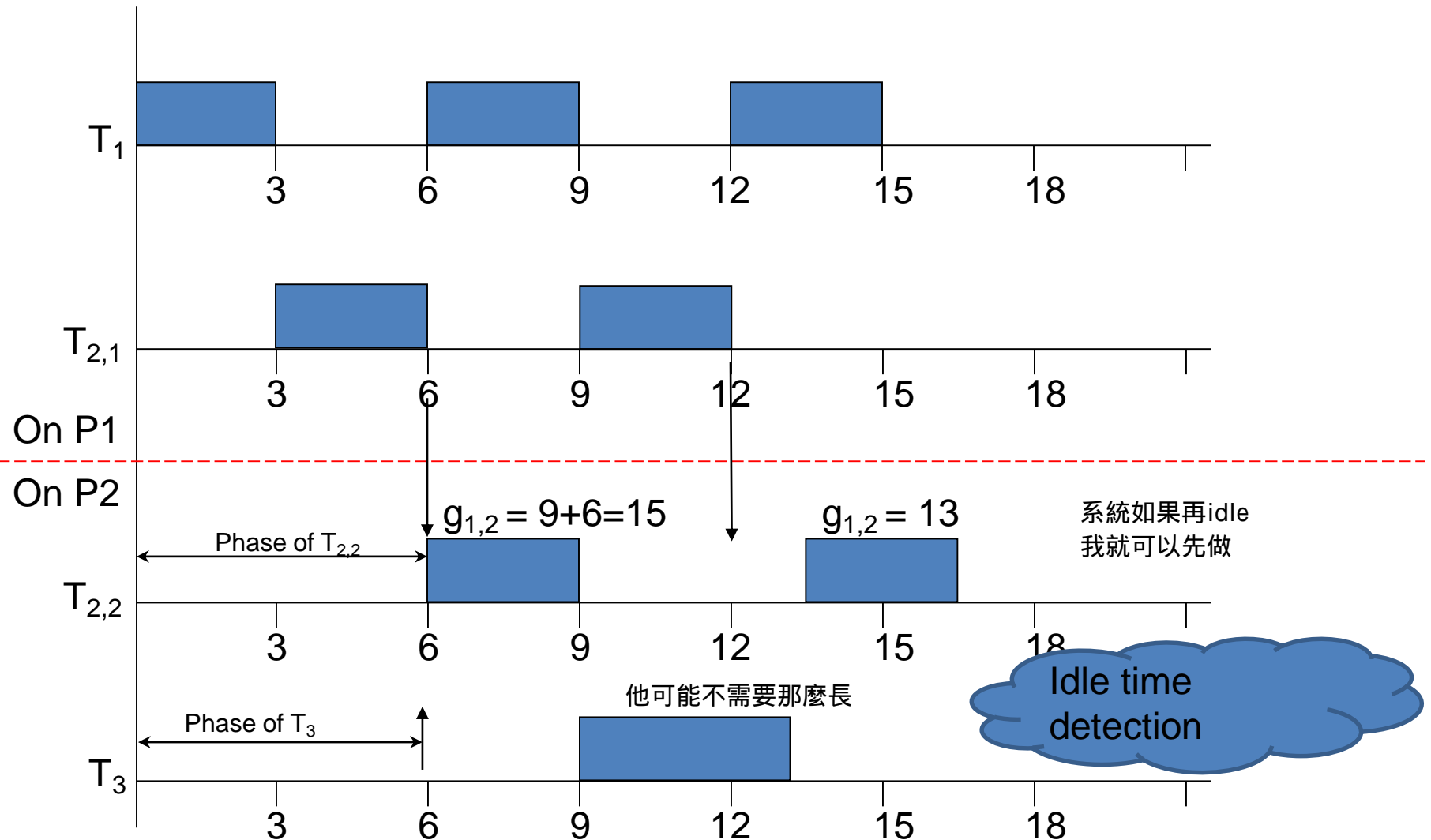
Release Guard Protocol

- A guard variable – *release guard* - associated with each subtask
- Release guard used to control release of each subtask
 - Contains next release time of subtask
- Synchronization signals as MPM
- Release guard updated
 - On getting synchronization signal
 - During idle time

Release Guard Protocol



Release Guard Protocol



Release Guard Protocol - Analysis

- Shares the same advantages as MPM
- Upper bound on EER still the same as MPM
 - Since upper bound on release time enforced by release guard
- Lower bound on EER less than that of MPM
 - If there are idle times
 - Results in lower average EER (end-to-end response time)

Schedulability Analysis

可排程分析

An upper bound W_i to the end-to-end response time of any periodic task T_i in a fixed-priority system synchronized according to the MPM protocol or the RG protocol is given by

$$W_i = \sum_{k=1}^{n(i)} W_{i,k}$$

比我低block我的影響

and

自己執行時間

優先比我>=對我的影響

$$W_{i,k} = \frac{e_{i,k} + b_{i,k} + \sum_{\phi_{j,l} \leq \phi_{i,k} \text{ and } \tau_{j,l} \in V_{i,k}} e_{j,l}}{1 - \sum_{\phi_{j,l} < \phi_{i,k} \text{ and } \tau_{j,l} \in V_{i,k}} u_{j,l}}$$

同一個processer上
1-優先比我高的人的U
通通不用比我高的資源

where $n(i)$ is the number of subtasks in T_i , $\phi_{i,k}$ is the priority of $\tau_{i,k}$, and the upper bound $W_{i,k}$ to the response time of every subtask $T_{i,k}$ is obtained by considering only subtasks on the same processor $V_{i,k}$, and by treating every such subtask $T_{j,l}$ as periodic task whose period is equal to the period p_j of the parent task T_j .

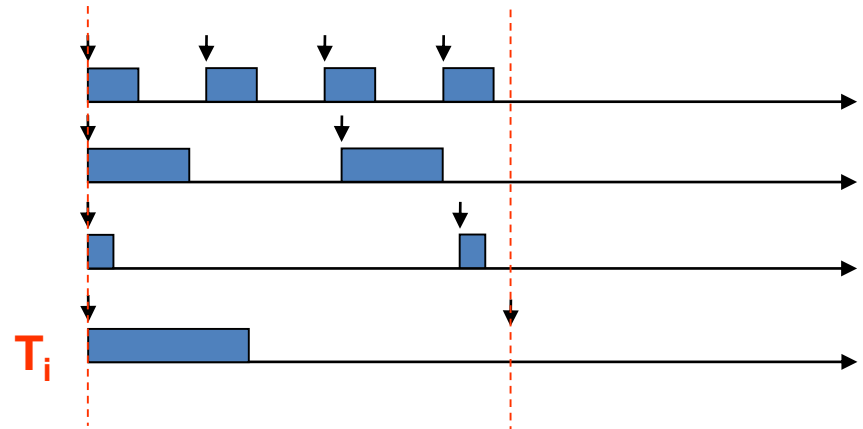
比較悲觀

General Scheduling Test (GST)

- Response time analysis
 - The response time of the job of T_i at critical instant can be calculated by the following recursive function

$$r_0 = \sum_{\forall i} c_i$$

$$r_n = \sum_{\forall i} c_i \left\lceil \frac{r_{n-1}}{p_i} \right\rceil$$



- Observation: the sequence of r_x , $x \geq 0$ may or may not converge

General Scheduling Test (GST)

- Example: $T1=(2,5)$, $T2=(2,7)$, $T3=(3,8)$
 - T1:
 - $R_0=2 \leq 5$ ok
 - T2:
 - $R_0=2+2=4 \leq 7$
 - $R_1=2 * \lceil 4/5 \rceil + 2 * \lceil 4/7 \rceil = 4 \leq 7$ ok
 - T3:
 - $R_0=2+2+3=7 \leq 8$
 - $R_1= 2 * \lceil 7/5 \rceil + 2 * \lceil 7/7 \rceil + 3 * \lceil 7/8 \rceil = 9 > 8$ failed
 - Note: each task succeeds \rightarrow the task set succeeds

Example

$$W12=(1+0+2)/(1-0)=3$$

$$W13=(2+1+1)/(1-0)=4$$

$$W21=(4+0+(1+2))/(1-1/15-2/15)=8.75$$

$$W31=(1+0+0)/(1-0)=1$$

$$W12=(2+1+1)/(1-1/2)=8$$

$$W41=(5+0+(1+2))/(1-1/2-2/15)=21.8$$

$$W12 R0=1+2=3$$

$$W12 R1=1*\text{ceil}(3/2)+2*\text{ceil}(3/15) = 4$$

$$W12b R0=1+2+1=4$$

$$W12b R1=1*\text{ceil}(4/2)+3*\text{ceil}(4/15) = 5$$

$$W12b R2=1*\text{ceil}(5/2) + 3*\text{ceil}(5/15) = 6$$

| $T_{i,k}$ | $V_{i,k}$ | p_i | $e_{i,k}$ | $UD_{i,k}$ | $b_{i,k}$ | $W_{i,k}$ | $W_{i,k}(GST)$ |
|-----------|-----------|-------|-----------|------------|-----------|-----------|----------------|
| $T_{1,1}$ | P_1 | 15 | 1 | 15 | 0 | 3 | 3 |
| $T_{1,3}$ | P_1 | 15 | 2 | 15 | 1 | 4 | 3(4) |
| $T_{2,1}$ | P_1 | 20 | 4 | 20 | 0 | 8.75 | 7 |
| $T_{3,1}$ | P_2 | 2 | 1 | 2 | 0 | 1 | 1 |
| $T_{1,2}$ | P_2 | 15 | 2 | 15 | 1 | 8 | 4(6) |
| $T_{4,1}$ | P_2 | 20 | 5 | 20 | 0 | 21.8 | 14 |

$$W41 R0=1+2+5=8$$


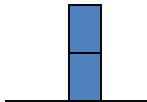
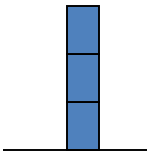

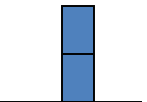
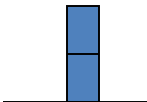

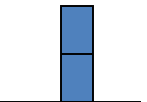

$$W41 R1=1*\text{ceil}(8/2)+2*\text{ceil}(8/15)+5*\text{ceil}(8/20) = 11$$

$$W41 R2=1*\text{ceil}(11/2)+2*\text{ceil}(11/15)+5*\text{ceil}(11/20)=13$$

$$W41 R3=1*\text{ceil}(13/2)+2*\text{ceil}(13/15)+5*\text{ceil}(13/20)=14$$

$$W41 R4=1*\text{ceil}(14/2)+2*\text{ceil}(14/15)+5*\text{ceil}(14/20)=14.3$$

Comparison of Protocols

| | DS | PM | MPM | RG |
|-----------------------------|--|--|---|---|
| Implementation complexity | Synch interrupts | Timer interrupts clock synchronization | Synch & timer interrupts | Synch & timer interrupts |
| Run-time overhead |  |  | |  |
| Average EER |  |  |  |  |
| Estimated worst case EER |  |  | | |
| Inherently missed deadlines | Yes | No | | |

Reference

- Real-time Systems, Jane Liu
- Bettati, R., ``End-to-end scheduling to meet deadlines in distributed systems,” Ph.D. thesis, University of Illinois at Urbana-Champaign
- Sun, J., ``Fixed-Priority Scheduling of Periodic Tasks With End-to-End Deadlines,” Ph.D. thesis, University of Illinois at Urbana-Champaign