
Problem Set 1

All parts are due Tuesday, September 13 at 11PM.

Name: Kevin Jiang

Collaborators:

Problem 1-1.

(a)

$$\begin{aligned}f_1 &= 20n + 18 = \Theta(n) \\f_2 &= 20n \cdot 18 = \Theta(n) \\f_3 &= 20n^{18} = \Theta(n^{18}) \\f_4 &= \log_{20}(n^{18}) = 18 \log_{20}(n) = \Theta(\log(n)) \\f_5 &= (\log_{18}(n))^{20} = \Theta(\log(n)^{20})\end{aligned}$$

$$\Rightarrow (f_4, f_5, \{f_1, f_2\}, f_3)$$

(b)

$$\begin{aligned}f_1 &= n^{2 \log n} = \Theta(n^{\log n^2}) \\f_2 &= 2^{2^{\log n}} = 2^n = \Theta(2^n) \\f_3 &= 2^{(\log n)^2} = (2^{\log n})^{\log n} = n^{\log n} = \Theta(n^{\log n}) \\f_4 &= \Theta(n^{\log n}) \\f_5 &= \Theta(2^{\log(\log n)})\end{aligned}$$

$$\Rightarrow (f_5, \{f_3, f_4\}, f_1, f_2)$$

(c)

$$\begin{aligned}f_1 &= \Theta(2^{n^3}) \\f_2 &= \Theta(2^{(n+1)^3}) \\f_3 &= \Theta(n^{n^2}) \\f_4 &= \Theta(4^{2^n}) \\f_5 &= \Theta(3^{2^n})\end{aligned}$$

$$\Rightarrow (f_3, f_1, f_2, f_5, f_4)$$

(d)

$$f_1 = (2n)! \approx \sqrt{4\pi n} \left(\frac{2n}{e}\right)^{2n} = \Theta\left(\left(\frac{2}{e}\right)^{2n} \cdot \sqrt{n} \cdot n^{2n}\right)$$

$$f_2 = \frac{(2n)!}{n! \cdot n!} \approx \frac{\sqrt{4\pi n} \left(\frac{2n}{e}\right)^{2n}}{2\pi n \cdot \left(\frac{n}{e}\right)^{2n}} = \Theta\left(\frac{1}{\sqrt{n}} \cdot 4^n\right)$$

$$f_3 = \Theta(4^n)$$

$$f_4 = \Theta(2^n \cdot n^n)$$

$$f_5 = \Theta((n^n)^2)$$

$$\Rightarrow (f_2, f_3, f_4, f_1, f_5)$$

Problem 1-2.

- (a) i.
- ii.
- (b) i.
- ii.
- iii.
- iv.

TREES

Problem 1-3.

- (a) For each of the n rows, we apply binary search to see if v is present in the row. Since there are m elements in each row, this takes $O(n \log m)$ time.
- (b) For each of the m elements, we apply binary search to see if v is present in the column. Since there are n rows, this takes $O(m \log n)$ time.
- (c) For each of the m elements, we apply binary search to see if v is present in the column. Since there are n rows, this takes $O(m \log n)$ time.
- (d) Submit your implementation to `alg.mit.edu`.