Poset and Lattice

Poset (Partial ordered set):-

A relation on set A is called partial order relation if it is

I) Reflexive
II) Anti-symmetric
III) Transitive

And set A together with the partial order relation R is called partial order set . It is denoted by (A,R)

Steps to draw Hasse Diagram :-

Step 1:- Delete the loops at each vertex

Step 2:- Delete the edges implied by transitivity

Step 3:- Rearrange the diagraph if necessary , such that all arrow go upwards.

Step 4:- Replace all arrows by edges and vertices by dots.

Points to remember in Hasse diagram :-

I) It never have horizontal line
II) Relation is always in upwards direction

1) Comparable elements :- In mathematics, any two elements x and y of a set P that is partially ordered by a binary relation $\leq$ are comparable when either $x \leq y$ or $y \leq x$. If it is not the case that x and y are comparable, then they are called incomparable.

2) Maximal Element :- In mathematics, especially in order theory, a maximal element of a subset S of some partially ordered set (poset) is an element of S that is not smaller than any other element in S. ... By contrast, neither a maximum nor a minimum exists for S.

3) Minimal Element :- A minimal element of a subset S of some partially ordered set is defined dually as an element of S that is not greater than any other element in S. The notions of maximal and minimal elements are weaker than those of greatest element and least element which are also known, respectively, as maximum and minimum.

4) Greatest Element :- In mathematics, especially in order theory, the greatest element of a subset S of a partially ordered set (poset) is an element of S that is greater than every other element of S. The term least element is defined dually, that is, it is an element of S that is smaller than every other element of S.

5) Least element :- In mathematics, especially in order theory, the greatest element of a subset S of a partially ordered set (poset) is an element of S that is greater than every other element of S. The term least element is defined dually, that is, it is an element of S that is smaller than every other element of S.

6) Unit Element :- In mathematics, especially in order theory, the greatest element of a subset S of a partially ordered set (poset) is an element of S that is greater than every other element of S. The term least element is defined dually, that is, it is an element of S that is smaller than every other element of S.

7) Zero element :- To any poset with a least element 0 we define the zero-divisor graph, denoted by , of as follows: its vertices are just the elements of , and $x , y \in G (P)$ are connected by an edge if and only if $\{ x , y \} \ell = \{ 0 \}$.

8) Upper bound :- Let S be a subset of A in the poset (A, R). If there exists an element a in A such that sRa for all s in S, then a is called an upper bound. Similarly for lower bounds. ... Definition: If a is an upper bound for S which is related to all other upper bounds then it is the least upper bound, denoted lub(S).

9) Lower bound :- An element $z \in A$ is called a lower bound of B if $z \leq x$ for every $x \in B$.

10) Greatest Lower Bound :- In the poset above, {a, b, c}, is an upper bound for all other subsets. ∅ is a lower bound for all other subsets. Definition: If a is an upper bound for S which is related to all other upper bounds then it is the least upper bound, denoted lub(S). Similarly for the greatest lower bound, glb(S).

11) Least Upper Bound :- If a is an upper bound for S which is related to all other upper bounds then it is the least upper bound, denoted lub(S).

Join (V) :- LUB of {a,b} is denoted by aVb and it is read as " a join b".

Meet (∩) :- GLB of {a,b} is denoted by a ∩ b and it read as "a meet b".

Bounded Lattices:- A lattice L is called a bounded lattice if it has greatest element 1 and a least element 0. Example: The power set P(S) of the set S under the operations of intersection and union is a bounded lattice since ∅ is the least element of P(S) and the set S is the greatest element of P(S).

A sublattice of a lattice L is a subset of L that is a lattice with the same meet and join operations as L. That is, if L is a lattice and M is a subset of L such that for every pair of elements a, b in M both $a \wedge b$ and $a \vee b$ are in M, then M is a sublattice of L.

A complemented lattice is a bounded lattice (with least element 0 and greatest element 1), in which every element a has a complement, i.e. an element b such that. a $\vee$ b = 1 and a $\wedge$ b = 0. In general an element may have more than one complement.

In mathematics, a distributive lattice is a lattice in which the operations of join and meet distribute over each other. The prototypical examples of such structures are collections of sets for which the lattice operations can be given by set union and intersection.

Links that are mentioned in the reference video :-

Link for NPTEL course :- https://nptel.ac.in/courses/106/106/106106183/

YouTube video links :-

https://youtu.be/3u1AudDafXA

https://youtu.be/KVdzsIjLTQk

https://youtu.be/b5sDjo9tfE8?t=18

https://youtu.be/xu51YdEN1uc

https://youtu.be/_g-s_SRzPxM

https://youtu.be/7lYk-5otTV4

https://youtu.be/dNXX8jBK39M

https://youtu.be/KsU6jRwrLj0

https://youtu.be/4cHA8AMPGhA

https://youtu.be/AU1ECbWq6A0