LOGIC

*Logic of statement

• Statement1: If today is Monday, then Mr. X gets |5. Statement2: Today is Monday. Statementc: (Therefore,) Mr. X gets |5.

• Statement1: If today is Monday, then Mr. X gets |5. Statement2: Mr. X gets |5. Statementc: (Therefore,) Today is Monday.

• Statement1: If today is Monday, then Mr. X gets |5. Statement2: Today is Tuesday. Statementc: (Therefore,) Mr. X gets |5.

• Statement1: If today is Monday, then Mr. X gets |5. Statement2: Today is Tuesday. Statementc: (Therefore,) Mr. X does not get |5.

*Formulas for truth table

F1: Each atomic formula is a formula.

F2: If x is a formula, then (¬x) is a formula.

F3: If x and y are formulas, then (x ∧ y), (x ∨ y), (x → y) and (x ↔ y) are formulas. The connective that has been introduced last in the process of generation of the formula is called the principal connective in that formula.

1. The expression (¬p5) is a formula. Ans: Since p5 ∈ A, by (F1), it is a formula. By (F2), (¬p5) is a formula. The principal connective in the formula is ¬.

2. The expression (¬(p3 ∧ (¬p4))) is a formula. Ans: p3, p4 ∈ A; by (F1), these are formulas. By (F2), (¬p4) is a formula. By (F3), (p3∧(¬p4)) is a formula. Next, by (F2), (¬(p3∧(¬p4))) is a formula. The principal connective in the formula is ¬.

2. The expression (p1 → (p1 ∨ p1)) is a formula. Ans: By (F1), p1 is a formula. By (F3), (p1 ∨ p1) is a formula. Once more, by (F3), (p1 → (p1 ∨ p1)) is a formula. The principal connective in the formula is →.

1. ¬ has the highest precedence.

2. ∧ and ∨ have the next precedence.

2. → and ↔ have the least precedence.

1. For an atomic variable pi , either f(pi) = T or f(pi) = F. For formulas p and q,
2.f(¬p) = F if f(p) = T, and f(¬p) = T if f(p) = F.
3. f(p ∧ q) = T if f(p) = f(q) = T, and (p ∧ q) = F otherwise.
4. f(p ∨ q) = F if f(p) = f(q) = F, and f(p ∨ q) = T otherwise.
5. f(p → q) = F if f(p) = T, f(q) = F, and f(p → q) = T otherwise.
6. f(p ↔ q) = T if f(p) = f(q), and f(p ↔ q) = F otherwise

*Laws of logic

1. [Commutativity] p ∨ q ≡ q ∨ p, p ∧ q ≡ q ∧ p

2. [Associativity] p ∨ (q ∨ r) ≡ (p ∨ q) ∨ r, p ∧ (q ∧ r) ≡ (p ∧ q) ∧ r

3. [Distributivity] p ∧ (q ∨ r) ≡ (p ∧ q) ∨ (p ∧ r), p ∨ (q ∧ r) ≡ (p ∨ q) ∧ (p ∨ r) DRAFT 7.3. EQUIVALENCE AND NORMAL FORMS IN SL 139

4. [De Morgan] ¬(p ∨ q) ≡ ¬p ∧ ¬q, ¬(p ∧ q) ≡ ¬p ∨ ¬q

5. [Idempotence] p ∨ p ≡ p, p ∧ p ≡ p

6. [Constants] $\bot \vee p \equiv p$, $\bot \wedge p \equiv \bot$, $\top \vee p \equiv \top$, $\top \wedge p \equiv p$, $p \vee \neg p \equiv \top$, $p \wedge \neg p \equiv \bot$, where $\bot$ denotes contradiction and $\top$ denotes tautology.

7. [Double Negation] $\neg(\neg p) \equiv p$

8. [Absorption] $p \vee (p \wedge q) \equiv p$, $p \wedge (p \vee q) \equiv p$

9. [Implication] $p \rightarrow q \equiv \neg p \vee q$, $\neg(p \rightarrow q) \equiv p \wedge \neg q$

10. [Contraposition] $p \rightarrow q \equiv \neg q \rightarrow \neg p$, $p \rightarrow \neg q \equiv q \rightarrow \neg p$

10. [Biconditional] $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$, $p \leftrightarrow q \equiv (\neg p \vee q) \wedge (p \vee \neg q)$, $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

*Links for reference

Mathematical Induction links

https://onlinecourses.nptel.ac.in/noc20_cs82/unit?unit=235&lesson=238
https://onlinecourses.nptel.ac.in/noc20_cs82/unit?unit=235&lesson=239
https://onlinecourses.nptel.ac.in/noc20_cs82/unit?unit=235&lesson=240
https://onlinecourses.nptel.ac.in/noc20_cs82/unit?unit=235&lesson=241

Quantifiers links
https://www.youtube.com/watch?v=K3TlN2wJyAI
https://www.youtube.com/watch?v=lxNaLc8riHY
https://www.youtube.com/watch?v=DpcUJrYTduc
https://www.youtube.com/watch?v=u9cTpIymSHI

Normal form links
https://www.youtube.com/watch?v=M9YyEwvBgeM
https://www.youtube.com/watch?v=rzxJ1nN_J4g
https://www.youtube.com/watch?v=hTt2pI_6RQY
https://www.youtube.com/watch?v=1k99s-u2z4g
https://www.youtube.com/watch?v=kIeQAvgAiXg