

Chapter: Graph Theory

Concepts and Formulae

Key Concepts

- 1) **A graph (denoted as $G=(V,E)$) consists of a non-empty set of vertices or nodes V and a set of edges E .**
- 2) **Degree of a Vertex** – The degree of a vertex V of a graph G (denoted by $\deg(V)$) is the number of edges incident with the vertex V .
- 3) **Even and Odd Vertex** – If the degree of a vertex is even, the vertex is called an even vertex and if the degree of a vertex is odd, the vertex is called an odd vertex.
- 4) **Degree of a Graph** – The degree of a graph is the largest vertex degree of that graph. For the above graph the degree of the graph is 3.
- 5) **The Handshaking Lemma** – In a graph, the sum of all the degrees of all the vertices is equal to twice the number of edges.

Types of Graphs

There are different types of graphs, which we will learn in the following section.

Null Graph-

A null graph has no edges. The null graph of n vertices is denoted by N_n

Simple Graph-

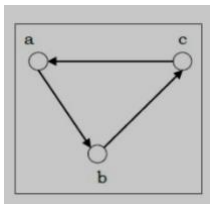
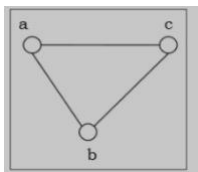
A graph is called simple graph/strict graph if the graph is undirected and does not contain any loops or multiple edges.

Multi-Graph-

If in a graph multiple edges between the same set of vertices are allowed, it is called Multigraph. In other words, it is a graph having at least one loop or multiple edges

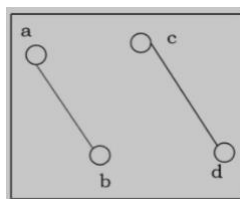
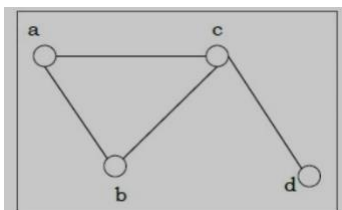
- **Directed and Undirected Graph**

A graph $G=(V,E)$ is called a directed graph if the edge set is made of ordered vertex pair and a graph is called undirected if the edge set is made of unordered vertex pair.



- **Connected and Disconnected Graph**

A graph is connected if any two vertices of the graph are connected by a path; while a graph is disconnected if at least two vertices of the graph are not connected by a path. If a graph G is disconnected, then every maximal connected subgraph of G is called a connected component of the graph G .

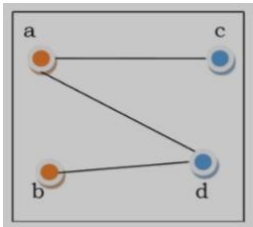


Regular Graph-

A graph is regular if all the vertices of the graph have the same degree. In a regular graph G of degree r , the degree of each vertex of G is r .

Bipartite Graph-

If the vertex-set of a graph G can be split into two disjoint sets, V_1 and V_2 , in such a way that each edge in the graph joins a vertex in V_1 to a vertex in V_2 , and there are no edges in G that connect two vertices in V_1 or two vertices in V_2 , then the graph G is called a bipartite graph.



Representation of Graphs

There are mainly two ways to represent a graph –

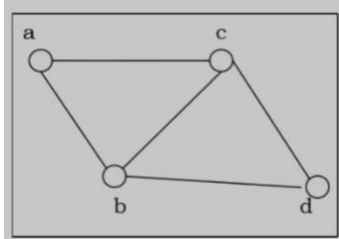
Adjacency Matrix

Adjacency List

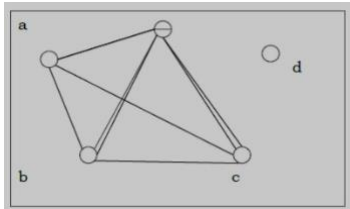
- Adjacency Matrix $A[V][V]$ is a 2D array of size $V \times V$ where V is the number of vertices in an undirected graph. If there is an edge between V_x to V_y then the value of $A[V_x][V_y] = 1$ and $A[V_y][V_x] = 1$, otherwise the value will be zero
- adjacency list, an array $(A[V])$ of linked lists is used to represent the graph G with V number of vertices. An entry $A[V_x]$ represents the linked list of vertices adjacent to the V_x -th vertex.

Planar vs. Non-planar graph

Planar graph – A graph G is called a planar graph if it can be drawn in a plane without any edges crossed. If we draw graph in the plane without edge crossing, it is called embedding the graph in the plane.



Non-planar graph – A graph is non-planar if it cannot be drawn in a plane without graph edges crossing.



Isomorphism

If two graphs G and H contain the same number of vertices connected in the same way, they are called isomorphic graphs (denoted by $G \cong H$).

Homomorphism

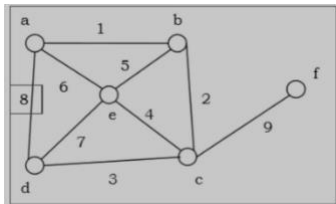
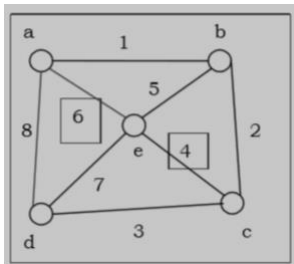
A homomorphism from a graph G to a graph H is a mapping (May not be a bijective mapping) $h: G \rightarrow H$ such that – $(x,y) \in E(G) \rightarrow (h(x), h(y)) \in E(H)$. It maps adjacent vertices of graph G to the adjacent vertices of the graph H .

Euler Graphs-

A connected graph G is called an Euler graph, if there is a closed trail which includes every edge of the graph G .

Hamiltonian Graphs

A connected graph G is called Hamiltonian graph if there is a cycle which includes every vertex of G and the cycle is called Hamiltonian cycle.



Go through this following link for the following topic to understand more better.

Link for NPTEL course :- <https://nptel.ac.in/courses/111/106/111106086/>

- 1) **Intro Graph theory**
<https://youtu.be/ruOPZSISb00>
- 2) **Representation of Graph**
https://youtu.be/5hPfm_uqXmw
<https://youtu.be/4R7chuhzq7k>
- 3) **Regular and bipartite graph**
https://youtu.be/gvQQ7f_BapE
<https://youtu.be/yelUNp4I740>
- 4) **Eular path and planner graph**
<https://youtu.be/kIE263bDtZs>
<https://youtu.be/5M-m62qTR-s>
- 5) **Planet and Non planer graph**
<https://youtu.be/Ald-ey0dBu4>
<https://youtu.be/w7-QjzJLF98>
- 6) **Isomorphic Graph**
<https://youtu.be/RbDne2Qm3YA>
<https://youtu.be/beTi3K01enE>
- 7) **Handshaking lemma**
<https://youtu.be/RBhqV0ZXYi0>
<https://youtu.be/4uilSwVeg0g>
- 8) **Hamilton Graph**
https://youtu.be/4IrESjFU_MU
<https://youtu.be/6QFSkhcHLiA>
https://youtu.be/IADKmt_fxM