Assignment 1

Mathematical Statistics
MS1413

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Introduction

Consider an urn with N identical balls marked with the numbers 1, 2, ..., N. The goal is to estimate N, the number of numbered balls in the urn. To achieve this, a given quantity n (sample size) is observed with replacement. Notice that associated with the i-th observation, we have the following random variable:

$$X_i : \text{the number of the } i\text{-th ball drawn from the urn,}$$

for $i = 1, 2, \ldots, n$. In this case, we have that:

$$P(X_i = k) = \frac{1}{N}, \quad k = 1, 2, \ldots, N.$$

Consider the following estimators for N:

$$\hat{N}_1 = X_{(n)} = \max(X_1, X_2, \ldots, X_n),$$
$$\hat{N}_2 = \frac{n+1}{n} X_{(n)},$$
$$\hat{N}_3 = 2\bar{X} - 1,$$

where $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$.

Task

- Find $\hat{N}_1$, $\hat{N}_2$, and $\hat{N}_3$. For that:

  1. Consider different sample sizes, such as $n = 10$, $n = 20$, $n = 50$, $n = 90$, and $n = 140$. For the parameter N, work with two scenarios: for instance, $N = 30$ and $N = 150$ (one smaller than the largest n and the other larger).

  2. Repeat the experiment 5 times and analyze the results to draw conclusions about the performance of $\hat{N}_1$, $\hat{N}_2$, and $\hat{N}_3$.

Assignment Rules

1. You may work either alone or in groups of two (2) students.

2. The goal of this assignment is to estimate the number of balls in the urn.

3. Submit a brief written report (PDF format) via the course page on Canvas by the deadline (as posted on Canvas).

4. Include your code in the report.

5. Ensure that:

   - You include your **names** and **email addresses** in the report.
   - The report is logically and clearly **structured**, with written explanations.
   - You have thoroughly checked the report for **spelling and grammar mistakes**.
   - The report is written in **English**.

Grading

This assignment will be graded as `G/Ux/U`.

Important!

Failure to meet any of the above requirements may result in a failing grade, requiring you to revise and resubmit the report by a later deadline.

Good luck!