Assignment 2

Mathematical Statistics
MS1413

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Introduction

Let Y be a random variable with a Rayleigh distribution. Its mean-based probability density function (pdf) is given by

$$f(y;\mu) = \frac{\pi y}{2\mu^2} \exp\left(-\frac{\pi y^2}{4\mu^2}\right), \quad y \geq 0,$$

where $\mu > 0$ is the mean parameter.
Your task is to estimate the parameters of this Rayleigh distribution, with μ chosen at your discretion. The following steps outline the process:

1. Apply the maximum likelihood estimation method.
2. Utilize the `optim` function for optimization.
3. Compare results obtained using both analytical and numerical gradients.
4. Perform Monte Carlo simulations with various sample sizes ($n = 10, 20, 50, 90, 140$), and compare the outcomes.
5. Ensure that only converged models are included in your analysis.
6. Calculate key performance metrics for the estimated parameters: mean, bias, mean squared error (MSE), skewness, and kurtosis.
7. Summarize and discuss your findings.

Tip

To generate pseudo-random samples using the inversion method, use the following function:

```
# Inversion method
rr = function(mu, n) {
    u = runif(n)
    y = 2 * mu * sqrt(-log(1 - u) / pi)
    y
}
```

Here, `mu` is the parameter value, and `n` is the sample size.

Assignment Rules

1. You may work either alone or in groups of two (2) students.
2. The goal of this assignment is to estimate the parameters of the Rayleigh distribution using the R software.
3. Submit a brief written report (PDF format) via the course page on Canvas by the deadline (as posted on Canvas).
4. Include your code in the report.
5. Ensure that:
    - You include your **names** and **email addresses** in the report.
    - The report is logically and clearly **structured**, with written explanations.
    - You have thoroughly checked the report for **spelling and grammar mistakes**.
    - The report is written in **English**.

Grading

This assignment will be graded as `G/Ux/U`.

Important!

Failure to meet any of the above requirements may result in a failing grade, requiring you to revise and resubmit the report by a later deadline.

Good luck!