Tautologies

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1. Expression 1

$$
\begin{aligned}
&\equiv \neg((c \vee 1) \implies (a \wedge \neg((b \vee \neg b) \wedge (a \wedge (a \vee b))))) \\
&\equiv (c \vee 1) \wedge \neg(a \wedge \neg((b \vee \neg b) \wedge (a \wedge (a \vee b))))) \\
&\equiv 1 \wedge \neg(a \wedge \neg((b \vee \neg b) \wedge (a \wedge (a \vee b))))) \\
&\equiv \neg(a \wedge \neg((b \vee \neg b) \wedge (a \wedge (a \vee b)))) \\
&\equiv \neg a \vee \neg\neg((b \vee \neg b) \wedge (a \wedge (a \vee b))) \\
&\equiv \neg a \vee ((b \vee \neg b) \wedge (a \wedge (a \vee b))) \\
&\equiv \neg a \vee (1 \wedge (a \wedge (a \vee b))) \\
&\equiv \neg a \vee (a \wedge (a \vee b)) \\
&\equiv \neg a \vee a \\
&\equiv 1
\end{aligned}
$$

2. Expression 2

$$
\begin{aligned}
&\equiv (((d \wedge (d \wedge (d \vee a))) \vee (\neg c \vee c)) \implies (c \wedge \neg c)) \implies (b \wedge \neg b) \\
&\equiv (\neg((d \wedge (d \wedge (d \vee a))) \vee (\neg c \vee c)) \vee (c \wedge \neg c)) \implies (b \wedge \neg b) \\
&\equiv (\neg((d \wedge (d \wedge (d \vee a))) \vee (\neg c \vee c)) \vee 0) \implies (b \wedge \neg b) \\
&\equiv \neg((d \wedge (d \wedge (d \vee a))) \vee (\neg c \vee c)) \implies (b \wedge \neg b) \\
&\equiv \neg\neg((d \wedge (d \wedge (d \vee a))) \vee (\neg c \vee c)) \vee (b \wedge \neg b) \\
&\equiv (b \wedge \neg b) \vee \neg\neg((d \wedge (d \wedge (d \vee a))) \vee (\neg c \vee c)) \\
&\equiv 0 \vee \neg\neg((d \wedge (d \wedge (d \vee a))) \vee (\neg c \vee c)) \\
&\equiv \neg\neg((d \wedge (d \wedge (d \vee a))) \vee (\neg c \vee c)) \\
&\equiv (d \wedge (d \wedge (d \vee a))) \vee (\neg c \vee c) \\
&\equiv (d \wedge d) \vee (\neg c \vee c) \\
&\equiv d \vee (\neg c \vee c) \\
&\equiv d \vee 1 \\
&\equiv 1
\end{aligned}
$$

3. Expression 3

$$
\begin{aligned}
&\equiv \neg\neg((b \vee \neg b) \wedge (a \wedge (a \vee b))) \\
&\equiv ((b \vee \neg b) \wedge (a \wedge (a \vee b))) \\
&\equiv (1 \wedge (a \wedge (a \vee b))) \\
&\equiv (a \wedge (a \vee b)) \\
&\equiv a
\end{aligned}
$$

4. Expression 4

$$
\begin{aligned}
&\equiv (c \vee (b \vee \neg b)) \vee ((\neg(c \wedge (c \vee b)) \implies (c \vee 1)) \wedge (a \wedge \neg a)) \\
&\equiv (c \vee (b \vee \neg b)) \vee ((\neg\neg(c \wedge (c \vee b)) \vee (c \vee 1)) \wedge (a \wedge \neg a)) \\
&\equiv (c \vee (b \vee \neg b)) \vee ((\neg\neg(c \wedge (c \vee b)) \vee 1) \wedge (a \wedge \neg a)) \\
&\equiv (c \vee (b \vee \neg b)) \vee (\neg\neg(c \wedge (c \vee b)) \wedge (a \wedge \neg a)) \\
&\equiv (c \vee 1) \vee (\neg\neg(c \wedge (c \vee b)) \wedge (a \wedge \neg a)) \\
&\equiv (c \vee 1) \vee (\neg\neg(c \wedge (c \vee b)) \wedge 0) \\
&\equiv (c \vee 1) \vee (\neg\neg c \wedge 0) \\
&\equiv (c \vee 1) \vee (c \wedge 0) \\
&\equiv 1 \vee 0 \\
&\equiv 1
\end{aligned}
$$