Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
F		2108685

Checking the Pulse and Temperature of Higher Education

Summary

It is very important to evaluate the development level of a country's higher education system and its healthy trend. However, the health of a country's higher education system is often affected by many factors such as the level of economic development, the enrollment rate, the government's investment in education and so on. Different factors have different degrees of influence. Therefore, we need to train the original data set to reasonably determine the weight of the corresponding factors on the basis of determining several factors that affect the level of higher education system, so as to establish a mathematical model to evaluate the situation of a country's higher education system.

We set up the basic model of TOPSIS to evaluate the current situation of higher education system in various countries, and look for the influence degree of various factors and the cost of improvement. In view of the unbalanced development of higher education in the world, we evaluate four countries' higher education system. For Mexico,, it has also put forward various suggestions, including improving the scale of running schools, enhancing citizens' awareness of entering colleges and universities, increasing the graduation rate, formulating a series of incentive measures for scientific researchers, and increasing exchanges between universities and other universities in the world on the basis of evaluating its higher education.

In addition to evaluating the current health status of national higher education, we also established the corresponding grey prediction model and linear regression model, so as to predict whether the development of higher education in the next few years will show a positive side.

Keywords: Higher education, TOPSIS, Gray model, Linear regression

Contents

1. Problem restatement

1.1 Background

1.1.1 An Overview of the Problem

The higher education system is an important part of a country to improve the knowledge and cultural quality of its citizens. The long-term sound development of the country cannot do without a healthy and sustainable higher education system. Each country's higher education method is different, and each method has its advantages and disadvantages. Now there are two problems that need to be solved. The first is how to assess whether a country's higher education system is healthy, and the country's corresponding higher education Is the education system sustainable? The second is if the corresponding higher education system can be improved, how to formulate some policies and reform schedules to optimize the higher education system according to the actual situation of the country.

1.1.2 Factors for evaluating higher education

1. Higher education enrollment rate: The enrollment rate is used to measure and obtain the ratio of the number of enrolled students to the number of residents eligible for enrollment in higher education. This can be used to reflect the enrollment scale of a country's colleges and universities, student graduation status, etc.

2. Per capita government expenditures of higher education students (accounting for per capita GDP): a direct reflection of the government's capital investment in higher education, so as to understand the country's emphasis on higher education.

3. The proportion of the number of people receiving higher education to the total number of people (over 25 years old): Through this data, we can know whether the country's previous higher education system has been established properly and the popularization of national higher education.

4. The country's scientific and technical articles published in recent years: understand the country's higher education development level and reflect the scientific research capabilities of citizens with higher education.

5. Higher education graduation rate: Through the graduation situation of students, we can know the learning situation of students and the requirements of colleges and universities for students.

1.2 Task

- Develop and validate a model or suite of models that allow you to assess the health of any nation's system of higher education;
- Apply your model to several countries, and then select a nation whose system of higher education has room for improvement based on your analysis;
- Propose an attainable and reasonable vision for your selected nation's system that supports a healthy and sustainable system of higher education;
- Use your model to measure the health of both the current system and proposed, healthy, sustainable system for your selected nation;
- Propose targeted policies and an implementation timeline that will support the migration from the current state to your proposed state;
- Use your model(s) to shape and/or assess the effectiveness of your policies;
- Discuss the real-world impacts (e.g., on students, on faculty, on schools, on communities, on the nation) of implementing your plan both during the transition and in the end state, acknowledging the reality that change is hard.

2. Model Assumptions and Notations

2.1 Assumptions and Justifications

2.2 Notations

Notations	Meaning
ER	Higher education enrollment rate
GE	Government expenditure per student, tertiary (% of GDP per capita)
EA	Educational attainment, at least completed short-cycle tertiary, population 25+, total (%) (cumulative)
NA	Number of scientific and technical articles published each year
GR	Gross graduation ratio for tertiary education
X	Input
X_min	Small-better index
X_internal	Interval-better index
Z^+	the optimal target
Z^-	the worst target
D_i^+	the closeness of each evaluation object to the best scheme
D_i^-	the closeness of each evaluation object to the worst scheme
w_j	weight
$C.I.$	Consistency Index
λ_{max}	Maximum eigenvalue

3. The Basic Model of Security Checkpoints

3.1 The Design of the Model

The task of this topic is to evaluate the higher education in the world, which involves various factors. Our group first made a reasonable choice of the model, which is TOPSIS[1].

The model deals with different characteristics of different parameters which may own different index. All of them need to be translated to benefit index.

The model input data set is as follows:

$$Input: \quad X = \{x1, x2 \cdots x5\}$$

The properties of the parameters are as follows:

ER	GE	EA	NA	GR
Benefit index	Benefit index	Benefit index	Benefit index	Benefit index

Small-better index：

$$X\_min = \frac{1}{x}$$

Interval-better index：

$$X\_interval = \begin{Bmatrix} 1 - \dfrac{a - x}{a - a^*}, & (x < a) \\ 1, & (a \le x \le b) \\ 1 - \dfrac{x - b}{b^* - b}, & (x > b) \end{Bmatrix}$$

The data matrix is as follows:

$$X = \begin{bmatrix} X_{11} & \cdots & X_{1j} \\ \vdots & \ddots & \vdots \\ X_{i1} & \cdots & X_{ij} \end{bmatrix}$$

Normalization treatment:

$$Z_{ij} = \frac{X_{ij}}{\sqrt{\sum_{i=1}^{n} X_{ij}^2}}$$

Normalized matrix:

$$Z = \begin{bmatrix} Z_{11} & \cdots & Z_{1j} \\ \vdots & \ddots & \vdots \\ Z_{i1} & \cdots & Z_{ij} \end{bmatrix}$$

Determine the optimal target:

$$Z^+ = (\max\{Z_{11},\cdots Z_{i1}\},\ \cdots \max\{Z_{1j},\cdots Z_{ij}\})$$

Determine the worst target:

$$Z^- = (\min\{Z_{11},\cdots Z_{i1}\},\ \cdots \min\{Z_{1j},\cdots Z_{ij}\})$$

Calculate the closeness of each evaluation object to the best scheme and the worst scheme:

$$D_i^+ = \sqrt{\sum_{j=1}^{m} w_j (Z_i^+ - Z_{ij})^2}$$

$$D_i^- = \sqrt{\sum_{j=1}^{m} w_j (Z_i^- - Z_{ij})^2}$$

Get the score::

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-}$$

The closer the above data are to 1, the better their education level is.

The purpose of the model is to make full use of the information of the original data[2], and the results can accurately reflect the gap between higher education in different countries. And then reflect the health status of higher education in a country.

At the same time, because different factors have different influence on higher education, it is necessary to distribute the weight reasonably[3]。

We adopt the eigenvalue method:

$$C.I. = \frac{\lambda_{max} - n}{n - 1}$$

$$\lambda_{max} = \frac{1}{n}\left(\sum_{i=1}^{n} \frac{(BW)_i}{w_i}\right)$$

Considering that the larger n is, the more difficult it is for the judgment matrix B to satisfy the consistency, so different error limits should be given to the matrices of different orders

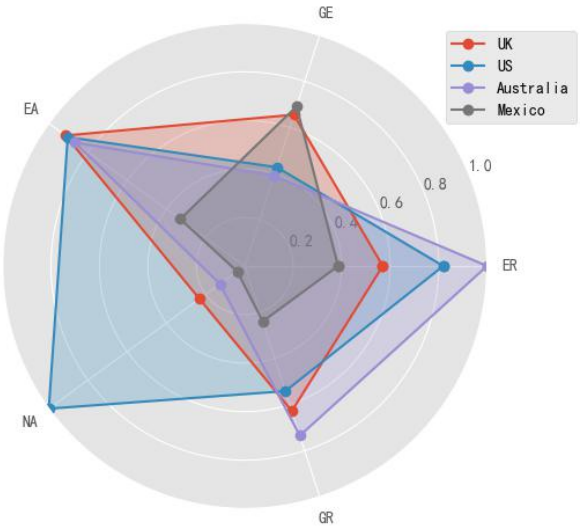
$$RI = [0, 0, 0.58, 0.90, 1.12, 1.24, 1.32, 1.41, 1.45, 1.49, 1.51]$$

At the same time, in order to predict the national higher education for a period of time in the future, we have carried out linear fitting and grey model prediction for each parameter of the target country to predict the future development of higher education.

3.2 The Result of the Model

We chose the data of 2018 to analyze four cities, namely, the United Kingdom, the United States, Australia and Mexico.

The initial data is shown in the figure



below:

Figure 1: Distribution of education in four countries

The weight distribution of each factor estimated by eigenvalue method is as follows：

ER	GE	EA	NA	GR
0.31507588	0.13507739	0.17161303	0.13372624	0.24450746

Table 3: Weight distribution

Based on this weight, evaluate the four countries by TOPSIS

United Kingdom	United States	Australia	Mexico
0.25545325	0.36971876	0.27400263	0.10082535

Table 4: score

To sum up, Mexico has the lowest score, and some problems can be clearly seen. For example, compared with other countries, Mexico has the highest Ge value, but the values of other indicators are the lowest. That is to say, in any way, Mexico's higher education related output is not proportional to its extremely high per capita government expenditure on higher education Obviously, there is a problem. It can be seen that its higher education situation needs to be improved.

Therefore, in the above four countries, we chose Mexico for further analysis and suggestions.

4. Recommendations and effectiveness evaluation for target countries

4.1 Recommendations

4.1.1 Visualization of 5 factors over the years

Combining the existing model results, we analyzed the historical data of these four countries from 2013 to 2018. For the sake of intuition, we visualized it.

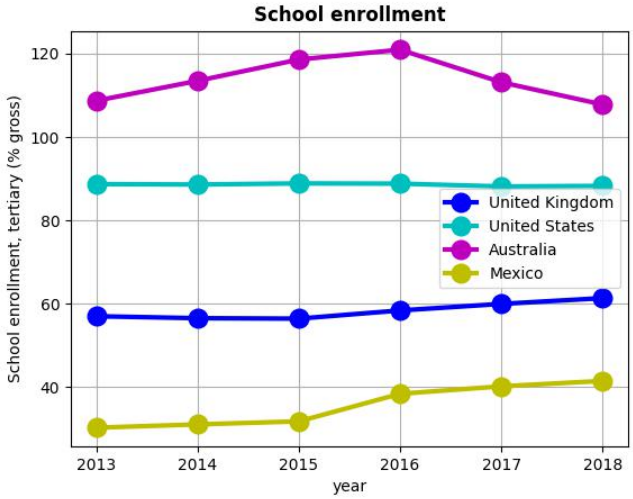


figure 2:School enrollment, tertiary (% gross)

Note: Since the higher education enrollment rate[4] is calculated as the number of higher education students in school/the number of corresponding age groups, its value may exceed 100%.

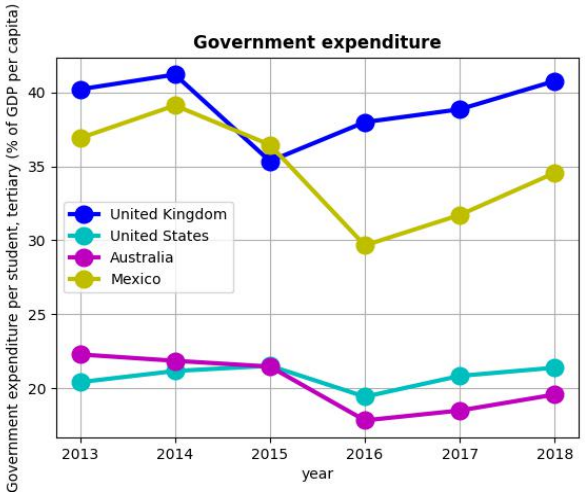


figure 3: Government expenditure per student, tertiary (% of GDP per capita)

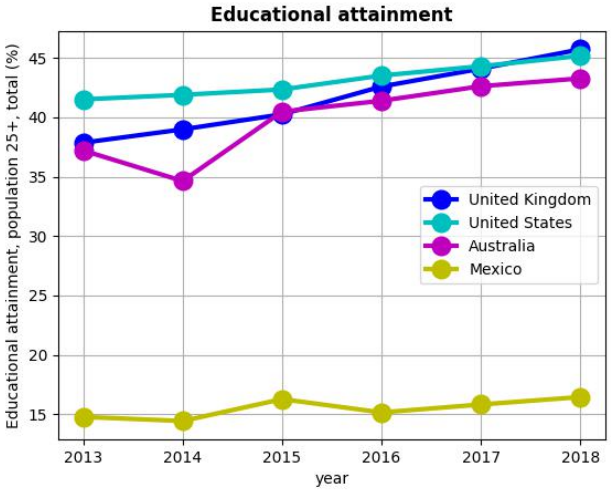


figure 4: Educational attainment, at least completed short-cycle tertiary, population 25+, total (%) (cumulative)

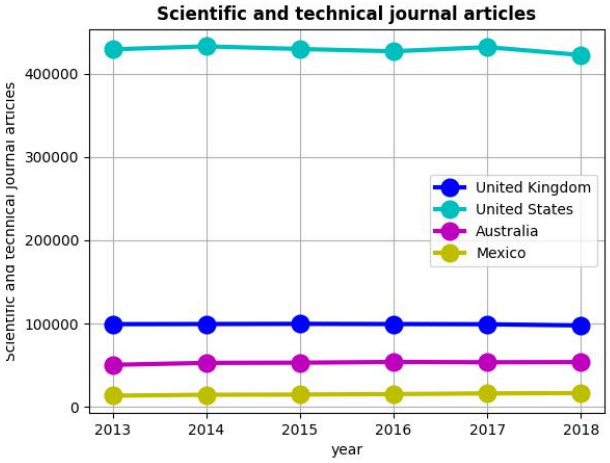


figure 5: Number of scientific and technical articles published each year

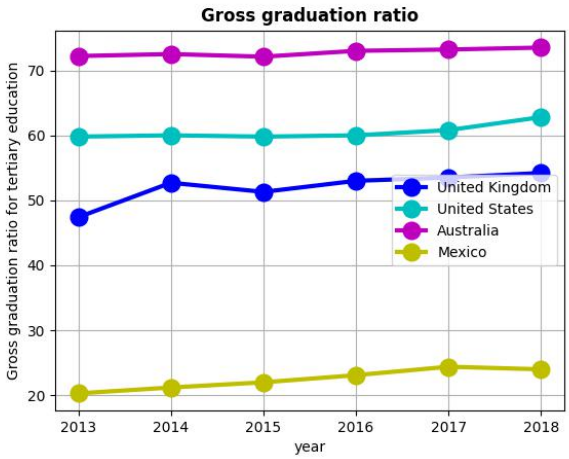


figure 6: Gross graduation ratio for tertiary education

Summarize the situation reflected by the above data, in the enrollment rate, the proportion of the number of people with higher education to the total number (over 25), the scientific and technical articles published in recent years in each country, and the graduation rate of higher education. Among the influencing factors, there is a clear gap between Mexico and other countries.

4.1.2 Percentage of international students

In addition to the above work, we also found the proportion of international students in higher education institutions in these four countries in 2013 and 2017.

	2013	2017
United Kingdom	18.98	21.14
United States	3.86	5.345
Australia	20.86	23.79
Mexico	0.20	0.46

Table 5: Percentage of international students

From the proportion of international students in these four countries, Mexico has the lowest proportion of international students. He is 20 times different from the United States, which ranks slightly higher than him, in 2013, and the difference is about 10 times in 2017. However, given the number of higher education institutions in the United States It is very large, and the United States was the country that most international students choose to go to during the 7 years from 2013 to 2019[2], so Mexico's higher education institutions are not highly recognized worldwide. Compared with the other three schools, its integration is far from enough.

4.1.3 Grey model and linear regression model prediction

To put forward some suggestions for a country's higher education to improve its health and sustainability, not only need to pay attention to the status quo of the country's higher education, but also to combine the changes in various indicators related to the country's higher education over the years. Therefore, we used the gray model and linear regression model to predict the changes in various indicators in Mexico in the next few years.

Linear simulation of the R^2 returned in the result of 5 factors(the coefficient of determination R^2 of the prediction) as follows：

factor	R^2
ER	0.9122706075142042

GE	0.38338717286571045
EA	0.5501590235813075
NA	0.9761462040246754
GR	0.9369670329670388

Table 6: the value of R^2

Remove the fitting results with R^2 less than 0.9 (the closer the R^2 is to 1, the better the fitting results), and use the Gray Model to predict.

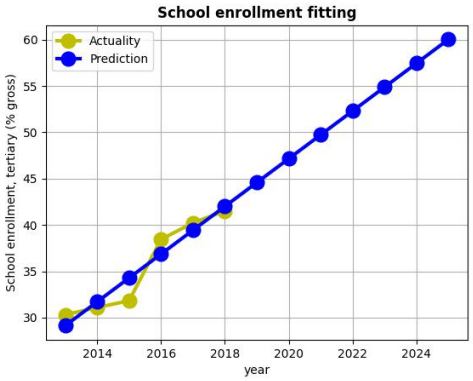


figure 7: Enrollment rate fitting

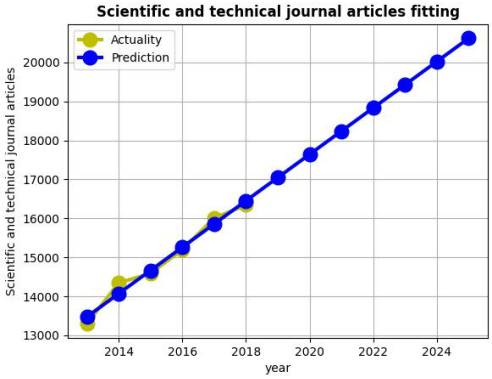


figure 8: Scientific and technical artical fitting

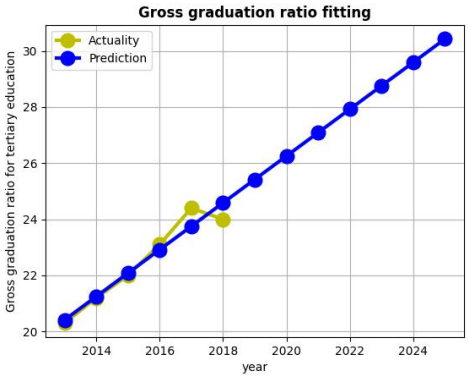


figure 9: Graduation rate fitting

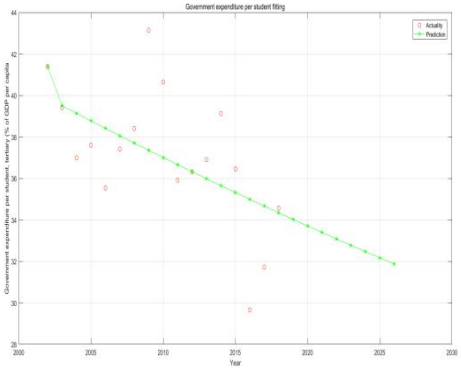


figure 10: Government expenditure fitting

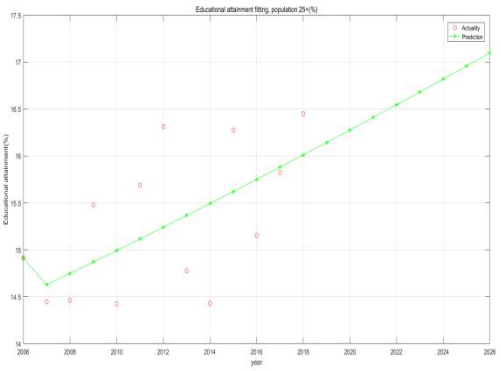


figure 11: Education penetration

4.1.4 Recommendations

Based on the above six factors, we give the following suggestions:

1. Increase the admission rate. This is the most basic and very important point for increasing higher education talents and enhancing the sustainability of higher education. Specific measures and policies can include:

1.1 Increase the number of domestic universities and provide more opportunities for further studies;

1.2 Relaxation of relevant restrictions on university entrance examinations, such as age, etc., or increase the size of adult universities and other possibilities;

1.3 It is necessary to reform middle school education and even more basic education;

2. Reform university education. Suggestions include:

2.1 Establish an incentive system, such as the establishment of bonuses, to vigorously support science and technology-related research, and reward the publication of science and technology-related articles;

2.2 Improve the quality of teaching and pay attention to the cultivation of higher education students' ability to acquire knowledge to increase student competitiveness and graduation rate.

2.3 Seize the opportunity to promote your university to the world, such as encouraging students to participate in international competitions, providing students with a high platform to encourage students to go out, which will not only exercise students, but also attract more international students.

Increase government investment in higher education or at least maintain the current level, because from the forecast situation, the value of the GE indicator in Mexican higher education will show a downward trend in the next few years.

This is obviously unreasonable. Sufficient investment in education can meet expectations. The necessary guarantee of output.

4.2 Effectiveness evaluation

On the basis of the above policies and recommendations, combined with the previous predictions of future changes, we hope that the values of various indicators will have more reasonable changes than the predicted values by 2025, that is, by 2025, we generally expect each The indicators were reached:

ER	GE	EA	NA	GR
70%	35%	25%	50000	50%

Table 7: Predictive value (2025, Mexico)

Comparing the data in 2018 with the data in 2025, you can see the changes intuitively:

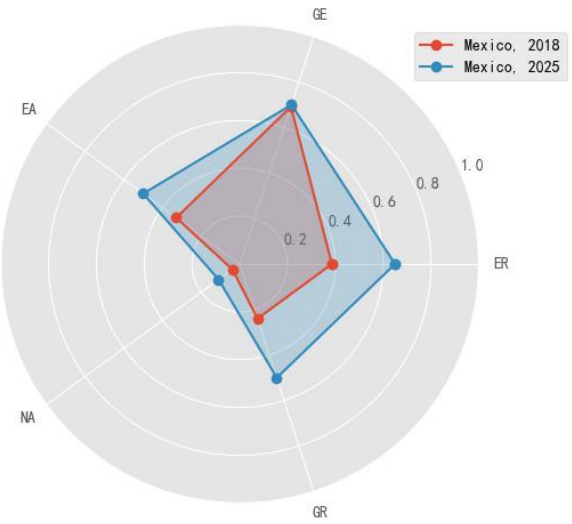


figure 12: Data comparison(2018 & 2025, Mexico)

It can be seen that with the support of the above policies, the health of higher education in Mexico has been improved considerably by 2025. Therefore, we can fully expect that under the influence of the above recommended policies, in ten years or even two Ten years later, the higher education situation in Mexico will be able to change for the better.

5. Conclusion

5.1 Strengths and weakness

5.1.1 Strengths

We compared the higher education systems of four countries: Britain in Europe, Australia in Oceania, and the United States and Mexico in North America. Through the establishment of a topsis model to analyze and compare their higher education enrollment rate, per capita government expenditure of higher education students (accounting for per capita GDP), the proportion of the number of people with higher education to the total number (over 25 years old), and the published science of each country in recent years And technical articles, higher education graduation rate. This model can intuitively compare and reflect the data of various countries, the results are displayed concisely and clearly, and it can be adjusted flexibly according to actual changes, etc.

5.1.2 Weakness

The problems of the model itself mainly include: the indicators we consider and choose may not be very comprehensive, and cannot particularly accurately reflect some of the real situation of the country's higher education; the calculation method of weights may find better alternatives.

5.2 Feature Outlook

We hope that in the next time we can collect more comprehensive data, add more consideration indicators for a more comprehensive comprehensive consideration, and apply this modified model to every country (if the data set supports it) , In order to raise the analysis work from the national level to the world level. If feasible, it may be possible to build a big data platform that encompasses the whole world.

Appendices

TOPSIS CODE

Language: Python

```
def maxtrixNormalize(data):
    K = np.power(np.sum(pow(data,2),axis =1),0.5)
    for i in range(0,K.size):
        for j in range(0,data[i].size):
            data[i,j] = data[i,j] / K[i]
```

```
    return data

def topsis(data,weight):
    listMax = np.array(
        [np.max(data[0, :]), np.max(data[1, :]), np.max(data[2, :]), np.max(data[3, :]),
np.max(data[4, :])])
    listMin = np.array(
        [np.min(data[0, :]), np.min(data[1, :]), np.min(data[2, :]), np.min(data[3, :]),
np.min(data[3, :])])
    maxList = []
    minList = []
    scoreList = []
    print(np.size(data, axis=1))
    for k in range(0, np.size(data, axis=1)):
        maxSum = 0
        minSum = 0
        for q in range(0, 5):
            maxSum += np.power(data[q, k] - listMax[q], 2) * weight[q]
            minSum += np.power(data[q, k] - listMin[q], 2) * weight[q] -
        maxList.append(pow(maxSum, 0.5))
        minList.append(pow(minSum, 0.5))
        scoreList.append(minList[k] / (minList[k] + maxList[k]))
    answer = np.array(scoreList)
    return (answer / np.sum(answer))
```

FITTING CODE

```
import matplotlib.pyplot as plt

import pandas as pd




def fitAndPredict(y, x, fig_path, ind, title, ylabel):

    '''

    create a model and fit it

    y = a + bx

    '''
```

```
    model = LinearRegression()
    model = model.fit(x, y)

    score = model.score(x, y)
    a = model.intercept_    # a
    b = model.coef_    # b

    # future
    x_append = x
    for j in range(2019, 2026):
        x_append = np.append(x_append, [[j]], axis=0)

    y_fit = model.predict(x_append)

    plt.plot(x, y, label='Actuality', linewidth=3, color='y', marker='o', markersize=12)
    plt.plot(x_append, y_fit, label='Prediction', linewidth=3, color='b', marker='o', markersize=12)

    plt.legend()    # display label
    plt.grid()
    plt.title(title[ind], fontsize='large', fontweight='bold')
    plt.xlabel('year')
    plt.ylabel(ylabel[ind])
    plt.savefig(fig_path[ind])
    plt.show()
    return a, b, score
```

```
path_data = '../dataset/data.xlsx'


df_data = pd.read_excel(path_data)
x = list(df_data.columns[2:])
x = np.array(x).reshape((-1, 1))
fig_path = [
        '../images/fit_school_enrollment.png',
        '../images/fit_government_expenditure.png',
        '../images/fit_educational_attainment.png',
        '../images/fit_journal_articles.png',
        '../images/fit_gross_graduation_ratio.png'
]
title = [
        'School enrollment fitting',
        'Government expenditure fitting',
        'Educational attainment fitting',
        'Scientific and technical journal articles fitting',
        'Gross graduation ratio fitting'
]
ylabel = [
        'School enrollment, tertiary (% gross)',
        'Government expenditure per student, tertiary (% of GDP per capita)',
        'Educational attainment, population 25+, total (%)',
        'Scientific and technical journal articles',
        'Gross graduation ratio for tertiary education',
]


linear_val = []
```

```
for ind in range(5):
    y = list(df_data.loc[ind + 15][2:])
    linear_val.append(fitAndPredict(y, x, fig_path, ind, title, ylabel))


for i in range(5):
    print(linear_val[i][2])
```

References

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[4] https://en.wikipedia.org/wiki/Gross_enrolment_ratio

[5] https://www.iie.org/Research-and-Insights/Project-Atlas/Explore-Data/

[6] https://www.iie.org/Research-and-Insights/Project-Atlas/Explore-Data

[7] https://www.iie.org/Research-and-Insights/Project-Atlas/Explore-Data/

[8] https://nces.ed.gov/ipeds/SummaryTables/report/813?templateId=8130&year=2019&expand_by=0&tt=aggregate&instType=1

[9] https://nces.ed.gov/ipeds/Search?query=GDP%20expenditure&query2=GDP%20expenditure&resultType=table&page=1&sortBy=relevance&overlayDigestTableId=201952

[10] https://open.canada.ca/data/en/dataset/69c30b1e-c346-4b02-930c-5c15fb0ed1d2

[11] https://knoema.com/atlas/Mexico/topics/Education/

[12] https://knoema.com/atlas/Australia/topics/Education/Tertiary-Education/Gross-enrolment-ratio-in-tertiary-education