hw5

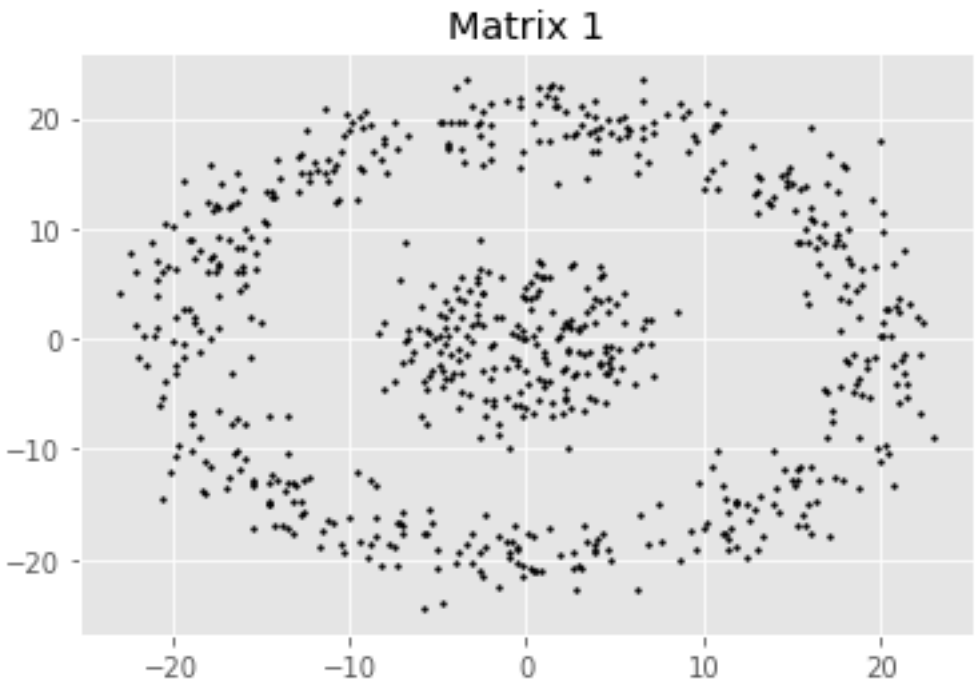
May 10, 2018

```
In [393]: import scipy.io
          import matplotlib.pyplot as plt
          import numpy as np
          import math
          import scipy

          %matplotlib inline
          plt.style.use('ggplot')
          np.random.seed(1)
```

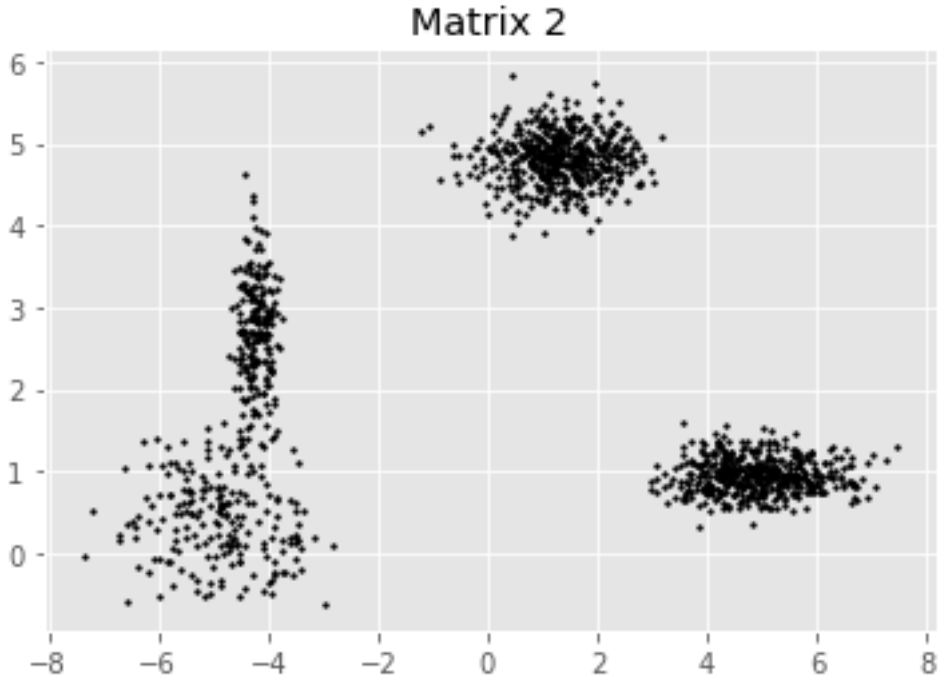
1 Datasets

We have two datasets, mat1 and mat2, both consisting of 2 dimensional samples (700, 1400 respectively). We plot these on an 2D plot to examine our data before clustering.

```
In [394]: mat1 = scipy.io.loadmat('data1.mat')['X']
          mat2 = scipy.io.loadmat('data2.mat')['X']

In [395]: mat1.shape, mat2.shape

Out[395]: ((2, 700), (2, 1400))

In [396]: plt.scatter(mat1[0], mat1[1], c='black', s=3)
          plt.title('Matrix 1')
          plt.show()
```



In [397]:
```
plt.scatter(mat2[0], mat2[1], c='black', s=3)
plt.title('Matrix 2')
plt.show()
```



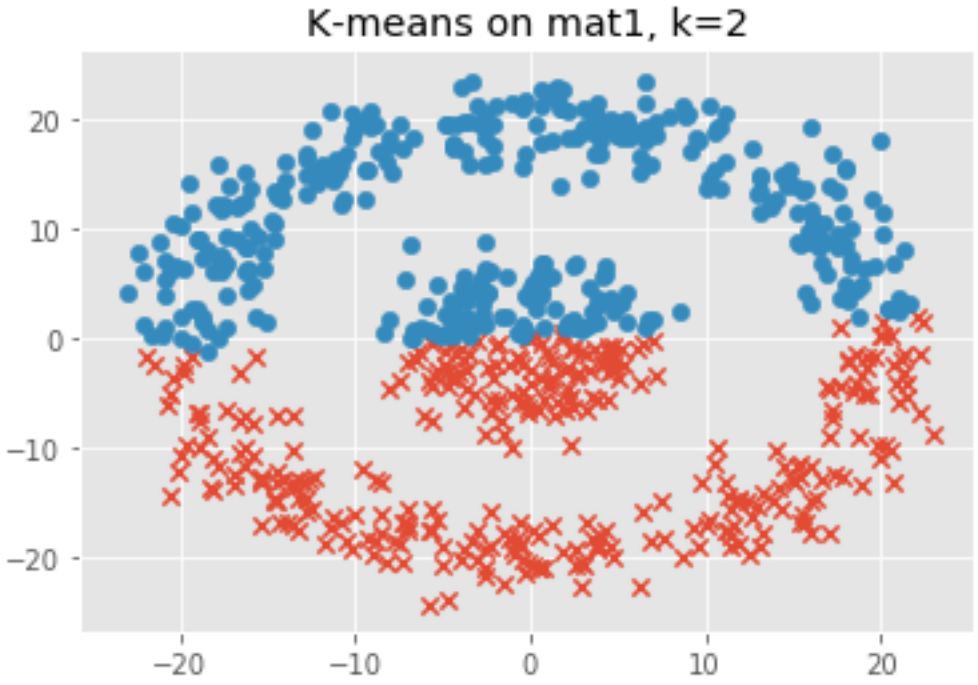
2 Problem 1, K-Means Clustering

We now try out K-means. K-means can only separate clusters by linear boundaries, so we expect mat1 to be unseparable by KMeans.

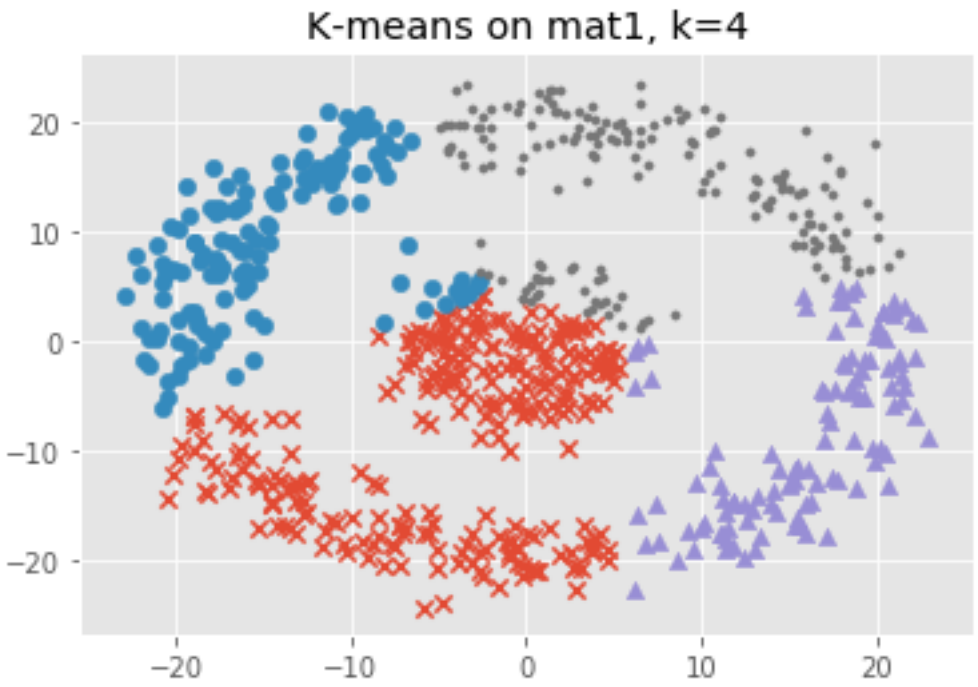
```
In [398]: def dist(a, b, ax=1):
              """ Euclidean distance """
              return np.linalg.norm(a-b, axis=ax)

In [399]: import copy

          def kmeans(X, k):
              C_x = np.random.uniform(0, np.max(X[0]), size=k)
              C_y = np.random.uniform(0, np.max(X[1]), size=k)
              C = np.array(list(zip(C_x, C_y)), dtype=np.float32)

              C_old = np.zeros(C.shape)
              clusters = np.zeros(X.shape[1])
              error = dist(C, C_old, None)

              flag = True

              while error != 0:
                  for i in range(X.shape[1]):
                      distances = dist((X[0][i], X[1][i]), C)
                      if numpy.isnan(distances).any() and flag:
                          flag = False
                          print(distances)
                          print("Coords: ")
                          print(X[0][i], X[1][i])
                          print("C: ")
                          print(C)
                      cluster = np.argmin(distances)
                      clusters[i] = cluster
                  C_old = copy.deepcopy(C)
                  for i in range(k):
                      points = np.array([(X[0][j], X[1][j]) for j in range(X.shape[1]) if clust
                      if len(points) == 0:
                          C[i] = 0
                      else:
                          C[i] = np.mean(points, axis=0)
                  error = dist(C, C_old, None)

              return clusters
```

In [400]:
```
def plot_clusters(X, clusters, k, title="Title here"):
    symbols = ['x', 'o', '^', '.', '1', '2', '3', '4']
    fig, ax = plt.subplots()
    for i in range(k):
        points = np.array([(X[0][j], X[1][j]) for j in range(X.shape[1]) if clusters
        if len(points) != 0:
            ax.scatter(points[:, 0], points[:, 1], s=40, marker=symbols[i])
    plt.title(title)
```

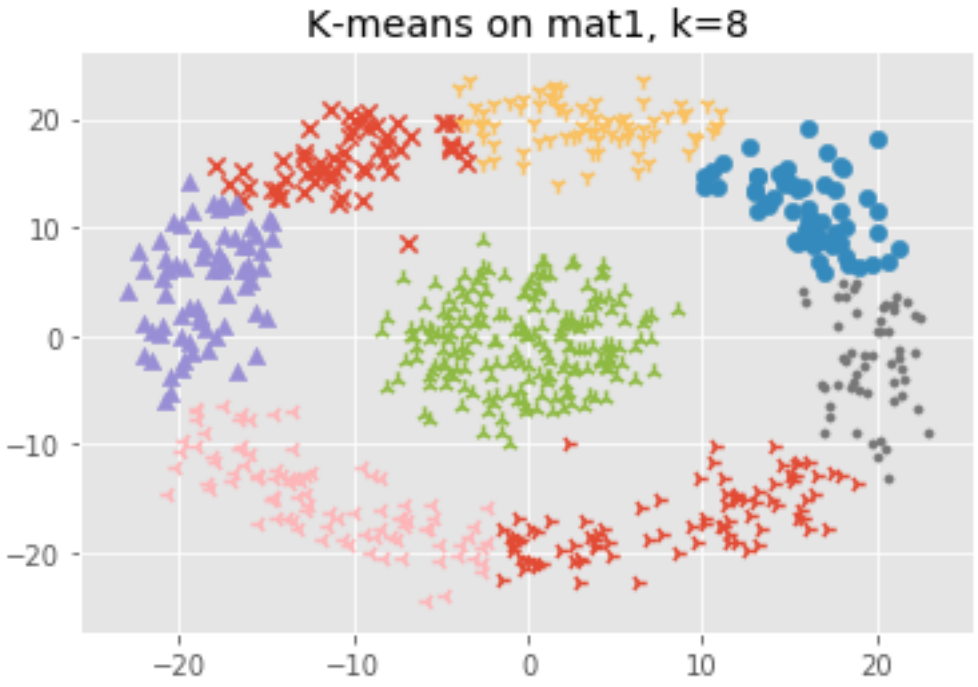
In [401]:
```
clusters = kmeans(mat1, k=2)
plot_clusters(mat1, clusters, 2, "K-means on mat1, k=2")
```



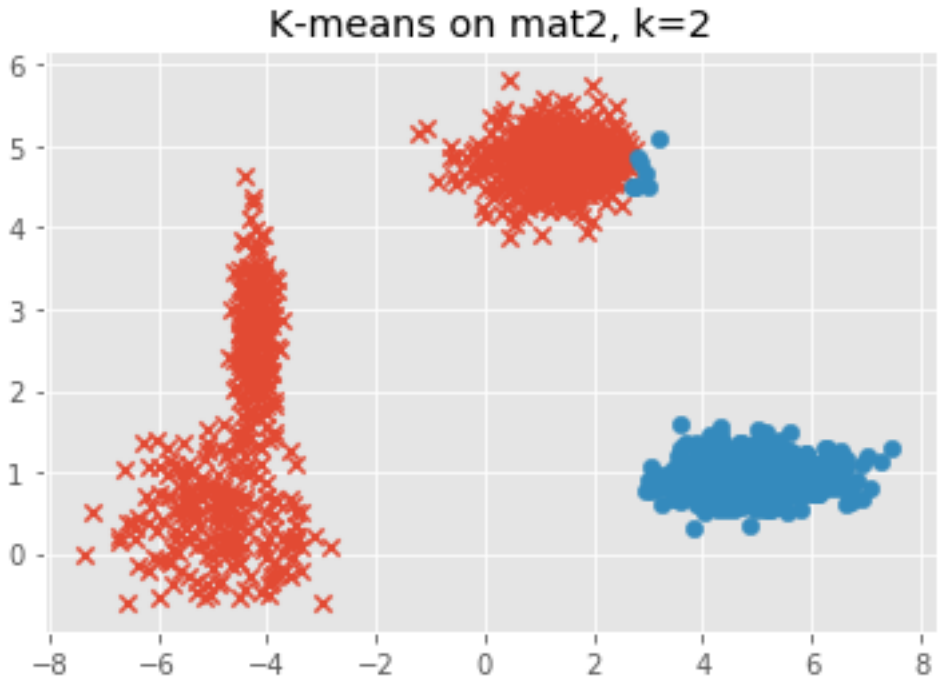
In [402]:
```
clusters = kmeans(mat1, k=4)
plot_clusters(mat1, clusters, 4, "K-means on mat1, k=4")
```



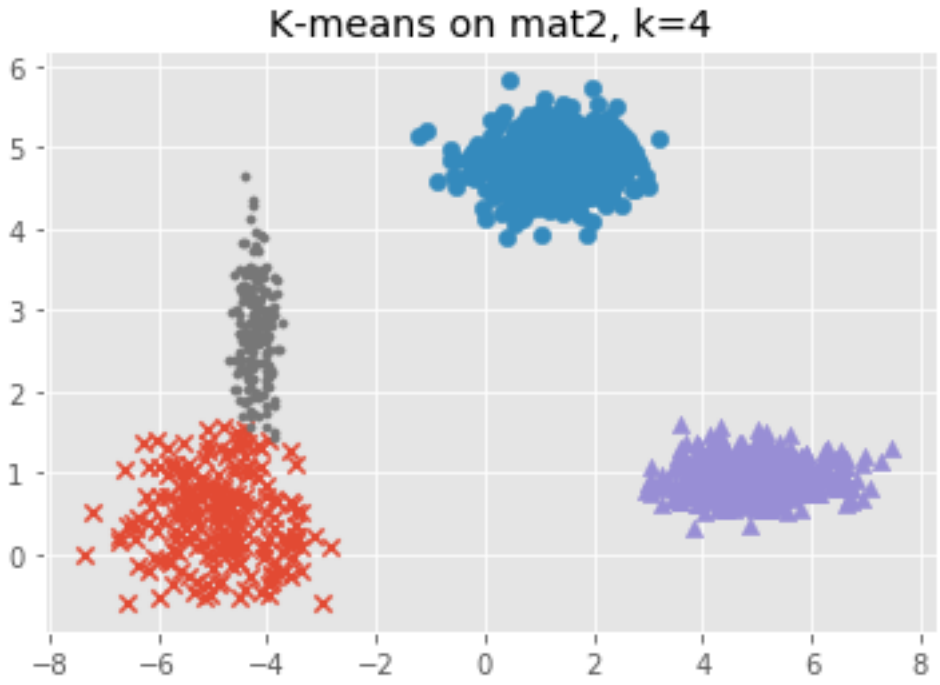
In [403]:
```
clusters = kmeans(mat1, k=8)
plot_clusters(mat1, clusters, 8, "K-means on mat1, k=8")
```



In [404]:
```
clusters = kmeans(mat2, k=2)
plot_clusters(mat2, clusters, 2, "K-means on mat2, k=2")
```

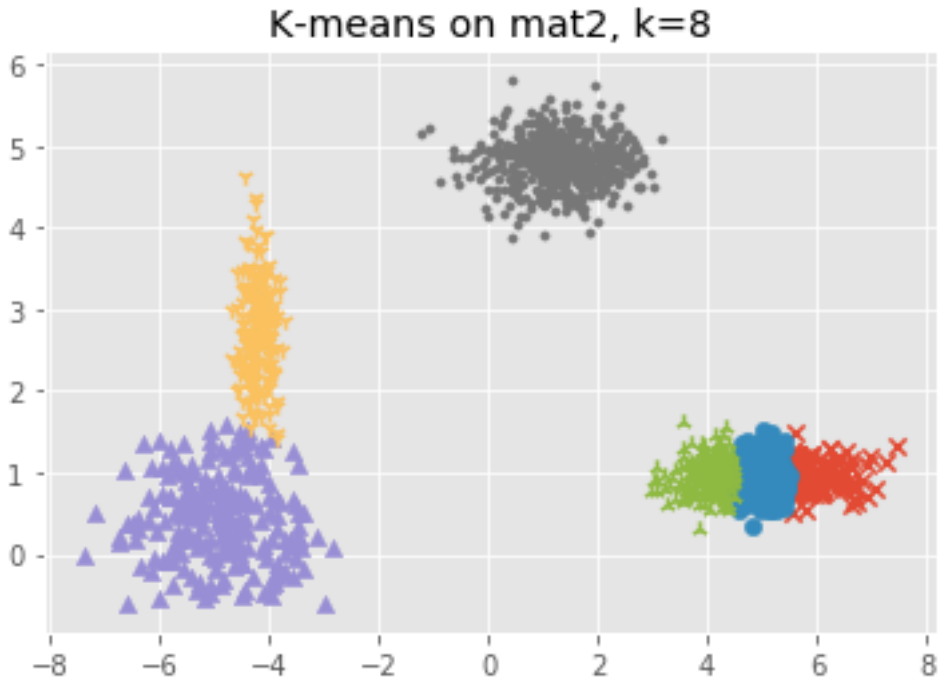


In [405]:
```
clusters = kmeans(mat2, k=4)
plot_clusters(mat2, clusters, 4, "K-means on mat2, k=4")
```



By inspection, we see that K-means clustering with 4 clusters successfully separates our mat2 data.

```
In [406]: clusters = kmeans(mat2, k=8)
          plot_clusters(mat2, clusters, 8, "K-means on mat2, k=8")
```



3 Problem 2, Spectral Clustering

Spectral clustering, or affinity-based clustering, can handle non linearly separable classes, so it may work on mat1.

```
In [407]: def similarity(a, b):
              return np.exp(-(abs(a-b)**2) / 10)
```

```
In [693]: def spectral_clustering(affinity, k):
              def laplacian(A):
                  D = np.zeros(A.shape)
                  w = np.sum(A, axis=0)
                  D.flat[::len(w) + 1] = w ** (-0.5)
                  return D.dot(A).dot(D)
              L = laplacian(affinity)
              _, eigvec = scipy.sparse.linalg.eigs(L, k)
```

```
            X = eigvec.real
            rows_norm = np.linalg.norm(X, axis=1, ord=2)
            labels = kmeans((X.T / rows_norm), k)
            return labels
```

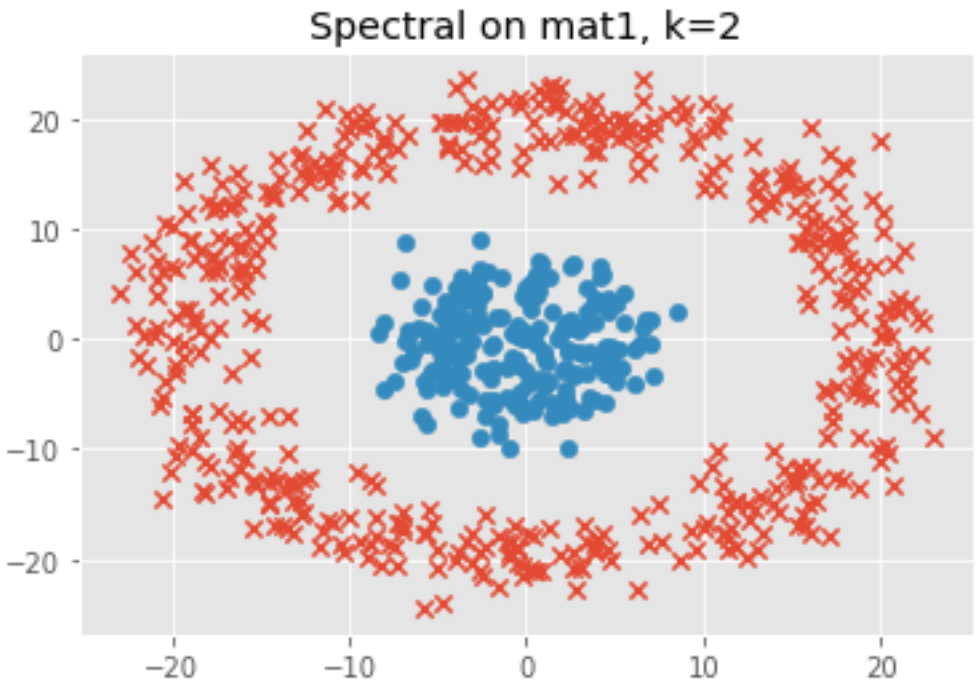
```
In [1133]: def compute_affinity(X):
               def squared_exponential(x, y, sig=0.8, sig2=1):
                   """ Models smooth functions

                   Function from previous spectral clustering project from tut """
                   norm = numpy.linalg.norm(x - y)
                   dist = norm * norm
                   return numpy.exp(- dist / (2 * sig * sig2))

               N = X.shape[0]
               res = np.zeros((N, N))
               sig = []
               for i in range(N):
                   dists = []
                   for j in range(N):
                       dists.append(np.linalg.norm(X[i] - X[j]))
                   dists.sort()
                   sig.append(np.mean(dists[:5]))

               for i in range(N):
                   for j in range(N):
                       res[i][j] = squared_exponential(X[i], X[j], sig[i], sig[j])
               return res
```

```
In [555]: affinity1 = compute_affinity(mat1.T)
```

```
In [556]: affinity1
```

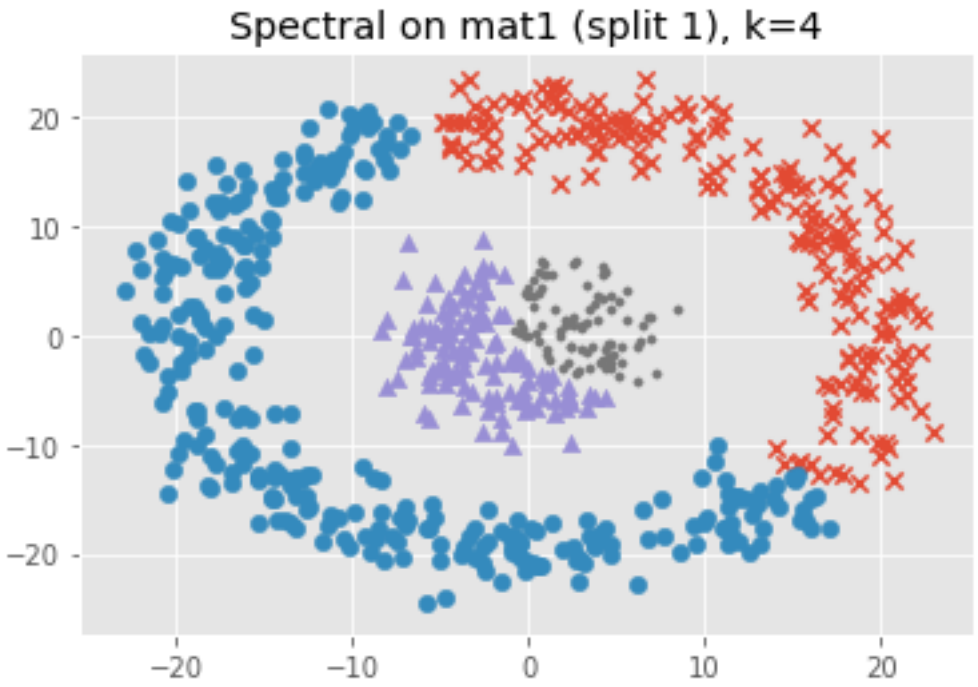
```
Out[556]: array([[1.00000000e+000, 9.07373112e-068, 2.13806994e-067, ...,
                  3.75536786e-156, 6.45797708e-058, 3.80635345e-104],
                 [9.07373112e-068, 1.00000000e+000, 1.28165649e-010, ...,
                  7.78220139e-177, 5.78066276e-003, 1.68477241e-160],
                 [2.13806994e-067, 1.28165649e-010, 1.00000000e+000, ...,
                  5.33745514e-118, 5.21226973e-015, 3.01382806e-099],
                 ...,
                 [3.75536786e-156, 7.78220139e-177, 5.33745514e-118, ...,
                  1.00000000e+000, 4.24887021e-210, 3.97603155e-019],
                 [6.45797708e-058, 5.78066276e-003, 5.21226973e-015, ...,
                  4.24887021e-210, 1.00000000e+000, 1.23857683e-179],
                 [3.80635345e-104, 1.68477241e-160, 3.01382806e-099, ...,
                  3.97603155e-019, 1.23857683e-179, 1.00000000e+000]])
```

```
In [697]: k1 = spectral_clustering(affinity1, 2)
          plot_clusters(mat1, k1, 2, "Spectral on mat1, k=2")
```



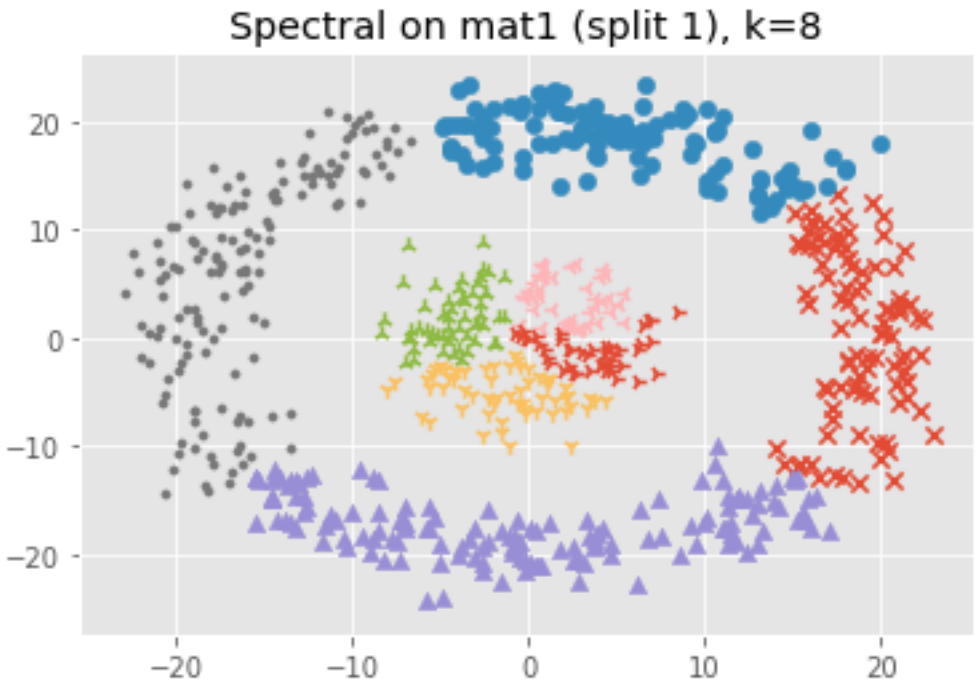
By inspection, we see that spectral clustering with 2 clusters successfully separates our mat1 data

```
In [521]: s1, s2 = mat1.T[[True if y==1 else False for y in Y]], mat1.T[[True if y==0 else Fals
          affinity1_1 = compute_affinity(s1)
          affinity1_2 = compute_affinity(s2)

In [522]: s1_labels = spectral_clustering(affinity1_1, 2)
          s2_labels = spectral_clustering(affinity1_2, 2)

          k2_labels = np.append(s1_labels, s2_labels+2)
          k2 = np.concatenate((s1, s2))
          plot_clusters(k2.T, k2_labels, 4, "Spectral on mat1 (split 1), k=4")
```



In [528]:
```
s3, s4, s5, s6 = k2[[True if y==0 else False for y in k2_labels]], k2[[True if y==1 e
affinity1_3 = compute_affinity(s3)
affinity1_4 = compute_affinity(s4)
affinity1_5 = compute_affinity(s5)
affinity1_6 = compute_affinity(s6)
```

In [535]:
```
s3_labels = spectral_clustering(affinity1_3, 2)
s4_labels = spectral_clustering(affinity1_4, 2)
s5_labels = spectral_clustering(affinity1_5, 2)
s6_labels = spectral_clustering(affinity1_6, 2)

k8_labels = np.concatenate((s3_labels, s4_labels+2, s5_labels+4, s6_labels+6))
k8 = np.concatenate((s3, s4, s5, s6))
plot_clusters(k8.T, k8_labels, 8, "Spectral on mat1 (split 2), k=8")
```



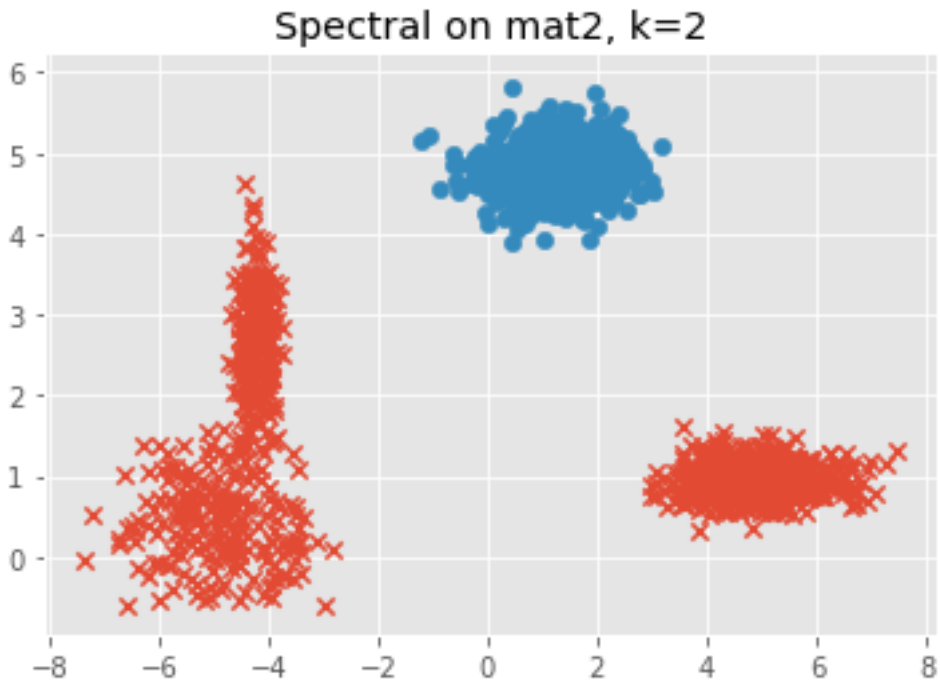
Spectral Wrapper Functions

```
In [592]: def spectral(M):
              affinity = compute_affinity(M.T)
              return spectral_clustering(affinity, 2), M

          def spectral4(M):
              Y, X = spectral(M)
              s1, s2 = M.T[[True if y==1 else False for y in Y]], X.T[[True if y==0 else False
              affinity1_1 = compute_affinity(s1)
              affinity1_2 = compute_affinity(s2)
              s1_labels = spectral_clustering(affinity1_1, 2)
              s2_labels = spectral_clustering(affinity1_2, 2)
              k2_labels = np.append(s1_labels, s2_labels+2)
              k2 = np.concatenate((s1, s2))
              return Y, X, k2_labels, k2

          def spectral8(M):
              Y1, X1, Y, X = spectral4(M)
              s3, s4, s5, s6 = X[[True if y==0 else False for y in Y]], X[[True if y==1 else Fa
              affinity1_3 = compute_affinity(s3)
              affinity1_4 = compute_affinity(s4)
              affinity1_5 = compute_affinity(s5)
              affinity1_6 = compute_affinity(s6)
```

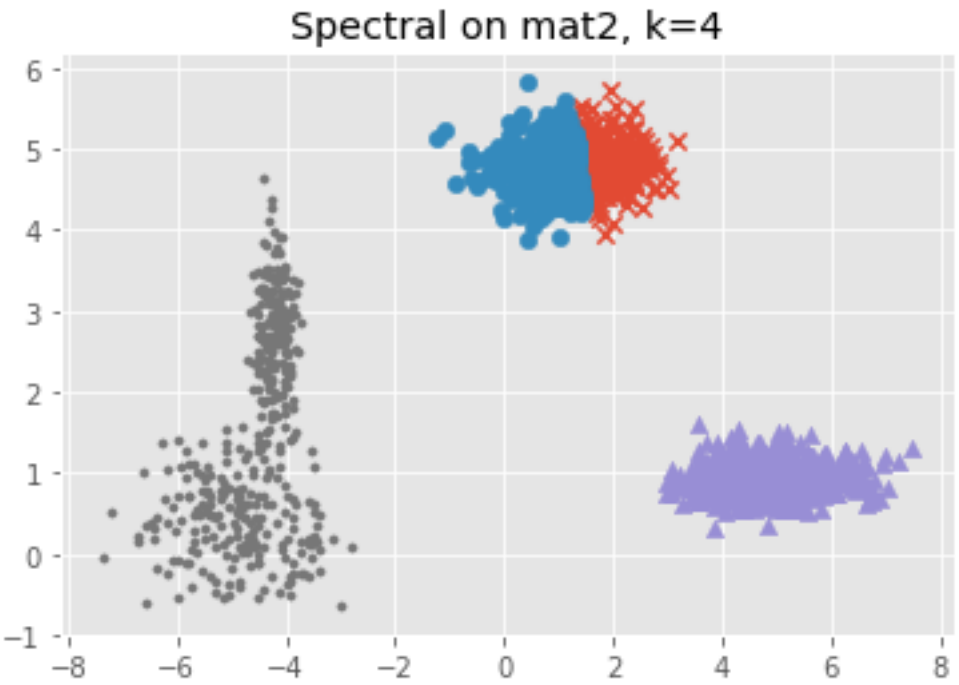
```
    s3_labels = spectral_clustering(affinity1_3, 2)
    s4_labels = spectral_clustering(affinity1_4, 2)
    s5_labels = spectral_clustering(affinity1_5, 2)
    s6_labels = spectral_clustering(affinity1_6, 2)

    k8_labels = np.concatenate((s3_labels, s4_labels+2, s5_labels+4, s6_labels+6))
    k8 = np.concatenate((s3, s4, s5, s6))
    return Y1, X1, Y, X, k8_labels, k8
```

In [593]:
```
Y1, X1, Y4, X4, Y8, X8 = spectral8(mat2)
```

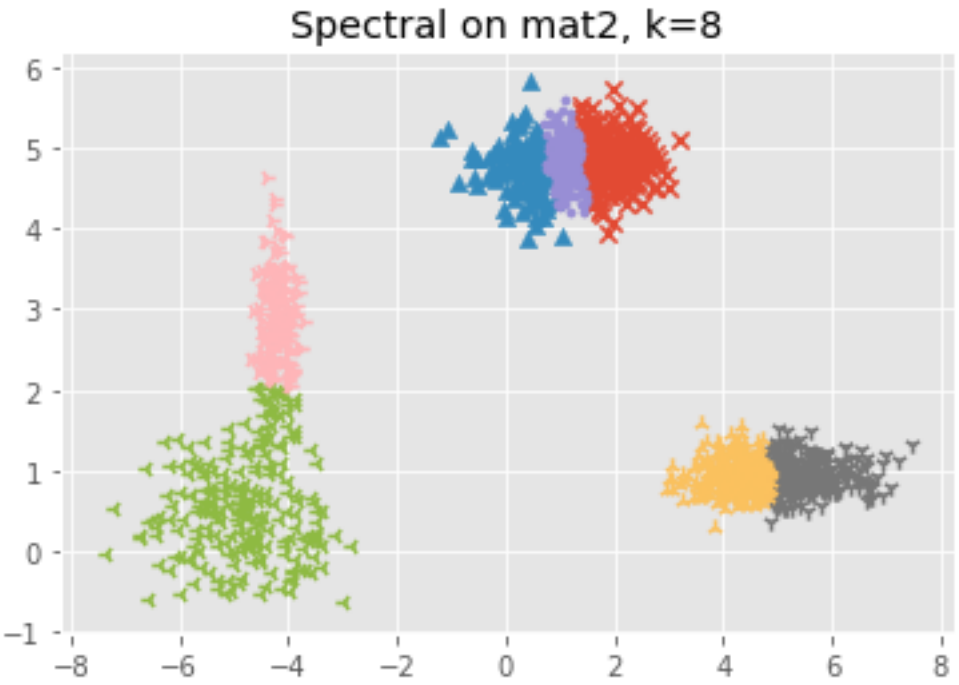
In [595]:
```
plot_clusters(X1, Y1, 2, "Spectral on mat2, k=2")
```



In [598]:
```
plot_clusters(X4.T, Y4, 4, "Spectral on mat2, k=4")
```




```
In [600]: plot_clusters(X8.T, Y8, 8, "Spectral on mat2, k=8")
```



4 Problem 3: Comparison with Cost Functions

In [677]:
```
def sum_squared_error(classes):
    Je = 0
    for cl in classes:
        mean = np.mean(cl, axis=0)
        for x in cl:
            Je += np.inner(x-mean, x-mean)
    return Je
```

In [724]:
```
def determinant(classes):
    S_W = 0
    for cl in classes:
        mean = np.mean(cl, axis=0)
        S_i = 0
        for i in range(np.array(cl).shape[0]):
            S_i += np.outer(cl[:][i]-mean, cl[:][i]-mean)
        S_W += S_i
    return np.linalg.det(S_W)
```

In [751]:
```
import itertools as it

def min_variance(classes):
    Je = 0
    for cl in classes:
        si = np.sum([np.inner(x-y,x-y) for x,y in it.combinations(cl, 2)])
        Je += si/np.array(cl).shape[0]
    return Je
```

In [737]:
```
def transform(X, Y, n):
    arr = [[] for i in range(n)]
    for i in range(len(Y)):
        arr[int(Y[i])].append(X[i])
    return arr
```

```
/usr/local/lib/python3.6/site-packages/numpy/core/fromnumeric.py:2957: RuntimeWarning: Mean of
  out=out, **kwargs)
/usr/local/lib/python3.6/site-packages/numpy/core/_methods.py:80: RuntimeWarning: invalid valu
  ret = ret.dtype.type(ret / rcount)
```

In [744]:
```
M2C2KMeans = transform(mat2.T, kmeans(mat2, k=2), 2)
M2C4KMeans = transform(mat2.T, kmeans(mat2, k=4), 4)
M2C8KMeans = transform(mat2.T, kmeans(mat2, k=8), 8)
M2C2Spectral = transform(X1.T, Y1, 2)
M2C4Spectral = transform(X4, Y4, 4)
M2C8Spectral = transform(X8, Y8, 8)

M1C2KMeans = transform(mat1.T, kmeans(mat1, k=2), 2)
```

```
M1C4KMeans = transform(mat1.T, kmeans(mat1, k=4), 4)
M1C8KMeans = transform(mat1.T, kmeans(mat1, k=8), 8)
M1C2Spectral = transform(mat1.T, k1, 2)
M1C4Spectral = transform(k2, k2_labels, 4)
M1C8Spectral = transform(k8, k8_labels, 8)
```

In [743]:
```
e1 = sum_squared_error(M2C2KMeans)
e2 = sum_squared_error(M2C4KMeans)
e3 = sum_squared_error(M2C8KMeans)
e4 = sum_squared_error(M2C2Spectral)
e5 = sum_squared_error(M2C4Spectral)
e6 = sum_squared_error(M2C8Spectral)

e7 = sum_squared_error(M1C2KMeans)
e8 = sum_squared_error(M1C4KMeans)
e9 = sum_squared_error(M1C8KMeans)
e10 = sum_squared_error(M1C2Spectral)
e11 = sum_squared_error(M1C4Spectral)
e12 = sum_squared_error(M1C8Spectral)
```

```
/usr/local/lib/python3.6/site-packages/numpy/core/fromnumeric.py:2957: RuntimeWarning: Mean of
  out=out, **kwargs)
/usr/local/lib/python3.6/site-packages/numpy/core/_methods.py:80: RuntimeWarning: invalid valu
  ret = ret.dtype.type(ret / rcount)
```

4.0.1 Sum Squared Error

In [738]:
```
print("Sum Squared Error, Dataset 2")
print("C=2 | KMeans: " + str(e1) + " Spectral: " + str(e4))
print("C=4 | KMeans: " + str(e2) + " Spectral: " + str(e5))
print("C=8 | KMeans: " + str(e3) + " Spectral: " + str(e6))
```

```
Sum Squared Error, Dataset 2
C=2 | KMeans: 11523.462641572416 Spectral: 12980.544476534591
C=4 | KMeans: 1308.589852762342 Spectral: 1367.4907262152626
C=8 | KMeans: 1249.690428419723 Spectral: 556.4301312921771
```

In [739]:
```
print("Sum Squared Error, Dataset 1")
print("C=2 | KMeans: " + str(e7) + " Spectral: " + str(e10))
print("C=4 | KMeans: " + str(e8) + " Spectral: " + str(e11))
print("C=8 | KMeans: " + str(e9) + " Spectral: " + str(e12))
```

```
Sum Squared Error, Dataset 1
C=2 | KMeans: 131904.90415142267 Spectral: 207710.95870316675
C=4 | KMeans: 67243.97450008814 Spectral: 129584.95960087515
C=8 | KMeans: 20491.671457244076 Spectral: 44623.695784612944
```

Discussion These results do not compare well with the visual observations given by the above clustering algorithms. When we increase the number of clusters, the numerical error obviously decreases. However, we see that dataset 1 (mat1) obviously has only 2 classes, so k=2 is the best (and is visually so, with the spectral clustering), but the numerical error decreases as we increase k. For dataset 2 which is linearly separable, KMeans and Spectral almost perform the same. This is not the case for dataset 1, where with K=2 Spectral successfully separates the clusters but KMeans does not. And yet, Spectral's sum squared error is 207710 > KMean's SSE of 139335.

4.0.2 Determinant

```
In [745]: e1 = determinant(M2C2KMeans)
          e2 = determinant(M2C4KMeans)
          e3 = determinant(M2C8KMeans)
          e4 = determinant(M2C2Spectral)
          e5 = determinant(M2C4Spectral)
          e6 = determinant(M2C8Spectral)

          e7 = determinant(M1C2KMeans)
          e8 = determinant(M1C4KMeans)
          e9 = determinant(M1C8KMeans)
          e10 = determinant(M1C2Spectral)
          e11 = determinant(M1C4Spectral)
          e12 = determinant(M1C8Spectral)
```

```
/usr/local/lib/python3.6/site-packages/numpy/core/fromnumeric.py:2957: RuntimeWarning: Mean of
  out=out, **kwargs)
/usr/local/lib/python3.6/site-packages/numpy/core/_methods.py:80: RuntimeWarning: invalid valu
  ret = ret.dtype.type(ret / rcount)
```

```
In [746]: print("Determinant, Dataset 2")
          print("C=2 | KMeans: " + str(e1) + " Spectral: " + str(e4))
          print("C=4 | KMeans: " + str(e2) + " Spectral: " + str(e5))
          print("C=8 | KMeans: " + str(e3) + " Spectral: " + str(e6))
```

```
Determinant, Dataset 2
C=2 | KMeans: 8107128.413448382 Spectral: 26358947.80734672
C=4 | KMeans: 145456.4670150636 Spectral: 446531.7034244581
C=8 | KMeans: 84989.73206552764 Spectral: 72199.32401656003
```

```
In [747]: print("Determinant, Dataset 1")
          print("C=2 | KMeans: " + str(e7) + " Spectral: " + str(e10))
          print("C=4 | KMeans: " + str(e8) + " Spectral: " + str(e11))
          print("C=8 | KMeans: " + str(e9) + " Spectral: " + str(e12))
```

```
Determinant, Dataset 1
C=2 | KMeans: 3285283551.736198 Spectral: 10751492347.651451
C=4 | KMeans: 975897318.337423 Spectral: 2557848611.074802
C=8 | KMeans: 120141496.7612552 Spectral: 494209586.2569458
```

4.0.3 Min Variance

```
In [753]: e1 = min_variance(M2C2KMeans)
          e2 = min_variance(M2C4KMeans)
          e3 = min_variance(M2C8KMeans)
          e4 = min_variance(M2C2Spectral)
          e5 = min_variance(M2C4Spectral)
          e6 = min_variance(M2C8Spectral)

          e7 = min_variance(M1C2KMeans)
          e8 = min_variance(M1C4KMeans)
          e9 = min_variance(M1C8KMeans)
          e10 = min_variance(M1C2Spectral)
          e11 = min_variance(M1C4Spectral)
          e12 = min_variance(M1C8Spectral)
```

```
/usr/local/lib/python3.6/site-packages/ipykernel_launcher.py:7: RuntimeWarning: invalid value e
  import sys
```

```
In [756]: print("Min Variance, Dataset 2")
          print("C=2 | KMeans: " + str(e1) + " Spectral: " + str(e4))
          print("C=4 | KMeans: " + str(e2) + " Spectral: " + str(e5))
          print("C=8 | KMeans: " + str(e3) + " Spectral: " + str(e6))
```

```
Min Variance, Dataset 2
C=2 | KMeans: 11523.46264157244 Spectral: 12980.544476534602
C=4 | KMeans: 955.5036139177143 Spectral: 1367.4907262152633
C=8 | KMeans: 637.8077128578185 Spectral: nan
```

```
In [757]: print("Min Variance, Dataset 1")
          print("C=2 | KMeans: " + str(e7) + " Spectral: " + str(e10))
          print("C=4 | KMeans: " + str(e8) + " Spectral: " + str(e11))
          print("C=8 | KMeans: " + str(e9) + " Spectral: " + str(e12))
```

```
Min Variance, Dataset 1
C=2 | KMeans: 131839.5046818803 Spectral: 207710.95870316686
C=4 | KMeans: 63969.713173109674 Spectral: 129584.9596008751
C=8 | KMeans: 22066.58996087212 Spectral: 44623.69578461293
```

Discussion We also note that Min Variance and Sum Squared Error are equivalent in this case.

5 Problem 4: Expectation Maximization

We use expectation maximization with gaussian assumption (gaussian mixture model) to find parameters (means, covariances, and priors) for the gaussian PDF. Once we converge (or in this case, run for 100 iterations), we use the last cluster result as our clustering.

This should be the slower of the three algorithms on this dataset without algorithmic optimization.

```
In [761]: mat2.shape

Out[761]: (2, 1400)

In [781]: for i in range(2,9):
              # find priors, means, and covariances
              print(i)

2
3
4
5
6
7
8


In [1081]: from scipy.stats import multivariate_normal

           def prob(val, mean, cov, prior):
               p = prior
               for i in range(len(val)):
                   try:
                       p *= multivariate_normal.pdf(val[i], mean[i], cov[i][i])
                   except IndexError:
                       p *= 1.0
               return p
```

5.0.1 Part A: Expectation Maximization

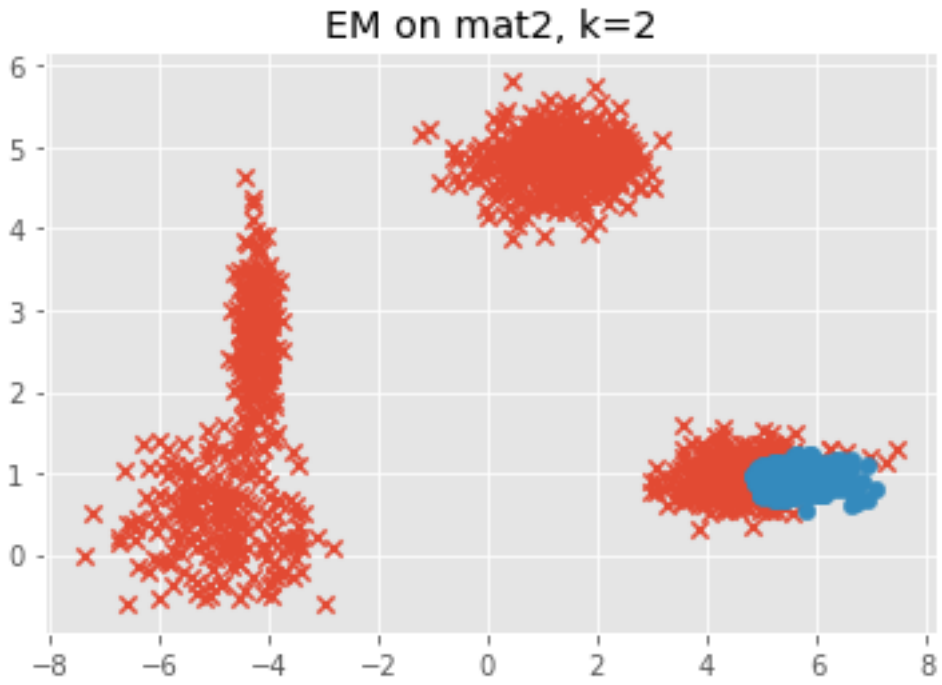
```
In [1094]: C_results = dict()

           for C in [2,4,8]:
               # initial parameters
               means = np.array([np.random.uniform(-5,5,2) for i in range(C)])
               covs = np.array([np.eye(2) for i in range(C)])
               priors = np.array([1.0/c] * C)

               clusters = []
               for i in range(10):
                   # evaluate most likely clusters for every point, given current parameters
                   clusters[:] = []
                   for i in range(mat2.shape[1]):
                       point = mat2[0][i], mat2[1][i]
                       cluster = np.argmax([prob(point, means[cl], covs[cl], priors[cl]) for cl
                       clusters.append(cluster)
```

```
            # reestimate parameters given current clusters
            assignments = np.array([np.concatenate(np.argwhere(np.array(clusters)==i)) i
            populations = np.array([len(i) for i in assignments])
            ratios = populations / np.sum(populations)

            means = np.array([np.mean(np.squeeze(np.dstack((mat2[0][assignments[i]], mat
            covs = np.array([np.cov(np.squeeze(np.dstack((mat2[0][assignments[i]], mat2
            priors = np.array(ratios)

        # get final clusters
        for i in range(mat2.shape[1]):
            point = mat2[0][i], mat2[1][i]
            cluster = np.argmax([prob(point, means[cl], covs[cl], priors[cl]) for cl in
            clusters.append(cluster)
        C_results[C] = clusters, means, covs, priors
```

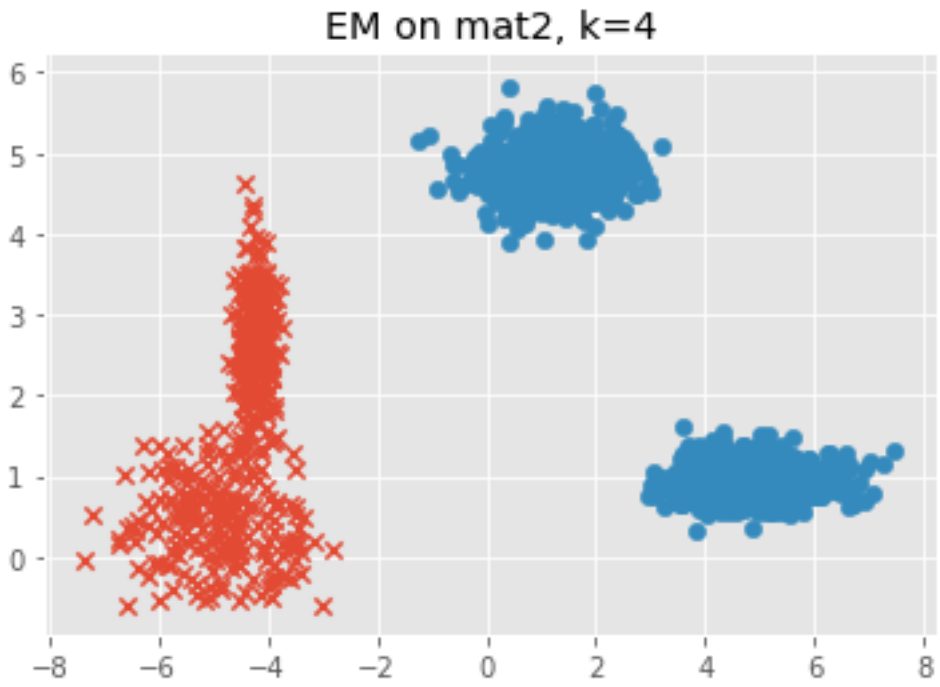
We use 10 iterations instead of 100 because of computer heating issues, and because of reasonable convergence with 10 iterations.

5.0.2 Part B: Clustering

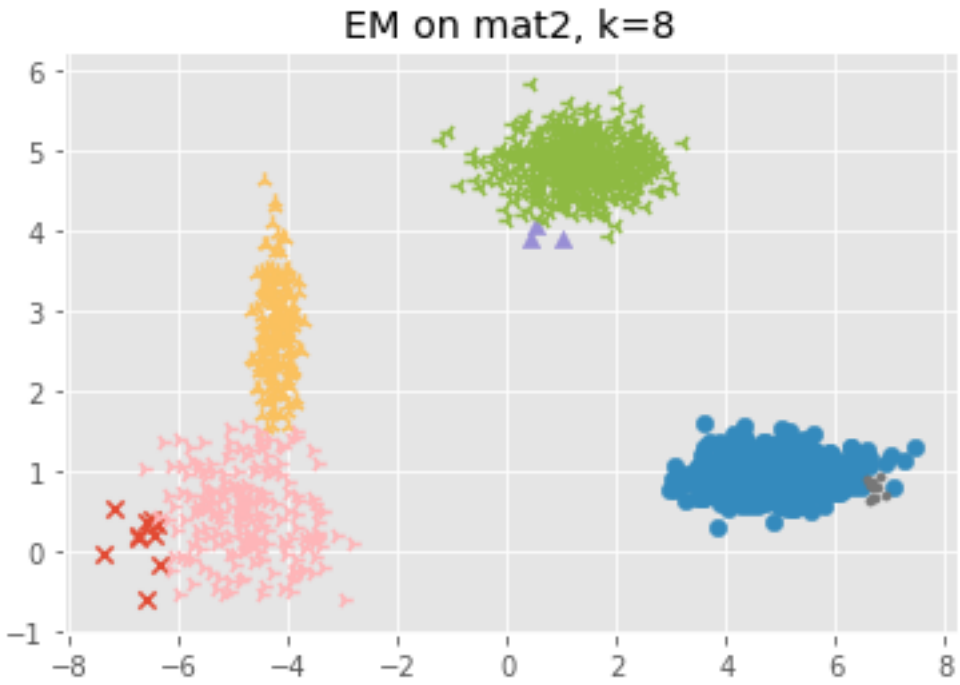
```
In [1095]: plot_clusters(mat2, C_results[2][0], 2, "EM on mat2, k=2")
```




```
In [1096]: plot_clusters(mat2, C_results[4][0], 4, "EM on mat2, k=4")
```



In [1097]: `plot_clusters(mat2, C_results[8][0], 8, "EM on mat2, k=8")`



5.0.3 Comparison with Cost Functions

```
In [1116]: M2C2EM = transform(mat2.T, C_results[2][0][:1400], 2)
           M2C4EM = transform(mat2.T, C_results[4][0][:1400], 4)
           M2C8EM = transform(mat2.T, C_results[8][0][:1400], 8)
```

```
In [1130]: e1 = sum_squared_error(M2C2KMeans)
           e2 = sum_squared_error(M2C4KMeans)
           e3 = sum_squared_error(M2C8KMeans)
           e4 = sum_squared_error(M2C2Spectral)
           e5 = sum_squared_error(M2C4Spectral)
           e6 = sum_squared_error(M2C8Spectral)
           e7 = sum_squared_error(M2C2EM)
           e8 = sum_squared_error(M2C4EM)
           e9 = sum_squared_error(M2C8EM)
```

```
/usr/local/lib/python3.6/site-packages/numpy/core/fromnumeric.py:2957: RuntimeWarning: Mean of
  out=out, **kwargs)
/usr/local/lib/python3.6/site-packages/numpy/core/_methods.py:80: RuntimeWarning: invalid valu
  ret = ret.dtype.type(ret / rcount)
```

```
In [1118]: print("Sum Squared Error, Dataset 2")
           print("C=2 | KMeans: " + str(e1) + " Spectral: " + str(e4) + " EM: " + str(e7))
           print("C=4 | KMeans: " + str(e2) + " Spectral: " + str(e5) + " EM: " + str(e8))
           print("C=8 | KMeans: " + str(e3) + " Spectral: " + str(e6) + " EM: " + str(e9))
```

```
Sum Squared Error, Dataset 2
C=2 | KMeans: 11523.462641572429 Spectral: 12980.544476534591 EM: 19650.16853656148
C=4 | KMeans: 955.5036139177154 Spectral: 1367.4907262152626 EM: 8523.805188355976
C=8 | KMeans: 637.8077128578192 Spectral: 556.4301312921771 EM: 877.5695135612715
```

```
In [1126]: e1 = min_variance(M2C2KMeans)
           e2 = min_variance(M2C4KMeans)
           e3 = min_variance(M2C8KMeans)
           e4 = min_variance(M2C2Spectral)
           e5 = min_variance(M2C4Spectral)
           e6 = min_variance(M2C8Spectral)
           e7 = min_variance(M2C2EM)
           e8 = min_variance(M2C4EM)
           e9 = min_variance(M2C8EM)
```

```
/usr/local/lib/python3.6/site-packages/ipykernel_launcher.py:7: RuntimeWarning: invalid value e
  import sys
```

```
In [1131]: print("Min Variance, Dataset 2")
           print("C=2 | KMeans: " + str(e1) + " Spectral: " + str(e4) + " EM: " + str(e7))
           print("C=4 | KMeans: " + str(e2) + " Spectral: " + str(e5) + " EM: " + str(e8))
           print("C=8 | KMeans: " + str(e3) + " Spectral: " + str(e6) + " EM: " + str(e9))
```

```
Min Variance, Dataset 2
C=2 | KMeans: 11523.462641572429 Spectral: 12980.544476534591 EM: 19650.16853656148
C=4 | KMeans: 955.5036139177154 Spectral: 1367.4907262152626 EM: 8523.805188355976
C=8 | KMeans: 637.8077128578192 Spectral: 556.4301312921771 EM: 877.5695135612715
```

```
In [1128]: e1 = determinant(M2C2KMeans)
           e2 = determinant(M2C4KMeans)
           e3 = determinant(M2C8KMeans)
           e4 = determinant(M2C2Spectral)
           e5 = determinant(M2C4Spectral)
           e6 = determinant(M2C8Spectral)
           e7 = determinant(M2C2EM)
           e8 = determinant(M2C4EM)
           e9 = determinant(M2C8EM)
```

```
/usr/local/lib/python3.6/site-packages/numpy/core/fromnumeric.py:2957: RuntimeWarning: Mean of
  out=out, **kwargs)
/usr/local/lib/python3.6/site-packages/numpy/core/_methods.py:80: RuntimeWarning: invalid valu
  ret = ret.dtype.type(ret / rcount)
```

```
In [1129]: print("Min Variance, Dataset 2")
           print("C=2 | KMeans: " + str(e1) + " Spectral: " + str(e4) + " EM: " + str(e7))
           print("C=4 | KMeans: " + str(e2) + " Spectral: " + str(e5) + " EM: " + str(e8))
           print("C=8 | KMeans: " + str(e3) + " Spectral: " + str(e6) + " EM: " + str(e9))
```

```
Min Variance, Dataset 2
C=2 | KMeans: 8107128.413448382 Spectral: 26358947.80734672 EM: 64168696.66829248
C=4 | KMeans: 145456.4670150636 Spectral: 446531.7034244581 EM: 6951116.3744544815
C=8 | KMeans: 84989.73206552764 Spectral: 72199.32401656003 EM: 128323.23851540437
```

6 Problem 5: HMMs

Transitions P(S1 | S1) = 0.8,P(S2 | S1) = 0.2,P(S1 | S2) = 0.2,P(S2 | S2) = 0.8

Emissions P(A | S1) = 0.4,P(C | S1) = 0.1,P(G | S1) = 0.4,P(T | S1) = 0.1
P(A | S2) = 0.1,P(C | S2) = 0.4,P(G | S2) = 0.1,P(T | S2) = 0.4

Priors P(S1) = 0.5, P(S2) = 0.5
Please see the `P5.pdf`