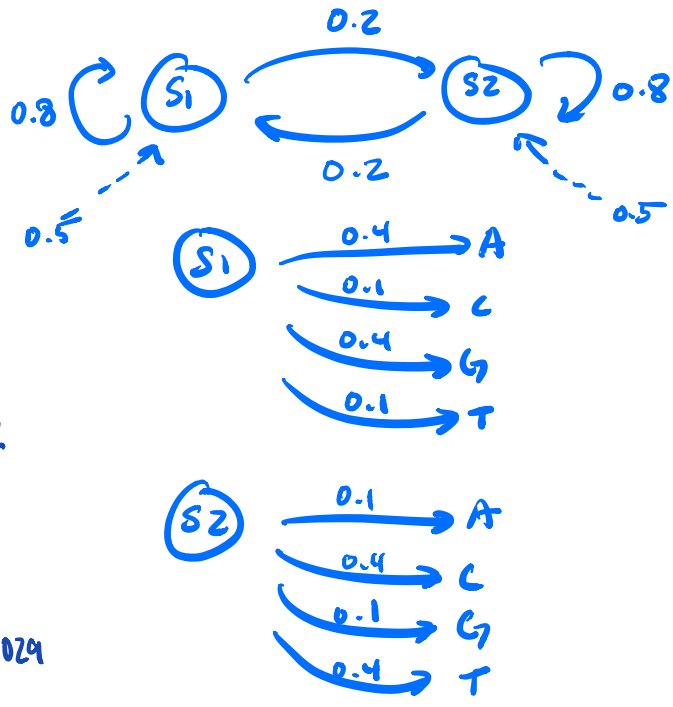
PART A



$P(\check{C}\check{G}\check{T}\check{C}\check{A}\check{G}) = ?$

$\alpha_1(1) = \pi_1 b_1(C) = (0.5)(0.1) = 0.05$
$\alpha_1(2) = \pi_2 b_2(C) = (0.5)(0.4) = 0.2$

$\alpha_2(1) = b_1(G)(\alpha_1(1)a_{11} + \alpha_1(2)a_{12})$
$= (0.4)((0.05)(0.8) + (0.2)(0.2)) = .032$
$\alpha_2(2) = b_2(G)(\alpha_1(1)a_{12} + \alpha_1(2)a_{22})$
$= (0.1)((0.05)(0.2) + (0.2)(0.8)) = .017$

$\alpha_3(1) = b_1(T)(\alpha_2(1)a_{11} + \alpha_2(2)a_{12})$
$= (0.1)((0.032)(0.8) + (0.017)(0.2)) = .0029$
$\alpha_3(2) = b_2(T)(\alpha_2(1)a_{12} + \alpha_2(2)a_{22})$
$= (0.4)((0.032)(0.2) + (0.017)) = .02$

$\alpha_4(1) = b_1(C)(\alpha_3(1)a_{11} + \alpha_3(2)a_{21})$
$= (0.1)((0.029)(0.8) + (0.02)(0.2)) = .00272$
$\alpha_4(2) = b_2(C)(\alpha_3(1)a_{12} + \alpha_3(2)a_{22})$
$= (0.4)((.0029)(0.2) + (.02)(0.8)) = .064232$

$\alpha_5(1) = b_1(A)(\alpha_4(1)a_{11} + \alpha_4(2)a_{21})$
$= (0.4)((0.00272)(0.8) + (0.064232)(0.2)) = .006 \rightarrow$
$\alpha_5(2) = b_2(A)(\alpha_4(1)a_{12} + \alpha_4(2)a_{22})$
$= (0.1)((0.00272)(0.2) + (0.064232)(0.8)) = .005 \rightarrow$

$\alpha_6(1) = b_1(G)(\alpha_5(1)a_{11} + \alpha_5(2)a_{21})$
$= (0.4)((0.006)(0.8) + (0.005)(0.2)) = .00232$
$\alpha_6(2) = b_2(G)(\alpha_5(1)a_{12} + \alpha_5(2)a_{22})$
$= (0.1)((0.006)(0.2) + (0.005)(0.8)) = .00052$

$p(C..G) = \alpha_6(1) + \alpha_6(2) \approx .00232 + .00052 = .00284 \approx \boxed{0.3\%}$

PART B

$P(q_1 = S_1 | O, \lambda) = \pi_1 = 0.5$

$p(q_2 = S_1 | O, \lambda) = p(q_2 = S_1 | q_1 = S_1)\, p(q_1 = S_1) + p(q_2 = S_1 | q_1 = S_2)\, p(q_1 = S_2) = (0.8)(0.5) + (0.2)(0.5) = 0.5$

$p(q_t = S_1 | O, \lambda) = \boxed{0.5}$

PART C

$$\delta_{t+1}(i) = \max_j \{\delta_t(j) * a_{ji} * b_i(O_{t+1})\}$$

① $\alpha_1(2) = 0.2$ $\quad\quad$ $\alpha_1(1) = 0.05$

② $p_1(G,2) = (0.4)\max\{p_1(C,1)a_{11}, p_2(C,1)a_{21}\} = 0.4\max\{(.05)(.8), (.2)(.2)\}$
$= 0.4(.04) = 0.016$

$p_2(G,2) = (0.1)\max\{p_1(C,1), a_{12}, p_2(C,1), a_{22}\} = 0.1\max\{(.05)(.2), (.2)(.8)\}$
$= 0.1(.16) = .016$

	C	G	T	C	A	G
S_1	.05	.016	.00128	.000 1204	.00 13 467	.0001346 x.4x.8
S_2	.20	.016	.00512	.0016 384	.000 13467	

③ $p_1(T,3) = (0.1)\max\{p_1(G,2)a_{11}, p_2(G,2)a_{21}\} = 0.1\max\{(.016)(.8), (.16)(.2)\}$
$= 0.1(.0128) = .00128$

$p_2(T,3) = (0.4)\max\{p_1(G,2)a_{12}, p_2(G,2)a_{22}\} = 0.4\max\{(.016)(.2), (.016)(.8)\}$
$= .00512$

④ $p_1(C,4) = (0.1)\max\{p_1(T,3)a_{11}, p_1(T,3)a_{21}\} = (0.1)\max\{(.00128)(.8), (.00512)(.2)\}$
$= (0.1)(.001024) = .0001024$

$p_2(C,4) = (0.4)\max\{p_1(T,3)a_{12}, p_1(T,3)a_{22}\} = (0.4)\max\{(.00128)(.2), (.00512)(.8)\}$
$= 0.4(.00512)(.8) = .0016384$

⑤ $p_1(A,5) = (0.4)\max\{p_1(C,4)a_{11}, p_2(C,4)a_{21}\} = (0.4)\max\{(.0001024)(.8), (.0016384)(.2)\}$
$= (0.4)(.00032768)$
$= .00013107$

$p_2(A,5) = (0.1)\max\{p_1(C,4)a_{12}, p_2(C,4)a_{22}\} = (0.1)\max\{(.0001024)(.2), (.0016384)(.8)\}$
$= (0.1)(.0013467)$
$= .00013467$

⑥ $p_1(G,6) = (0.4)\max\{p_1(A,5)a_{11}, p_1(A,5)a_{21}\} = (0.4)\max\{(.00013107)(.8), (.00013467)(.2)\}$
$= 0.4(.00013107)(.8)$

$p_2(G,6) = (0.1)\max\{p_1(A,5)a_{12}, p_2(A,5)a_{22}\} = (0.1)\max\{(.00013107)(.2), (.00013467)(.8)\}$
$= < p_1(G,6)$

Most probable path is S2 - S2 - S2 - S2 - S1 - S1.