Trabalho 2

Resolução de Sistemas Lineares e Não-lineares

- Iterações

Matriz escolhida:

$$Q = \begin{bmatrix} 8 & -4 & 0 & -1 & 0 & 0 & 20 \\ 0 & -2.5 & 4.5 & 0 & 0 & -2 & 14 \\ 0 & -5 & 0 & -2 & 8.5 & -1.5 & -30 \\ -4 & 11.5 & -2.5 & 0 & -5 & 0 & -12 \\ -1 & 0 & 0 & 3 & -2 & 0 & 8 \\ 0 & 0 & -2 & 0 & -1.5 & 8 & 0 \end{bmatrix}$$

→ A matriz Q deve ser pivotada.

- Método de Jacobi

{Solução inicial

$x_i^{(1)} = 0 \quad , i = 1, 2, \ldots, 6$

{1ª iteração

{$i = 1$

$$x_1^{(2)} = \frac{1}{a_{11}}\left[b_1 - \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{1j} x_j^{(1)}\right] = \frac{1}{8}\left[20 - \sum_{\substack{j=1 \\ j \neq i}}^{6} a_{1j} x_j^{(1)}\right]$$

{$i = 2$

$$= \frac{20}{8} = 2{,}5$$

$$x_2^{(2)} = \frac{1}{a_{22}}\left[b_2 - \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{2j} x_j^{(1)}\right] = \frac{1}{11.5}\left[-12 - 0\right]$$

{$i = 3$

$$= -1.043478$$

$$x_3^{(2)} = \frac{1}{a_{33}}\left[b_3 - \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{3j} x_j^{(1)}\right] = \frac{1}{4.5}\left[14 - 0\right] = 3{,}111111$$

{$i = 4$

$$x_4^{(2)} = \frac{1}{a_{44}}\left[b_4 - \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{4j} x_j^{(1)}\right] = \frac{1}{3}\left[8 - 0\right] = 2{,}666666$$

{$i = 5$

$$x_5^{(2)} = \frac{1}{a_{55}}\left[b_5 - \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{5j} x_j\right] = \frac{1}{8.5}\left[-30 - 0\right] = -3.529411$$

{ i = 6

$$x_6^{(2)} = \frac{1}{a_{66}}\left[b_6 - \sum_{\substack{j=1 \\ j\neq i}}^{m} a_{6j}\overset{0}{x_j}\right] = \frac{1}{8}\left[0 - 0\right] = 0$$

$$\left[x_1^{(2)}, x_2^{(2)}, x_3^{(2)}, x_4^{(2)}, x_5^{(2)}, x_6^{(2)}\right] = [2.5, -1.043478, 3.111111, 2.666666, -3.52941 1, 0]$$

{ 2ª iteração

{ i = 1

$$x_1^{(3)} = \frac{1}{a_{11}}\left[b_1 - \sum_{\substack{j=1 \\ j\neq i}}^{m} a_{1j}x_j^{(2)}\right] = \frac{1}{8}\left[20 - 1.507248\right] = 2.311594$$

{ i = 2

$$x_2^{(3)} = \frac{1}{a_{22}}\left[b_2 - \sum_{\substack{j=1 \\ j\neq i}}^{m} a_{2j}x_j^{(2)}\right] = \frac{1}{11.5}\left[-12 + 0.130723\right] = -1.032111$$

{ i = 3

$$x_3^{(3)} = \frac{1}{a_{33}}\left[b_3 - \sum_{\substack{j=1 \\ j\neq i}}^{m} a_{3j}x_j^{(2)}\right] = \frac{1}{4.5}\left[14 - 2.60837\right] = 2.531400$$

{ i = 4

$$x_4^{(3)} = \frac{1}{a_{44}}\left[b_4 - \sum_{\substack{j=1 \\ j\neq i}}^{m} a_{4j}x_j^{(2)}\right] = \frac{1}{3}\left[8 - 4.558826\right] = 1.147058$$

{ i = 5

$$x_5^{(3)} = \frac{1}{a_{55}}\left[b_5 - \sum_{\substack{j=1 \\ j\neq i}}^{m} a_{5j}x_j^{(2)}\right] = \frac{1}{8.5}\left[-30 + 0.1159\right] = -3.515771$$

{ i = 6

$$x_6^{(3)} = \frac{1}{a_{66}}\left[b_6 - \sum_{\substack{j=1 \\ j\neq i}}^{m} a_{6j}x_j^{(2)}\right] = \frac{1}{8}\left[0 + 0.928104\right] = 0.116013$$

{3ª iteração

{i = 1

$$x_1^{(4)} = \frac{1}{a_{11}}\left[b_1 - \sum_{\substack{j=1 \\ j\neq i}}^{m} a_{1j}x_j\right] = \frac{1}{8}\left[20 - 2.981391\right] = 2.127326$$

{i = 2

$$x_2^{(4)} = \frac{1}{a_{22}}\left[b_2 - \sum_{\substack{j=1 \\ j\neq i}}^{m} a_{2j}x_j\right] = \frac{1}{11,5}\left[-12 - 2.003575\right] = -1.217737$$

{i = 3

$$x_3^{(4)} = \frac{1}{a_{33}}\left[b_3 - \sum_{\substack{j=1 \\ j\neq i}}^{m} a_{3j}x_j\right] = \frac{1}{4.5}\left[14 - 2.348253\right] = 2.589277$$

{i = 4

$$x_4^{(4)} = \frac{1}{a_{44}}\left[b_4 - \sum_{\substack{j=1 \\ j\neq i}}^{m} a_{4j}x_j\right] = \frac{1}{8}\left[8 - 4.719950\right] = 1.093350$$

{i = 5

$$x_5^{(4)} = \frac{1}{a_{55}}\left[b_5 - \sum_{\substack{j=1 \\ j\neq i}}^{m} a_{5j}x_j\right] = \frac{1}{8.5}\left[-30 - 2.692419\right] = -3.846167$$

{i = 6

$$x_6^{(4)} = \frac{1}{a_{66}}\left[b_6 - \sum_{\substack{j=1 \\ j\neq i}}^{m} a_{6j}x_j\right] = \frac{1}{8}\left[0 - 0.210848\right] = -0.026356$$

{4ª iteração

{i = 1

$$x_1^{(5)} = \frac{1}{a_{11}}\left[b_1 - \sum_{\substack{j=1 \\ j\neq i}}^{m} a_{1j}x_j\right] = \frac{1}{8}\left[20 - 3777\,59\right] = 2.027800$$

{i = 2

$$x_2^{(5)} = \frac{1}{a_{22}}\left[b_2 - \sum_{\substack{j=1 \\ j\neq i}}^{m} a_{2j}x_j\right] = \frac{1}{11,5}\left[-12 - 4.241827\right] = -1.412898$$

$\{i = 3$

$$x_3^{(5)} = \frac{1}{a_{33}}\left[b_3 - \sum_{\substack{j=1 \\ j \neq i}}^{m} a_{3j}x_j\right] = \frac{1}{4.5}\left[14 - 3.096608\right] = 2.422976$$

$\{i = 4$

$$x_4^{(5)} = \frac{1}{a_{44}}\left[b_4 - \sum_{\substack{j=1 \\ j \neq i}}^{m} a_{4j}x_j\right] = \frac{1}{3}\left[8 - 5.550074\right] = 0.816642$$

$\{i = 5$

$$x_5^{(5)} = \frac{1}{a_{55}}\left[b_5 - \sum_{\substack{j=1 \\ j \neq i}}^{m} a_{5j}x_j\right] = \frac{1}{8.5}\left[-30 - 3.941519\right] = -3.993120$$

$\{i = 6$

$$x_6^{(5)} = \frac{1}{a_{66}}\left[b_6 - \sum_{\substack{j=1 \\ j \neq i}}^{m} a_{6j}x_j\right] = \frac{1}{8}\left[0 - 0.590658\right] = -0.073837$$

$\{5^a$ iteração

$\{i = 1$

$$x_1^{(6)} = \frac{1}{a_{11}}\left[b_1 - \sum_{\substack{j=1 \\ j \neq i}}^{m} a_{1j}x_j\right] = \frac{1}{8}\left[20 - 4.839936\right] = 1.895008$$

$\{i = 2$

$$x_2^{(6)} = \frac{1}{a_{22}}\left[b_2 - \sum_{\substack{j=1 \\ j \neq i}}^{m} a_{2j}x_j\right] = \frac{1}{11.5}\left[-12 - 5.797204\right] = -1.547583$$

$\{i = 3$

$$x_3^{(6)} = \frac{1}{a_{33}}\left[b_3 - \sum_{\substack{j=1 \\ j \neq i}}^{m} a_{3j}x_j\right] = \frac{1}{4.5}\left[14 - 3.679924\right] = 2.293350$$

$i = 4$

$$x_4^{(6)} = \frac{1}{a_{44}}\left[b_4 - \sum_{\substack{j=1 \\ j\neq i}}^{n} a_{4j}x_j\right] = \frac{1}{3}\left[8 - 5.958443\right] = 0.680519$$

$i = 5$

$$x_5^{(6)} = \frac{1}{a_{55}}\left[b_5 - \sum_{\substack{j=1 \\ j\neq i}}^{n} a_{5j}x_j\right] = \frac{1}{8.5}\left[-30 - 5.551913\right] = -4.182578$$

$i = 6$

$$x_6^{(6)} = \frac{1}{a_{66}}\left[b_6 - \sum_{\substack{j=1 \\ j\neq i}}^{n} a_{6j}x_j\right] = \frac{1}{8}\left[0 - 1.14392\right] = -0.142990$$

- Método de Gauss - Seidel.

{ 1ª iteração

Solução inicial

$x_i = 0$, para $i = 1, 2, \ldots, 6$

$i = 1$

$$x_1^{(1)} = \frac{1}{8}\left[20 - 0\right] = 2.5$$

$i = 2$

$$x_2^{(1)} = \frac{1}{11.5}\left[-12 - (-10)\right] = -0.173913$$

$i = 3$

$$x_3^{(1)} = \frac{1}{4.5}\left[14 - 0.434782\right] = 3.014492$$

$i = 4$

$$x_4^{(1)} = \frac{1}{3}\left[8 - (-2.5)\right] = 3.5$$

{ i = 5

$$x_5^{(2)} = \frac{1}{8.5}\left[-30 - (-6.130434)\right] = -2.808184$$

{ i = 6

$$x_6^{(2)} = \frac{1}{8}\left[0 - (-1.816709)\right] = 0.227088$$

{ 2ª situação

{ i = 1

$$x_1^{(3)} = \frac{1}{8}\left[20 - (-2.804347)\right] = 2.850543$$

{ i = 2

$$x_2^{(3)} = \frac{1}{11.5}\left[-12 - (-4.897485)\right] = -0.617609$$

{ i = 3

$$x_3^{(3)} = \frac{1}{4.5}\left[14 - (1.089847)\right] = 2.868922$$

{ i = 4

$$x_4^{(3)} = \frac{1}{3}\left[8 - (2.765824)\right] = 1.744725$$

{ i = 5

$$x_5^{(3)} = \frac{1}{8.5}\left[-30 - (-0.742033)\right] = -3.442113$$

{ i = 6

$$x_6^{(3)} = \frac{1}{8}\left[0 - (-0.574674)\right] = 0.071834$$

{ 3ª situação

{ i = 1

$$x_1^{(4)} = \frac{1}{8}\left[20 - 0.728714\right] = 2.409285$$

$\{ i = 2$

$$x_2^{(4)} = \frac{1}{11.5}\left[-12 - 0.401119\right] = -1.078358$$

$\{ i = 3$

$$x_3^{(4)} = \frac{1}{4.5}\left[14 - 2.552226\right] = 2.543949$$

$\{ i = 4$

$$x_4^{(4)} = \frac{1}{3}\left[8 - 4.474941\right] = 1.175019$$

$\{ i = 5$

$$x_5^{(4)} = \frac{1}{8.5}\left[-30 - 2.934000\right] = -3.874588$$

$\{ i = 6$

$$x_6^{(4)} = \frac{1}{8}\left[0 - 0.723983\right] = -0.090497$$

$\{$ 4ª iteração

$i = 1$

$$x_1^{(5)} = \frac{1}{8}\left[20 - 3.138413\right] = 2.107698$$

$\{ i = 2$

$$x_2^{(5)} = \frac{1}{11.5}\left[-12 - 4.382274\right] = -1.441936$$

$\{ i = 3$

$$x_3^{(5)} = \frac{1}{4.5}\left[14 - 3.785838\right] = 2.269813$$

$\{ i = 4$

$$x_4^{(5)} = \frac{1}{3}\left[8 - 5.641478\right] = 0.786173$$

$\{ i = 5$

$$x_5^{(5)} = \frac{1}{8.5}\left[-30 - 5.773083\right] = -4.208598$$

$\{ i = 6$

$$x_6^{(5)} = \frac{1}{8}\left[0 - 1.773269\right] = -0.221658$$

{5ª iteração

{i=1

$$x_1^{(6)} = \frac{1}{8}\left[20 - 4.981573\right] = 1.877303$$

{i=2

$$x_2^{(6)} = \frac{1}{11.5}\left[-12 - 7.859242\right] = -1.726890$$

{i=3

$$x_3^{(6)} = \frac{1}{4.5}\left[14 - 4.760544\right] = 2.053212$$

{i=4

$$x_4^{(6)} = \frac{1}{3}\left[8 - 6.539892\right] = 0.486702$$

{i=5

$$x_5^{(6)} = \frac{1}{8.5}\left[-30 - 1.993536\right] = -4.469827$$

{i=6

$$x_6^{(6)} = \frac{1}{8}\left[0 - 2.598316\right] = -0.324789$$