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CS325-400

Homework #2

1.) $T(n) = 3T(n-1) + 1$

$= 3(3T(n-2)+1) + 1 = 9T(n-2) + 3 + 1 = 9T(n-2) + 4$

$= 9(3T(n-3)+1) + 4 = 27T(n-3) + 9 + 3 + 1 = 27T(n-3) + 13$

$= 27(3T(n-4)+1) + 13 = 81T(n-4) + 27 + 9 + 3 + 1 = 81T(n-4) + 40$

$= \sum_{i=0}^{n} 3^i = \Theta(3^i)$

2.) a.) Ternary search first checks to see if the right index is greater than or equal 1. If it were less than 1, that would mean that the search element was not found, and -1 would be returned. Next, it finds the 2 midpoints that split the data into approximate thirds. It checks the 2 midpoints for equality on the search element. If found, it returns the index of the search element. If the search element is less than the first midpoint, or in other words, in the first third part of the sorted array, ternary search is called again on the first third part of the sorted array. If the search element is greater than the second midpoint, or in other words, in the last third part of the sorted array, ternary search is called again on the last part of the sorted array. If the search element is in neither the first or last thirds of the sorted array, then ternary search is called on the middle part of the sorted array.

```
PSEUDOCODE
ternarySearch(array, Lindex, Rindex, x)
    if Rindex >= 1
        mid1 = Lindex + (Rindex - Lindex) / 3
        mid2 = Rindex - (Rindex - Lindex) / 3
        if array[mid1] == x
            return mid1
        if array[mid2] == x
            return mid2
        if x < array[mid1]
            return ternarySearch(array, Lindex, mid1 - 1, x)
        else if x > array[mid2]
            return ternarySearch(array, mid2 + 1, Rindex, x)
        else
            return ternarySearch(array, mid1 + 1, mid2 - 1, x)
     return -1
```

b.) $T(n) = T\left(\frac{n}{3}\right) + \Theta(1)$

c.) Master Method:

a = 1, b = 3, $\log_3 1 = 0, n^0 = 1$

Case 2:

$T(n) = \Theta(\lg n)$

The running times of Binary and Ternary search are both $\Theta(\lg n)$.

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3.) a.) If the array size is only 1, then return the 1 element as the max and min. If the array size is 2, then make one comparison to see which element is bigger and smaller. If the array is greater than 2, then recursively split the array in half to see what the max and min are from each half. Compare the 2 halves min and max to see the absolute min and max and return the max, min pair.
The following pseudocode based on the code from:
https://www.geeksforgeeks.org/maximum-and-minimum-in-an-array/

```
PSEUDOCODE
min_and_max(array, Lindex, Rindex)
   if arraySize == 1
      return element as (max, min)
   else if arraySize == 2
      if array[0] > array[1]
        max = array[0]
        min = array[1]
      else
        max = array[1]
        min = array[0]
      return (max, min)
   else
      min_and_max(array, Lindex, (Lindex+Rindex/2))
      min_and_max(array, (Lindex+Rindex/2), Lindex)
      compare 2 max's to see which is bigger
      compare 2 min's to see which is smaller
      return (max, min)
```

b.) $T(n) = 2T(n/2) + \Theta(1)$

c.) Master Method:

a = 2, b = 2, $\log_2 2 = 1, n^1 = n$

Case 1:

$T(n) = \Theta(n)$

The running times of recursive min_and_max algorithm and an iterative algorithm for finding min and max are both $\Theta(n)$.

4.) a.)

```
PSEUDOCODE
If array is greater than 1, sort. Otherwise, no need to sort
    Split the array into 4 quarters

    Keep splitting arrays into quarters until only one or less element
    in each quarter(recursion)

    If there are elements not yet compared in the first two quarters,
    compare and put lower/equal value in next spot in left array

    If there are still elements in the q1 array
    Put them in the next spot in left array

    If there are still elements in the q2 array
    Put them in the next spot in left array

    If there are elements not yet compared in the last two quarters,
    compare and put lower/equal value in next spot in right array

    If there are still elements in the q3 array
    Put them in the next spot in right array
```

```
There are still elements in the q4 array
Put them in the next spot in right array

If there are elements not yet compared in both halves, compare and
put lower/equal value in next spot in array

If there are still elements in the left array
Put them in the next spot in array

If there are still elements in the right array
Put them in the next spot in array
```

b.) $T(n) = 4T(n/4) + \Theta(3n)$

c.) Master Method:

a = 4, b = 4, $\log_4 4 = 1, n^1 = n$

$f(n) = 3n = \Theta(\mathrm{n})$

Case 2:

$T(n) = \Theta(\mathrm{nlgn})$

5.) b+c.)

Merge Sort 4	
n	T(n)
20000	0.152
40000	0.302
60000	0.436
80000	0.572
100000	0.816
120000	1.028
140000	1.498
160000	1.644
180000	1.992
200000	2.58

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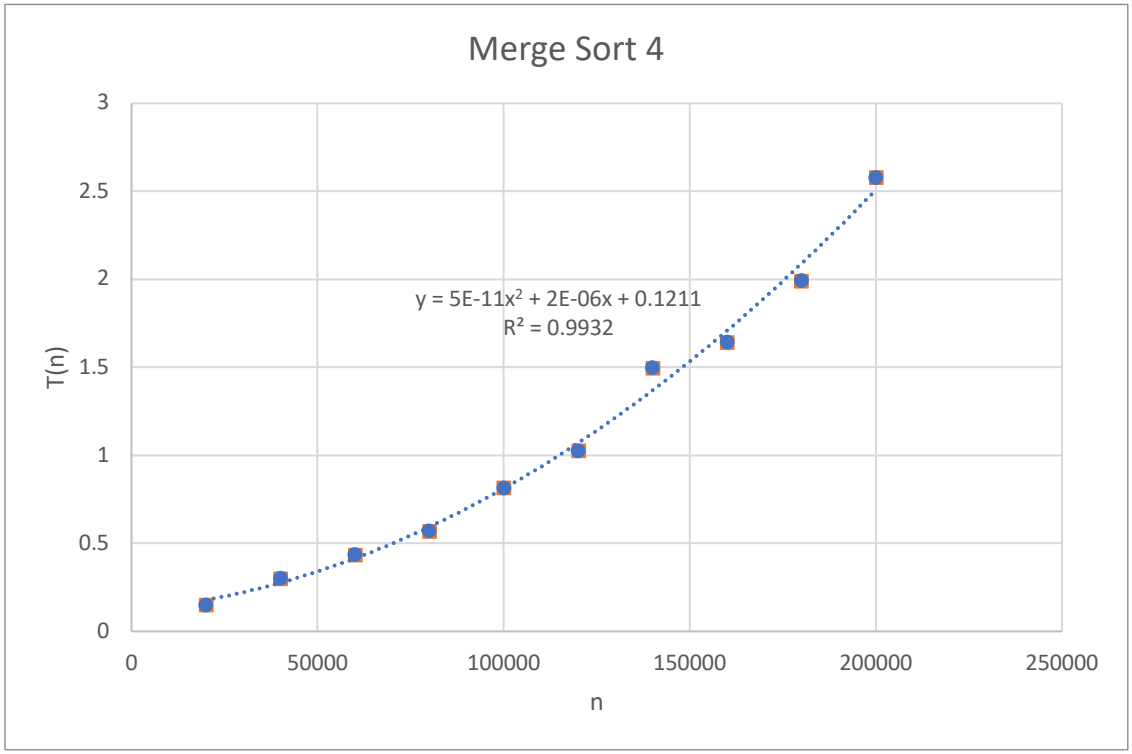
d.) Interestingly, the trendline that best fit Merge Sort 4 was quadratic. This defies my expectation that worst-case scenario of the recurrence would be $\Theta(\text{nlgn})$. However, the difference may be due to server demand due to other students using it.



e.)

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