Kevin Harvell
CS325-400

Homework #1

1a.) $f(n) = \Omega(g(n))$ because $\lim_{x\to\infty}\left(\frac{x^{0.75}}{\sqrt{x}}\right) = \infty$

1b.) $f(n) = \Theta(g(n))$ because $\lim_{x\to\infty}\left(\frac{\ln(x)}{\log_e(x)}\right) = 1$

1c.) $f(n) = O\big(g(n)\big)$ because $\lim_{x\to\infty}\left(\frac{x\log_{10}(x)}{x\sqrt{x}}\right) = 0$

1d.) $f(n) = O\big(g(n)\big)$ because $\lim_{x\to\infty}\left(\frac{e^n}{3^n}\right)$ = 0

1e.) $f(n) = \Theta(g(n))$ because $\lim_{x\to\infty}\left(\frac{2^x}{2^{x-1}}\right) = 2$

1f.) $f(n) = O(g(n))$ because $\lim_{x\to\infty}\left(\frac{4^x}{x!}\right) = 0$

2a.) If f1(n) = Ω(g(n)) and f2(n) = O(g(n)) then f1(n)= Θ (f2(n))

Prove false with counter-example:
Let $f_1(n) = n^3$, $f_2(n) = n$, and $g(n) = n^2$
$f_1(n) = \Omega\big(g(n)\big)$ and $f_2(n) = O(g(n))$
BUT, $f_1(n) \neq \Theta f_2(n)$
Because $\lim_{x\to\infty}\left(\frac{x^3}{x}\right) = \infty$

2b.) If f1(n) = O(g1(n)) and f2(n) = O(g2(n)) then f1(n)+ f2(n)= O(g1(n) + g2(n))

Prove true:
If $f_1(n) = O(g_1(n))$, then there exists constants c_1& n_1 such that
I. $0 < f_1(n) \leq c_1 g_1(n)$ for all $n \geq n_1$and
since $f_2(n) = O(g_2(n))$, there exists constants c_2& n_2 such that
II. $0 < f_2(n) \leq c_2 g_2(n)$ for all $n \geq n_2$
Let $g = g_1(n) + g_2(n)$
III. $f_1(n) \leq g$ for all $n \geq n_1 + n_2$
IV. $f_2(n) \leq g$ for all $n \geq n_1 + n_2$
By adding III & IV,
$0 < f_1(n) + f_2(n) \leq g + g$
$0 < f_1(n) + f_2(n) \leq 2g$ for all $n \geq n_1 + n_2$
$c = 2$
Therefore,
$f_1(n) + f_2(n) \leq cg(n)$ and
$f_1(n) + f_2(n) = O(g(n))$ or
$f_1(n) + f_2(n) = O(g_1(n) + g_2(n))$

```
4a.)
#!/usr/bin/python3

'''
Name: Kevin Harvell
Date: 1/9/19
About: This program has a function called mergeSort that recursively splits
an array in half until 1 element in each array. Then builds sorted arrays
by combining the smaller arrays and comparing the values.
It takes an input file and sorts using the insertionSort function, and creates
an output file.
It creates an array of random numbers between 1 and 10000 of size n and
times how long it takes to sort the array

# The following code is based on pseudocode from Introduction to Algorithms – 3rd
Edition p.26
'''

import random
import time

def insertionSort(arr):
    for j in range(1, len(arr)):
        key = arr[j]
        # Insert arr[j] into the sorted sequence
        i = j
        while i > 0 and arr[i - 1] > key:
            arr[i] = arr[i - 1]
            i = i - 1
        arr[i] = key
    return arr


unsorted = []
n = input("Enter the number of elements in the array: ")
for x in range(n):
    unsorted.append(random.randint(0, 10000))
print(unsorted)
t0 = time.time()
print(insertionSort(unsorted))
t1 = time.time()
print(t1 - t0)



#!/usr/bin/python3

'''
Name: Kevin Harvell
Date: 1/9/19
About: This program has a function called mergeSort that recursively splits
an array in half until 1 element in each array. Then builds sorted arrays
by combining the smaller arrays and comparing the values.
It creates an array of random numbers between 1 and 10000 of size n and
times how long it takes to sort the array

# The following code is based on code from
# http://interactivepython.org/courselib/static/pythonds/SortSearch/TheMergeSort.html
'''

import random
```

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```python
import time

def mergeSort(arr):
    # If array is greater than 1, sort. Otherwise, no need to sort
    if len(arr) > 1:
        # Split the array into 2 halves
        mid = len(arr) // 2
        left = arr[:mid]
        right = arr[mid:]
        # Keep splitting arrays into halves until only one element in both halves
        mergeSort(left)
        mergeSort(right)

        i = 0
        j = 0
        k = 0
        # If there are elements not yet compared in both halves, compare and
        # put lower/equal value in next spot in array
        while i < len(left) and j < len(right):
            if left[i] < right[j]:
                arr[k] = left[i]
                i = i + 1
            else:
                arr[k] = right[j]
                j = j + 1
            k = k + 1
        # There are still elements in the left array
        # Put them in the next spot in array
        while i < len(left):
            arr[k] = left[i]
            i = i + 1
            k = k + 1
        # There are still elements in the right array
        # Put them in the next spot in array
        while j < len(right):
            arr[k] = right[j]
            j = j + 1
            k = k + 1
    return arr

unsorted = []
n = input("Enter the number of elements in the array: ")
for x in range(n):
    unsorted.append(random.randint(0, 10000))
print(unsorted)
t0 = time.time()
print(mergeSort(unsorted))
t1 = time.time()
print(t1 - t0)
```

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4b.)

Insertion Sort	
n	T(n)
625	0.04333333
1250	0.49
2500	0.57333333
5000	1.79333333
7500	3.23333333
10000	6.22
12500	7.72666667
15000	13.45
20000	24.6166667
25000	39.97

Merge Sort	
n	T(n)
20000	0.19666667
40000	0.44333333
60000	0.61666667
80000	0.84
100000	0.81333333
120000	0.99666667
140000	1.08
160000	1.14333333
180000	1.67333333
200000	2.09

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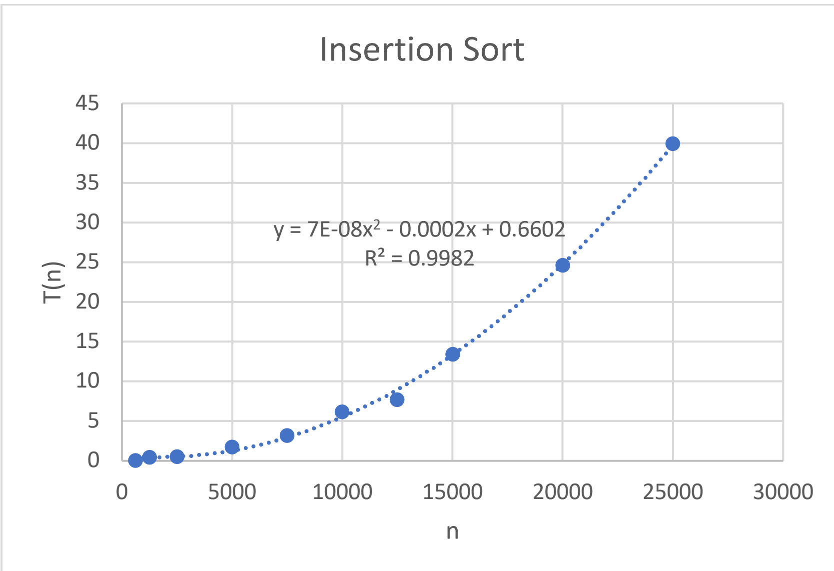
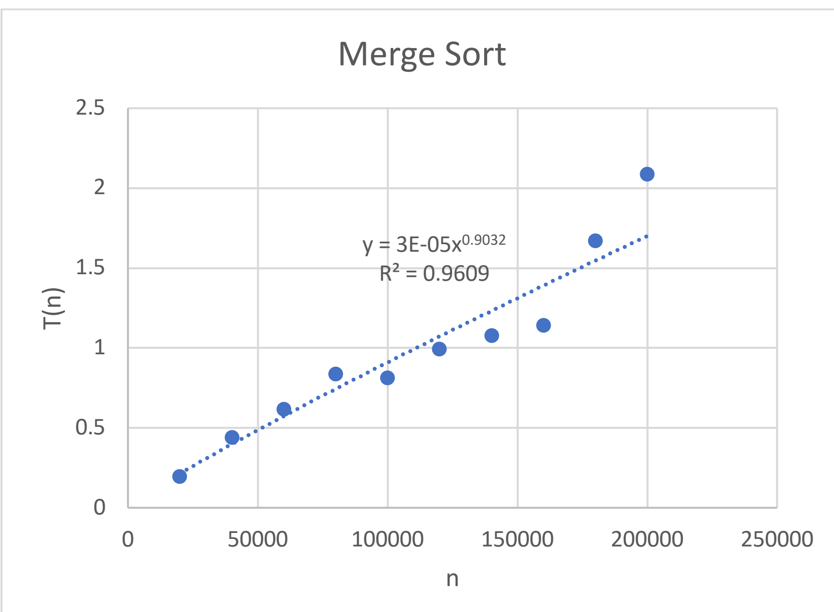
4c.) As expected, a quadratic regression best fit the insertion sort data. The equation for the regression line is featured below on the graph with an R^2 value.
Unexpectedly, a power regression best fit the merge sort data. The equation for the regression line is featured below on the graph with an R^2 value. I attribute this unexpected outcome to flip server irregularities. In a perfect world, my server would not be used by many other students. Even though I took several tests and averaged the results, the data did not fit a logarithmic regression the best as I would expect theoretically.





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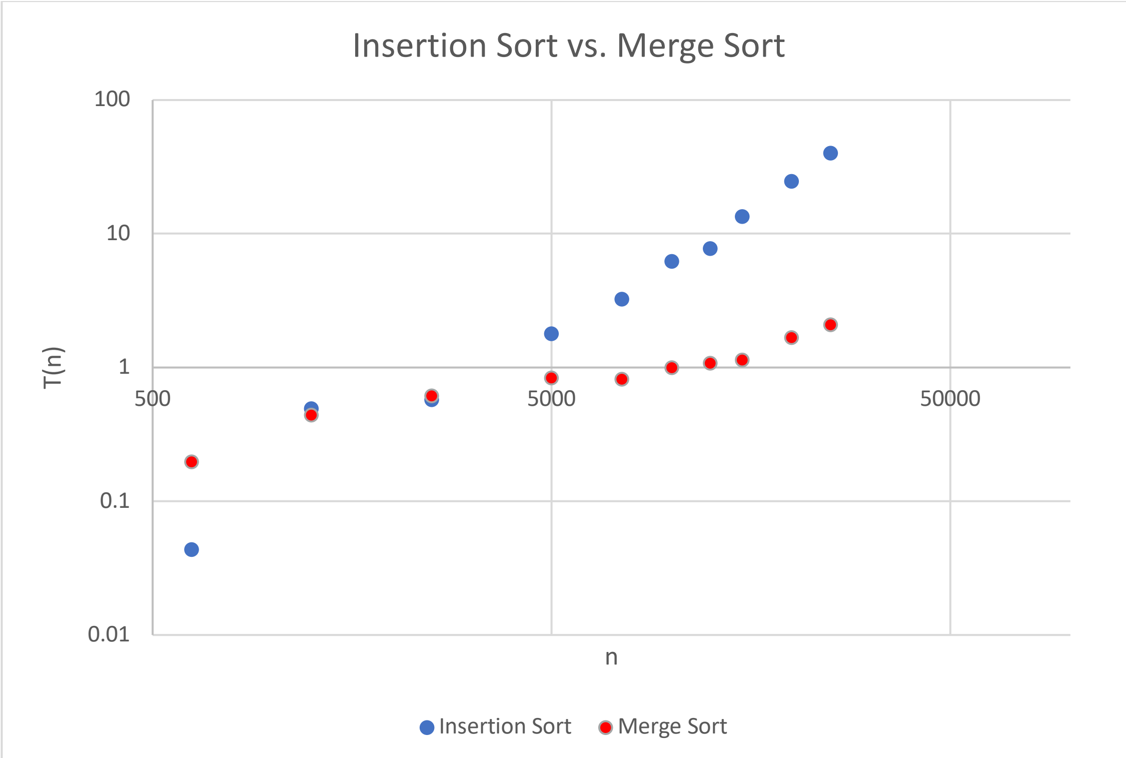
4d.)



4e.) I am struggling to compare experimental running times to theoretical running times because for my experimental running times, there is a known computer processing speed whereas theoretically, there is no known computer processing speed.
In theory, insertion sort would have a worst-case scenario of O(n^2). If I were to take n=625, where experimentally I got T(n) = 0.04, and put it theoretically into f(n) = n^2, that would yield f(n) = 390625. I am not sure how to compare 0.04 with 390625. 390625 would in theory be multiplied by some constant based on the processing speed of the theoretical computer.
I have similar confusion for merge sort which would have a worst-case scenario of O(nlogn). Similarly, if I were to take n=20000, where experimentally I got T(n)=0.20, and put it theoretically into f(n)=nlgn, that would yield f(n) = 285754. Again, I am not sure how to compare 0.20 with 285754. 285754 would in theory be multiplied by some constant based on the processing speed of the theoretical computer.

All that said, it is clear to me that the experimental running times for insertion sort are clearly quadratic making the theory of it being O(n^2) very apparent.
However, for merge sort, the experimental running times created a trendline that was considered a Power function. This is probably due to inconsistencies in the flip server caused by the ebb and flow of other users using it, even though I ran several test cases and averaged the results. Interestingly, from n = 20000 to n = 160000 the merge sort experimental running times did appear to be close to the theoretical O(nlgn).