Kevin Harvell

Assignment 3 – Part 2

Exercise Set 6.1 p.378 - 3, 7, 13, 18, 33, 34

3.a.) No, because there is an element of R that is not in T. For example, 2 is in R but is not in T.

3.b.) Yes, because any integer that is divisible by 6 is also divisible by 2. For example, $z=6k$ for some integer k. $z=2(3k)$ meaning it is divisible by 2.

3.c.) Yes, because any integer that is divisible by 6 is also divisible by 3. For example, $z=6k$ for some integer k. $z=3(2k)$ meaning it is divisible by 3.

7.) Let A={x ∈ Z | x=6a+4 for some integer a}, B={y ∈ Z | y=18b−2 for some integer b}, and C={z ∈ Z | z=18c+16 for some integer c}.Prove or disprove each of the following statements.

a. A ⊆ B

This statement is false because there is an element of A that is not an element in B. For example, let $x = 10$. $x \in A$ because there is an integer a such that $10 = 6a + 4$. In this case, $a = 1$. However, $x \notin B$ because there is no integer b such that $10 = 18b - 2$. In this case $b = \frac{2}{3}$ which is not an integer. Thus $10 \in A$ but $10 \notin B$, and so $A \nsubseteq B$.

b. B ⊆ A

Proof:

Suppose n is a particular but arbitrarily chosen element of B. [We must show that $n \in A$. By definition of A, this means we must show that $n = 6 \cdot (some\ integer) + 4$.]

By definition of B, there is an integer b such that $n = 18b - 2$. [Given that $n = 18b - 2$, is there an integer, say a, such that $18b - 2 = 6a + 4$? Solve for a to obtain $a = 3b - 1$.]

Let $a = 3b - 1$. Then a is an integer because products and differences of integers are integers.

Also $6a + 4 = 6(3b - 1) + 4 = 18b - 6 + 4 = 18b - 2 = n$

Thus, by definition of A, *n* is an element of A [which is what was to be shown.]

c. B=C

Proof that $B \subseteq C$:

Suppose m is a particular but arbitrarily chosen element of B. By definition of B, there is an integer *b* such that $m = 18b - 2$. [Is there an integer, say *c*, such that $18b - 2 = 18c + 16$? Solve for *c* to obtain $c = \frac{18b-18}{18} = b - 1$.]

Let $c = b - 1$.

Then *c* is an integer because it is the difference of integers.

Also $18c + 16 = 18(b - 1) + 16 = 18b - 18 + 16 = 18b - 2 = m$,

Thus, by definition of *C*, *m* is an element of *C* [which is what was to be shown.]

Proof that $C \subseteq B$:

Suppose q is a particular but arbitrarily chosen element of C. By definition of C, there is an integer *c* such that $q = 18c + 16$. [Is there an integer, say *b*, such that $18c + 16 = 18b - 2$? Solve for *b* to obtain $b = \frac{18c+18}{18} = c + 1$.]

Let $b = c + 1$.

Then *b* is an integer because it is the sum of integers.

Also $18b - 2 = 18(c + 1) - 2 = 18c + 18 - 2 = 18c + 16 = q$,

Thus by definition of *B*, *q* is an element of *B* [which is what was to be shown.]

Kevin Harvell
Assignment 3 – Part 2

13.a.) True
13.b.) False
13.c.) False
13.d.) False
13.e.) True
13.f.) True
13.g.) True
13.h.) True
13.i.) False

18.a.) No, because ∅ has no elements.
18.b.) No, because ∅ is just an empty set whereas {∅} is a set with an empty set inside it.
18.c.) Yes, ∅ is an element inside {∅}.
18.d.) Yes, every set is an element in itself.

33.a.) $\mathcal{P}(\emptyset) = \{\emptyset\}$
33.b.) $\mathcal{P}\big(\mathcal{P}(\emptyset)\big) = \{\emptyset, \{\emptyset\}\}$
33.c.) $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset))) = \{\emptyset, \{\emptyset\}, \big\{\{\emptyset\}\big\}, \{\emptyset, \{\emptyset\}\}\}$

34.a.) $\{\big(1, (u, m)\big), \big(2, (u, m)\big), \big(3, (u, m)\big), \big(1, (u, n)\big), \big(2, (u, n)\big), \big(3, (u, n)\big),$
$\big(1, (v, m)\big), \big(2, (v, m)\big), \big(3, (v, m)\big), \big(1, (v, n)\big), \big(2, (v, n)\big), \big(3, (v, n)\big)\}$

34.b.) $A_1 \times A_2 = \{(1, u), (2, u), (3, u), (1, v), (2, v), (3, v)\}$
$(A_1 \times A_2) \times A_3 = \{\big((1, u), m\big), \big((1, u), n\big), \big((2, u), m\big), \big((2, u), n\big), \big((3, u), m\big), \big((3, u), n\big),$
$\big((1, v), m\big), \big((1, v), n\big), \big((2, v), m\big), \big((2, v), n\big), \big((3, v), m\big), \big((3, v), n\big)\}$

34.c.) $A_1 \times A_2 \times A_3 = \{(1, u, m), (1, u, n), (1, v, m), (1, v, n), (2, u, m), (2, u, n), (2, v, m),$
$(2, v, n), (3, u, m), (3, u, n), (3, v, m), (3, v, n)\}$