**Assignment 6 – Part 2**
**Set 5.9 - 6, 10, 13.b, 16**

6. Define a set $S$ recursively as follows:
   I. BASE: $a \in S$
   II. RECURSION: If $s \in S$, then,
   a. $sa \in S$ b. $sb \in S$
   III. RESTRICTION: Nothing is in $S$ other than objects defined in I and II above.

   Use structural induction to prove that every string in $S$ begins with an $a$.

**The only object in the base for $S$ is $a$, which begins with an $a$.**
**The recursion for $S$ consists of two rules denoted II(a) and II(b). When rule II(a) is applied to the base the result is $aa \in S$, which begins with an $a$. Rule II(a) can be applied over and over and the result will always begin with an $a$. Similarly, rule II(b) applied to the base results in $ab \in S$, which also begins with an $a$. Rule II(b) can also be applied over and over with the result always beginning with an $a$.**
**Because no objects other than those obtained through the base and recursion conditions are contained in $S$, it must be the case that every object in $S$ satisfies the property that every string begins with an $a$ [what we needed to show.]**

*H* **10.** Define a set $S$ recursively as follows:

I. BASE: $0 \in S$, $5 \in S$

II. RECURSION: If $s \in S$ and $t \in S$ then

a. $s + t \in S$ b. $s - t \in S$

III. RESTRICTION: Nothing is in $S$ other than objects defined in I and II above.

Use structural induction to prove that every integer in $S$ is divisible by 5.

**The base objects of $\mathbf{0} \in \boldsymbol{S}$ and $\mathbf{5} \in \boldsymbol{S}$ are both divisible by 5.**
**The recursion for $\boldsymbol{S}$ consists of two rules denoted II(a) and II(b).**
**By II(a),**
$\mathbf{0}+\mathbf{0} \in \boldsymbol{S}, \mathbf{0}+\mathbf{5} \in \boldsymbol{S}, \mathbf{5}+\mathbf{0} \in \boldsymbol{S}, \boldsymbol{and}\ \mathbf{5}+\mathbf{5} \in \boldsymbol{S}$
**These sums are all divisible by 5. If we were to keep adding up these sums, we would always get a result divisible by 5.**
**By II(b),**
$\mathbf{0}-\mathbf{0} \in \boldsymbol{S}, \mathbf{0}-\mathbf{5} \in \boldsymbol{S}, \mathbf{5}-\mathbf{0} \in \boldsymbol{S}, \mathbf{5}-\mathbf{5} \in \boldsymbol{S}$
**These differences are all divisible by 5. If we were to keep subtracting these differences, we would always get a result divisible by 5.**
**Because no objects other than those obtained through the base and recursion conditions are contained in $\boldsymbol{S}$, it must be the case that every object in $\boldsymbol{S}$ is divisible by 5.**

13. Consider the set $P$ of parenthesis structures defined in Example 5.9.4. Give derivations showing that each of the following is in $P$.

**a.** ()(()) b. (())(())

**(1) By I, () is in P.**
**(2) By (1) and II(a), (()) is in P.**
**(3) By (2), (1), and II(b), (())(()) is in P.**

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Assignment 6 – Part 2

16. Give a recursive definition for the set of all strings of 0's and 1's for which all the 0's precede all the 1's.

**I. BASE: $\lambda$ is in $S$, where $\lambda$ is the null string.**

**II. RECURSION: If $s \in S$, then**

**(a) $0s \in S$ and (b) $s1 \in S$,**

**where $0s$ and $s1$ are the concatenations of $s$ with $0$ and $1$ respectively.**

**III. RESTRICTION: Nothing is in $S$ other than objects defined in I and II above.**