

Assignment 8 – Part 1  
Set 9.2- 32.c, 33, 36 , 39-b,d , Set 9.5 - 7-b, 14, 20

32. a. How many ways can the letters of the word *ALGORITHM* be arranged in a row?
- b. How many ways can the letters of the word *ALGORITHM* be arranged in a row if *A* and *L* must remain together (in order) as a unit?
- c. How many ways can the letters of the word *ALGORITHM* be arranged in a row if the letters *GOR* must remain together (in order) as a unit?

32.c.)  $7! = 5040$

33. Six people attend the theater together and sit in a row with exactly six seats.
- a. How many ways can they be seated together in the row?
- b. Suppose one of the six is a doctor who must sit on the aisle in case she is paged. How many ways can the people be seated together in the row with the doctor in an aisle seat?
- c. Suppose the six people consist of three married couples and each couple wants to sit together with the husband on the left. How many ways can the six be seated together in the row?

33.a.)  $6! = 720$

33.b.)  $5! = 120$

33.c.)  $3! = 6$

36. Write all the 3-permutations of  $\{s, t, u, v\}$ .

$$36.) P(4, 3) = \frac{4!}{(4-3)!} = \frac{4!}{1!} = 24$$

stu	stv	suv	sut	svt	svu
tsu	tsv	tus	tuv	tv�	tvu
ust	usv	uts	utv	uvs	uvt
vst	vsu	vts	vtu	vus	vut

39. **a.** How many ways can three of the letters of the word *ALGORITHM* be selected and written in a row?
- b. How many ways can six of the letters of the word *ALGORITHM* be selected and written in a row?
- c.** How many ways can six of the letters of the word *ALGORITHM* be selected and written in a row if the first letter must be *A*?
- d. How many ways can six of the letters of the word *ALGORITHM* be selected and written in a row if the first two letters must be *OR*?

$$39.b.) P(9, 6) = \frac{9!}{(9-6)!} = 9 * 8 * 7 * 6 * 5 * 4 = 60480$$

$$39.d.) P(7, 4) = \frac{7!}{(7-4)!} = 7 * 6 * 5 * 4 = 840$$

7. A computer programming team has 13 members.
- How many ways can a group of seven be chosen to work on a project?
  - Suppose seven team members are women and six are men.
    - How many groups of seven can be chosen that contain four women and three men?
    - How many groups of seven can be chosen that contain at least one man?
    - How many groups of seven can be chosen that contain at most three women?

$$7.b.i.) \binom{7}{4} \binom{6}{3} = \frac{7!}{4!3!} \cdot \frac{6!}{3!3!} = \frac{7 \cdot 6 \cdot 5 \cdot 6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = 7 \cdot 5 \cdot 5 \cdot 4 = 700$$

$$7.b.ii.) \binom{13}{7} - \binom{7}{7} = \frac{13!}{7!6!} - \frac{7!}{7!0!} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} - 1 = \frac{13 \cdot 11 \cdot 9 \cdot 8}{6} - 1 = 1715$$

$$7.b.iii.) \binom{7}{1} \binom{6}{6} + \binom{7}{2} \binom{6}{5} + \binom{7}{3} \binom{6}{4} = \frac{7!}{1!6!} \cdot \frac{6!}{6!0!} + \frac{7!}{2!5!} \cdot \frac{6!}{5!1!} + \frac{7!}{3!4!} \cdot \frac{6!}{4!2!} = 7 + \frac{7 \cdot 6 \cdot 6}{2 \cdot 1} + \frac{7 \cdot 6 \cdot 5 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} \\ = 7 + 7 \cdot 6 \cdot 3 + 7 \cdot 5 \cdot 3 \cdot 5 = 7 + 126 + 525 = 658$$

14. a. How many 16-bit strings contain exactly seven 1's?  
 b. How many 16-bit strings contain at least thirteen 1's?  
 c. How many 16-bit strings contain at least one 1?  
 d. How many 16-bit strings contain at most one 1?

$$14.a.) \binom{16}{7} = \frac{16!}{7!9!} = \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 11440$$

$$14.b.) \binom{16}{13} + \binom{16}{14} + \binom{16}{15} + \binom{16}{16} = \frac{16!}{13!3!} + \frac{16!}{14!2!} + \frac{16!}{15!1!} + \frac{16!}{16!0!} \\ = \frac{16 \cdot 15 \cdot 14}{3 \cdot 2 \cdot 1} + \frac{16 \cdot 15}{2 \cdot 1} + \frac{16}{1} + 1 = 560 + 120 + 17 = 697$$

$$14.c.) 2^{16} - 1 = 65535$$

$$14.d.) \binom{16}{0} + \binom{16}{1} = \frac{16!}{0!16!} + \frac{16!}{1!15!} = 1 + \frac{16}{1} = 17$$

20. a. How many distinguishable ways can the letters of the word *MILLIMICRON* be arranged in order?
- b. How many distinguishable orderings of the letters of *MILLIMICRON* begin with *M* and end with *N*?
- c. How many distinguishable orderings of the letters of *MILLIMICRON* contain the letters *CR* next to each other in order and also the letters *ON* next to each other in order?

$$\begin{aligned}
 20.a.) & \binom{11}{3} \binom{8}{2} \binom{6}{2} \binom{4}{1} \binom{3}{1} \binom{2}{1} \binom{1}{1} \\
 &= \frac{11!}{3!8!} \cdot \frac{8!}{2!6!} \cdot \frac{6!}{2!4!} \cdot \frac{4!}{1!3!} \cdot \frac{3!}{1!2!} \cdot \frac{2!}{1!1!} \cdot \frac{1!}{1!} \\
 &= \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1} \cdot \frac{8 \cdot 7}{2 \cdot 1} \cdot \frac{6 \cdot 5}{2 \cdot 1} \cdot \frac{4}{1} \cdot \frac{3}{1} \cdot \frac{2}{1} \cdot \frac{1}{1} = 6652800
 \end{aligned}$$

$$20.b.) \frac{9!}{1!3!2!1!1!1!0!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 30240$$

$$20.c.) \frac{9!}{2!3!2!1!1!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 15120$$