Assignment 7 – Part 2
Set 9.4 - 6, 7, 16, 27

6. a. Given any set of seven integers, must there be two that have the same remainder when divided by 6? Why?
   b. Given any set of seven integers, must there be two that have the same remainder when divided by 8? Why?

6.a.) Yes. There are 7 objects and only 6 possible remainders.

6.b.) No. There are 7 objects and 8 possible remainders. Counterexample: 0, 1, 2, 3, 4, 5, 6. None have the same remainder when divided by 8.

H 7. Let $S = \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. Suppose six integers are chosen from S. Must there be two integers whose sum is 15? Why?

7.) Partition set S into four disjoint subsets:
$\{3, 12\}, \{4, 11\}, \{5, 10\}, \{6, 9\}, \{7, 8\}$
Yes. Each of the integers in S occurs in exactly one of the 5 subsets and the sum of the integers in each subset is 15. Thus, if six integers from S are chosen, two must be from the same subset. It follows that the sum of these two integers is 15.

16. How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?

16.) 20 integers are divisible by 5 from 1 through 100. 80 are not. It is possible to pick 80 integers not divisible by 5, but then it follows that the next integer picked will be divisible by 5. Thus, 81 integers must be picked.

27. In a group of 2,000 people, must at least 5 have the same birthday? Why?

27.) $\frac{2000}{365} = 5.48$
$ceil(5.48) = 6$
Thus, at least 6 people have the same birthday.