Assignment 5 – Part 2
p305 Set 5.4 - 2, 10

2. Suppose $b_1, b_2, b_3, \ldots$ is a sequence defined as follows:

$$b_1 = 4, \; b_2 = 12$$
$$b_k = b_{k-2} + b_{k-1} \quad \text{for all integers } k \geq 3.$$

Prove that b_n is divisible by 4 for all integers $n \geq 1$.

2.) Let $P(n)$ be $b_n = b_{n-2} + b_{n-1}$ is divisible by 4 for all integers $n \geq 1$
Show that P(0) and P(1) are true:
P(0)=4, which is divisible by 4 because 4 = 4*1
P(1)=12, which is divisible by 4 because 12 = 4*3
Suppose $P(i)$ is divisible by 4 for all integers i from 1 through k:
$b_i = b_{i-2} + b_{i-1}$ is divisible by 4 for all integers i with $1 \leq i \leq k$
By definition of divisibility
$b_{i-2} + b_{i-1} = 4r$ where r is some integer.
We must show that
$b_{k+1} = b_{k-1} + b_k$ is divisible by 4
By substitution
$b_{k+1} = b_{k-1} + 4r$
By our assumption we know that $b_{k-1}$is also divisible by 4. Thus by substitution
$b_{k+1} = 4r + 4r$
$b_{k+1} = 4(2r)$
Thus we have shown that $b_{k+1}$is divisible by 4. Since we have proved the basis step and the inductive step, we conclude that the proposition is true.

H 10. The problem that was used to introduce ordinary mathematical induction in Section 5.2 can also be solved using strong mathematical induction. Let $P(n)$ be "any collection of n coins can be obtained using a combination of 3¢ and 5¢ coins." Use strong mathematical induction to prove that $P(n)$ is true for all integers $n \geq 14$.

10.) Show that $P(14), P(15), and\ P(16)$ are true:

$P(14) = 3₵ + 3₵ + 3₵ + 5₵$
$P(15) = 5₵ + 5₵ + 5₵$
$P(16) = 3₵ + 3₵ + 5₵ + 5₵$

[Suppose $P(k)$ is true for a particular but arbitrarily chosen integer $k \geq 14$. That is:]
k₵ can be obtained using 3₵ and 5₵ coins.
[We must show that $P(k+1)$ is true. That is:] We must show that
$(k+1)$₵ can be obtained using 3₵ and 5₵ coins.
Case 1 (There is a 5₵ coin among those used to make up the k₵.)
In this case replace the 5₵ coin by two 3₵ coins; the result will be $(k+1)$₵
Case 2 (There is not a 5₵ coin among those used to make up the k₵.)
In this case, because $k \geq 14$, at least three 3₵ coins must have been used. So remove three 3₵ coins and replace them by two 5₵ coins; the result will be $(k+1)$₵.
Thus in either case $(k+1)$₵ can be obtained using 3₵ and 5₵ coins [as was to be shown.]