Kevin Harvell

Quiz over Week 6 and 7

Use iteration to guess an explicit formula for the sequence -

$d_k = 2d_{k-1} + 3$, for all integers $k \geq 1$

$d_0 = 2$

1.) $d_1 = 2(2) + 3 = 2^2 + 3$
$d_2 = 2(2^2 + 3) + 3 = 2^3 + 2 \cdot 3 + 3$
$d_3 = 2(2^3 + 2 \cdot 3 + 3) + 3 = 2^4 + 2^2 \cdot 3 + 2 \cdot 3 + 3$
$d_4 = 2(2^4 + 2^2 \cdot 3 + 2 \cdot 3 + 3) + 3 = 2^5 + 2^3 \cdot 3 + 2^2 \cdot 3 + 2 \cdot 3 + 3$
Guess:
$d_n = 2^{n+1} + 2^{n-1} \cdot 3 + 2^{n-2} \cdot 3 + \cdots + 2^2 \cdot 3 + 2 \cdot 2 + 3$

$$= 2^{n+1} + 3\sum_{i=0}^{n-1} 2^i$$

$$= 2^{n+1} + 3 \cdot \frac{2^n - 1}{2 - 1}$$

$= 2^{n+1} + 3 \cdot (2^n - 1)$
$= 2 \cdot 2^n + 3 \cdot 2^n - 3$
$= 5 \cdot 2^n - 3$

a) Give a recursive definition of the set of positive integers that are multiple of 6.

b) Give a recursive definition of the set of binary strings that have even length. Please remember that empty string λ has length 0 and hence has even length.

2.a.) Basis step: $6 \in S$
Recursive step: $If\ x \in S, then\ x + 6 \in S$
2.b.) Basis step: $\lambda \in S$
Recursive step: $If\ w \in S, then\ 00w \in S, 01w \in S, 10w \in S\ and\ 11w \in S$

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Define a set S recursively as follows:

I. BASE: $0 \in S$, $5 \in S$

II. RECURSION: If $s \in S$, and $t \in S$ then

a. $s + t \in S$ b. $s - t \in S$

III. RESTRICTION: Nothing is in S other than objects defined in I and II above. Use structural induction to prove that every integer in S is divisible by 5.

3.) The base objects of $0 \in S$ and $5 \in S$ are both divisible by 5.

The recursion for S consists of two rules denoted II(a) and II(b).

By II(a),

$0 + 0 \in S, 0 + 5 \in S, 5 + 0 \in S, and\ 5 + 5 \in S$

These sums are all divisible by 5. If we were to keep adding up these sums, we would always get a result divisible by 5.

By II(b),

$0 - 0 \in S, 0 - 5 \in S, 5 - 0 \in S, 5 - 5 \in S$

These differences are all divisible by 5. If we were to keep subtracting these differences, we would always get a result divisible by 5.

Because no objects other than those obtained through the base and recursion conditions are contained in S, it must be the case that every object in S is divisible by 5.

a) In a group of 2,000 people, must at least 5 have the same birthday? Why?

b) Show that if seven integers are selected from the first 10 positive integers there must be at least two pairs of these integers with the sum 11.

4.a.) 2000/365 = 5.4

By generalized pigeon hold principle, there are at least 5 people that have the same birthday.

4.b.) We can divide the first ten positive integers into the following subsets:

$\{1,10\}, \{2,9\}, \{3,8\}, \{4,7\}, \{5,6\}$

The sum of each of these subsets is 11.

Since we choose 7 numbers from 10, 3 numbers are unchosen. That means that there is a maximum of 3 groups chosen. Thus, at least 2 groups are chosen with a sum 11.

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a) According to the Inclusion/Exclusion Rule for Two Sets

"If A and B are finite sets, then N(A ∪ B) = N(A) + N(B) − N(A ∩ B)"

Now, how many integers from 1 through 100 are neither multiples of 2 nor multiples of 3 ?

b) How many strings are there of four lowercase letters that have the letter 's' in them?

5.a.) Every second integer from 2 to 100 is a multiple of 2, and each can be represented in the form 2j for some integer j from 1 to 50. Thus, there are 50 multiples of 2 in this range.
Every third integer from 3 to 99 is a multiple of 3, and each can be represented in the form 3k for some integer k from 1 to 33. Thus, there are 33 multiples of 3 in this range.
However, some integers are both multiples of 2 and 3. In other words, they are divisible by 6. Specifically, every sixth integer from 6 to 96 is a multiple of 6, and each can be represented in the form 6m for some integer m from 1 to 16. Thus, there are 16 multiples of 6 in this range.
$N(A \cup B) = 50 + 33 - 16 = 67$
This number represents the numbers from 1 to 100 that are divisible by either 2 or 3.
$100\ numbers - 67\ that\ are\ divisible\ by\ 2\ or\ 3$
$= 33\ numbers\ that\ are\ neither\ multiples\ of\ 2\ or\ 3$

5.b.) There are 26^4 strings in all and 25^4 that do not have the letter s. Thus, there are $26^4 - 25^4 = 66{,}351$ that have the letter s.