Kevin Harvell

Assignment 4 – Part 1

P378 Set 6.1: 12, 16, Set 6.2 : 4, 10, 14, Set 6.3: 12, 37, 42

Set 6.1: 12, 16

12.a) $[-3, 2)$
12.b) $(-1, 0]$
12.c) $(-\infty, -3) \cup (0, \infty)$
12.d) $[-3, 0] \cup (6, 8]$
12.e) $\emptyset$
12.f) $(-\infty, -1) \cup (2, \infty)$
12.g) $(-\infty, -3) \cup (2, \infty)$
12.h) $(-\infty, -1) \cup (0, \infty)$
12.i) $(-\infty, -1] \cup (0, \infty)$
12.j) $(-\infty, -3) \cup [2, \infty)$

16.a) $A \cup (B \cap C) = \{a, b, c\}$, $(A \cup B) \cap C = \{b, c\}$, $(A \cup B) \cap (A \cup C) = \{a, b, c\}$
$A \cup (B \cap C)$ and $(A \cup B) \cap (A \cup C)$ are equal

16.b) $A \cap (B \cup C) = \{b, c\}$, $(A \cap B) \cup C = \{b, c, e\}$, $(A \cap B) \cup (A \cap C) = \{b, c\}$
$A \cap (B \cup C) = \{b, c\}$ and $(A \cap B) \cup (A \cap C)$ are equal

16.c) $(A - B) - C = \{a\}$, $A - (B - C) = \{a, b, c\}$
These sets are not equal.

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Set 6.2 : 4, 10, 14

4. The following is a proof that for all sets A and B, if $A \subseteq B$, then $A \cup B \subseteq B$. Fill in the blanks.

Proof: Suppose A and B are any sets and $A \subseteq B$. *[We must show that* $\underline{\quad (a) \quad}$*.]* Let $x \in \underline{\quad (b) \quad}$. *[We must show that* $\underline{\quad (c) \quad}$*.]* By definition of union, $x \in \underline{\quad (d) \quad}$ $\underline{\quad (e) \quad}$ $x \in \underline{\quad (f) \quad}$. In case $x \in \underline{\quad (g) \quad}$, then since $A \subseteq B$, $x \in \underline{\quad (h) \quad}$. In case $x \in B$, then clearly $x \in B$. So in either case, $x \in \underline{\quad (i) \quad}$ *[as was to be shown].*

4.a) $A \cup B \subseteq B$
4.b) $A \cup B$
4.c) $x \in B$
4.d) A
4.e) or
4.f) B
4.g) A
4.h) B
4.i) B

10. For all sets A, B, and C,

$$(A - B) \cap (C - B) = (A \cap C) - B.$$

10.) **Proof that $(A - B) \cap (C - B) \subseteq (A \cap C) - B$:**

Suppose $x \in (A - B) \cap (C - B)$. By definition of intersection, $x \in (A - B)$ and $x \in (C - B)$. Since $x \in (A - B)$, then $x \in A$ and $x \notin B$. Since $x \in (C - B)$, then $x \in C$ and $x \notin B$. Hence, $x \in A$ and $x \in C$, which means $x \in (A \cap C)$ by definition of intersection. Since $x \notin B$, $(A - B) \cap (C - B) \subseteq (A \cap C) - B$ [what we needed to show.]

Proof that $(A \cap C) - B \subseteq (A - B) \cap (C - B)$:

Suppose $x \in (A \cap C) - B$. It follows that $x \in A \cap C$ and $x \notin B$.
By definition of intersection, if $x \in A \cap C$, then $x \in A$ and $x \in C$. Hence, $x \in A$, $x \in C$, and $x \notin B$.
For $x \in (A - B)$, $x \in A$ and $x \notin B$, which we've proven to be the case.
For $x \in (C - B)$, $x \in C$ and $x \notin B$, which we've proven to be the case.
By definition of intersection, $x \in (A - B) \cap (C - B)$
Thus, $(A \cap C) - B \subseteq (A - B) \cap (C - B)$ [what we needed to show.]

14. For all sets A, B, and C, if $A \subseteq B$ then $A \cup C \subseteq B \cup C$.

14.) **Proof:**

Suppose A and B are sets with $A \subseteq B$. [We must show that $A \cup C \subseteq B \cup C$.] Let $x \in A$. $x \in A \cup C$ by definition of union. If $x \in A$ then $x \in B$ by definition of subsets. Because $x \in B$, $x \in B \cup C$ by definition of union. Therefore, because $x \in A \cup C$ and $x \in B \cup C$, $A \cup C \subseteq B \cup C$ [what we needed to show.]

P400 Set 6.3: 12, 37, 42

H 12. For all sets A, B, and C,

$$A \cap (B - C) = (A \cap B) - (A \cap C).$$

12.) **Proof that $A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$:**

Suppose $x \in A \cap (B - C)$. Then $x \in A$ and $x \in B$ and $x \notin C$. So it is true that $x \in A$ and $x \in B$ and that $x \in A$ and $x \notin C$. Thus, $A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$ [what we needed to show.]

Proof that $(A \cap B) - (A \cap C) \subseteq A \cap (B - C)$:

Suppose $x \in (A \cap B) - (A \cap C)$. Then $x \in A$ and $x \in B$, but $x \notin (A \cap C)$, and so $x \notin C$. Thus, $(A \cap B) - (A \cap C) \subseteq A \cap (B - C)$ [what we needed to show.]

37. For all sets A and B, $(B^c \cup (B^c - A))^c = B$.

37.)

$$\begin{aligned}
(B^C \cup (B^C - A))^C &= (B^C)^C \cap (B^C - A)^C && \text{by De Morgan's Laws} \\
&= B \cap (B^C - A)^C && \text{by Double Complement Law} \\
&= B \cap (B^C \cap A^C)^C && \text{by Set Difference Law} \\
&= B \cap ((B^c)^c \cup (A^C)^C) && \text{by De Morgan's Laws} \\
&= B \cap (B \cup A) && \text{by Double Complement Law} \\
&= B && \text{by Absorption Laws}
\end{aligned}$$

42. $(A - (A \cap B)) \cap (B - (A \cap B))$

42.)

$$\big(A - (A \cap B)\big) \cap \big(B - (A \cap B)\big)$$

$= (A \cap (A \cap B)^C) \cap (B \cap (A \cap B)^C)$	by Set Difference Laws
$= (A \cap (A^C \cup B^C)) \cap (B \cap (A^C \cup B^C))$	by De Morgan's Laws
$= ((A \cap A^C) \cup (A \cap B^C)) \cap ((B \cap A^C) \cup (B \cap B^C))$	by Distributive Laws
$= (\emptyset \cup (A \cap B^C)) \cap ((B \cap A^C) \cup \emptyset)$	by Complement Laws
$= (\emptyset \cup (A \cap B^C)) \cap (\emptyset \cup (B \cap A^C))$	by Commutative Laws
$= ((\emptyset \cup A) \cap (\emptyset \cup B^C)) \cap ((\emptyset \cup B) \cap (\emptyset \cup A^C))$	by Distributive Laws
$= ((A \cup \emptyset) \cap (B^C \cup \emptyset)) \cap ((B \cup \emptyset) \cap (A^C \cup \emptyset))$	by Commutative Laws
$= (A \cap B^C) \cap (B \cap A^C)$	by Identity Laws
$= (A \cap A^C) \cap (B \cap B^C)$	by Commutative & Associative Laws
$= \emptyset \cap \emptyset$	by Complement Laws
$= \emptyset$	by Idempotent Laws