Kevin Harvell
Week 2 Quiz

1.) In the domain of all students, we define predicates

M(x) : x is a math major

C(x) : x is a computer science major

A(x) : x is required to take CS 225 .

Express each of the following English sentences in terms of M(x), C(x), A(x), quantifiers, and logical connectives.

(a) Some computer science majors are required to take CS 225.

$$\exists \boldsymbol{x} | \boldsymbol{C}(\boldsymbol{x}) \wedge \boldsymbol{A}(\boldsymbol{x})$$

(b) Not all math majors are computer science majors.

$$\sim\forall \boldsymbol{x}(\boldsymbol{M}(\boldsymbol{x}) \wedge \boldsymbol{C}(\boldsymbol{x}))$$

(c) All students who are both computer science and math majors are not required to take CS 225.

$$\forall \boldsymbol{x}((\boldsymbol{C}(\boldsymbol{x}) \wedge \boldsymbol{M}(\boldsymbol{x})) \rightarrow \sim\boldsymbol{A}(\boldsymbol{x}))$$

(d) There is a student who is neither a math major and is not required to take CS 225.

$$\exists \boldsymbol{x} | \sim\boldsymbol{M}(\boldsymbol{x}) \wedge \sim\boldsymbol{A}(\boldsymbol{x})$$

2.) Let B(x), S(x), and A(x) be the predicates

B(x) : x is a good basketball player

S(x) : x is a good soccer player

A(x) : x is a good athlete

Translate each of the following quantified logic expressions (provided in the file) into English considering the domain to consist of all people .

1.) For all people, if someone is a good athlete, then he/she is a good basketball and soccer player.

2.) Not everyone is a good soccer player.

3.) There is a person who is a good soccer and basketball player or not a good athlete.

4.) There is a person who is not both a good soccer player and not a good basketball player.

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3.) Negate each of the following statements:

1) Every real number is positive or negative or zero .

There is no number that is positive or negative or zero

2) Some people are not honest .

All people are honest.

3) Not all young people are good athletes.

All young people are good athletes.

4.) Prove or disprove that $\forall x\ (P(x) \rightarrow Q(x))$ and $\sim\exists x \sim(\sim Q(x) \rightarrow \sim P(x))$ are logically equivalent. (Hint: Start with the double negation rule $\sim(\sim(\forall x\ (P(x) \rightarrow Q(x)))$

$\forall x\ (P(x) \rightarrow Q(x)) \equiv \sim(\sim(\forall x\ (P(x) \rightarrow Q(x)))$ **by Double Negation**

$\equiv \sim\exists \boldsymbol{x}\sim(\boldsymbol{P}(\boldsymbol{x}) \rightarrow \boldsymbol{Q}(\boldsymbol{x}))$ **by DeMorgan's Law for quantifiers**

$\equiv \sim\exists \boldsymbol{x}\sim(\sim\boldsymbol{Q}(\boldsymbol{x}) \rightarrow \sim\boldsymbol{P}(\boldsymbol{x}))$ **by contrapositive**

Therefore, $\forall x\ (P(x) \rightarrow Q(x))$ and $\sim\exists x \sim(\sim Q(x) \rightarrow \sim P(x))$ are logically equivalent

5.) True or false: For the set of all negative integers, $\exists x\ (x + 1 < - x)$

TRUE. Take the case of x = -6. $-6 + 1 < -(-6)$

$-5 < 6$

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6.) By using direct proof method, show that if k is any odd integer and m is any even integer, then, $k^2 + m^2$ is odd.

Proof:

Suppose *k* is any odd integer and *m* is any even integer. [We must show that $\boldsymbol{k^2 + m^2}$ is odd.] By definition of odd, *k = 2a + 1* where *a* is some integer. By definition of even, *m = 2b* where *b* is some integer. By substitution and algebra,

$$\boldsymbol{k^2 + m^2 = (2a+1)^2 + (2b)^2}$$

$$\boldsymbol{= 4a^2 + 4a + 1 + 4b^2}$$

$$\boldsymbol{= 2\left(2a^2 + 2a + 2b^2\right) + 1}$$

Assume $\boldsymbol{t = \left(2a^2 + 2a + 2b^2\right)}$, which makes *t* an integer because it is the sum of the product of integers. Thus, $\boldsymbol{k^2 + m^2 = 2t + 1}$, and by definition of odd, $\boldsymbol{k^2 + m^2}$ is odd [what we needed to show.]

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7.) Prove by contraposition that for all integers m and n, if m + n is even then m and n are both even or m and n are both odd.

If m + n is even, then ((m and n are both even) OR (m and n are both odd)

Contraposition: If ~(m and n are both odd) AND ~(m and n are both even), then m + n is odd.

Proof (by contraposition):

Suppose *m* and *n* are not both odd nor both even. [We must show that *m + n* is odd.] There are two cases when *m* and *n* are not both odd nor both even. One case is when *m* is even and *n* is odd, and the other case is when *m* is odd and *n* is even.

Case 1 (*m* is odd, *n* is even):

Suppose m is odd and n is even. [We must show that *m + n* is odd.] By definition of odd, $m = 2j + 1$ where *j* is some integer. By definition of even, $n = 2k$ where *k* is some integer. By substitution and algebra,

$$m + n = 2j + 1 + 2k$$

$$= 2(j + k) + 1$$

Assume $t = j + k$, which makes *t* an integer because it is the sum of integers. Thus, $m + n = 2t + 1$, and by definition of odd, $m + n$ is odd [as we needed to show].

Case 2 (*m* is even, *n* is odd):

Suppose *m* is even, and *n* is odd. [We must show that *m + n* is odd.] By definition of even, $m = 2a$ where *a* is some integer. By definition of odd, $n = 2b + 1$ where *b* is some integer. By substitution and algebra,

$$m + n = 2a + 2b + 1$$

$$= 2(a + b) + 1$$

Assume $q = a + b$, which makes *q* an integer because it is the sum of integers.
Thus, $m + n = 2q + 1$, and by definition of odd, $m + n$ is odd [as we needed to show].

In both cases of *m* and *n* not being both odd nor both even, we found that *m + n* is odd [as we needed to show].