Kevin Harvell

Week 3 Quiz

1.) Use proof by contradiction to show that the set of prime numbers is infinite.

Proof(by contradiction):

Suppose not. That is, suppose that the set of prime numbers is finite. This statement implies that there is a largest prime number. Let the list of all prime numbers be represented by $p_1, p_2, p_3, \ldots p_k$ where p_k is the largest prime number.
Multiplying all the primes together can be represented by $p_1 \cdot p_2 \cdot p_3 \cdot \ldots p_k$
Let $m = p_1 \cdot p_2 \cdot p_3 \cdot \ldots p_k + 1$
m cannot be evenly divided by any of these prime numbers. For example,
$\frac{m}{p_1} = p_2 \cdot p_3 \cdot \ldots p_k + \frac{1}{p_1}$
Thus, $\frac{1}{p_1}$ does not evenly divide, which implies m is a prime number bigger than p_k. This is true for p_2 and so on as well. Therefore, because m is not divisible by any of p_1 through p_k, and m is bigger than p_k we have a contradiction [what we needed to show.]

2.) Suppose a ∈ Z. If a^3 is even, then a is even. Use proof by contradiction.

Let P = "a^3 is even", and Q = "a is even". Assume for the sake of contradiction $\sim(P \to Q)$ or $P \wedge \sim Q$ meaning that a^3 is even and a is not even, which makes a odd.
By definition of even, $a^3 = 2k$ for some integer k, and by definition of odd, $a = 2j + 1$ for some integer j. a is an integer because it is the product and sum of integers.
By substitution,

$$a^3 = (2j+1)^3 = (2j+1)(2j+1)(2j+1)$$

By definition in Example 4.2.3, an odd integer times another odd integer is odd. The odd result multiplied by another odd integer is still odd. Thus,
$(2j+1)(2j+1)(2j+1)$ is odd, which contradicts our assumption that a^3 is even. [Hence the supposition is false and the theorem is true.]

3.) Let A ={x ∈ Z | x =10a+7 for some integer a }
and B={y ∈ Z | y = 10b − 3 for some integer b}.

Prove or disprove that B ⊆ A

Suppose m is a particular but arbitrarily chosen element of B. [We must show that $x \in A$. By definition of A, this means we must show that $m = 10 \times (some\ integer) + 7$.] By definition of B, there is an integer such that $m = 10b - 3$. [Given that $m = 10b - 3$, can m also be expressed as $10 \times (some\ integer) + 7$? I.e., is there an integer, say a such that $10b - 3 = 10a + 7$? Solve for a to obtain $a = \frac{10b-10}{10} = b - 1$. Check to see if this works.] Let $a = b - 1$. [First check that a is an integer.] Then a is an integer because it is the difference of integers. [Then check that $m = 10a + 7$.]
Also $10a + 7 = 10(b-1) + 7 = 10b - 10 + 7 = 10b - 3 = m$.
Thus, by definition of A, m is an element of A [which is what was to be shown.]

4.) False

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5.) False

6.) False

7.) True

8.) $A = \{-6, -4, -1, 0\}$

$$\mathcal{P}(A) = \left\{ \begin{matrix} \emptyset, \{-6\}, \{-4\}, \{-1\}, \{0\}, \{-6,-4\}, \{-6,-1\}, \{-6,0\}, \{-4,-1\}, \{-4,0\}, \{-1,0\}, \{-6,-4,-1\}, \{-6,-4,0\}, \\ \{-6-1,0\}, \{-4,-1,0\}, \{-6,-4,-1,0\} \end{matrix} \right\}$$

9.) Let, A={{1}} , B={1, {2}} and C = {{1,2}}. Find (A X B) X C .

$$(A \times B) \times C = \{\big((\{1\},1),\{1,2\}\big),\big((\{1\},\{2\}),\{1,2\}\big)\}$$