Assignment 6 – Part 1
Assignment: Set 5.6 : 2 , 4 & Set 5.7: 4, 6, 7

Find the first four terms of each of the recursively defined sequences in 1–8.

2. $b_k = b_{k-1} + 3k$, for all integers $k \geq 2$
   $b_1 = 1$

$\boldsymbol{b_1 = 1}$
$\boldsymbol{b_2 = 1 + 3(2) = 7}$
$\boldsymbol{b_3 = 7 + 3(3) = 16}$
$\boldsymbol{b_4 = 16 + 3(4) = 28}$
$\boldsymbol{b_5 = 28 + 3(5) = 43}$

4. $d_k = k(d_{k-1})^2$, for all integers $k \geq 1$
   $d_0 = 3$

$\boldsymbol{d_0 = 3}$
$\boldsymbol{d_1 = 1(3)^2 = 9}$
$\boldsymbol{d_2 = 2(9)^2 = 162}$
$\boldsymbol{d_3 = 3(162)^2 = 78732}$
$\boldsymbol{d_4 = 4(78732)^2 = 24794911296}$

In each of 3–15 a sequence is defined recursively. Use iteration to guess an explicit formula for the sequence. Use the formulas from Section 5.2 to simplify your answers whenever possible.

4. $b_k = \dfrac{b_{k-1}}{1 + b_{k-1}}$, for all integers $k \geq 1$
$b_0 = 1$

$$\boldsymbol{b_1 = \frac{1}{1+1} = \frac{1}{2\cdot 1} = \frac{1}{2}}$$
$$\boldsymbol{b_2 = \frac{\frac{1}{2}}{1+\frac{1}{2}} = \frac{1}{2}\cdot\frac{2}{3} = \frac{1}{3}}$$
$$\boldsymbol{b_3 = \frac{\frac{1}{3}}{1+\frac{1}{3}} = \frac{1}{3}\cdot\frac{3}{4} = \frac{1}{4}}$$
$$\boldsymbol{b_4 = \frac{\frac{1}{4}}{1+\frac{1}{4}} = \frac{1}{4}\cdot\frac{4}{5} = \frac{1}{5}}$$

Guess:

$$\boldsymbol{b_n = \frac{1}{n+1}}$$

H **6.** $d_k = 2d_{k-1} + 3$, for all integers $k \geq 2$
$d_t = 2$

$$\boldsymbol{d_{t+1} = 2\cdot 2 + 3 = 2^2 + 1\cdot 3}$$
$$\boldsymbol{d_{t+2} = 2\cdot(2\cdot 2 + 3) + 3 = 2^3 + 3\cdot 3}$$
$$\boldsymbol{d_{t+3} = 2\cdot\left(2^3 + 3\cdot 3\right) + 3 = 2^4 + 7\cdot 3}$$
$$\boldsymbol{d_{t+4} = 2\cdot\left(2^4 + 7\cdot 3\right) + 3 = 2^5 + 15\cdot 3}$$

Guess:

$$\boldsymbol{d_n = 2^n + 3(2^{n-1} - 1)}$$

7. $e_k = 4e_{k-1} + 5$, for all integers $k \geq 1$
$e_0 = 2$

$$\boldsymbol{e_1 = 4\cdot 2 + 5}$$
$$\boldsymbol{e_2 = 4(4\cdot 2 + 5) + 5 = 16\cdot 2 + 5\cdot 5}$$
$$\boldsymbol{e_3 = 4(16\cdot 2 + 5\cdot 5) + 5 = 64\cdot 2 + 21\cdot 5}$$
$$\boldsymbol{e_4 = 4(64\cdot 2 + 21\cdot 5) + 5 = 256\cdot 2 + 85\cdot 5}$$

Guess:

$$\boldsymbol{e_n = 4^n\cdot 2 + (4^{n-1} + 4^{n-2} + \cdots + 4^0)\cdot 5}$$