Kevin Harvell
Week 4 - Quiz

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Let A = {x | -2 <x ≤ 5}, B = {x | -9≤ x ≤ 2} and C = {x | 2 ≤x< 4}, where x represents an integer number. Determine the sets $(A - C) \cup A$, $(A \cap B) - C$ and $B \cap C^c$.

1.) $\boldsymbol{A = \{-1, 0, 1, 2, 3, 4, 5\}}$ $\boldsymbol{B = \{-9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2\}}$ $\boldsymbol{C = \{2, 3\}}$
$\boldsymbol{(A - C) \cup A = \{-1, 0, 1, 4, 5\} \cup \{-1, 0, 1, 2, 3, 4, 5\} = \{-1, 0, 1, 2, 3, 4, 5\}}$
$\boldsymbol{(A \cap B) - C = \{-1, 0, 1, 2\} - \{2, 3\} = \{-1, 0, 1\}}$
$\boldsymbol{B \cap C^C = \{-9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1\}}$

Use an element argument to prove the statement:

For all sets A, B and C, $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

2.) Suppose $x \in (A \cup B) \cap (A \cup C)$. By definitions of union and intersect , $(x \in A$ or $x \in B)$ AND $(x \in A$ or $x \in C)$. This states that x is an element in sets A or B, and also x is an element in set A or C. It follows that x must be an element in A or an element in B and C.
On the other hand, $x \in A \cup (B \cap C)$ implies that $x \in A$ or $(x \in B$ AND $x \in C)$. This states that x is an element in A or an element in B and C.
Therefore, $x \in (A \cup B) \cap (A \cup C)$ follows that $x \in A \cup (B \cap C)$. Hence we can conclude that $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$ [what we needed to show.]

Construct an algebric proof for all sets A , B and C,

$$A \cup (B - C) = (A \cup B) - (C - A)$$

3.)

$\boldsymbol{A \cup (B - C) = A \cup (B \cap C^C)}$	**by Set Difference law**
$\boldsymbol{= (A \cup B) \cap (A \cup C^C)}$	**by Distributive law**
$\boldsymbol{= (A \cup B) \cap (A^c \cap C)^C}$	**by DeMorgan's law**
$\boldsymbol{= (A \cup B) \cap (C \cap A^C)^C}$	**by Commutative law**
$\boldsymbol{= (A \cup B) \cap (C - A)^C}$	**by Set Difference law**
$\boldsymbol{= (A \cup B) - (C - A)}$	**by Set Difference law**

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What are the terms a_0, a_1, a_2 and a_3 of the sequence $\{a_n\}$, where a_n equals:

1) $a_n = (-1)^{n+2} * n^3$, For all n> =0

2) $a_n = 2$

4.) 1.) $a_0 = (-1)^{0+2} * 0^3 = 0$

$a_1 = (-1)^{1+2} * 1^3 = -1$

$a_2 = (-1)^{2+2} * 2^3 = 8$

$a_3 = (-1)^{3+2} * 3^3 = -27$

2.) $a_0 = 2$

$a_1 = 2$

$a_2 = 2$

$a_3 = 2$

Given that, $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$. Use this identity to find a simple expression for $\sum_{k=1}^{n-1} \frac{1}{k(k+1)}$

5.) $\frac{1}{k} - \frac{1}{k+1} + \frac{1}{k+1} - \frac{1}{k+2} + \cdots + \frac{1}{n-1} - \frac{1}{n} = \frac{1}{k} - \frac{1}{n}$

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Compute the following summations. (Instructions: Showing your work is necessary and to receive credit you **must** make use of the formula provided in the attached document summation_formula.pdf . An intermediate form will be acceptable, so you don't need to calculate the final result.)

a) $\sum_{i=3}^{7} (3i + 5)$

b) $\sum_{k=0}^{3} 2^{k+3}$

c) $\sum_{j=3}^{5} 4 \cdot (-1)^j$

6.a.) $\sum_{i=3}^{7}(3i+5) = \sum_{1}^{7}(3i+5) - \sum_{1}^{2}(3i+5)$

$$= \left(\sum_{1}^{7} 3i + \sum_{1}^{7} 5\right) - \left(\sum_{1}^{2} 3i + \sum_{1}^{2} 5\right)$$

$$= \left(3\sum_{1}^{7} i + \sum_{1}^{7} 5\right) - \left(3\sum_{1}^{2} i + \sum_{1}^{2} 5\right)$$

$$= \left(3 \cdot \frac{7(7+1)}{2} + 7 \cdot 5\right) - \left(3 \cdot \frac{2(2+1)}{2} + 2 \cdot 5\right)$$

6.b.) $\sum_{k=0}^{3} 2^{k+3} = \sum_{k=0}^{3} 2^k \cdot 2^3$

$$= 8 \sum_{k=0}^{3} 2^k$$

$$= 8 \cdot \frac{2^{3+1} - 1}{2 - 1}$$

6.c.) $\sum_{j=3}^{5} 4 \cdot (-1)^j = \sum_{j=0}^{5} 4 \cdot (-1)^j - \sum_{j=0}^{2} 4 \cdot (-1)^j$

$$= \frac{4 \cdot (-1)^{5+1} - 4}{-1 - 1} - \frac{4 \cdot (-1)^{2+1} - 4}{-1 - 1}$$