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1) max $10x_1 + 10x_2 + 20x_3 + 20x_4$

s.t. $12x_1 + 8x_2 + 6x_3 + 4x_4 \le 210$

$3x_1 + 6x_2 + 12x_3 + 24x_4 \le 210.$

$x_1, x_2, x_3, x_4 \ge 0$

a) min $210y_1 + 210y_2.$

s.t. $12y_1 + 3y_2. \ge 10$

$6y_1 + 8y_2. \ge 10.$

$6y_1 + 12y_2 \ge 20.$

$4y_1 + 24y_2. \ge 20.$

$y_1, y_2 \ge 0.$

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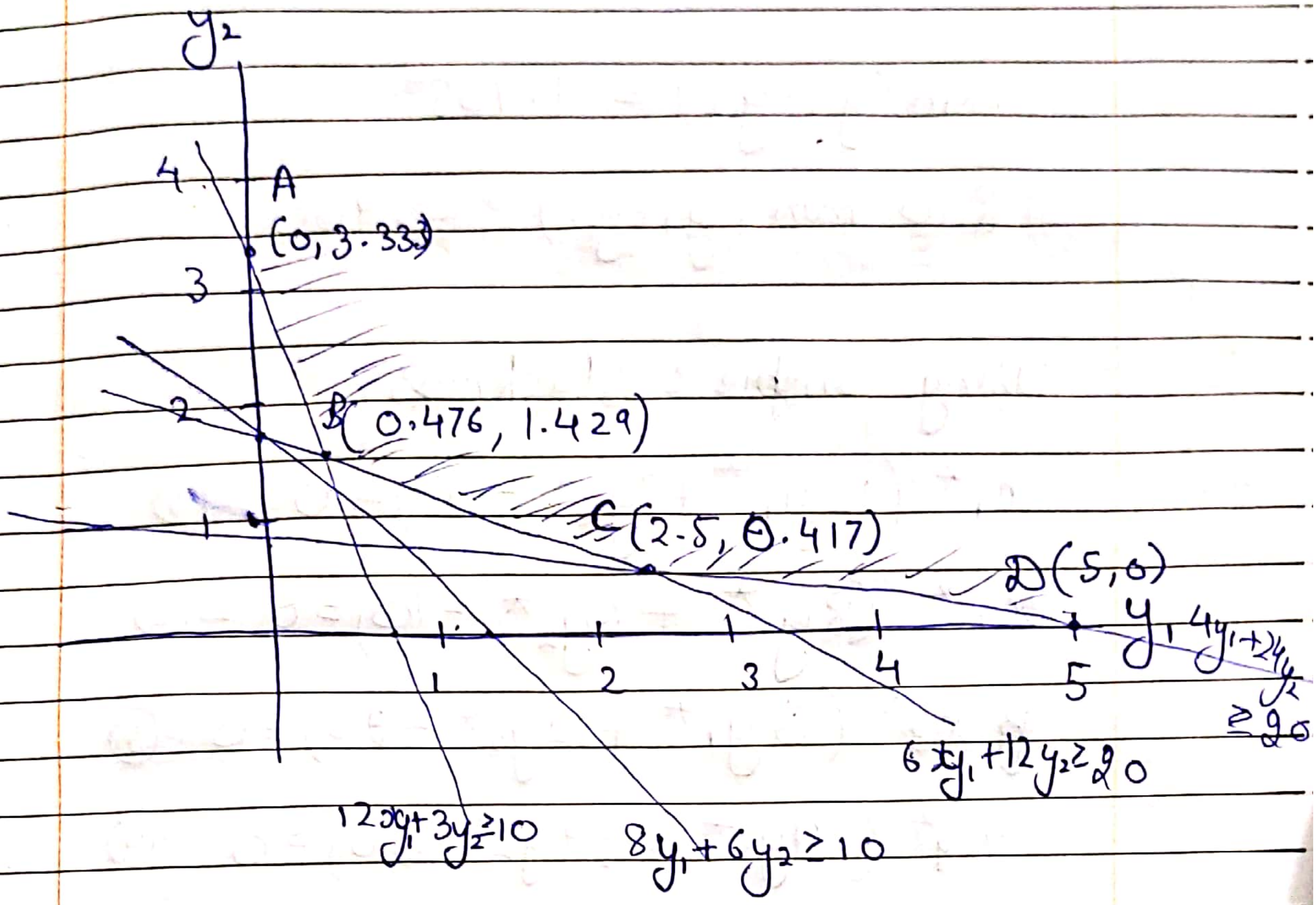

For A, B, C, D.

$\min 210\,(y_1+y_2)$

$= 210 \min (y_1+y_2).$

$\min\ y_1+y_2$

for A $= 3.33$

for B $= 1.905$

for C $= 2.917$

for D $= 5$

$\min(y_1+y_2) = 1.905$

$210 \min(y_1+y_2) = 400.$

Using Complete Slackness,

$x_1^*(12y_1^* + 3y_2^* - 10) = 0$ — (1)

$x_2^*(8y_1^* + 6y_2^* - 10) = 0$ — (2)

$x_3^*(6y_1^* + 12y_2^* - 20) = 0$ — (3)

$x_4^*(4y_1^* + 24y_2^* - 20) = 0$ — (4)

As y_1^* & $y_2^* \in B$.

So, for (2) & (4),
x_2^* & x_4^* must be 0.

So, $x_2^* = x_4^* = 0$

c) max $10x_1 + 20x_3$

s.t. $12x_1 + 6x_3 \le 210$

$3x_1 + 12x_3 \le 210$.

$x_1, x_3 \ge 0$.

Corner points $= A\left(0, \frac{35}{2}\right), B\left(\frac{35}{2}, 0\right), C(10, 15)$.

At A 350

At B 175.

At C 400

$x_1 = 10$

$x_2 = 0$

$x_3 = 15$ is optimal primal sol^n

$x_4 = 0$

2) a) $\min \; c_1^T x_1 + c_2^T x_2 + c_3^T x_3$

s.t. $Ax_1 \geq b_1$

$Bx_2 = b_2$

$Cx_3 \leq b_3 \qquad x_1, x_2, x_3 \geq 0$

$\Downarrow$

$Ax_1 \geq b_1$

$Bx_2 = b_2$

$-Cx_3 \geq -b_3$

$x_1, x_2, x_3 \geq 0$

Dual:-

a) $\max \; b_1^T y_1 + b_2^T y_2 - b_3^T y_3$

$A^T y_1 \leq c_1$

$B^T y_2 \leq c_2$

$-C^T y_3 \leq c_3$

$y_1, y_3 \geq 0$

b).

$c_1^T x_1 \geq y_1 A^T x_1 \geq b_1^T y_1$ —(i)

$c_2^T x_2 \geq y_2 B^T x_2 = b_2^T y_2$ —(ii)

$c_3^T x_3 \geq -y_3 C^T x_3 \geq -b_3^T y_3$

~~$c_3^T x_3 \geq$~~ let $y_3 = -y_3$

$y_3 \leq 0$

$c_3^T x_3 \geq y_3 C^T x_3 \geq b_3^T y_3$ —(iii)

from i), ii) & iii)

$c_1^T x_1 \geq b_1^T y_1$

$c_2^T x_2 \geq b_2^T y_2$

$c_3^T x_3 \geq b_3^T y_3$

$$[c_1^T \; c_2^T \; c_3^T] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \geq [b_1^T \; b_2^T \; b_3^T] \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\bar{c}^T \bar{x} \geq \bar{b}^T \bar{y}$$

3)a)

$$\min_{y} \sum_{i \in J : (i,j) \in E} y_i + \sum_{j \in C : (i,j) \in E} y_j$$

s.t.

$$y_i + y_j \geq p_{ji} \quad \forall j \in C, i \in J : (i,j) \in E$$

$$y_i \geq 0 \quad y_j \geq 0 \quad \forall (i,j) \in E$$

c). As the ~~value~~ optimal for both primal & dual $= 5$

So, strong duality holds.

d) In decreasing order,

$p_{ij} = \{ (2,4) : 3, (2,6) : 2, (1,4) : 2, (3,4) : 1, (3,5) : 1 \}$

~~Inside~~

From greedy approach, we will choose max possible reward.

So, first candidate 2 chooses 4 profit $= 3$.

Now, only candidate 3 can choose 5. total profit $= 4$

~~as other possible attempts~~

So, from greedy approach our profit is 4 which is less than optimal.

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4).

a) $\max\ 13x_1 + 16x_2 + 16x_3 + 14x_4 + 39x_5$

s.t.

$11x_1 + 53x_2 + 5x_3 + 5x_4 + 29x_5 \leq 40$

$3x_1 + 6x_2 + 5x_3 + 1x_4 + 34x_5 \leq 20$

$x_i \geq 0 \quad \forall i \in \{1,2,3,4,5\}.$

$x_i \leq 1$

d)

$\min\ 40y_1 + 20y_2 + y_3 + y_4 + y_5 + y_6 + y_7$

s.t.
$$\begin{aligned}
11y_1 + 3y_2 + y_3 &\geq 13 \\
53y_1 + 6y_2 + y_4 &\geq 16 \\
5y_1 + 5y_2 + y_5 &\geq 16 \\
5y_1 + y_2 + y_6 &\geq 14 \\
29y_1 + 34y_2 + y_7 &\geq 39
\end{aligned}$$

$y_i \geq 0 \quad \forall i \in \{1,2,3,4,5,6,7\}.$

Optimal value is same as in part c).

3)b)

```
Thread count was 1 (of 8 available processors)

Solution count 1: 5

Optimal solution found (tolerance 1.00e-04)
Best objective 5.000000000000e+00, best bound 5.000000000000e+00, gap 0.0000%
Candidate  1  took job  4
Candidate  2  took job  6
Candidate  3  took job  5
max profit :  5.0
PS C:\Users\vivek\Documents\code\algotithm> 
```

c)

```
Solution count 2: 5 6

Optimal solution found (tolerance 1.00e-04)
Best objective 5.000000000000e+00, best bound 5.000000000000e+00, gap 0.0000%
Value of y 1 is 0.0
Value of y 2 is 2.0
Value of y 3 is 1.0
Value of y 4 is 2.0
Value of y 5 is 0.0
Value of y 6 is 0.0
min sum :  5.0
PS C:\Users\vivek\Documents\code\algotithm> 
```

4)b)

```
Solved in 2 iterations and 0.03 seconds (0.00 work units)
Optimal objective  5.744901720e+01
x[1] 1.0
x[2] 0.20085995085995084
x[3] 1.0
x[4] 1.0
x[5] 0.2880835380835381
max :  57.449017199017206
PS C:\Users\vivek\Documents\code\algotithm> 
                                                  Ln 17, Col 31   Spaces: 4   UTF-8
```

c)

```
Solved in 2 iterations and 0.01 seconds (0.00 work units)
Optimal objective  5.744901720e+01
x[1] 1.0
x[2] 0.20085995085995084
x[3] 1.0
x[4] 1.0
x[5] 0.2880835380835381
max :  57.449017199017206
[0.1904176904176904, 0.9846437346437346, 7.951474201474202, 0.0, 10.124692874692876, 12.063267813267814, 0.0]
PS C:\Users\vivek\Documents\code\algotithm> 
                              Ln 15, Col 26   Spaces: 4   UTF-8   CRLF   Python   3.8.8 ('base': conda)
```

d) and e)

```
Optimal objective  5.744901720e+01
min :  57.4490171990172
y[1] 0.1904176904176904
y[2] 0.9846437346437348
y[3] 7.951474201474201
y[4] 0.0
y[5] 10.124692874692874
y[6] 12.063267813267814
y[7] 0.0
[1.0, 0.20085995085995084, 1.0, 1.0, 0.2880835380835381]
PS C:\Users\vivek\Documents\code\algotithm> 
```

We get same result from Pi and our dual program also strong duality holds