MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.

   (a) Reasons why the government issues bond rather than printing money include: 1. printing money could cause inflation and lead to the loss of value in the currency. 2. Issuing bond allows the government to borrow money from a large pool of investors allowing for a more stable source of raising money. 3. Bonds can be traded in the market allowing for more liquidity.

   (b) The flattening interest rate might be the result of the long-term interest rate falling more than the short-term interest rate or the result of investors expecting inflation to decrease.

   (c) Quantitative easing is when the central bank purchases securities from the market to reduce the interest rate and provide more liquidity to encourage lending and investing when the economy is in recession. Since the beginning of COVID-19, the US FED become the largest buyer of Treasury Securities in the market by purchasing US Treasuries and mortgage-backed bonds reducing the interest rate to nearly zero percent.

2. For this assignment I decided to choose the following bonds: "CAN 1.5 June 1, CAN1 June1, CAN 2 June 1, CAN 2.25 June 1, CAN 0.5 Dec 1, CAN 2.5 Mar 1, CAN 1 Sep 1, CAN 1.25 Mar 1, CAN 1.5, Dec 1 " The reasons that I pick these bonds are: 1) all of those bonds are paid semi-annually. 2) Similar coupon rates 3) Their maturity dates are evenly distributed, most of them are 6 months apart from each other.

3. The eigenvalues and eigenvectors of the covariance matrix describe the magnitude and direction of the variability of the stochastic process. The eigenvectors give the direction of maximum variability in the multi-dimensional data, and the eigenvalues correspond to the amount of variance in the direction given by each eigenvector[1].

Empirical Questions - 75 points

4.

   (a) I calculated the Yield to Maturity of each bond with the following equation YTM $= \frac{C+(FV-P)/T}{(FV+P)/2}$. I obtained a flat yield curve which could mean there is a small difference between short-term interest rate and long-term interest rate and the current market are expected to experience slow economic growth

[1] A step-by-step explanation of principal component analysis (PCA). Built In. (n.d.)

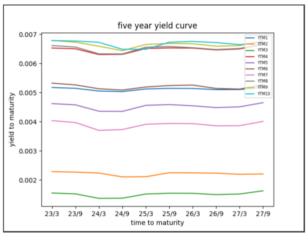

Figure 1: 5 year yield curve

(b) 1) Sort the collected bond by maturity so that we can build a spot curve starting from the shortest maturity to the longest maturity. 2) Calculate yield to maturity using the formula: YTM $= \frac{C+(FV-P)/T}{(FV+P)/2}$. 3) Using the ytms calculated above to obtain the present value of all cash flows for the bond using present value $= C*(1+ytm)^{(-t)} + FV*(1+ytm)^{(-T)}$ 4) Compare the present value with the market price of the bond if the present value 5) repeat step 1 to 4 for all bonds. 6) Interpolate the ytms of each bond to obtain continuous curves for the spot rate for terms ranging from 1 to 5 years.

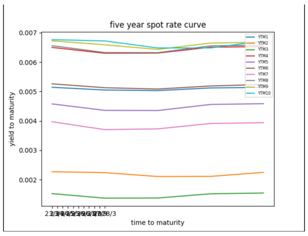

Figure 2: 5-year spot curve

(c) 1) first define a range of terms ranging from 2 to 5 years representing 1yr-1yr to 1 year-4year forward rate. 2) loops over the days of the data and calculates the forward rates for each day using the equation: forward rate = (future spot rate*term-current spot rate*(term -1))/term. 3)if the term is equal to 2 years then take the average of 1-year and 2-year spot rates and if it is greater than 2 years use the formula in step 2.

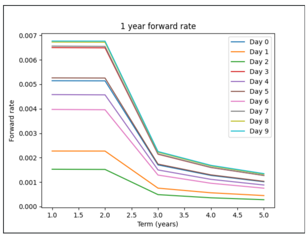

Figure 3: 1-year forward curve

5. The matrix of the forward curve is

$$M = \begin{pmatrix} 2.09673843e-06 & 1.49986757e-05 & 8.83356514e-06 & 9.03469135e-06 \\ 1.49986757e-05 & 1.07294058e-04 & 6.31943739e-05 & 6.46353989e-05 \\ 8.83356514e-06 & 6.31943739e-05 & 3.72227557e-05 & 3.80733453e-05 \\ 9.03469135e-06 & 6.46353989e-05 & 3.80733453e-05 & 3.89447489e-05 \end{pmatrix}$$

the matrix of daily log returns of yield is

$$M = \begin{bmatrix} 2.65131181e-05 & 1.34551771e-04 & 1.27614867e-05 & 3.05625606e-06 & 3.56737486e-05 \\ 1.34551771e-04 & 9.32274127e-04 & 1.65345154e-04 & .82481898e-05 & 1.99461185e-04 \\ 4.70311702e-04 & 4.02114328e-04 & 1.64813486e-04 & 2.48666929e-04 & 4.02114328e-04 \\ 8.06404891e-04 & 2.15475137e-04 & 4.93000412e-04 & 1.27614867e-05 & 1.65345154e-04 \\ 3.05625606e-06 & 3.82481898e-05 & 6.25606e-06 & 3.82481898e-05 & 1.49587181e-05 \end{bmatrix}$$

6. The eigenvalues and eigenvectors of forward rate covariance are given by :
1.85552042e-04, [0.106296 -0.18557134 -0.85451899 -0.47335155],
6.25883123e-09, [0.76041501 -0.58154269 0.27322045 -0.0944865],
5.34157753e-19, [0.44788729 0.26959596 -0.41524338 0.74450518],
6.01789830e-14, [0.45811259 0.74477473 0.15073424 -0.46121866]

   the eigenvalues and eigenvectors of daily log return matrix are :
2.43866986e-03, [0.08340674 0.07673677 0.21197051 0.56020902 0.25983825],
5.28244024e-04, [0.08340674 0.07673677 0.21197051 0.56020902 0.25983825],
6.08626918e-05, [0.34303935 0.55858285 0.04413709 0.33669955 0.26708259],
4.46670808e-05, [0.50665792 0.25040978 0.08629762 0.3840643 .39340638],
1.24693762e-05, [0.10666532 0.41396405 0.25501055 0.06990739 0.44328471]
The eigenvector that is associated with the largest eigenvalue gives you information about how much variability can be explained by that eigenvector. It explains the data's largest variation in that eigenvector's direction.

References and GitHub Link to Code

Github : https://github.com/kevin-00115/APM466-A1
A step-by-step explanation of principal component analysis (PCA). Built In. (n.d.). Retrieved February 13, 2023, from https://builtin.com/data-science/step-step-explanation-principal-component-analysis