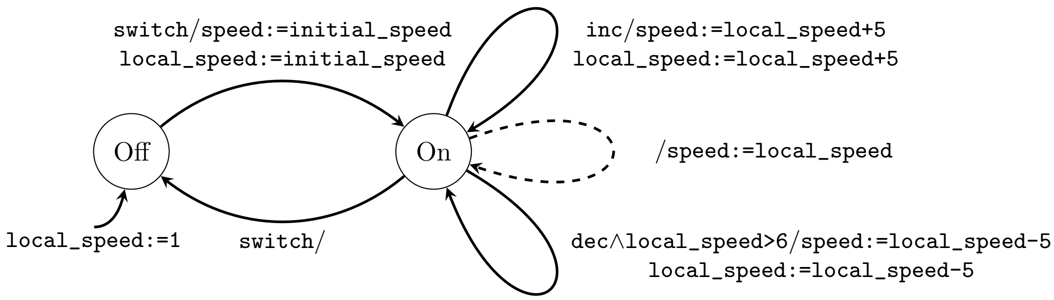
Question 1 [20 marks]

a)

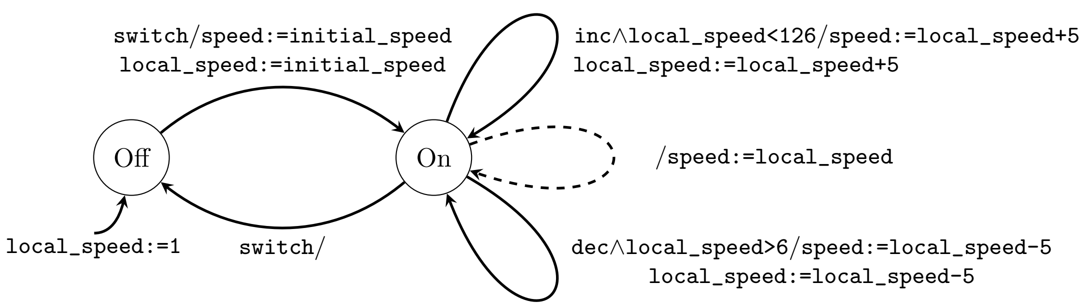
variable: `local_speed` : $\mathbb{N}$
inputs: `switch`, `inc`, `dec` : `pure`, `initial_speed` : $\mathbb{N}$
output: `speed` : $\mathbb{N}$



Rubric: correct states [1 mark], correct guards [2 marks], correct outputs [1 mark], correct variable update [1 mark], correct initialization [1 mark], infinitely many reachable states [1 marks]. If `local_speed>6` is missing from guard `dec∧local_speed>6` [-1 mark] as the output can no longer belong to $\mathbb{N}$. This subtraction is from the allocated mark to correct guards.

b)

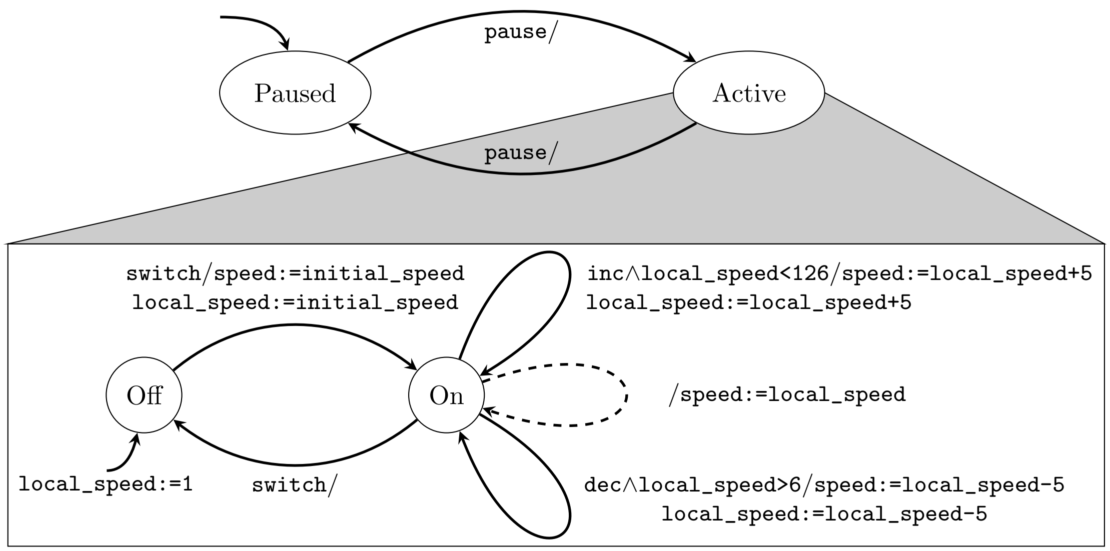
variable: `local_speed` : $\{1, 2, \ldots, 130\}$
inputs: `switch`, `inc`, `dec` : `pure`, `initial_speed` : $\{1, 2, \ldots, 130\}$
output: `speed` : $\mathbb{N}$



Rubric: correct modifications [2 marks], the number of states is $130 \times 2 = 260$ [2 marks].

c)

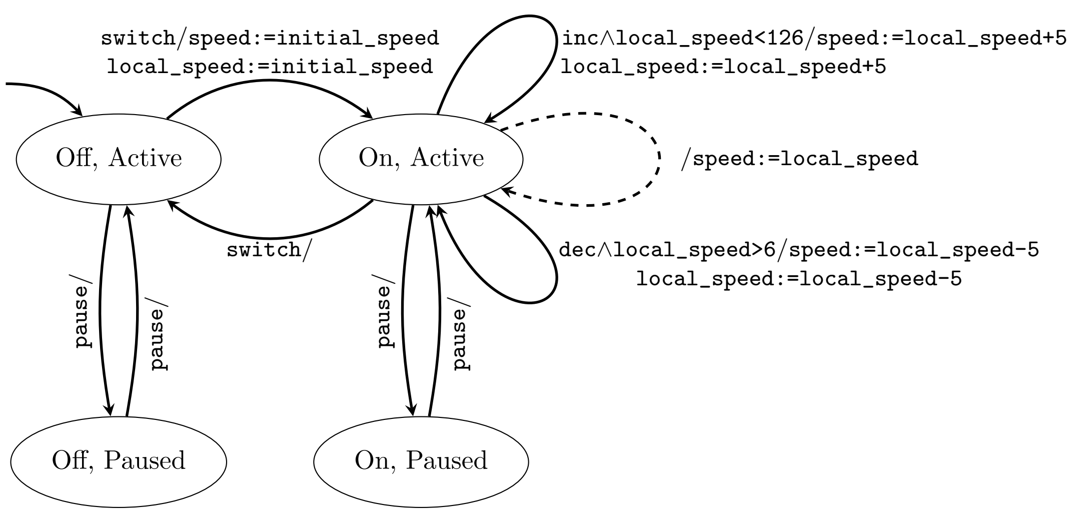
variable: `local_speed` : $\{1, 2, \ldots, 130\}$
inputs: `switch`, `inc`, `dec`, `pause` : `pure`, `initial_speed` : $\{1, 2, \ldots, 130\}$
output: `speed` : $\mathbb{N}$



Rubric: correct hierarchical FSM [2 marks], history transition and explanation [2 marks].

d)

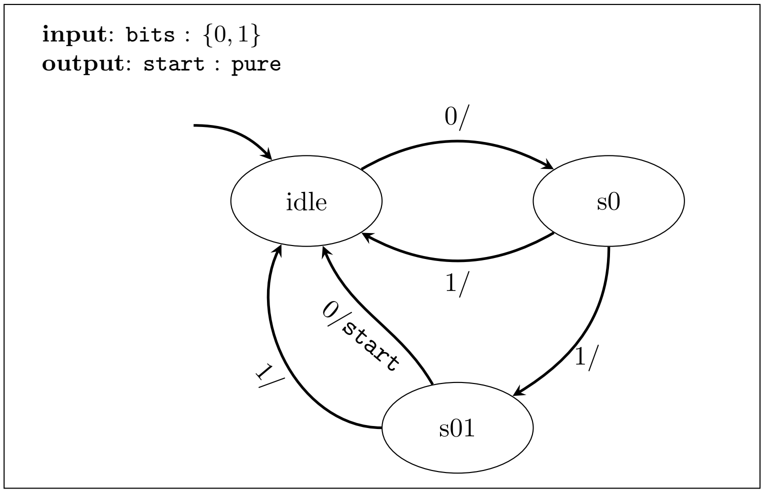
variable: `local_speed` : $\{1, 2, \ldots, 130\}$
inputs: `switch`, `inc`, `dec`, `pause` : `pure`, `initial_speed` : $\{1, 2, \ldots, 130\}$
output: `speed` : $\mathbb{N}$



Rubric: correct states [1 mark], correct guards [1 marks], correct outputs [1 mark], correct variable update [1 mark], correct initialization [1 mark].

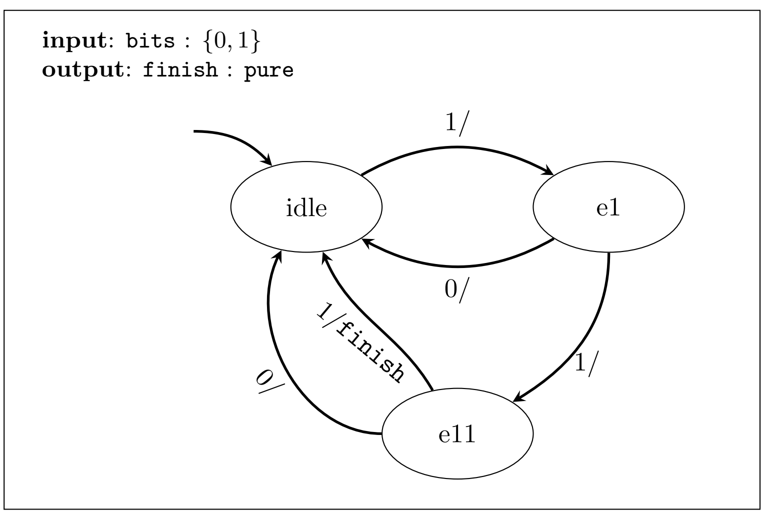
Question 2 [22 marks]

a)



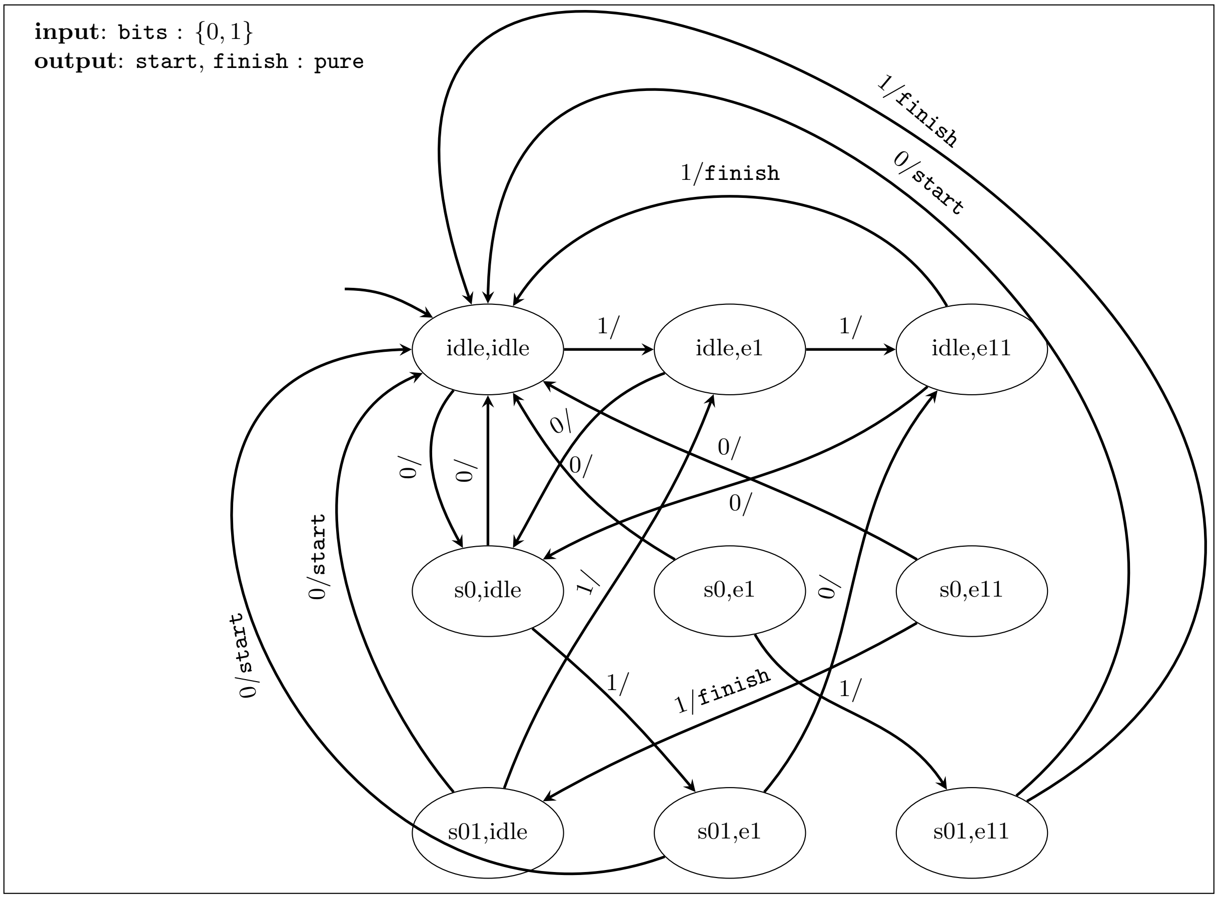
Rubric: correct states [1 mark], correct guards [1 marks], correct outputs [1 mark], correct variable update [1 mark], correct initialization [1 mark].

b)



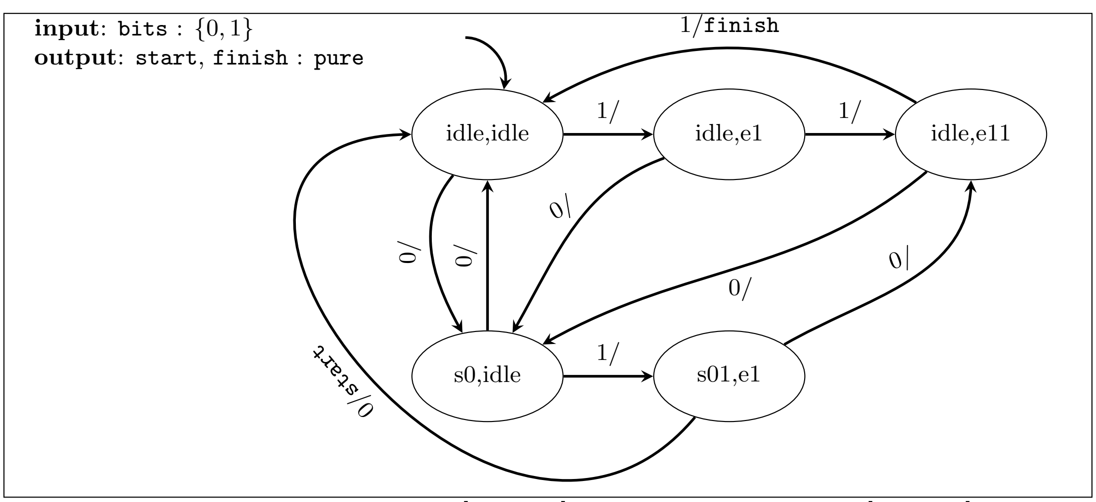
Rubric: correct states [1 mark], correct guards [1 marks], correct outputs [1 mark], correct variable update [1 mark], correct initialization [1 mark].

c)



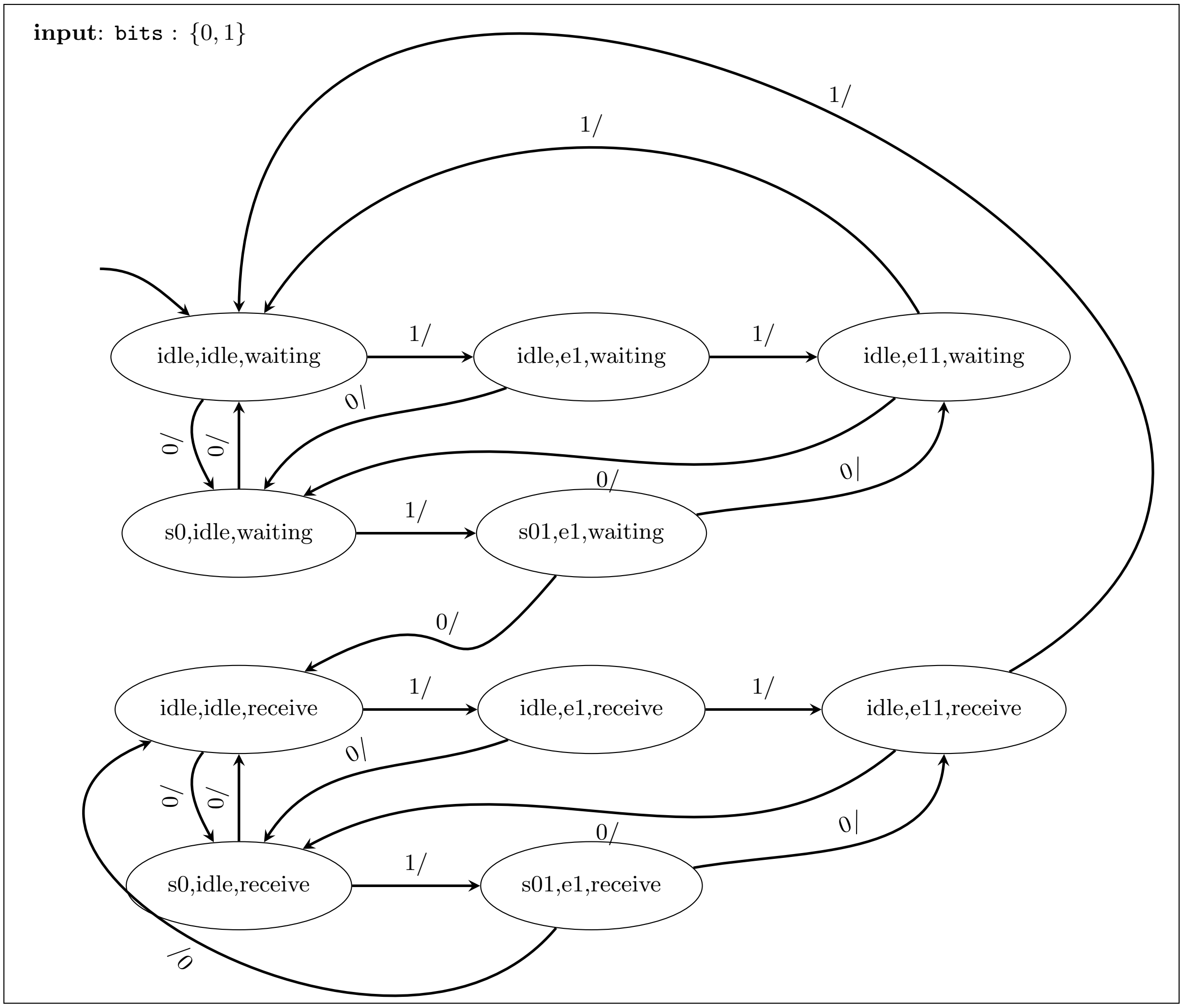
Rubric: correct states [1 mark], correct transitions [1 mark], correct guard [1 marks], correct output [1 mark], correct initialization [1 mark].

d)



Rubric: The FSM has 5 reachable states [1 marks], correct reachable states [1 marks].

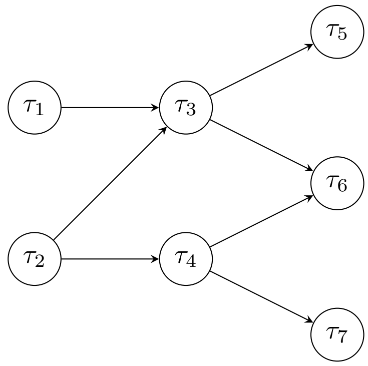
e)



Rubric: correct states [1 mark], correct transitions [1 mark], correct guard [1 marks], correct output [1 mark], correct initialization [1 mark].

Question 3 [15 marks]

a)



Rubric: Each mistake -1 mark

b)

The modified release times and deadlines are:

	A	B	C	D	E	F	G
r_i^*	0	0	3	2	7	7	5
d_i^*	11	11	15	15	20	20	20

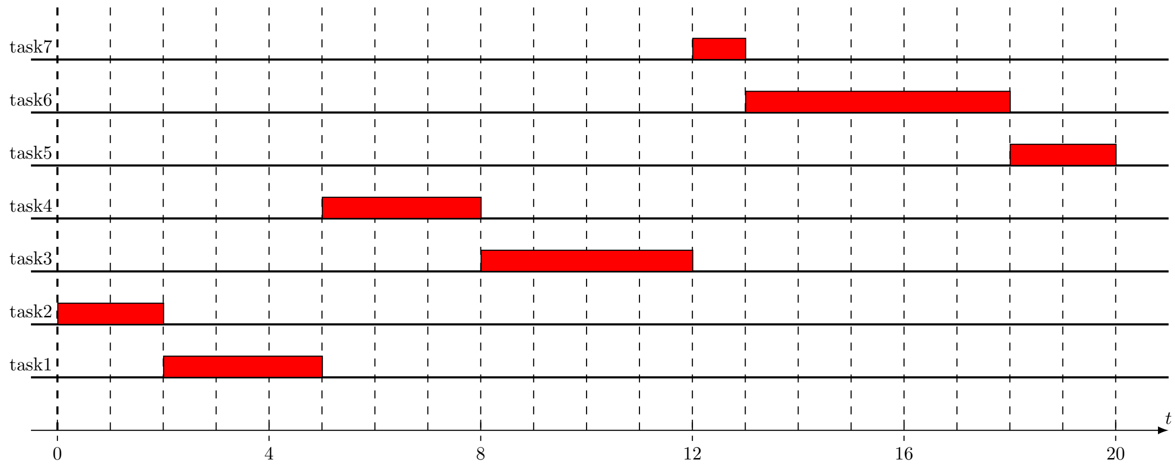
In particular, the release time modification proceeds as follows: [2 marks; -1 mark for each mistake]

$$\text{for root nodes } \tau_1 \text{ and } \tau_2 \text{ set } r_1^* = r_1 = 0, r_2^* = r_2 = 0$$
$$r_3^* = \max\{r_3, \max\{r_1^* + e_1, r_2^* + e_2\} = \max\{0, \max\{3, 2\}\} = 3$$
$$r_4^* = \max\{r_4, r_2^* + e_2\} = \max\{0, 0 + 2\} = 2$$
$$r_5^* = \max\{r_5, r_3^* + e_3, r_4^* + e_4\} = \max\{0, 3 + 4, 2 + 3\} = 7$$
$$r_6^* = \max\{r_6, r_3^* + e_3\} = \max\{0, 3 + 4\} = 7$$
$$r_7^* = \max\{r_7, r_4^* + e_4\} = \max\{0, 2 + 3\} = 5$$

The deadline modification proceeds as follows: [2 marks; -1 mark for each mistake]

$$\text{for the leaf nodes } \{\tau_5, \tau_6, \tau_7\} \text{ set } d_5^* = d_6^* = d_7^* = 20$$
$$d_3^* = \min\{d_3, d_5^* - C_E, d_F^* - C_F\} = \min\{20, 20 - 2, 20 - 5\} = 15$$
$$d_4^* = 15$$
$$d_1^* = 11$$
$$d_2^* = 11$$

One resulting EDF* schedule, based on the modified release times and deadlines, is depicted below. [4 marks; -1 mark for each mistake]



c) No [1 mark], the task set is no longer schedulable. Under the new conditions, the constraints among tasks τ_2, τ_3 and τ_5 introduce a cycle in the precedence graph. As a result, none of the three tasks can be executed as first and therefore, no feasible schedule exists. [2 marks]

Question 4 [13 marks]

a)

In s1, the firing function is given by

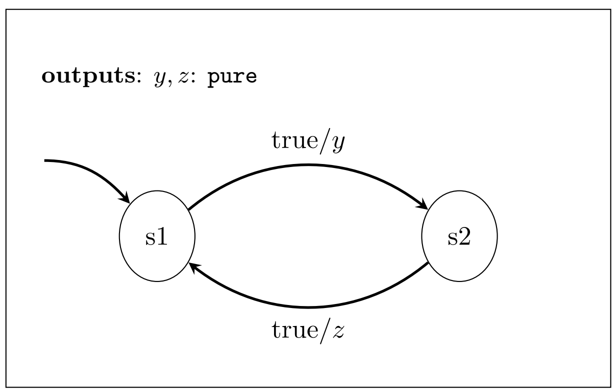
$$a_{\text{s1}}(x, w) = \begin{cases} (\text{present}, \text{absent}), & x = \text{present}, \\ (\text{present}, \text{present}), & x = \text{absent}. \end{cases}$$

This function admits a unique fixed point of (present, absent) [2 marks]. In s2, the firing function is given by

$$a_{\text{s2}}(x, w) = \begin{cases} (\text{absent}, \text{present}), & w = \text{present}, \\ (\text{present}, \text{present}), & w = \text{absent}. \end{cases}$$

This function admits a unique fixed point of (absent, present) [2 marks]. Therefore, this feedback connection is well-formed [2 marks]. The synchronous model is also constructive (following the algorithm in the textbook gives the same fixed points) [2 marks].

b)



Rubric: correct states [1 mark], correct transitions [1 mark], correct guard [1 mark], correct output [1 mark], correct initialization [1 mark]

Question 5 [20 marks]

a) The relationship is true [2 marks]. Consider any trace $q = q_1 q_2 q_3 \dots$. The proof [8 marks] follows from

$$
\begin{array}{ll}
 & q \models \psi \vee (\varphi \wedge \mathbf{X}(\varphi \mathbf{U} \psi)) \\
\text{if and only if} & (q \models \psi) \text{ or } (q \models \varphi \wedge \mathbf{X}(\varphi \mathbf{U} \psi)) \\
\text{if and only if} & (\psi \text{ holds for } q_1) \text{ or } ((q \models \varphi) \text{ and } (q \models \mathbf{X}(\varphi \mathbf{U} \psi)) \\
\text{if and only if} & (\psi \text{ holds for } q_1) \text{ or } ((\varphi \text{ holds for } q_1) \text{ and} \\
 & \qquad (\exists i \geq 2 \text{ such that } \psi \text{ holds for } q_i \text{ and } \varphi \text{ holds for } q_j, \forall j \in \{2, \dots, i\}) \\
\text{if and only if} & (\psi \text{ holds for } q_1) \text{ or } (\exists i \geq 2 \text{ such that } \psi \text{ holds for } q_i \text{ and } \varphi \text{ holds for } q_j, \forall j \in \{1, \dots, i\}) \\
\text{if and only if} & \exists i \geq 1 \text{ such that } \psi \text{ holds for } q_i \text{ and } \varphi \text{ holds for } q_j, \forall j \in \{1, \dots, i\} \\
\text{if and only if} & q \models \varphi \mathbf{U} \psi
\end{array}
$$

Rubric: In the above proof, each line is worth 1 or 2 marks. There are many ways to write down the proof. If the proof differs, each mistake should result in -1 or -2 marks depending on the severity.

b.1) $\mathbf{F}(\varphi_1 \vee \varphi_2 \vee \varphi_3 \vee \varphi_4 \vee \varphi_5 \vee \varphi_6)$ [2 marks]

b.2) $(\mathbf{F}\varphi_1) \wedge (\mathbf{F}\varphi_2) \wedge (\mathbf{F}\varphi_3) \wedge (\mathbf{F}\varphi_4) \wedge (\mathbf{F}\varphi_5) \wedge (\mathbf{F}\varphi_6)$ [2 marks]

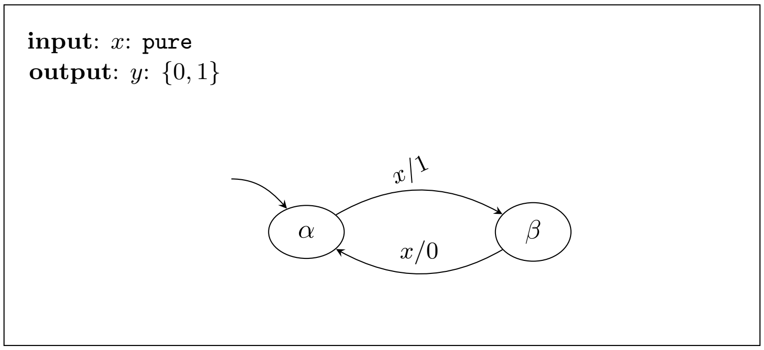
b.3) $\mathbf{F}(\varphi_1 \wedge \mathbf{F}(\varphi_3 \wedge \mathbf{F}(\varphi_5 \wedge \mathbf{F}(\varphi_2 \wedge \mathbf{F}(\varphi_4 \wedge \mathbf{F}\varphi_6)))))$ [2 marks]

b.4) $\mathbf{XXXX}\varphi_5$ or $\mathbf{X}^4\varphi_5$ [2 marks]

b.5) $\mathbf{G}(\varphi_3 \implies ((\mathbf{X}\varphi_5) \vee (\mathbf{G}\neg\varphi_5)))$ [2 marks]

Question 6 [7 marks]

A two-state bisimilar machine is shown below:



Rubric: correct states [1 mark], correct transitions [1 mark], correct guard [1 mark], correct output [1 mark], correct initialization [1 mark].
The bisimulation relation [2 marks] is

$$\{(\mathrm{A}, \alpha), (\mathrm{B}, \beta), (\mathrm{C}, \alpha), (\mathrm{D}, \beta), (\mathrm{E}, \alpha), (\mathrm{F}, \beta)\},$$

or equivalently

$$\{(\alpha, \mathrm{A}), (\beta, \mathrm{B}), (\alpha, \mathrm{C}), (\beta, \mathrm{D}), (\alpha, \mathrm{E}), (\beta, \mathrm{F})\}.$$

Question 7 [8 marks]

a) Let the affine model be $f(x) = ax + b$, where $f(x)$ is the output voltage of the sensor and x is the CO2 concentration. We have $f(1100) = 1.68$ and $f(1600) = 1.61$. We get $a = -1.4 \times 10^{-4}$ V/ppm [2 marks] and $b = 1.83$ V [2 marks].

b) Precision is given by $p = (1.7 - 1.5)/2^4 = 1.25 \times 10^{-2}$ [1 mark]. Dynamic range is given by $D_{dB} = 20 \log_{10}((1.7 - 1.5)/p) = 20 \log_{10}(2^4) = 24$ decibels [1 mark].

c) SNR$= 20 \log_{10}(1.65/0.1) = 24$ decibels [2 marks].