ELEN90055 Control Systems
Workshop 4 - *No hardware*
Balancing an actuated inverted pendulum *

Reports due 11.59pm AEDT, Sun 23 Oct

The aim of Workshop 4 is to design and implement a feedback controller to balance an actuated inverted pendulum. As with Workshop 3, this workshop is designed to use the LEGO MINDSTORMS EV3 robot. In this workshop, the aim is to balance the pendulum mounted on the front; see Figure 1. Specifically, the control system for the pendulum motor is required to maintain the pendulum in the upright position. To this end, a gyro measurement of the angular rate of the pendulum is available. The upright position is to be maintained in the presence of disturbances that act on the pendulum assembly. In particular, an impulsive disturbance applied manually to the top of the pendulum, and a disturbance that acts at the base of the pendulum due to the motion of the robot corresponding to reorientation of the heading (i.e. steering). In this 'No hardware' version, the hardware is modeled with a high fidelity Simulink block that captures most of the dynamics of the actual hardware, and also provides options to simulate the impulsive disturbances.

For your model of the system dynamics, the designed controller should achieve the following specifications:

1. internal stability with a phase margin of at least $\frac{40\pi}{180}$ radians;
2. the aforementioned regulation and disturbance rejection objectives; and
3. a complementary sensitivity function bandwidth of no more than 50 rad/s.

You can use the provided Simulink model '`~~LegoPendulumModel_NH~~.slx`' which requires an associated '`*.slxp`' file to simulate the pendulum; see Figure 2. Table 1 provides an overview of the functions of the four switches. The attached appendix may be used to as needed to assist in modelling and parameter identification.

To help facilitate modelling the simulink model has a *modeling mode*. In modeling mode the pendulum is simulated upside down with a pendulum angle of zero (0) corresponding to the pendulum facing directly down (as shown in Figure 3). In this mode, the pendulum starts in an horizontal position i.e., 90

*this is a modified version of workshop "Balancing an inverted pendulum on the EV3 robot" prepared by Michael Cantoni. This *No hardware* version prepared by Adair Lang, Amir Saberi and Girish Nair is purely simulation-based. Code modified by Armaghan Zafar.

Switch	Switch=`up`	Switch=`down`
`SW1`	Uses designed Feedback controller block to generate input	No input (free pendulum)
`SW2`	Pendulum model in *modeling mode*	Pendulum model in *normal mode*
`SW3`	No disturbances applied to bottom of pendulum	Two impulsive disturbances applied to bottom of pendulum
`SW4`	No disturbances applied to top of pendulum	Two impulsive disturbances applied to top of pendulum

Table 1: Description of Simulink model switch configurations.

degrees from 0. This mode may be useful to facilitate determining of model parameters. In the *normal* mode the pendulum starts in the upright position which is considered to be 0 angle.

Workshop 4 is to be completed over a 3-week period. It constitutes 10%of your total assessment. In the third week, a demonstrator will assess you individually. The structure of the group report is the same as that for Workshop 3 (except for consideration of open-loop control, which is not relevant in Workshop 4). Again the report page limit is 6 A4 pages, 11pt font, 1.5 spacing

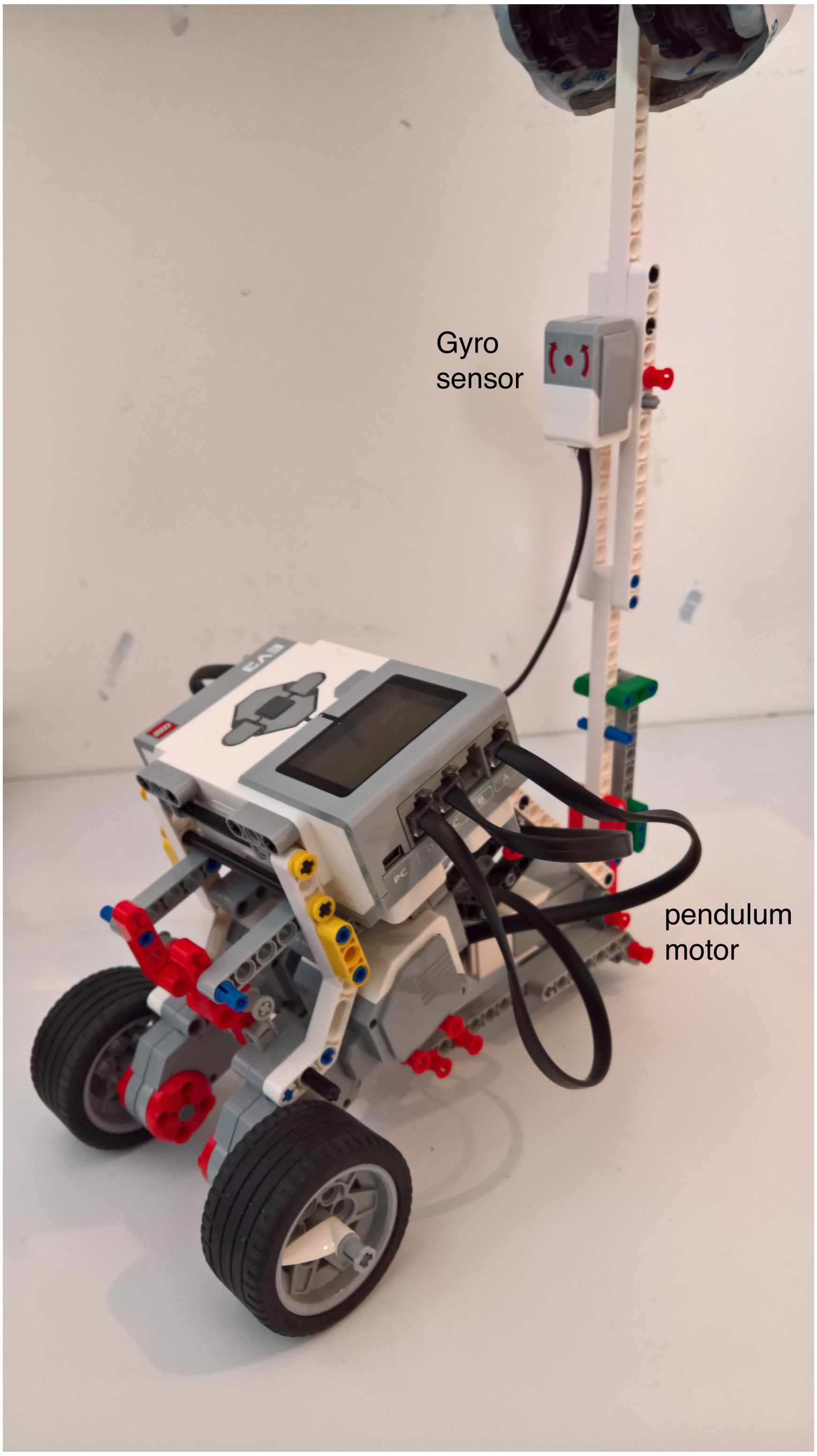

Figure 1: LEGO robot with inverted pendulum.

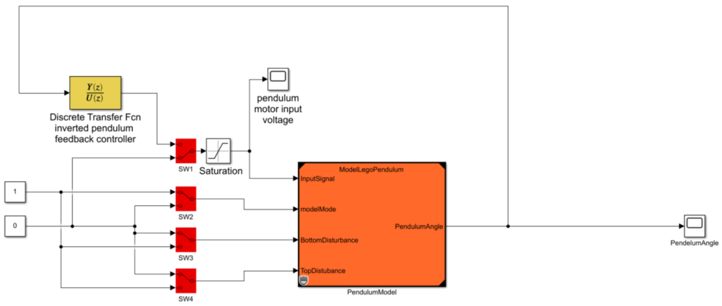

Figure 2: Simulink Model for workshop.

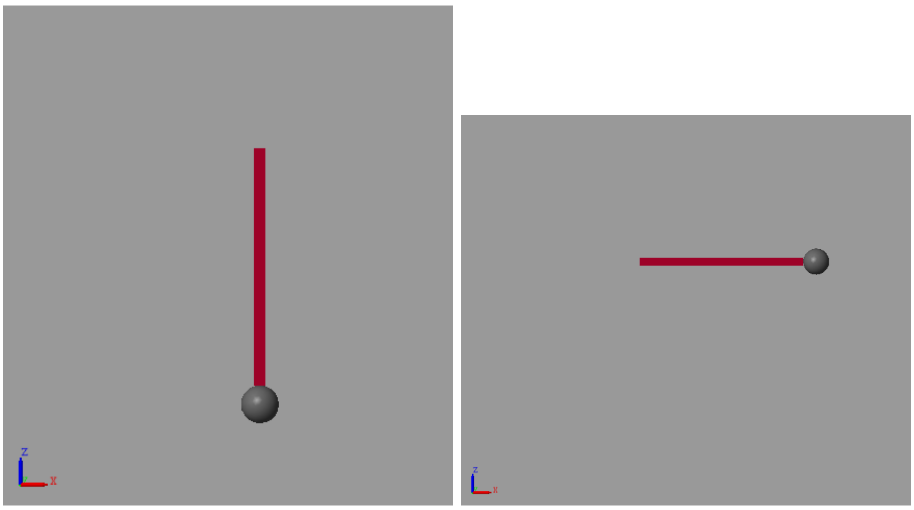

Figure 3: Left: Pendulum in modeling mode zero position, Right: Pendulum in modeling mode start position

Appendix: Pendulum actuator data and modelling

Motor data source: `http://www.philohome.com/motors/motorcomp.htm`

Stalled characteristics:
torque - $\tau = 15 \times 10^{-2}$ N·m;
current - $i = 0.78$ A;
voltage - $v = 9$ V.
No load characteristics:
speed - $\omega = 260 \times 2\pi/60 = 27.2$ rad/sec;
current - $i = 0.08$ A;
voltage - $v = 9$ V.

Using stalled characteristics, analysis of the electrical side (note that the back emf is zero when stalled) yields the following motor parameters:
$K_t = \tau/i = 15 \times 10^{-2}/0.78 = 0.192$ N·m/A;
$R = v/i = 9/0.78 = 11.5$ ohms;
$K_t/R = 1.66 \times 10^{-2}$ N·m/V.

Using the no load characteristics, analysis of the electrical side yields the following back emf constant:
$K_b = \frac{1}{\omega} \times (v - R \times i) = \frac{1}{27.2}(9 - 11.5 \times 0.08) = 0.297$ V·sec/rad.

The small signal dynamics about the pendulum in the upright position are given by

$$m\ell^2\ddot{\theta} = \tau - b\dot{\theta} + mg\ell\theta,$$

where m is the equivalent pendulum bob mass in kg, ℓ is the length in m, b is the coefficient of viscous damping in N·m·sec/rad (which can be identified from experimental data, obtained for example with the pendulum swinging in the down position and the motor not driven), τ is the torque of the motor in N·m, g is gravitational acceleration in m/sec^2, and θ is the angle in radians. With $\tau = K_t i = \frac{K_t}{R}(v - K_b\dot{\theta})$, taking Laplace transforms yields

$$\frac{\Theta(s)}{V(s)} = \frac{K_t/R}{(s^2 \times m\ell^2 + s \times (b + \frac{K_t K_b}{R}) - mg\ell)} \qquad \text{(rad/V)}.$$

Accounting for the "percentage full power" units of the motor command and the gyro sensor and motor encoder angle measurement units, the transfer function from the motor command to the pendulum angle in degrees is given by

$$\frac{\Theta_{\text{deg}}(s)}{V_{\%}(s)} = \frac{9}{100} \cdot \frac{180}{\pi} \cdot \frac{K_t/R}{(s^2 \times m\ell^2 + s \times (b + \frac{K_t K_b}{R}) - mg\ell)} \qquad \text{(deg/p.c.f.p.)}.$$

Note: to estimate l and b, you may use the small-signal differential equation above but with a negative sign in front of mgl\theta on the right-hand side, and with \tau = 0. In this case \theta = 0 corresponds to the downward position.