Control Systems WS4

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I Introduction

This workshop investigates the control system design process for an inverted pendulum mounted on a LEGO MINDSTORM EV3 robot. The goal is to have robustness to disturbances to the top and bottom of the pendulum, maintaining tracking of the reference angle which is let to be a constant 0 deg. - corresponding to the upright position.
This process was done through testing of a MATLAB protected system which could be tested on. A nominal plant had to be modelled to inform controller design choices.

The control system had to meet design specifications:

1. Internal stability with a phase margin of at least $\frac{40\pi}{180}$ radians (40 deg.);
2. Regulation and disturbance rejection;
3. Complementary sensitivity function bandwidth less than or equal to 50 rad/s

II System Modelling

It is given that the small signal dynamics of the pendulum (in the downright position) is given by the equation:

$$ml^2\ddot{\theta} = \tau - b\dot{\theta} - mgl\theta \tag{1}$$

where τ is given by,

$$\tau = \frac{K_t}{R}(v - K_b\dot{\theta})$$

and with stalled characteristics,

$$\frac{K_t}{R} = 0.192 \text{ and } K_b = 0.297$$

In order to model the system we create a system diagram as described in (1), setting $\tau = 0$ will simulate a pendulum free-fall (we also set the initial conditions: $\theta(0) = \frac{\pi}{2}, \dot{\theta}(0) = 0$ and $\ddot{\theta}(0)$). This nominal plant model diagram is shown below:

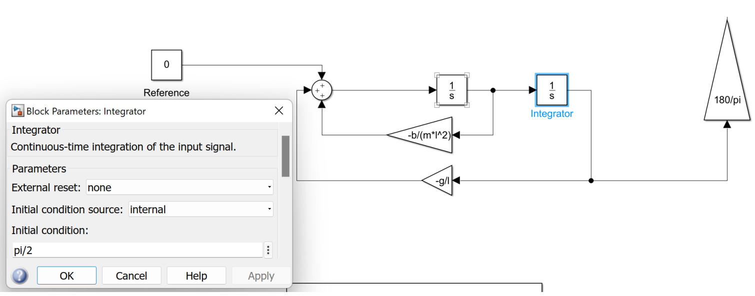

Figure 1: Plant model constructed from equation (1), $\tau = 0$

The variables b and l in (1) remain unknown and at our discretion. Making use of 'modelling mode' of the Simulink protected model, we freely vary these parameters until the gyroscope measurement scopes (angle vs time) show a satisfactory match in their output characteristics.
There is difficulty is this modelling process seen as the oscillation frequency of the actual model increases over time as seen in the scope measurements below:

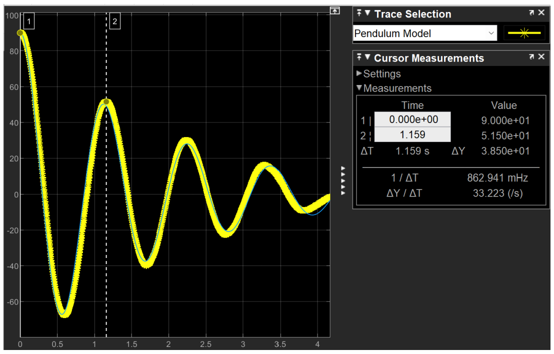

Figure 2: First time interval between peaks: $\Delta T = 1.159s$

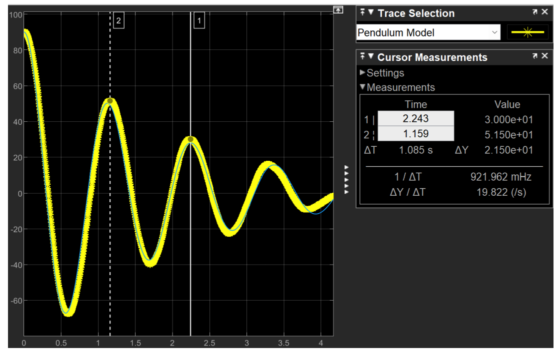

Figure 3: Second time interval between peaks: $\Delta T = 1.085s$

This change in oscillation frequency against time isn't captured in the small signal approximation. Through trial and error, we nominate the value pair: $(b, l) = (0.03, 0.31)$.
The two superimposed scopes show how the nominal plant (blue) matches to the real plant (yellow):

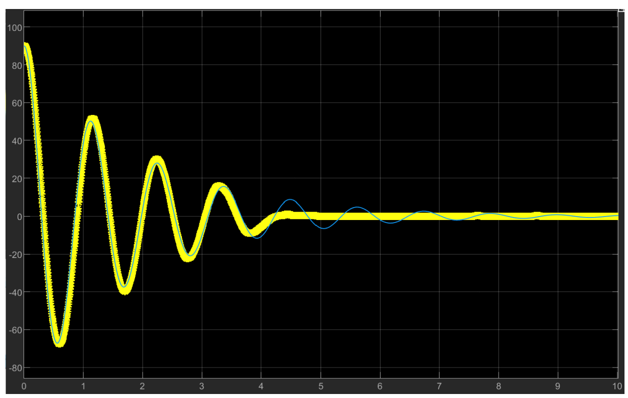

Figure 4: Nominal vs Real plant free-response behaviour

III Control Design

Satisfied with the nominated plant model we consider now the design of the controller.
The transfer function of our nominal plant (with m = 0.3) is given by:

$$G_0(s) = \frac{\Theta_{deg}(s)}{V_{\%}(s)} = \frac{0.0856}{0.02883s^2 + 0.0.03493 - 0.9114}$$

The poles of our plant are at $s = -6.2609$ and $s = 5.0493$. Immediately we notice that the latter pole is unstable as it lies in the RHP. In terms of Nyquist stability criteria, P = 1.

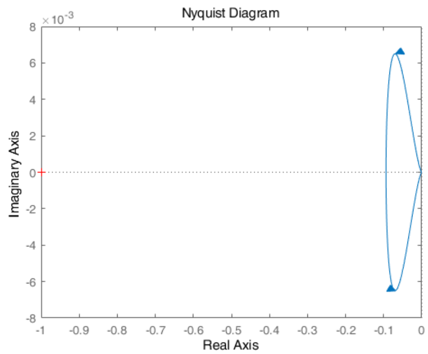

Figure 5: Nyquist plot of G_0

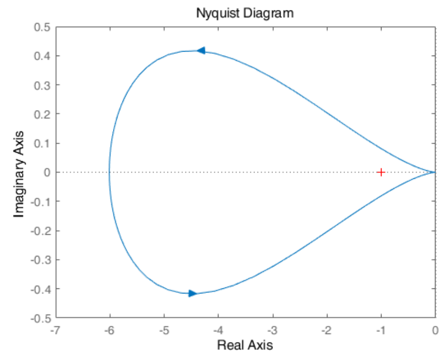

Figure 6: Zoom in of Nyquist plot of $\Lambda_0 = K_c G_0$ with $K_c = 64$, note it's an anticlockwise closed contour

The Nyquist plot of G_0 in Figure 5. shows an anticlockwise closed contour far from the critical point (N=0). Based on this figure, the high-gain controller should be greater than 10 in order to get one anticlockwise closed contour to stabilize the system. Figure 6. shows the Nyquist plot of the open-loop transfer function where a high-gain controller is implemented, moving the critical point to within the closed contour (N=1). Therefore the complementary sensitivity function, T_0 is stable for $K_c = 64$, $Z = N + P = 1$. Next, consider also the Magnitude and Phase plot of Λ_0:

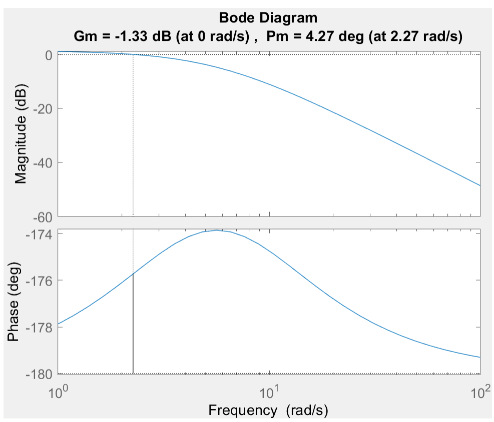

Figure 7: Magnitude and Phase plot of $\Lambda_0 = K_c G_0(s)$, $K_c = 64$

From the above we see we have very unhealthy phase margin, far from specifications and with a bandwidth of effectively 0 (to be expected as we have applied no gain to it).
We decide to introduce a lead controller to supply additional phase to meet our specified Phase Margin (40 deg.) and shift our gain crossover frequency higher, thereby increasing the system bandwidth:

$$C(s) = K\frac{\tau_z s + 1}{\tau_p s + 1}$$

Let's parameterise our time constants as such:

$$\tau_z = T, \tau_p = \alpha T$$

To achieve our desired Phase Margin we do the following steps:

$$\Phi_{max} = 40 + 10 - 4.27 \approx 45$$

$$\alpha = \frac{1 - sin(\Phi_{max})}{1 + sin(\Phi_{max})} \approx 0.08055$$

$$\alpha$$

After trial and error procedure to choose a appropriate maximum phase position, ω_{max} is determined to be 15 rad/s:

$$T = \frac{1}{\omega_{max}\sqrt{\alpha}} = 0.1609$$

$$\implies \tau_z = 0.1609 \text{ and } \tau_p = 0.0276$$

Let's look at,

$$C(s) = 64\frac{0.1609 + 1}{0.0276s + 1}$$

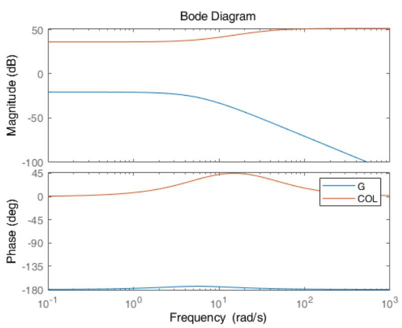

Figure 8: Nyquist plot of $G_0(s)$ (blue) and $C(s)$ (orange)

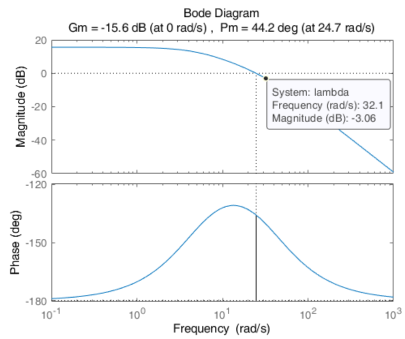

Figure 9: Nyquist plot of Λ_0

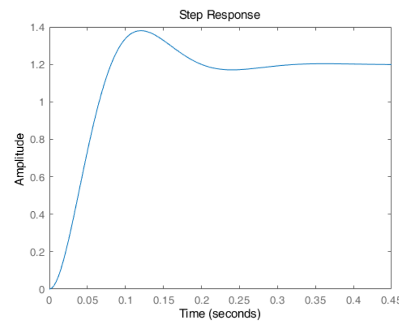

Figure 10: Step response of Λ_0

With this controller we see that our Phase Margin specification has been met and that we have a bandwidth less than 50 rad/s. Therefore with the introduction of a lead compensation control design, an adequate controller has been designed.

IV Implementation and Testing

Implementing the designed lead-compensation controller with the protected real plant model we get the angle vs time scopes:

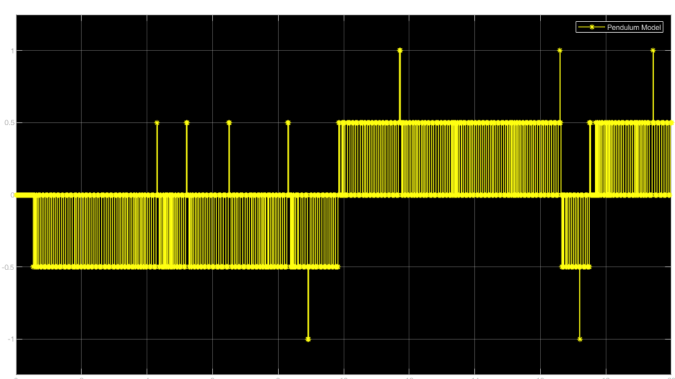

Figure 11: Scope of pendulum angle against time (no disturbances

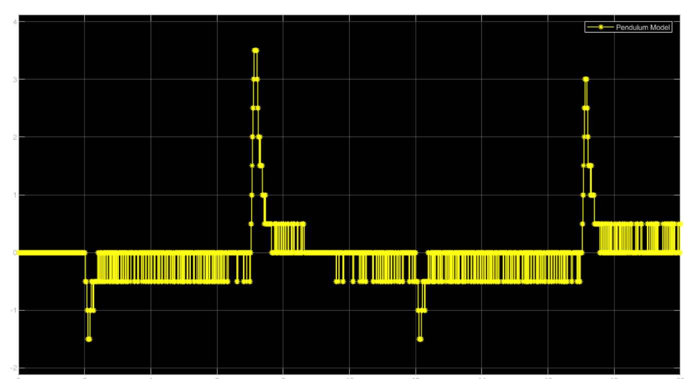

Figure 12: Scope of pendulum angle against time (with top and bottom disturbances

The maximum deviation from 0 where no disturbances are present is within ± 1 degree. Where both top and bottom disturbances are present there is maximum deviation of ± 3.5 degrees. Overall, we can be confident in the robustness of our system in the face of disturbances.

V Discussion

System Modelling

From figure 4, even though the nominal model perfectly overlap the real model at the beginning, it still has some oscillation at the end which can not be eliminated. There could be a few potential reasons. Firstly, in our nominal model, we approximate the $sin(\theta) = l$ in the small angle. This may be inappropriate when the angle varies between 0 to 90 degrees. Secondly, the effect of friction is ignored as well. Even though the coefficient of viscous damping is introduced, it may not sufficient to reflect the effect of resistance.

Control design

From figure 11 and 12, it shows that the system generally good since the maximum deviation is relatively small. But it also shows that the system has a problem of oscillation between the reference and a certain angle. This may because of the high constant gain, which reflects the error too much. However, due to the limitation of lead compensation and our plant, a high-gain controller is also required to stabilize the system and to reject the input disturbance. On the one hand, we should keep constant gain big to achieve low frequency disturbance rejection objective. On the other hand, we don't want the system to over-react to the error between the reference.

To improve the behaviour of the system, a lag compensation may be introduced to form a lead-lag compensation in order to achieve a small steady-state error. But it may be difficult to determine parameter values.

VI Conclusion

This report has detailed the considerations and design process in stabilising an inverted pendulum system. By first nominating a relatively accurate plant model, we were able to apply complex analytical methods to control and mitigate unideal behaviour. The plant model had incredibly poor phase margin so some type of lead compensation was necessary. Tuning this correctly resulted in a system that met phase margin and bandwidth specifications and is internally stable.