5 Composition of State Machines

State machines provide a convenient way to model behaviors of systems. One disadvantage that they have is that for most interesting systems, the number of states is very large, often even infinite. Automated tools can handle large state spaces, but humans have more difficulty with any direct representation of a large state space.

A time-honored principle in engineering is that complicated systems should be described as compositions of simpler systems. This chapter gives a number of ways to do this with state machines. The reader should be aware, however, that there are many subtly different ways to compose state machines. Compositions that look similar on the surface may mean different things to different people. The rules of notation of a model are called its **syntax**, and the meaning of the notation is called its **semantics**.

Example 5.1: In the standard syntax of arithmetic, a plus sign $+$ has a number or expression before it, and a number or expression after it. Hence, $1+2$, a sequence of three symbols, is a valid arithmetic expression, but $1+$ is not. The semantics of the expression $1+2$ is the addition of two numbers. This expression means "the number three, obtained by adding 1 and 2." The expression $2+1$ is syntactically different, but semantically identical (because addition is commutative).

The models in this book predominantly use a visual syntax, where the elements are boxes, circles, arrows, etc., rather than characters in a character set, and where the positioning of the elements is not constrained to be a sequence. Such syntaxes are less standardized than, for example, the syntax of arithmetic. We will see that the same syntax can have many different semantics, which can cause no end of confusion.

Example 5.2: A now popular notation for concurrent composition of state machines called Statecharts was introduced by Harel (1987). Although they are all based on the same original paper, many variants of Statecharts have evolved (von der Beeck, 1994). These variants often assign different semantics to the same syntax.

In this chapter, we assume an actor model for extended state machines using the syntax summarized in Figure 5.1. The semantics of a single such state machine is described in Chapter 3. This chapter will discuss the semantics that can be assigned to compositions of multiple such machines.

The first composition technique we consider is concurrent composition. Two or more state machines react either simultaneously or independently. If the reactions are simultaneous, we call it **synchronous composition**. If they are independent, then we call it

asynchronous composition. But even within these classes of composition, many subtle variations in the semantics are possible. These variations mostly revolve around whether and how the state machines communicate and share variables.

The second composition technique we will consider is hierarchy. Hierarchical state machines can also enable complicated systems to be described as compositions of simpler systems. Again, we will see that subtle differences in semantics are possible.

5.1 Concurrent Composition

To study concurrent composition of state machines, we will proceed through a sequence of patterns of composition. These patterns can be combined to build arbitrarily complicated systems. We begin with the simplest case, side-by-side composition, where the state machines being composed do not communicate. We then consider allowing communication through shared variables, showing that this creates significant subtleties that can complicate modeling. We then consider communication through ports, first looking at serial composition, then expanding to arbitrary interconnections. We consider both synchronous and asynchronous composition for each type of composition.

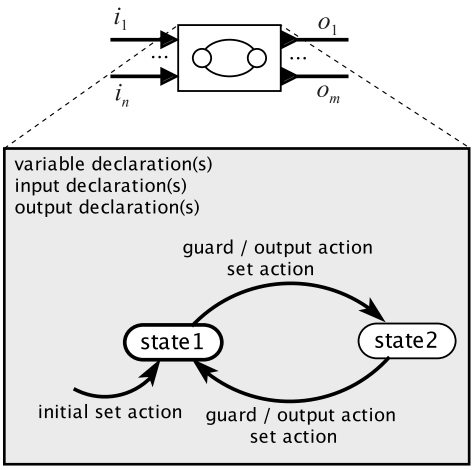


Figure 5.1: Summary of notation for state machines used in this chapter.

About Synchrony

The term **synchronous** means (1) occurring or existing at the same time or (2) moving or operating at the same rate. In engineering and computer science, the term has a number of meanings that are mostly consistent with these definitions, but oddly inconsistent with one another. In referring to concurrent software constructed using threads or processes, synchronous communication refers to a rendezvous style of communication, where the sender of a message must wait for the receiver to be ready to receive, and the receiver must wait for the sender. Conceptually, the two threads see the communication occurring at the same time, consistent with definition (1). In Java, the keyword `synchronized` defines blocks of code that are not permitted to execute simultaneously. Oddly, two code blocks that are synchronized *cannot* "occur" (execute) at the same time, which is inconsistent with both definitions.

In the world of software, there is yet a third meaning of the word synchronous, and it is this third meaning that we use in this chapter. This third meaning underlies the synchronous languages (see box on page 148). Two key ideas govern these languages. First, the outputs of components in a program are (conceptually) simultaneous with their inputs (this is called the **synchrony hypothesis**). Second, components in a program execute (conceptually) simultaneously and instantaneously. Real executions do not literally occur simultaneously nor instantaneously, and outputs are not really simultaneous with the inputs, but a correct execution must behave as if they were. This use of the word synchronous is consistent with *both* definitions above; executions of components occur at the same time and operate at the same rate.

In circuit design, the word synchronous refers to a style of design where a clock that is distributed throughout a circuit drives latches that record their inputs on edges of the clock. The time between clock edges needs to be sufficient for circuits between latches to settle. Conceptually, this model is similar to the model in synchronous languages. Assuming that the circuits between latches have zero delay is equivalent to the synchrony hypothesis, and global clock distribution gives simultaneous and instantaneous execution.

In power systems engineering, synchronous means that electrical waveforms have the same frequency and phase. In signal processing, synchronous means that signals have the same sample rate, or that their sample rates are fixed multiples of one another. The term synchronous dataflow, described in Section 6.3.2, is based on this latter meaning of the word synchronous. This usage is consistent with definition (2).

5.1.1 Side-by-Side Synchronous Composition

The first pattern of composition that we consider is **side-by-side composition**, illustrated for two actors in Figure 5.2. In this pattern, we assume that the inputs and outputs of the two actors are disjoint, i.e., that the state machines do not communicate. In the figure, actor A has input i_1 and output o_1, and actor B has input i_2 and output o_2. The composition of the two actors is itself an actor C with inputs i_1 and i_2 and outputs o_1 and o_2.[1]

In the simplest scenario, if the two actors are extended state machines with variables, then those variables are also disjoint. We will later consider what happens when the two state machines share variables. Under **synchronous composition**, a reaction of C is a simultaneous reaction of A and B.

Example 5.3: Consider FSMs A and B in Figure 5.3. A has a single pure output a, and B has a single pure output b. The side-by-side composition C has two pure outputs, a and b. If the composition is synchronous, then on the first reaction, a will be *absent* and b will be *present*. On the second reaction, it will be the reverse. On subsequent reactions, a and b will continue to alternate being present.

[1]The composition actor C may rename these input and output ports, but here we assume it uses the same names as the component actors.

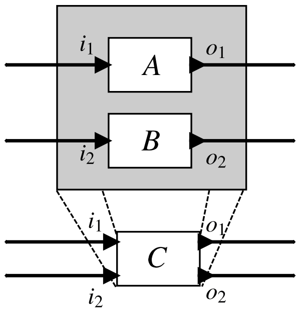


Figure 5.2: Side-by-side composition of two actors.

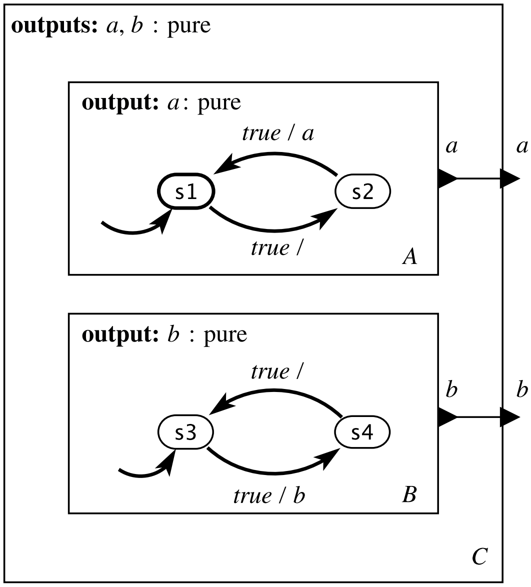


Figure 5.3: Example of side-by-side composition of two actors.

Synchronous side-by-side composition is simple for several reasons. First, recall from Section 3.3.2 that the environment determines when a state machine reacts. In synchronous side-by-side composition, the environment need not be aware that C is a composition of two state machines. Such compositions are **modular** in the sense that the composition itself becomes a component that can be further composed as if it were itself an atomic component.

Moreover, if the two state machines A and B are deterministic, then the synchronous side-by-side composition is also deterministic. We say that a property is **compositional** if a property held by the components is also a property of the composition. For synchronous side-by-side composition, determinism is a compositional property.

In addition, a synchronous side-by-side composition of finite state machines is itself an FSM. A rigorous way to give the semantics of the composition is to define a single state machine for the composition. Suppose that as in Section 3.3.3, state machines A and B

are given by the five tuples,

$$\begin{aligned} A &= (\mathit{States}_A, \mathit{Inputs}_A, \mathit{Outputs}_A, \mathit{update}_A, \mathit{initialState}_A) \\ B &= (\mathit{States}_B, \mathit{Inputs}_B, \mathit{Outputs}_B, \mathit{update}_B, \mathit{initialState}_B) \;. \end{aligned}$$

Then the synchronous side-by-side composition C is given by

$$\mathit{States}_C = \mathit{States}_A \times \mathit{States}_B \tag{5.1}$$

$$\mathit{Inputs}_C = \mathit{Inputs}_A \times \mathit{Inputs}_B \tag{5.2}$$

$$\mathit{Outputs}_C = \mathit{Outputs}_A \times \mathit{Outputs}_B \tag{5.3}$$

$$\mathit{initialState}_C = (\mathit{initialState}_A, \mathit{initialState}_B) \tag{5.4}$$

and the update function is defined by

$$\mathit{update}_C((s_A, s_B), (i_A, i_B)) = ((s'_A, s'_B), (o_A, o_B)),$$

where

$$(s'_A, o_A) = \mathit{update}_A(s_A, i_A),$$

and

$$(s'_B, o_B) = \mathit{update}_B(s_B, i_B),$$

for all $s_A \in \mathit{States}_A$, $s_B \in \mathit{States}_B$, $i_A \in \mathit{Inputs}_A$, and $i_B \in \mathit{Inputs}_B$.

Recall that Inputs_A and Inputs_B are sets of valuations. Each valuation in the set is an assignment of values to ports. What we mean by

$$\mathit{Inputs}_C = \mathit{Inputs}_A \times \mathit{Inputs}_B$$

is that a valuation of the inputs of C must include *both* valuations for the inputs of A and the inputs of B.

As usual, the single FSM C can be given pictorially rather than symbolically, as illustrated in the next example.

Example 5.4: The synchronous side-by-side composition C in Figure 5.3 is given as a single FSM in Figure 5.4. Notice that this machine behaves exactly as described in Example 5.3. The outputs a and b alternate being present. Notice further that $(\mathsf{s1}, \mathsf{s4})$ and $(\mathsf{s2}, \mathsf{s3})$ are not reachable states.

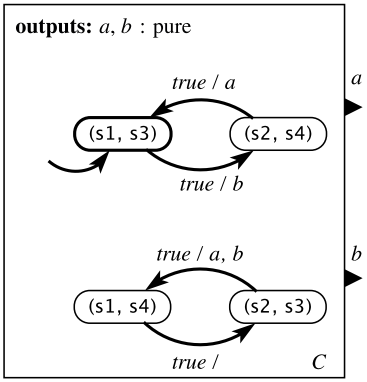


Figure 5.4: Single state machine giving the semantics of synchronous side-by-side composition of the state machines in Figure 5.3.

5.1.2 Side-by-Side Asynchronous Composition

In an **asynchronous composition** of state machines, the component machines react independently. This statement is rather vague, and in fact, it has several different interpretations. Each interpretation gives a semantics to the composition. The key to each semantics is how to define a reaction of the composition C in Figure 5.2. Two possibilities are:

- **Semantics 1.** A reaction of C is a reaction of one of A or B, where the choice is nondeterministic.
- **Semantics 2.** A reaction of C is a reaction of A, B, or both A and B, where the choice is nondeterministic. A variant of this possibility might allow *neither* to react.

Semantics 1 is referred to as an **interleaving semantics**, meaning that A or B never react simultaneously. Their reactions are interleaved in some order.

A significant subtlety is that under these semantics machines A and B may completely miss input events. That is, an input to C destined for machine A may be present in a reaction where the nondeterministic choice results in B reacting rather than A. If this is not desirable, then some control over scheduling (see sidebar on page 118) or synchronous composition becomes a better choice.

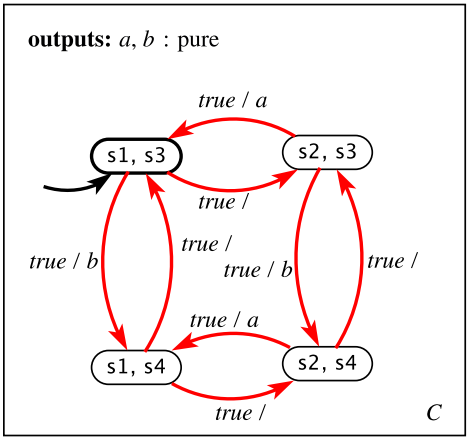


Figure 5.5: State machine giving the semantics of asynchronous side-by-side composition of the state machines in Figure 5.3.

Example 5.5: For the example in Figure 5.3, semantics 1 results in the composition state machine shown in Figure 5.5. This machine is nondeterministic. From state $(\mathsf{s1}, \mathsf{s3})$, when C reacts, it can move to $(\mathsf{s2}, \mathsf{s3})$ and emit no output, or it can move to $(\mathsf{s1}, \mathsf{s4})$ and emit b. Note that if we had chosen semantics 2, then it would also be able to move to $(\mathsf{s2}, \mathsf{s4})$.

For asynchronous composition under semantics 1, the symbolic definition of C has the same definitions of $States_C$, $Inputs_C$, $Outputs_C$, and $initialState_C$ as for synchronous composition, given in (5.1) through (5.4). But the update function differs, becoming

$$update_C((s_A, s_B), (i_A, i_B)) = ((s'_A, s'_B), (o'_A, o'_B)),$$

where either

$$(s'_A, o'_A) = update_A(s_A, i_A) \text{ and } s'_B = s_B \text{ and } o'_B = absent$$

or

$$(s'_B, o'_B) = update_B(s_B, i_B) \text{ and } s'_A = s_A \text{ and } o'_A = absent$$

for all $s_A \in \mathit{States}_A$, $s_B \in \mathit{States}_B$, $i_A \in \mathit{Inputs}_A$, and $i_B \in \mathit{Inputs}_B$. What we mean by $o'_B = \mathit{absent}$ is that all outputs of B are absent. Semantics 2 can be similarly defined (see Exercise 2).

Scheduling Semantics for Asynchronous Composition

In the case of semantics 1 and 2 given in Section 5.1.2, the choice of which component machine reacts is nondeterministic. The model does not express any particular constraints. It is often more useful to introduce some scheduling policies, where the environment is able to influence or control the nondeterministic choice. This leads to two additional possible semantics for asynchronous composition:

- **Semantics 3.** A reaction of C is a reaction of one of A or B, where the environment chooses which of A or B reacts.
- **Semantics 4.** A reaction of C is a reaction of A, B, or both A and B, where the choice is made by the environment.

Like semantics 1, semantics 3 is an interleaving semantics.

In one sense, semantics 1 and 2 are more compositional than semantics 3 and 4. To implement semantics 3 and 4, a composition has to provide some mechanism for the environment to choose which component machine should react (for scheduling the component machines). This means that the hierarchy suggested in Figure 5.2 does not quite work. Actor C has to expose more of its internal structure than just the ports and the ability to react.

In another sense, semantics 1 and 2 are less compositional than semantics 3 and 4 because determinism is not preserved by composition. A composition of deterministic state machines is not a deterministic state machine.

Notice further that semantics 1 is an abstraction of semantics 3 in the sense that every behavior under semantics 3 is also a behavior under semantics 1. This notion of abstraction is studied in detail in Chapter 14.

The subtle differences between these choices make asynchronous composition rather treacherous. Considerable care is required to ensure that it is clear which semantics is used.

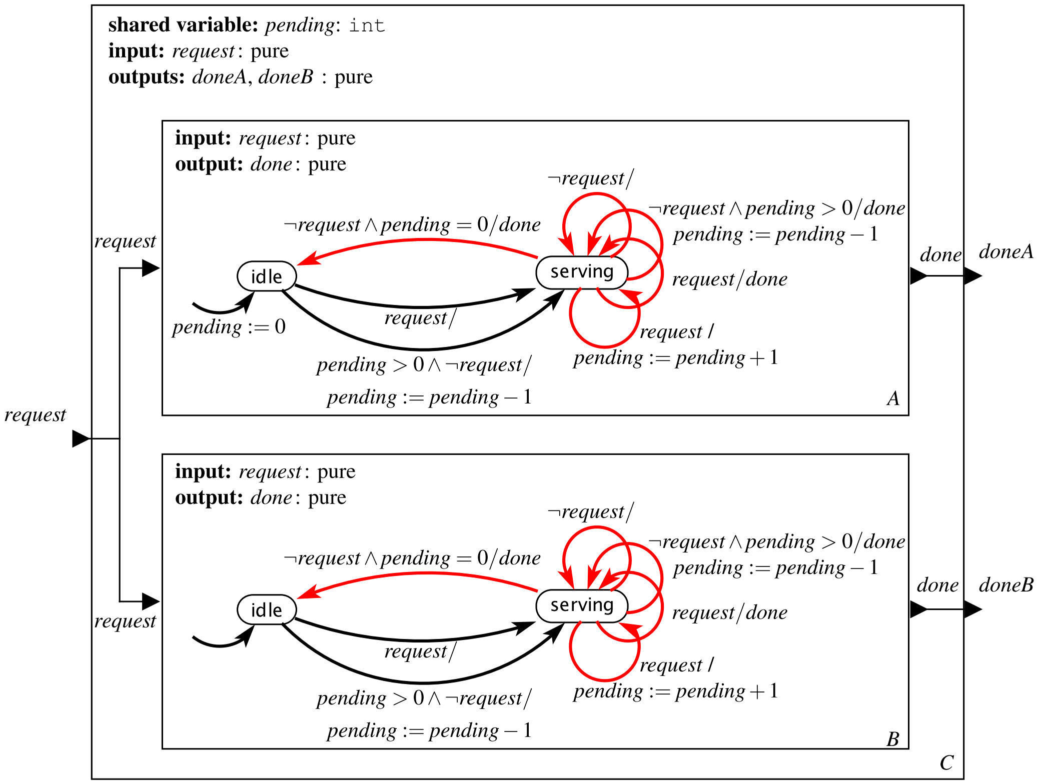


Figure 5.6: Model of two servers with a shared task queue, assuming asynchronous composition under semantics 1.

5.1.3 Shared Variables

An extended state machine has local variables that can be read and written as part of taking a transition. Sometimes it is useful when composing state machines to allow these variables to be shared among a group of machines. In particular, such shared variables can be useful for modeling interrupts, studied in Chapter 10, and threads, studied in Chapter 11. However, considerable care is required to ensure that the semantics of the model conforms with that of the program containing interrupts or threads. Many complications arise, including the memory consistency model and the notion of atomic operations.

Example 5.6: Consider two servers that can receive requests from a network. Each request requires an unknown amount of time to service, so the servers share a queue of requests. If one server is busy, the other server can respond to a request, even if the request arrives at the network interface of the first server.

This scenario fits a pattern similar to that in Figure 5.2, where A and B are the servers. We can model the servers as state machines as shown in Figure 5.6. In this model, a shared variable *pending* counts the number of pending job requests. When a request arrives at the composite machine C, one of the two servers is nondeterministically chosen to react, assuming asynchronous composition under semantics 1. If that server is idle, then it proceeds to serve the request. If the server is serving another request, then one of two things can happen: it can coincidentally finish serving the request it is currently serving, issuing the output *done*, and proceed to serve the new one, or it can increment the count of pending requests and continue to serve the current request. The choice between these is nondeterministic, to model the fact that the time it takes to service a request is unknown.

If C reacts when there is no request, then again either server A or B will be selected nondeterministically to react. If the server that reacts is idle and there are one or more pending requests, then the server transitions to serving and decrements the variable *pending*. If the server that reacts is not idle, then one of three things can happen. It may continue serving the current request, in which case it simply transitions on the self transition back to serving. Or it may finish serving the request, in which case it will transition to idle if there are no pending requests, or transition back to serving and decrement *pending* if there are pending requests.

The model in the previous example exhibits many subtleties of concurrent systems. First, because of the interleaving semantics, accesses to the shared variable are atomic operations, something that is quite challenging to guarantee in practice, as discussed in Chapters 10 and 11. Second, the choice of semantics 1 is reasonable in this case because the input goes to both of the component machines, so regardless of which component machine reacts, no input event will be missed. However, this semantics would not work if the two machines had independent inputs, because then requests could be missed. Semantics 2 can help prevent that, but what strategy should be used by the environment to determine which machine reacts? What if the two independent inputs both have requests present at the same reaction of C? If we choose semantics 4 in the sidebar on page 118 to allow both machines to react simultaneously, then what is the meaning when both machines update the shared variable? The updates are no longer atomic, as they are with an interleaving semantics.

Note further that choosing asynchronous composition under semantics 1 allows behaviors that do not make good use of idle machines. In particular, suppose that machine A is serving, machine B is idle, and a *request* arrives. If the nondeterministic choice results in machine A reacting, then it will simply increment *pending*. Not until the nondeterministic choice results in B reacting will the idle machine be put to use. In fact, semantics 1 allows behaviors that never use one of the machines.

Shared variables may be used in synchronous compositions as well, but sophisticated subtleties again emerge. In particular, what should happen if in the same reaction one machine reads a shared variable to evaluate a guard and another machine writes to the shared variable? Do we require the write before the read? What if the transition doing the write to the shared variable also reads the same variable in its guard expression? One possibility is to choose a **synchronous interleaving semantics**, where the component machines react in arbitrary order, chosen nondeterministically. This strategy has the disadvantage that a composition of two deterministic machines may be nondeterministic. An alternative version of the synchronous interleaving semantics has the component machines react in a fixed order determined by the environment or by some additional mechanism such as priority.

The difficulties of shared variables, particularly with asynchronous composition, reflect the inherent complexity of concurrency models with shared variables. Clean solutions require a more sophisticated semantics, to be discussed in Chapter 6. In that chapter, we will explain the synchronous-reactive model of computation, which gives a synchronous composition semantics that is reasonably compositional.

So far, we have considered composition of machines that do not directly communicate. We next consider what happens when the outputs of one machine are the inputs of another.

5.1.4 Cascade Composition

Consider two state machines A and B that are composed as shown in Figure 5.7. The output of machine A feeds the input of B. This style of composition is called cascade composition or **serial composition**.

In the figure, output port o_1 from A feeds events to input port i_2 of B. Assume the data type of o_1 is V_1 (meaning that o_1 can take values from V_1 or be *absent*), and the data type of i_2 is V_2. Then a requirement for this composition to be valid is that

$$V_1 \subseteq V_2 \; .$$

This asserts that any output produced by A on port o_1 is an acceptable input to B on port i_2. The composition **type check**s.

For cascade composition, if we wish the composition to be asynchronous, then we need to introduce some machinery for buffering the data that is sent from A to B. We defer discussion of such asynchronous composition to Chapter 6, where dataflow and process network models of computation will provide such asynchronous composition. In this chapter, we will only consider synchronous composition for cascade systems.

In synchronous composition of the cascade structure of Figure 5.7, a reaction of C consists of a reaction of both A and B, where A reacts first, produces its output (if any), and then B reacts. Logically, we view this as occurring in zero time, so the two reactions are in a sense **simultaneous and instantaneous**. But they are causally related in that the outputs of A can affect the behavior of B.

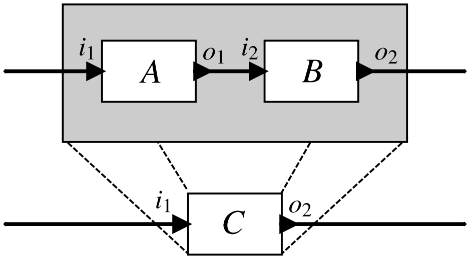


Figure 5.7: Cascade composition of two actors.

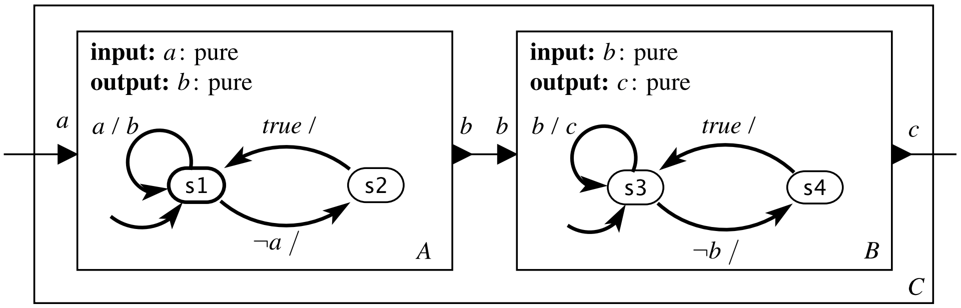


Figure 5.8: Example of a cascade composition of two FSMs.

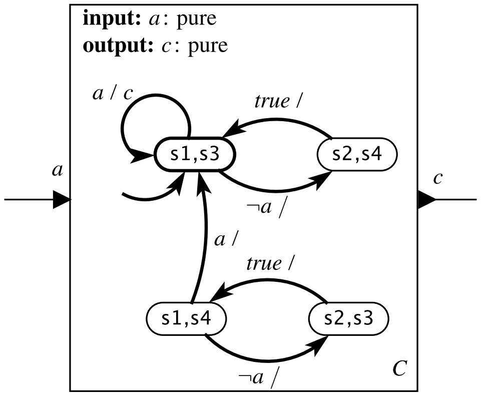


Figure 5.9: Semantics of the cascade composition of Figure 5.8, assuming synchronous composition.

Example 5.7: Consider the cascade composition of the two FSMs in Figure 5.8. Assuming synchronous semantics, the meaning of a reaction of C is given in Figure 5.9. That figure makes it clear that the reactions of the two machines are simultaneous and instantaneous. When moving from the initial state (s1, s3) to (s2, s4) (which occurs when the input a is absent), the composition machine C

does not pass through (s2, s3)! In fact, (s2, s3) is not a reachable state! In this way, a *single* reaction of C encompasses a reaction of both A *and* B.

To construct the composition machine as in Figure 5.9, first form the state space as the cross product of the state spaces of the component machines, and then determine which transitions are taken under what conditions. It is important to remember that the transitions are simultaneous, even when one logically causes the other.

Example 5.8: Recall the traffic light model of Figure 3.10. Suppose that we wish to compose this with a model of a pedestrian crossing light, like that shown in Figure 5.10. The output *sigR* of the traffic light can provide the input *sigR* of the pedestrian light. Under synchronous cascade composition, the meaning of the composite is given in Figure 5.11. Note that unsafe states, such as (green, green), which is the state when both cars and pedestrians have a green light, are not reachable states, and hence are not shown.

In its simplest form, cascade composition implies an ordering of the reactions of the

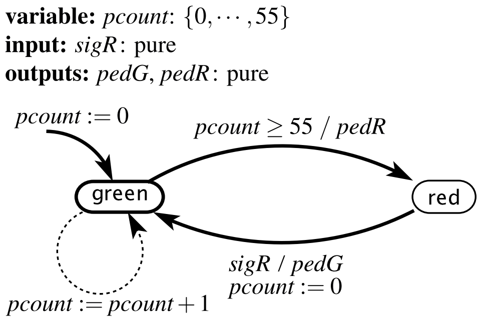

variable: $pcount$: $\{0, \cdots, 55\}$
input: $sigR$: pure
outputs: $pedG, pedR$: pure
$pcount := 0$
$pcount \geq 55$ / $pedR$
green
red
$sigR$ / $pedG$
$pcount := 0$
$pcount := pcount + 1$

Figure 5.10: A model of a pedestrian crossing light, to be composed in a synchronous cascade composition with the traffic light model of Figure 3.10.

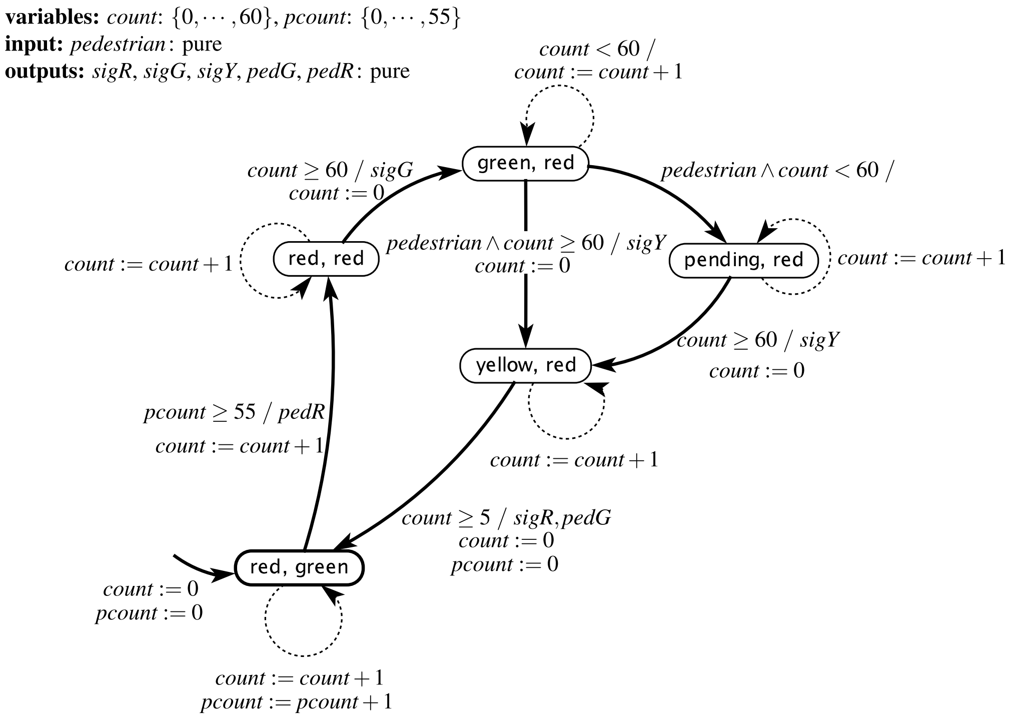

variables: $count$: $\{0, \cdots, 60\}$, $pcount$: $\{0, \cdots, 55\}$
input: $pedestrian$: pure
outputs: $sigR$, $sigG$, $sigY$, $pedG$, $pedR$: pure
green, red
red, red
pending, red
yellow, red
red, green
$count < 60$ / $count := count + 1$
$count \geq 60$ / $sigG$ $count := 0$
$pedestrian \wedge count < 60$ /
$pedestrian \wedge count \geq 60$ / $sigY$ $count := 0$
$count := count + 1$
$count := count + 1$
$count \geq 60$ / $sigY$ $count := 0$
$pcount \geq 55$ / $pedR$ $count := count + 1$
$count := count + 1$
$count \geq 5$ / $sigR, pedG$ $count := 0$ $pcount := 0$
$count := 0$ $pcount := 0$
$count := count + 1$ $pcount := pcount + 1$

Figure 5.11: Semantics of a synchronous cascade composition of the traffic light model of Figure 3.10 with the pedestrian light model of Figure 5.10.

components. Since this ordering is well defined, we do not have as much difficulty with shared variables as we did with side-by-side composition. However, we will see that in more general compositions, the ordering is not so simple.

5.1.5 General Composition

Side-by-side and cascade composition provide the basic building blocks for building more complex compositions of machines. Consider for example the composition in Figure 5.12. A_1 and A_3 are a side-by-side composition that together define a machine B. B and A_2 are a cascade composition, with B feeding events to A_2. However, B and A_2 are also a cascade composition in the opposite order, with A_2 feeding events to B. Cycles like

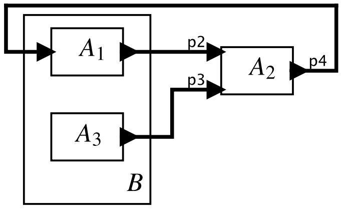


Figure 5.12: Arbitrary interconnections of state machines are combinations of side-by-side and cascade compositions, possibly creating cycles, as in this example.

this are called feedback, and they introduce a conundrum; which machine should react first, B or A_2? This conundrum will be resolved in the next chapter when we explain the synchronous-reactive model of computation.

5.2 Hierarchical State Machines

In this section, we consider **hierarchical FSMs**, which date back to Statecharts (Harel, 1987). There are many variants of Statecharts, often with subtle semantic differences between them (von der Beeck, 1994). Here, we will focus on some of the simpler aspects only, and we will pick a particular semantic variant.

The key idea in hierarchical state machines is state refinement. In Figure 5.13, state B has a refinement that is another FSM with two states, C and D. What it means for the machine to be in state B is that it is in one of states C or D.

The meaning of the hierarchy in Figure 5.13 can be understood by comparing it to the equivalent flattened FSM in Figure 5.14. The machine starts in state A. When guard g_2 evaluates to true, the machine transitions to state B, which means a transition to state C, the initial state of the refinement. Upon taking this transition to C, the machine performs action a_2, which may produce an output event or set a variable (if this is an extended state machine).

There are then two ways to exit C. Either guard g_1 evaluates to true, in which case the machine exits B and returns to A, or guard g_4 evaluates to true and the machine transitions to D. A subtle question is what happens if both guards g_1 and g_4 evaluate to true. Different

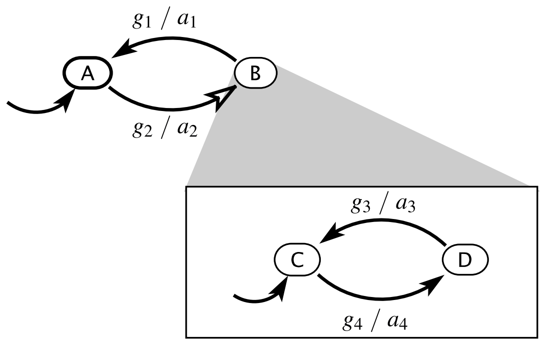


Figure 5.13: In a hierarchical FSM, a state may have a refinement that is another state machine.

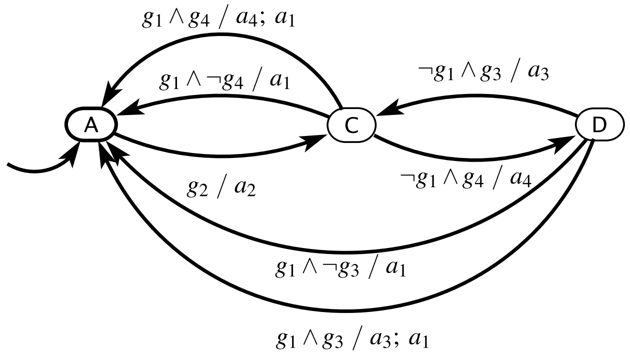

$g_1 \wedge g_4$ / a_4; a_1
$g_1 \wedge \neg g_4$ / a_1
$\neg g_1 \wedge g_3$ / a_3
A
C
D
g_2 / a_2
$\neg g_1 \wedge g_4$ / a_4
$g_1 \wedge \neg g_3$ / a_1
$g_1 \wedge g_3$ / a_3; a_1

Figure 5.14: Semantics of the hierarchical FSM in Figure 5.13.

variants of Statecharts may make different choices at this point. It seems reasonable that the machine should end up in state A, but which of the actions should be performed, a_4, a_1, or both? Such subtle questions help account for the proliferation of different variants of Statecharts.

We choose a particular semantics that has attractive modularity properties (Lee and Tripakis, 2010). In this semantics, a reaction of a hierarchical FSM is defined in a depth-first fashion. The deepest refinement of the current state reacts first, then its container state machine, then its container, etc. In Figure 5.13, this means that if the machine is in state

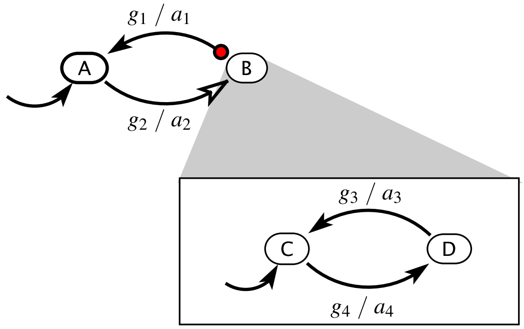


Figure 5.15: Variant of Figure 5.13 that uses a preemptive transition.

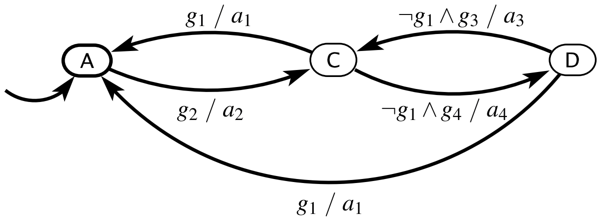


Figure 5.16: Semantics of Figure 5.15 with a preemptive transition.

B (which means that it is in either C or D), then the refinement machine reacts first. If it is C, and guard g_4 is true, the transition is taken to D and action a_4 is performed. But then, as part of the same reaction, the top-level FSM reacts. If guard g_1 is also true, then the machine transitions to state A. It is important that logically these two transitions are simultaneous and instantaneous, so the machine does not actually go to state D. Nonetheless, action a_4 is performed, and so is action a_1. This combination corresponds to the topmost transition of Figure 5.14.

Another subtlety is that if two (non-absent) actions are performed in the same reaction, they may conflict. For example, two actions may write different values to the same output port. Or they may set the same variable to different values. Our choice is that the actions are performed in sequence, as suggested by the semicolon in the action $a_4;\ a_1$. As in an imperative language like C, the semicolon denotes a sequence. If the two actions conflict, the later one dominates.

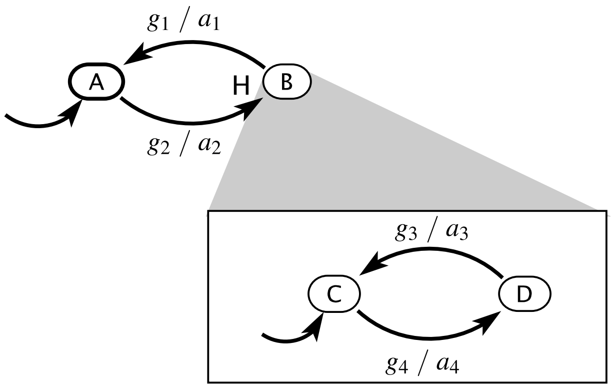


Figure 5.17: Variant of the hierarchical state machine of Figure 5.13 that has a history transition.

Such subtleties can be avoided by using a **preemptive transition**, shown in Figure 5.15, which has the semantics shown in Figure 5.16. The guards of a preemptive transition are evaluated *before* the refinement reacts, and if any guard evaluates to true, the refinement does not react. As a consequence, if the machine is in state B and g_1 is true, then neither action a_3 nor a_4 is performed. A preemptive transition is shown with a (red) circle at the originating end of the transition.

Notice in Figures 5.13 and 5.14 that whenever the machine enters B, it always enters C, never D, even if it was previously in D when leaving B. The transition from A to B is called a **reset transition** because the destination refinement is reset to its initial state, regardless of where it had previously been. A reset transition is indicated in our notation with a hollow arrowhead at the destination end of a transition.

In Figure 5.17, the transition from A to B is a **history transition**, an alternative to a reset transition. In our notation, a solid arrowhead denotes a history transition. It may also be marked with an "H" for emphasis. When a history transition is taken, the destination refinement resumes in whatever state it was last in (or its initial state on the first entry).

The semantics of the history transition is shown in Figure 5.18. The initial state is labeled (A, C) to indicate that the machine is in state A, and if and when it next enters B it will go

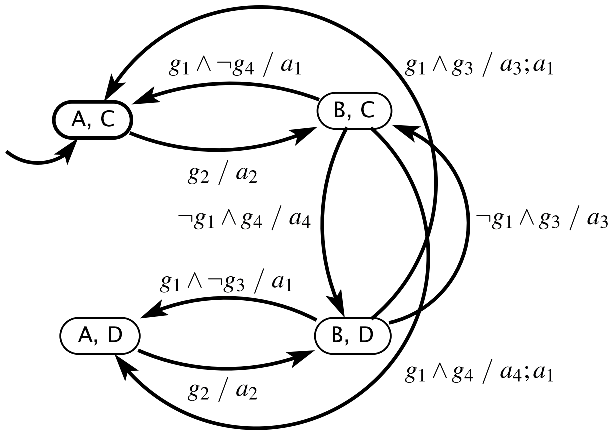

A, C
B, C
A, D
B, D
$g_1 \wedge \neg g_4 \,/\, a_1$
$g_1 \wedge g_3 \,/\, a_3; a_1$
$g_2 \,/\, a_2$
$\neg g_1 \wedge g_4 \,/\, a_4$
$\neg g_1 \wedge g_3 \,/\, a_3$
$g_1 \wedge \neg g_3 \,/\, a_1$
$g_1 \wedge g_4 \,/\, a_4; a_1$
$g_2 \,/\, a_2$

Figure 5.18: Semantics of the hierarchical state machine of Figure 5.17 that has a history transition.

to C. The first time it goes to B, it will be in the state labeled (B, C) to indicate that it is in state B and, more specifically, C. If it then transitions to (B, D), and then back to A, it will end up in the state labeled (A, D), which means it is in state A, but if and when it next enters B it will go to D. That is, it remembers its history, specifically where it was when it left B.

As with concurrent composition, hierarchical state machines admit many possible meanings. The differences can be subtle. Considerable care is required to ensure that models are clear and that their semantics match what is being modeled.

5.3 Summary

Any well-engineered system is a composition of simpler components. In this chapter, we have considered two forms of composition of state machines, concurrent composition and hierarchical composition.

For concurrent composition, we introduced both synchronous and asynchronous composition, but did not complete the story. We have deferred dealing with feedback to the next chapter, because for synchronous composition, significant subtleties arise. For asynchronous composition, communication via ports requires additional mechanisms that are not (yet) part of our model of state machines. Even without communication via ports, significant subtleties arise because there are several possible semantics for asynchronous composition, and each has strengths and weaknesses. One choice of semantics may be suitable for one application and not for another. These subtleties motivate the topic of the next chapter, which provides more structure to concurrent composition and resolves most of these questions (in a variety of ways).

For hierarchical composition, we focus on a style originally introduced by Harel (1987) known as Statecharts. We specifically focus on the ability for states in an FSM to have refinements that are themselves state machines. The reactions of the refinement FSMs are composed with those of the machine that contains the refinements. As usual, there are many possible semantics.

Exercises

1. Consider the extended state machine model of Figure 3.8, the garage counter. Suppose that the garage has two distinct entrance and exit points. Construct a side-by-side concurrent composition of two counters that share a variable c that keeps track of the number of cars in the garage. Specify whether you are using synchronous or asynchronous composition, and define exactly the semantics of your composition by giving a single machine modeling the composition. If you choose synchronous semantics, explain what happens if the two machines simultaneously modify the shared variable. If you choose asynchronous composition, explain precisely which variant of asynchronous semantics you have chosen and why. Is your composition machine deterministic?

2. For semantics 2 in Section 5.1.2, give the five tuple for a single machine representing the composition C,

$$(\mathit{States}_C, \mathit{Inputs}_C, \mathit{Outputs}_C, \mathit{update}_C, \mathit{initialState}_C)$$

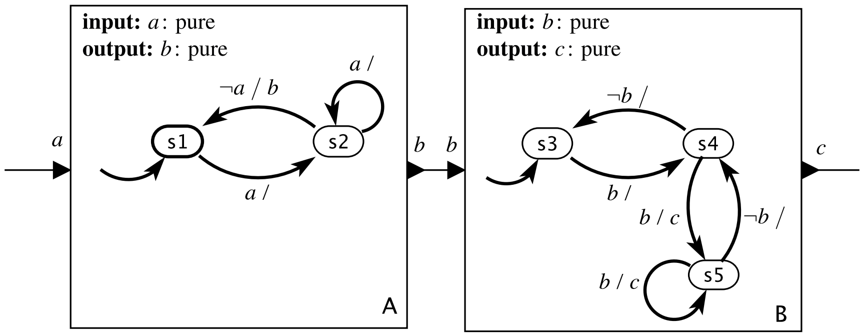
for the side-by-side asynchronous composition of two state machines A and B. Your answer should be in terms of the five-tuple definitions for A and B,

$$(\mathit{States}_A, \mathit{Inputs}_A, \mathit{Outputs}_A, \mathit{update}_A, \mathit{initialState}_A)$$

and

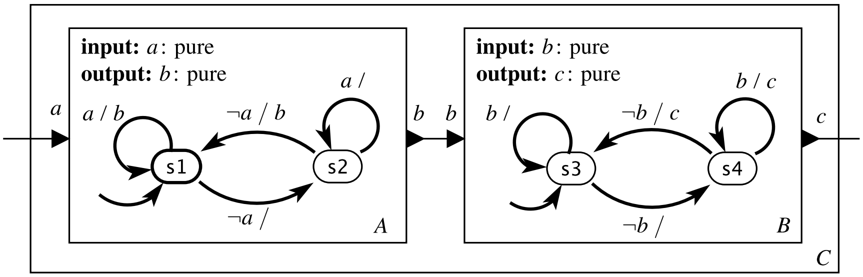
$$(\mathit{States}_B, \mathit{Inputs}_B, \mathit{Outputs}_B, \mathit{update}_B, \mathit{initialState}_B)$$

3. Consider the following synchronous composition of two state machines A and B:



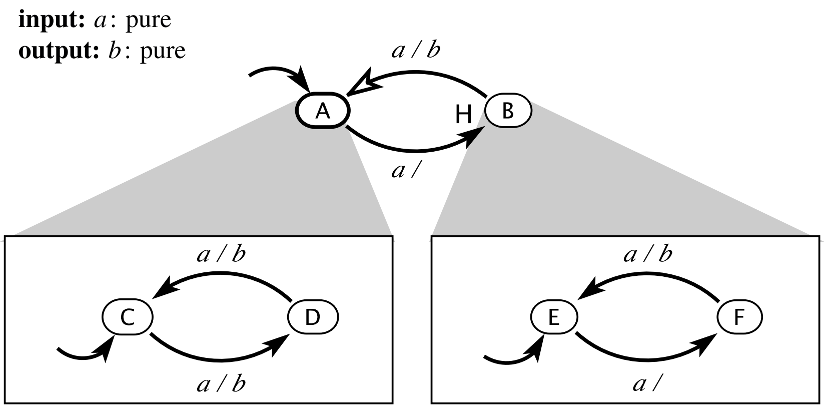
Construct a single state machine C representing the composition. Which states of the composition are unreachable?

4. Consider the following synchronous composition of two state machines A and B:



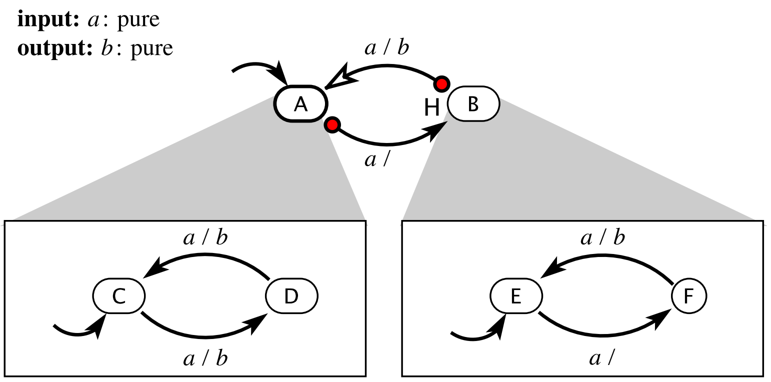
Construct a single state machine C representing the composition. Which states of the composition are unreachable?

5. Consider the following hierarchical state machine:

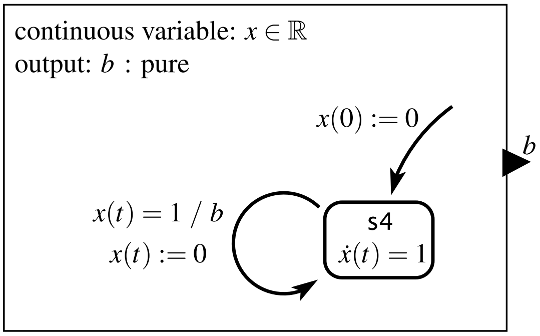


Construct an equivalent flat FSM giving the semantics of the hierarchy. Describe in words the input/output behavior of this machine. Is there a simpler machine that exhibits the same behavior? (Note that equivalence relations between state machines are considered in Chapter 14, but here, you can use intuition and just consider what the state machine does when it reacts.)

6. How many reachable states does the following state machine have?



7. Suppose that the machine of Exercise 8 of Chapter 4 is composed in a synchronous side-by-side composition with the following machine:



Find a tight lower bound on the time between events a and b. That is, find a lower bound on the time gap during which there are no events in signals a or b. Give an argument that your lower bound is tight.