Control Systems WS3

Daniel Terrett: 1080856
Hongbo Zhou: 1067814

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I Introduction

This workshop investigates the control system design process for a LEGO MINDSTORMS EV3 robot. Due to reasons outside of our control, this process has been done through simulation in MATLAB and Simulink as appropriate design tools.

The design process involves an in-depth look at the similarities and differences between open-loop and closed-loop control design - how they perform in ideal and non-ideal situations.

These system designs were implemented for 2 DC motors used to actuate the drive wheels of the robot. In an ideal setting where there are no disturbances open-loop control should theoretically have perfect tracking but introducing disturbance results in incredibly poor performance. In theoretical contrast, closed-loop control doesn't have perfect tracking where there are no disturbances but should be far more robust to disturbances.

II System Modelling

With reference to Figure 1. our DC motor model is approximated via linear models.

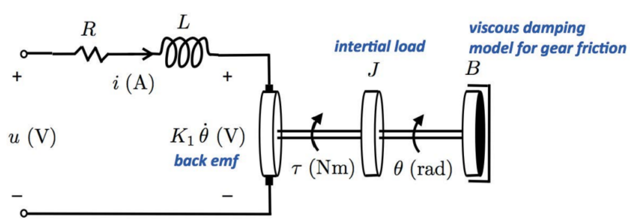

Figure 1: DC Motor model

We obtain a system of ODEs by applying Kirchhoff's Voltage Law to the electrical side of the motor, and Newton's Second Law to the mechanical side:

$$J\frac{d^2\theta}{dt^2}(t) = K_2 i(t) - B\frac{d\theta}{dt}(t) \tag{1}$$

$$u(t) = Ri(t) + L\frac{di}{dt}(t) + K_1\frac{d\theta}{dt}(t) \tag{2}$$

When taken to s-domain and assuming zero initial conditions and assuming L = 0, we obtain the plant model:

$$\frac{\Theta(s)}{U(s)} = G(s) := \frac{K_m}{s(T_m s + 1)} \tag{3}$$

The contants K_m and T_m are introduced for simplicity over carrying around K_1, K_2, B and R.

It is immediately obvious that both motors actually differ in their response to the same input signal, one exhibiting greater rotation than the other. We would expect to see that the LEGO robot would thus tend to favour one direction of motion than the other - simply, if intended to travel straight it would be slightly turning. The best way to compromise our plant design is to have it so that it matches each motor 'equally well' i.e. change the constants so that the plant model response is as close to a continuous plot of the midpoints across time.

After some investigation the set of values $V_m = 30$, $K_m = 6.95$, $T_m = 0.06$ produces a plot that matches both motors equally well.

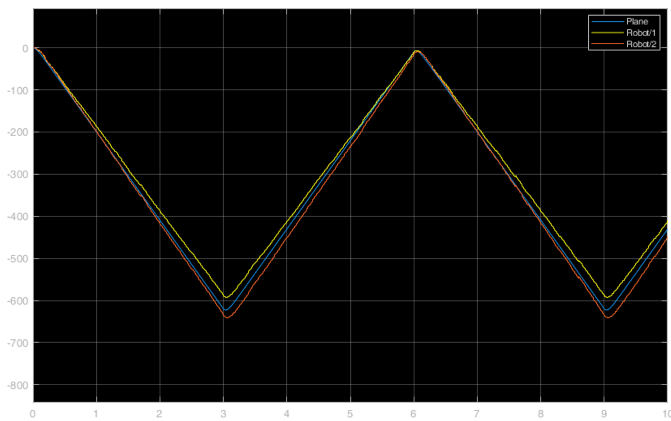

Figure 2: Plot of Angle (y-axis) against time (x-axis) for default $V_m = 30$, $K_m = 7.00$, $T_m = 0.05$

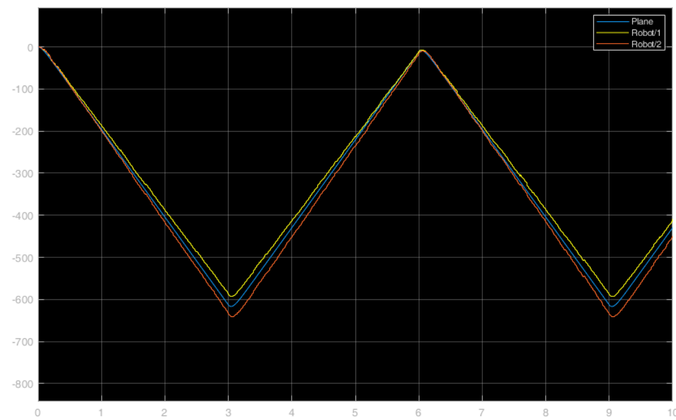

Figure 3: Plot of Angle (y-axis) against time (x-axis) for manipulated $V_m = 30$, $K_m = 6.95$, $T_m = 0.06$.

Playing with the parameters it can be see that:

V_m: controls the magnitude of all scope traces collectively.

K_m: controls the magnitude of the plant behaviour. By increasing it the plot will eventually stretch below the red plot, and if decreased too much will be above the yellow plot:

T_m: controls the strength of transient behaviour of the designed plant, increasing it's value results in a slower transient. It's effectively less responsive to change in input.

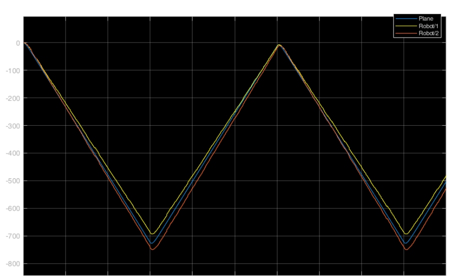

Figure 4: Plot of Angle (y-axis) against time (x-axis) for $V_m = 35$, $K_m = 7.00$, $T_m = 0.05$. V_m has been increased and see that the magnitudes have all increased to about 700.

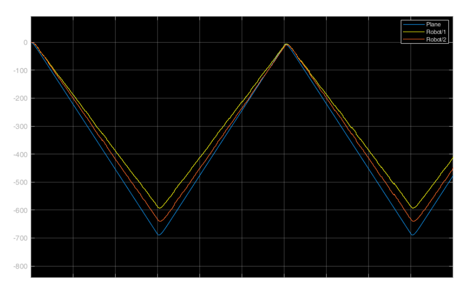

Figure 5: Plot of Angle (y-axis) against time (x-axis) for $V_m = 30$, $K_m = 7.75$, $T_m = 0.05$. K_m has been increased close to 700 while the others traces remain at default level.

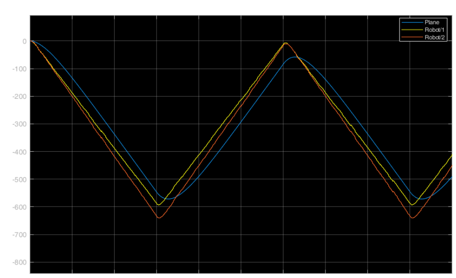

Figure 6: Plot of Angle (y-axis) against time (x-axis) for $V_m = 30$, $K_m = 7.00$, $T_m = 0.40$. As T_m has increased it is observed that the system is far less responsive.

III Control Design

Open-loop Control

Finding the Analytical Step Response

Let T_{OL} be the open-loop transfer function:

$$T_{OL} = G(s)C_{OL}(s) = \frac{K_m}{s(T_m s + 1)} \frac{s(T_{m0} s + 1)}{K_{m0}(T_o s + 1)^2} = \frac{K_m}{K_{m0}} \frac{T_{m0} s + 1}{T_m s + 1} \frac{1}{(T_o s + 1)^2}$$

Applying final value theorem:

$$FV = \lim_{s \to 0} T_{OL} = \frac{K_m}{K_{m0}}$$

See that under the conditions, $K_m = K_{m0}$ and $T_m = T_{m0}$ the expression simplifies down to:

$$T_{OL} = \frac{1}{(T_O s + 1)^2}, \quad FV = 1$$

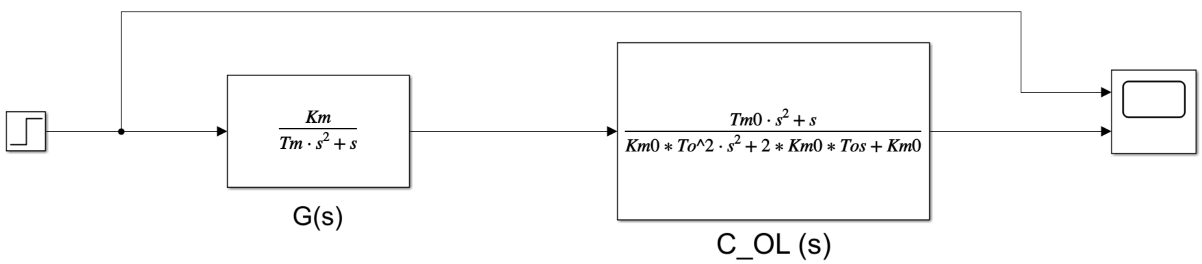

Figure 7: Simulink design of Open-loop control system

In the case where $K_m = K_{m0}$ and $T_m = T_{m0}$ we get near-perfect tracking in the step-response. Note that this is with $T_o = 0.01$ which causes transient behaviour to decay quickly. However, we can see it's effect by the small discrepancy near the positive edge of the step input.

Figure 8: Scope of the Open-loop step response and the step-input

To obtain the analytical step-response, we alter the relationships between parameter pairs: $K_m \longleftrightarrow K_{m0}$ and $T_m \longleftrightarrow T_{m0}$

K_{m0} controls the steady-state value of the output, if greater than K_m then after the rising edge it is visibly offset above the step input signal.

T_{m0} controls the dampening of the system From the expression for T_{OL} we can derive the relative-error transfer functions:

$$\frac{1}{T_{OL}}\frac{\partial T_{OL}}{\partial Km}\Big|_{(K_m,T_m)=(K_{m0},T_{m0})} = \frac{K_{m0}(T_O s+1)^2 s(T_m s+1)}{sK_m(T_{m0}s+1)}\frac{s(T_{m0}s+1)}{K_{m0}(T_O s+1)^2 s(T_m s+1)}\Big|_{(K_m,T_m)=(K_{m0},T_{m0})}$$
$$= \frac{1}{K_{m0}}$$

$$\frac{1}{T_{OL}}\frac{\partial T_{OL}}{\partial Tm}\Big|_{(K_m,T_m)=(K_{m0},T_{m0})} = \frac{K_{m0}(T_O s+1)^2 s(T_m s+1)}{sK_m(T_{m0}s+1)}\frac{-sK_m(T_{m0}s+1)}{K_{m0}(T_O s+1)^2 s(T_m s+1)^2}\Big|_{(K_m,T_m)=(K_{m0},T_{m0})}$$
$$= \frac{-s}{(T_{m0}s+1)^2}$$

Determine Parameter Values

Based on the derived formula of T_{OL}, we can confirm that poles are located at $-\frac{1}{T_m}$ and $-\frac{1}{T_o}$. Since T_m and T_o are all positive, the system is stable.

However, due to the limitation of having one controller attempting to control two different motors, the angular velocity differences between motors and controlled signal are not able to be completely removed. Therefore, the angle error increases over time. The second specification can not be met. Nonetheless that, we still want the motor angles are closed to the angle reference, such that the final value of step response of should be 1. In that case, according to the final value theorem, K_{m0} should equal to K_m.

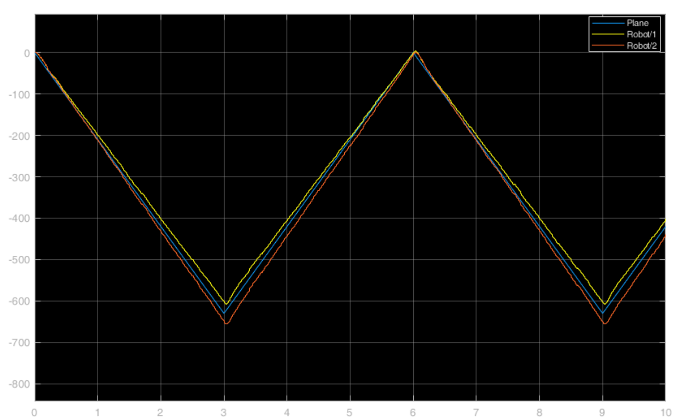

Figure 9: Plot of Angle(y-axis) against time (x-axis) for open loop controller with determined parameters

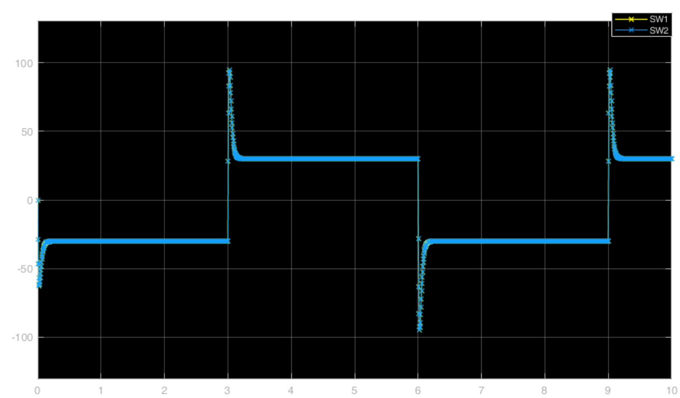

Figure 10: Motor input voltages for open loop controller with determined parameters.

In order to make motor voltage not saturate, we find that T_{M0} should keep small, but then the motor angles will not match the reference equally well. To solve this problem, T_o should be increased to increase the range of T_{M0} that the motor angles can match the reference well. After several attempts, the final parameter values are: $K_{m0} = 6.95$, $T_{m0} = 0.095$ and $T_o = 0.02$.

Feedback Control

Determine The Value of The Constant Gain

Let T_{CL} be the close-loop transfer function:

$$T_{CL} = \frac{C_{CL}G(s)}{1 + C_{CL}G(s)} = \frac{K_c K_m}{s(T_m s+1) + K_c K_m}$$

To make sure the system is still stable, root locus is plotted with respect to G(s) as shown in Figure 11. It shows that the system will always stable if Kc is positive. We see that, as Kc continues to grow larger, the magnitude of oscillation in the output voltage will become larger too. These oscillations will approach infinity if Kc is allowed to be unbounded. After trying several values, we decide to set Kc to be 4.

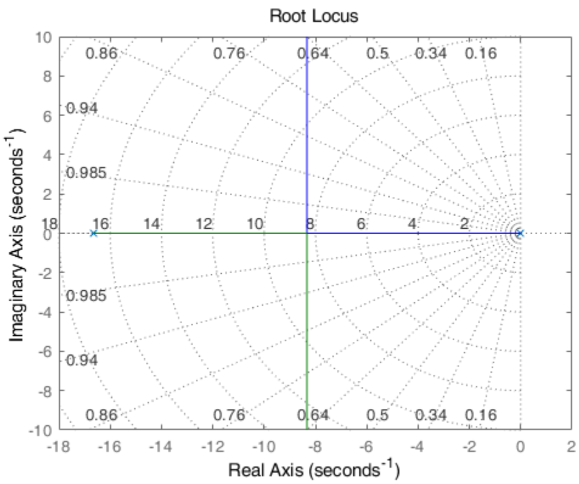

Figure 11: Root locus of constant gain Kc

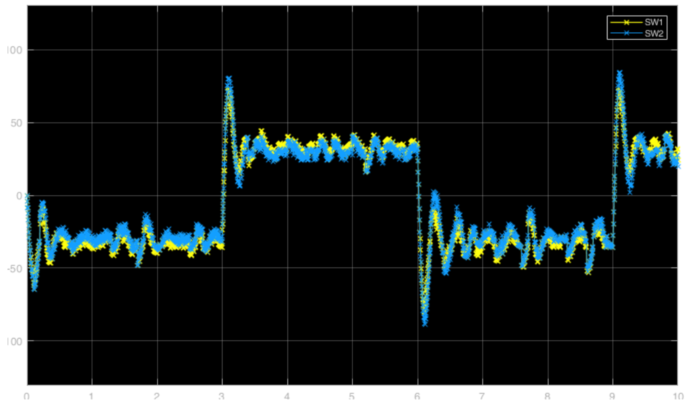

Figure 12: Motor input voltages for feedback loop with Kc = 4.

The following figures show that the angle error is very small, which is less than 5% of the reference angular velocity (10.5 degrees) in steady-state.

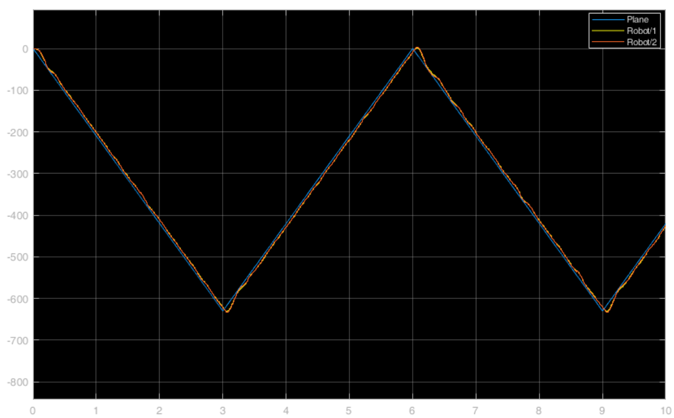

Figure 13: Motor input voltages for feedback loop with Kc = 4.

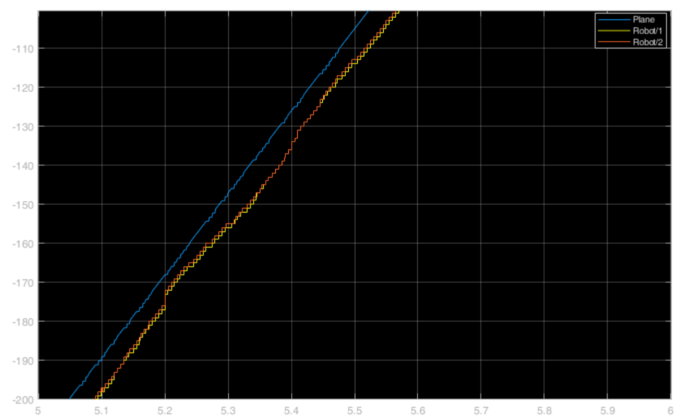

Figure 14: Close up of Figure 11

Error functions

$$\frac{1}{T_{CL}}\frac{\partial T_{CL}}{\partial Km}\bigg|_{(K_m,T_m)=(K_{m0},T_{m0})} = \frac{s(T_m s+1)+K_cK_m}{K_cK_m}\frac{K_c[s(T_m s+1)+K_cK_m]-K_c^2K_m}{[s(T_m s+1)+K_cK_m]^2}$$
$$= \frac{1}{K_m} - \frac{K_c}{s(T_m s+1)+K_cK_m}$$

$$\frac{1}{T_{CL}}\frac{\partial T_{CL}}{\partial Tm}\bigg|_{(K_m,T_m)=(K_{m0},T_{m0})} = \frac{s(T_m s+1)+K_cK_m}{K_cK_m}\frac{-K_cK_m s^2}{[s(T_m s+1)+K_cK_m]^2}$$
$$= -\frac{s^2}{s(T_m s+1)+K_cK_m}$$

IV Implementation and Testing

As seen from the plots below, closed-loop control showed far better tracking than open-loop control in the absence of disturbances. This might've been unexpected, however, considering the fact that we are approximating a nominal plant model for two motors that don't behave the same this introduces a latent degree of error that open loop control may not be able to control as effectively as feedback control. This error is that each wheel will rotate a different amount under the same input.

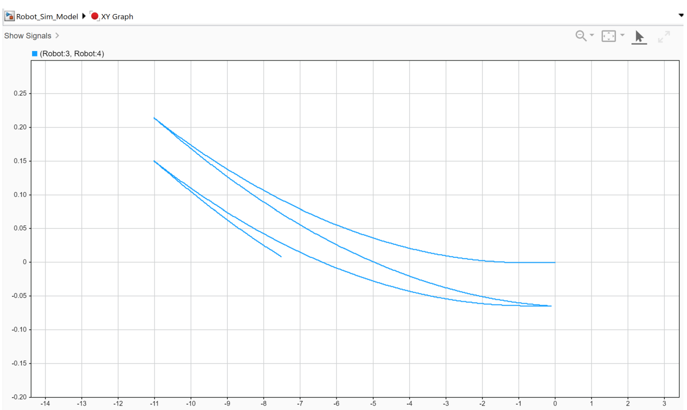

Figure 15: Simulated plot of the EV3 robot moving in an XY plane with open loop control

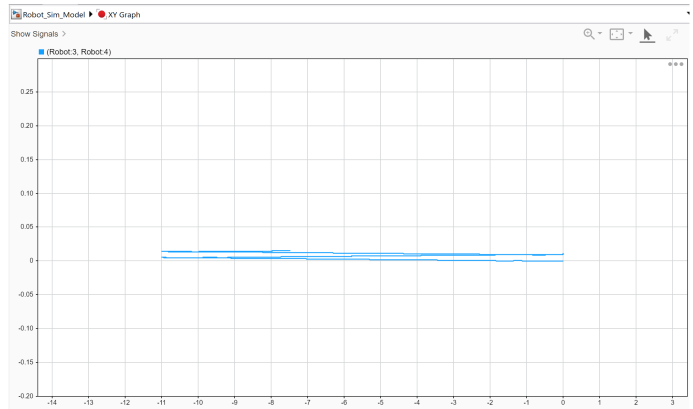

Figure 16: Simulated plot of the EV3 robot moving in an XY plane with closed loop control

The next plots show system behaviour when subject to disturbances, again feedback control performs far better than open-loop control relative to their no-disturbance counterparts.

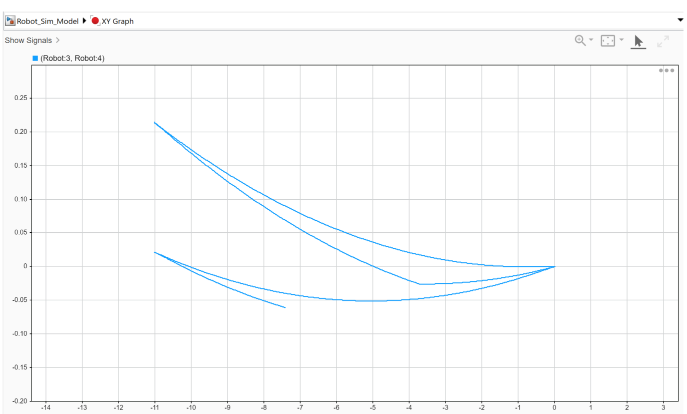

Figure 17: Simulated plot of the EV3 robot moving in an XY plane with open loop control with disturbances

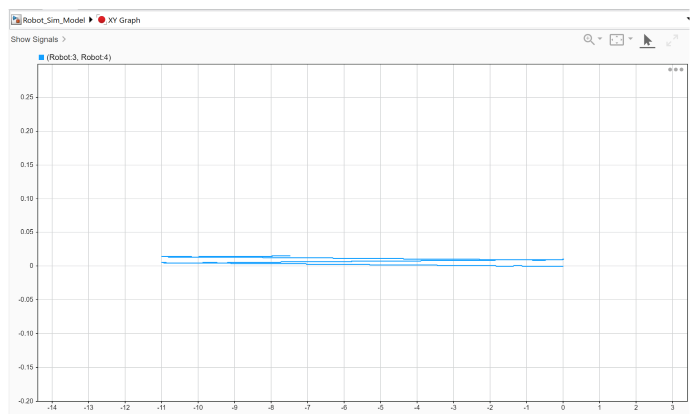

Figure 18: Simulated plot of the EV3 robot moving in an XY plane with closed loop control with disturbances

V Conclusion

In this report it was found and concluded that feedback control has far greater practical application than open-loop control. It performed more effectively both with and without disturbances, despite open-loop being expected to perform better in an undisturbed environment. For open loop control it is seen that nominating a plant of equal error-difference for two different motor systems is not sufficient. In order to improve the control design further, one suggestion could be to design a control system for each motor individually i.e. 2 controllers. Other than that more complex feedback design could improve overall controller performance.