Lecture 2

Open-loop* versus *closed-loop

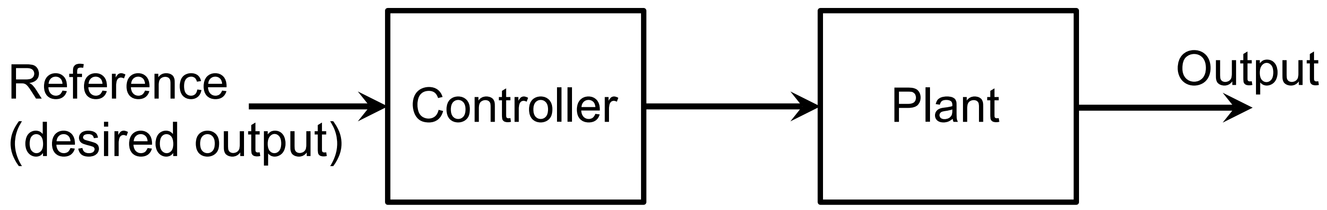
Outline

- Motivation
- Static example (car cruise control)
- Dynamic systems - general case
- Operational amplifier
- Conclusions

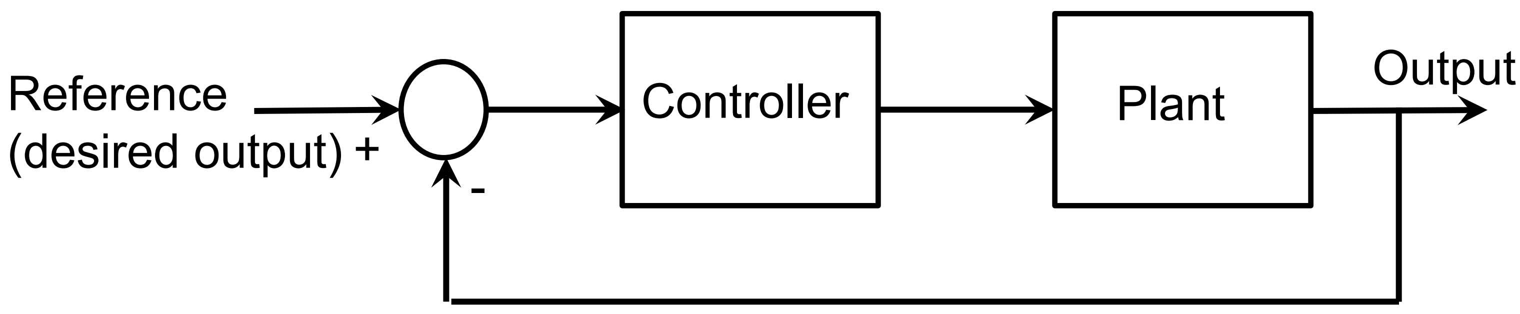
Motivation

- This subject is mainly about closed loop (feedback) control, but open loop control is sometimes useful.
- The considered examples capture the main differences between open and closed loop.
- More details will be given in the following lectures.

Open loop and closed loop



OPEN LOOP CONTROL



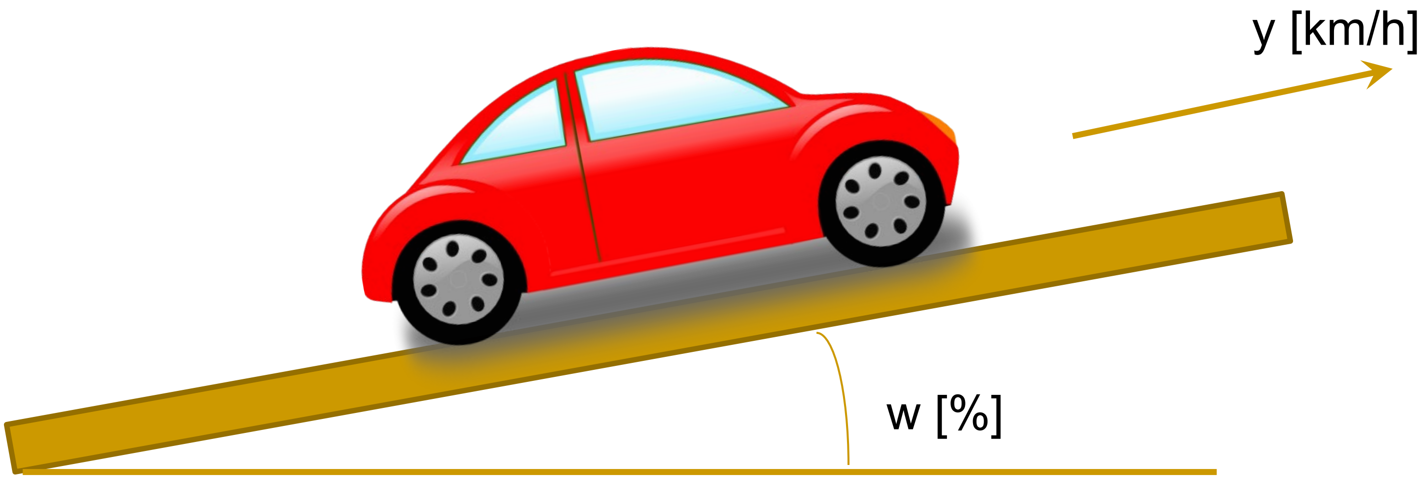
CLOSED LOOP CONTROL

Open or Closed Loop?

- Toaster
- Electric Kettle
- Hand drier
- Washing machine
- Dishwasher
- Power Steering

Open versus closed loop

- Cruise control problem (ignore dynamics):



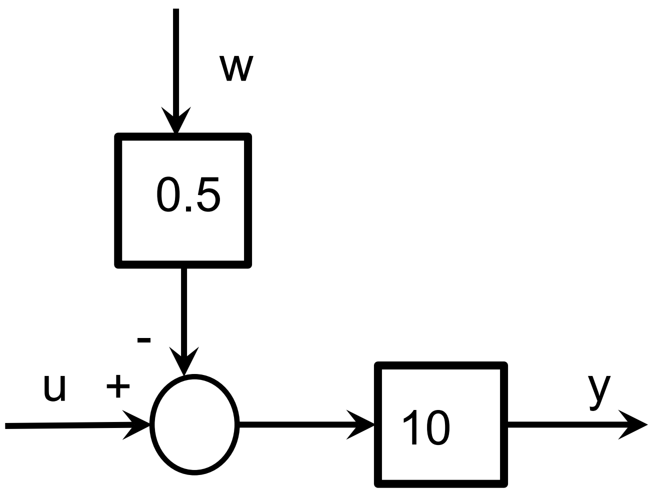
Model is found to be $y = 10 (u - 0.5 w)$

u [degrees] is the throttle angle

From *Feedback Control of Dynamic Systems*, G.F. Franklin et al

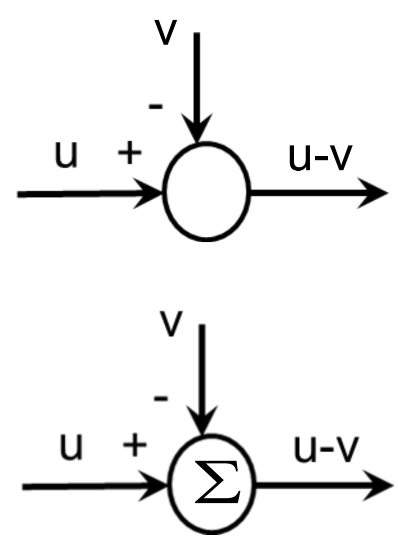
Block diagram

- We have the following block diagram:



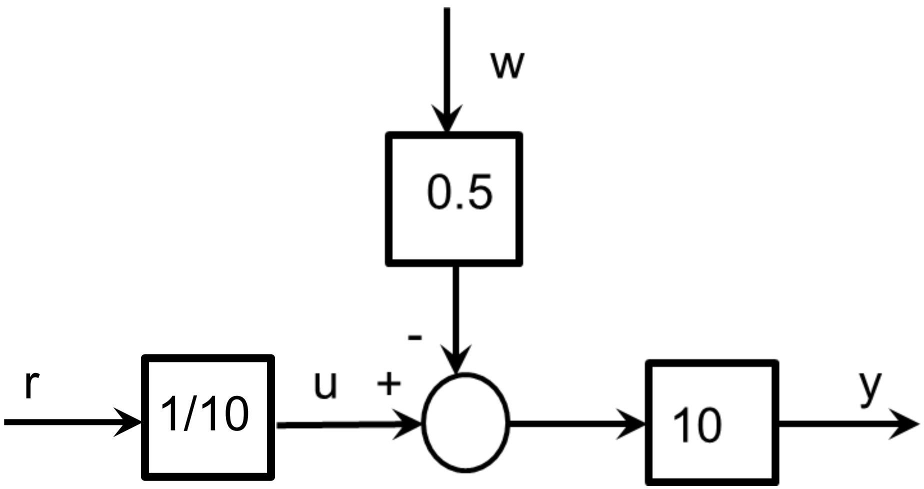
y=10(u-0.5w)

Comparator:



Open loop controller

- We have the following block diagram:



$$\begin{aligned} y_{ol} &= 10(u - 0.5w) \\ &= 10\left(\frac{r}{10} - 0.5w\right) \\ &= r - 5w \end{aligned}$$

$$e_{ol} = r - y_{ol} = 5w$$

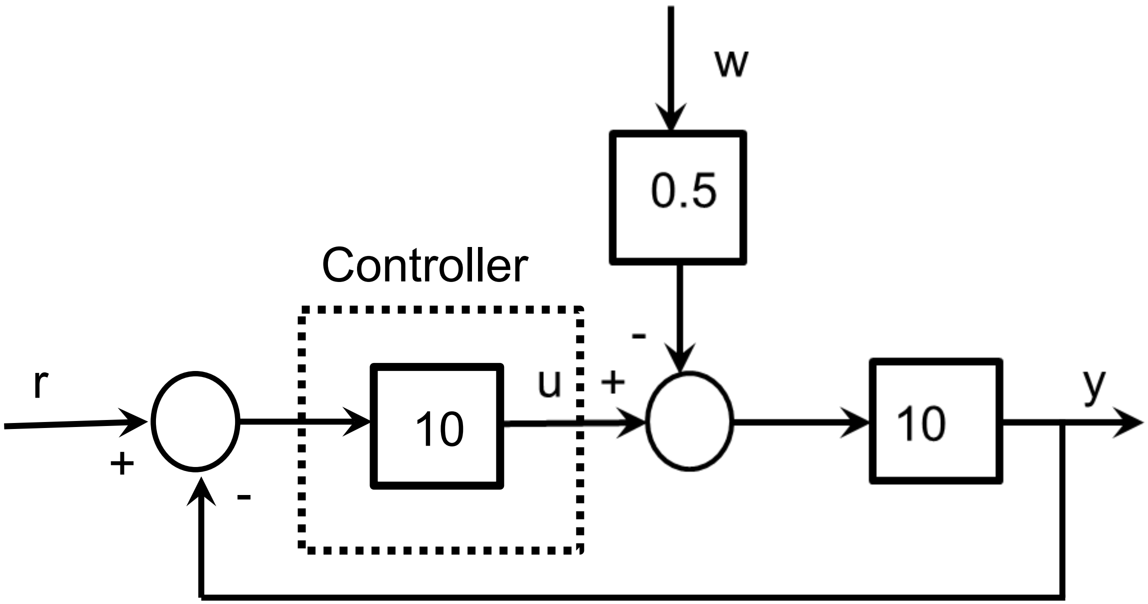
$$\%\text{error} = 500\frac{w}{r}$$

Comments on open loop

- *When disturbance is zero, we have perfect tracking* r=y or in other words the error is zero.
- When w=1, r=65, then y=60 and we get 7.69% error in speed.
- When w=2, r=65, then y=55 and we get 15.38% error in speed.
- We can say that *the scheme is not robust* with respect to disturbance.

Closed loop controller

- We have the following block diagram:



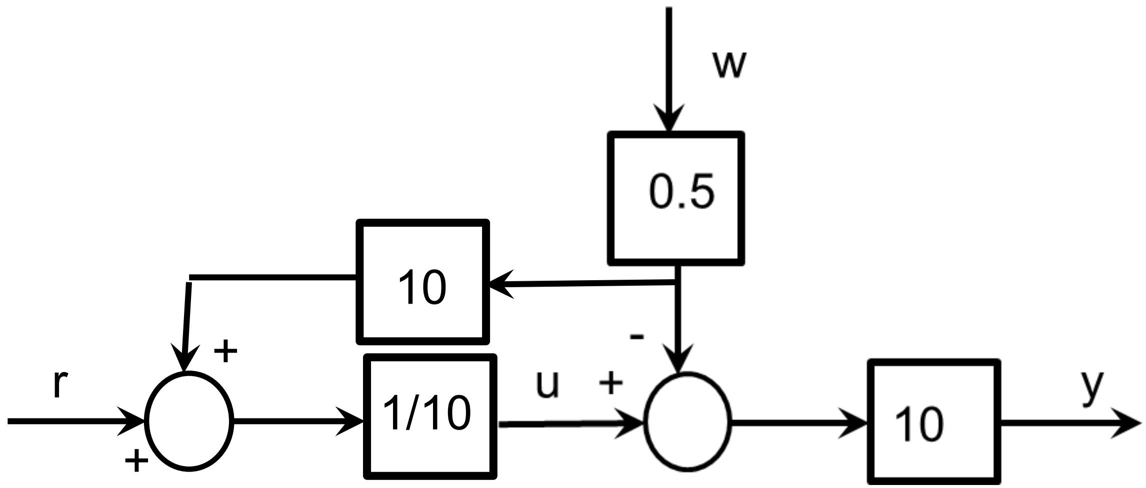
$$
\begin{aligned}
y_{cl} &= 10u - 5w \\
u &= 10(r - y_{cl})
\end{aligned}
$$

$$
\begin{aligned}
y_{cl} &= \frac{100}{101}r - \frac{5}{101}w \\
e_{cl} &= \frac{r}{101} + \frac{5w}{101}
\end{aligned}
$$

Comments on closed loop

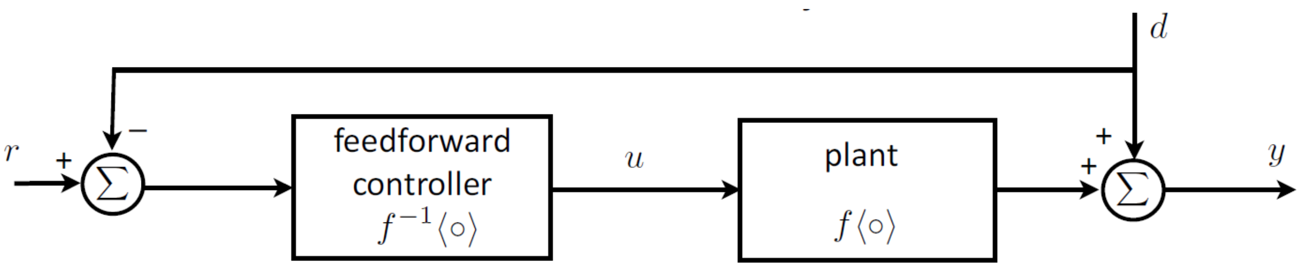
- *When disturbance is zero, we do not have perfect tracking! The error is 0.643%*
- If w=1, r=65, the error is 0.693%. Note that it is 10 times smaller than in the open loop case!
- We can say that *feedback improves robustness* of the system in general.

What if we measure disturbance?



$$y = 10(u - 0.5w) = 10\,[1/10(r+5w) - 0.5w] = r$$

General case open loop (we measure disturbance)



- We want

$$r = y = f\langle u\rangle + d \quad \Longleftarrow \quad u = f^{-1}\langle r - d\rangle$$

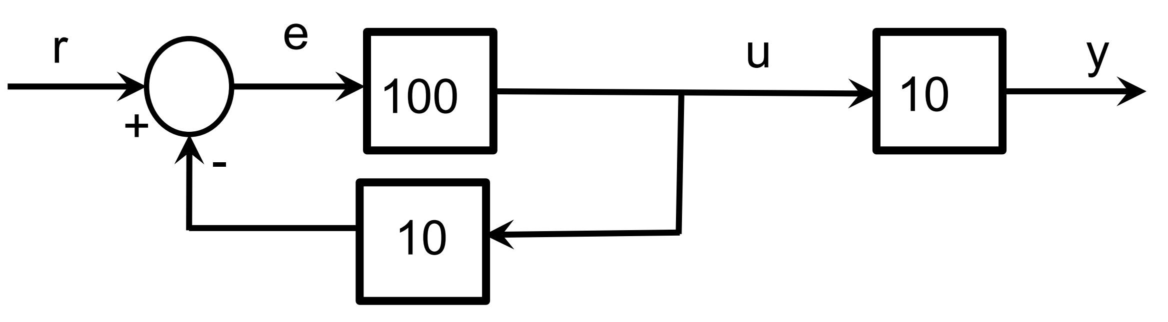
where $f\langle\circ\rangle$ is a causal mapping and $f^{-1}\langle\circ\rangle$ is its inverse.

Comments

- Open loop control gives perfect tracking in a perfect scenario (no disturbance, perfect model).
- Open loop control requires a lot:
  - The disturbance needs be measured
  - The model needs to be inverted exactly
  - The inverse may not be realisable
  - Model and inverse need to be “BIBO stable”

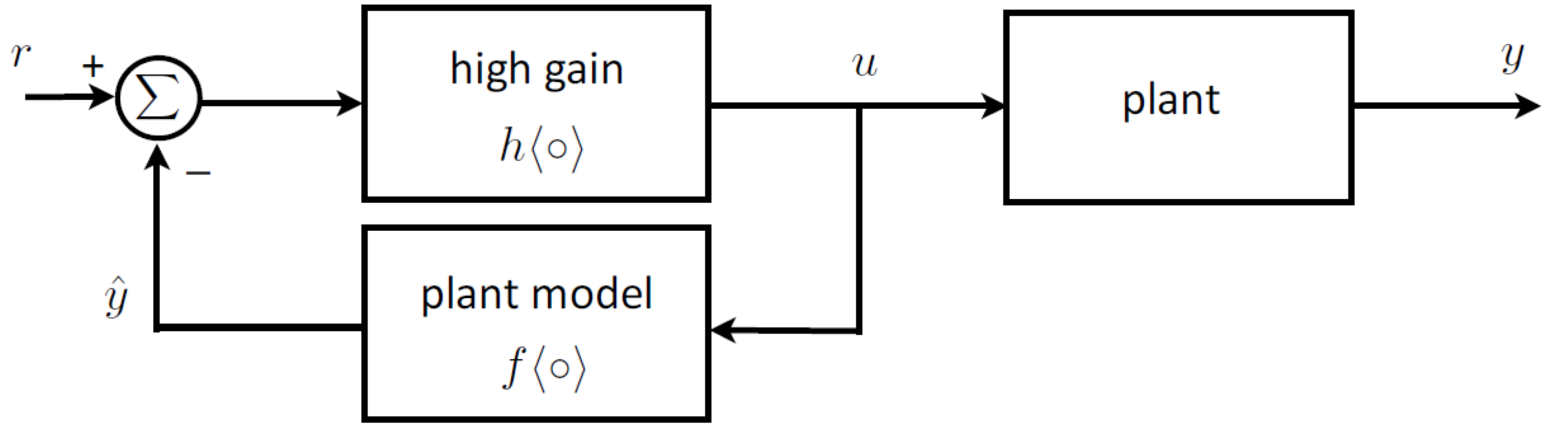
An alternative implementation (no disturbance)

u=100 e
e = r – 10u
u+1000u=100r
u=100 r/(1+1000) ≈ r/10



y = 10 u = 10 (100/(1+1000) r) ≈ 10 (1/10 r) = r

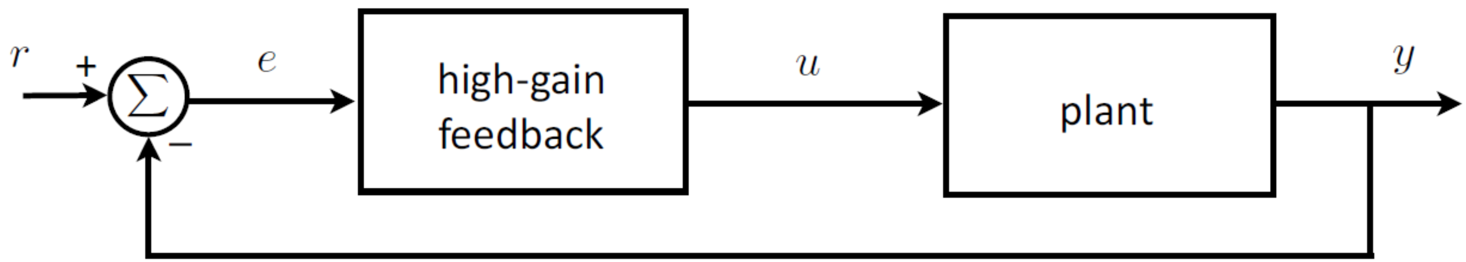
Approximate open-loop inversion..



- We have here

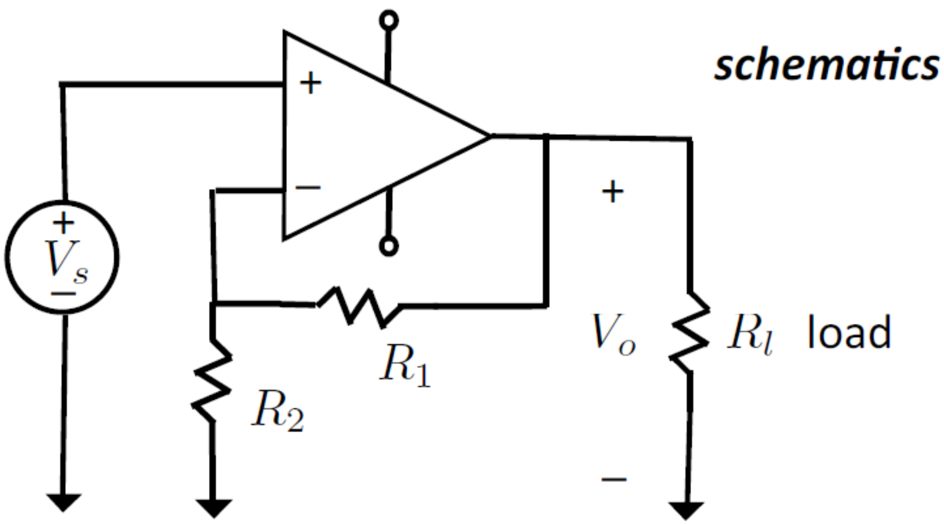
$$
\begin{aligned}
u = h\langle r - f\langle u\rangle\rangle \quad &\Longrightarrow \quad r - f\langle u\rangle = h^{-1}\langle u\rangle \\
&\Longrightarrow \quad u = f^{-1}\langle r - h^{-1}\langle u\rangle\rangle \overset{\text{for high-gain } h}{\approx} u = f^{-1}\langle r\rangle \quad \Longrightarrow \quad y \approx r
\end{aligned}
$$

→ High-gain closed-loop control!



- This scheme can be viewed as an alternative to approximate open loop control
- High-gain feedback controllers respond aggressively to small errors. May destabilise the system.
- But feedback can help make the system robust to disturbances and uncertainty

Example: Operational amplifier in feedback configuration

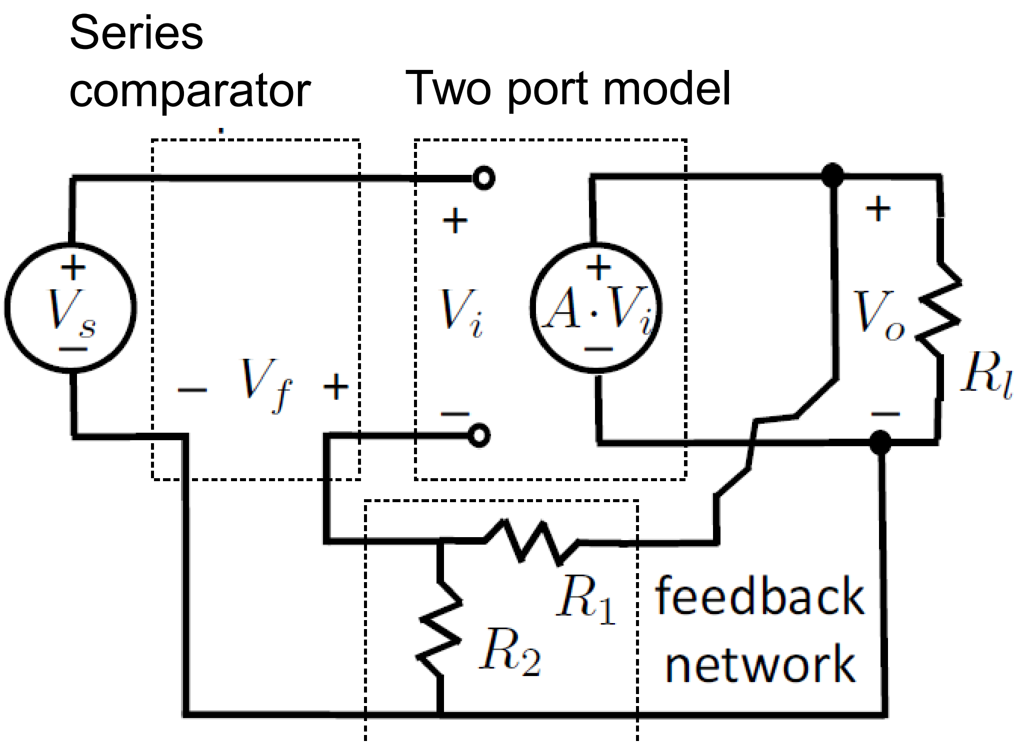


- High gain amplifiers are crucial in long-distance telecoms
- Major issue: very uncertain gain
- Feedback mitigates this uncertainty

Comments

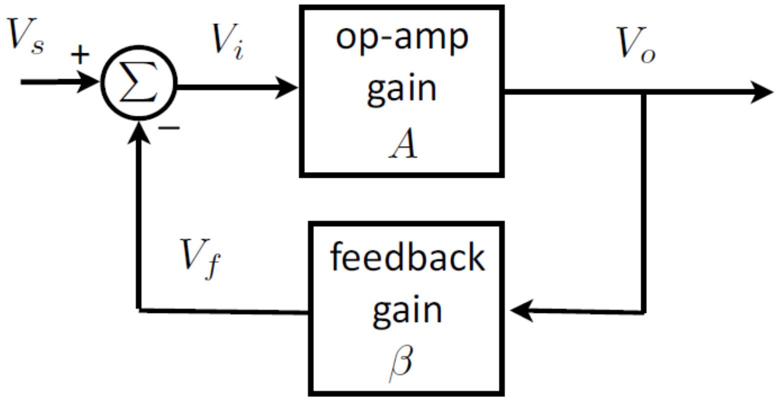
- Too high a gain may destabilise the system.
- But too low a gain can lead to poor robustness against disturbances
- Nyquist and Bode in Bell Labs formalised these trade-offs from 1920-40, sparking the "Golden Age of Invention" in the US, and the birth of feedback control theory.

Opamp model



$$\beta = R_2/(R_1 + R_2)$$

Simplified analysis



$$\beta = R_2/(R_1 + R_2)$$

$$\left.\begin{aligned} V_i &= V_s - V_f \\ V_o &= A \cdot V_i \\ V_f &= \beta \cdot V_o \end{aligned}\right\} \Rightarrow V_o = A \cdot (V_s - \beta \cdot V_o)$$

$$V_o = \left(\frac{A}{1 + \beta \cdot A}\right) \cdot V_s \overset{\text{for large } A}{\approx} \frac{1}{\beta} \cdot V_s$$

Summary

- Open loop control gives perfect tracking when there are no disturbances and model uncertainties. However, it is NOT robust to disturbances and modelling errors.

- Closed loop (feedback) may give imperfect tracking when there are no disturbances and model uncertainties. However, the scheme is robust to disturbances and modelling errors.