Lectures 17/18:
Invariants and Temporal Logic

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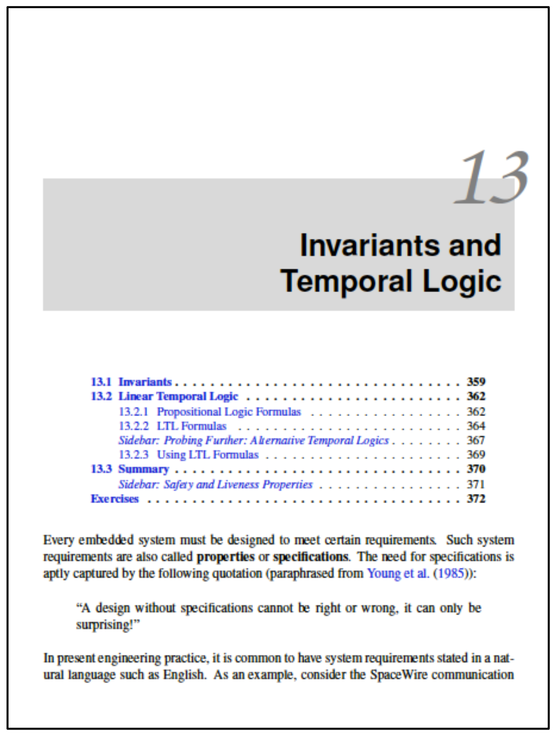
Outline

- Specification, verification, and control
- Temporal logic



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Invariants and Temporal Logic

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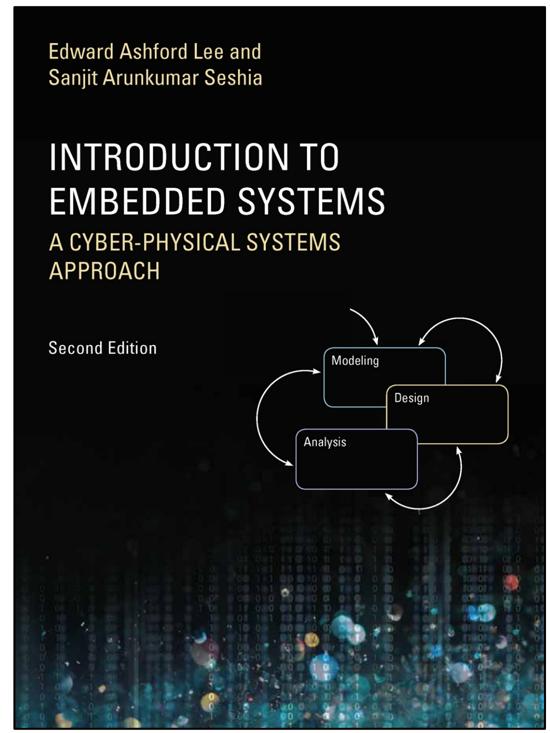
Every embedded system must be designed to meet certain requirements. Such system requirements are also called **properties** or **specifications**. The need for specifications is aptly captured by the following quotation (paraphrased from Young et al. (1985)):

> "A design without specifications cannot be right or wrong, it can only be surprising!"

In present engineering practice, it is common to have system requirements stated in a natural language such as English. As an example, consider the SpaceWire communication



When is a Design "Correct"?

- A design is correct when it meets its *specification* (requirements) in its operating environment

"A design without specification cannot be right or wrong, it can only be surprising!"

[paraphrased from Young et al., 1986]

- Simply running a few ad-hoc tests is not enough!

"For example, is not proof."

Yiddish Proverb

- Many embedded systems are deployed in safety-critical applications (avionics, automotive, medical, …)

The Challenge of Dependable Software in Cyber-Physical Systems

Today’s medical devices run on software… software defects can have life-threatening consequences.

[different device]

[From the Journal of Pacing and Clinical Electrophysiology, 2004]

“the patient collapsed while walking towards the cashier after refueling his car […] A week later the patient complained to his physician about an increasing feeling of unwell-being since the fall.”

“In 1 of every 12,000 settings, the software can cause an error in the programming resulting in the possibility of producing paced rates up to 185 beats/min.”

Specification, Verification, and Control

- **Specification**

  A mathematical statement of the design objective (desired properties of the system)

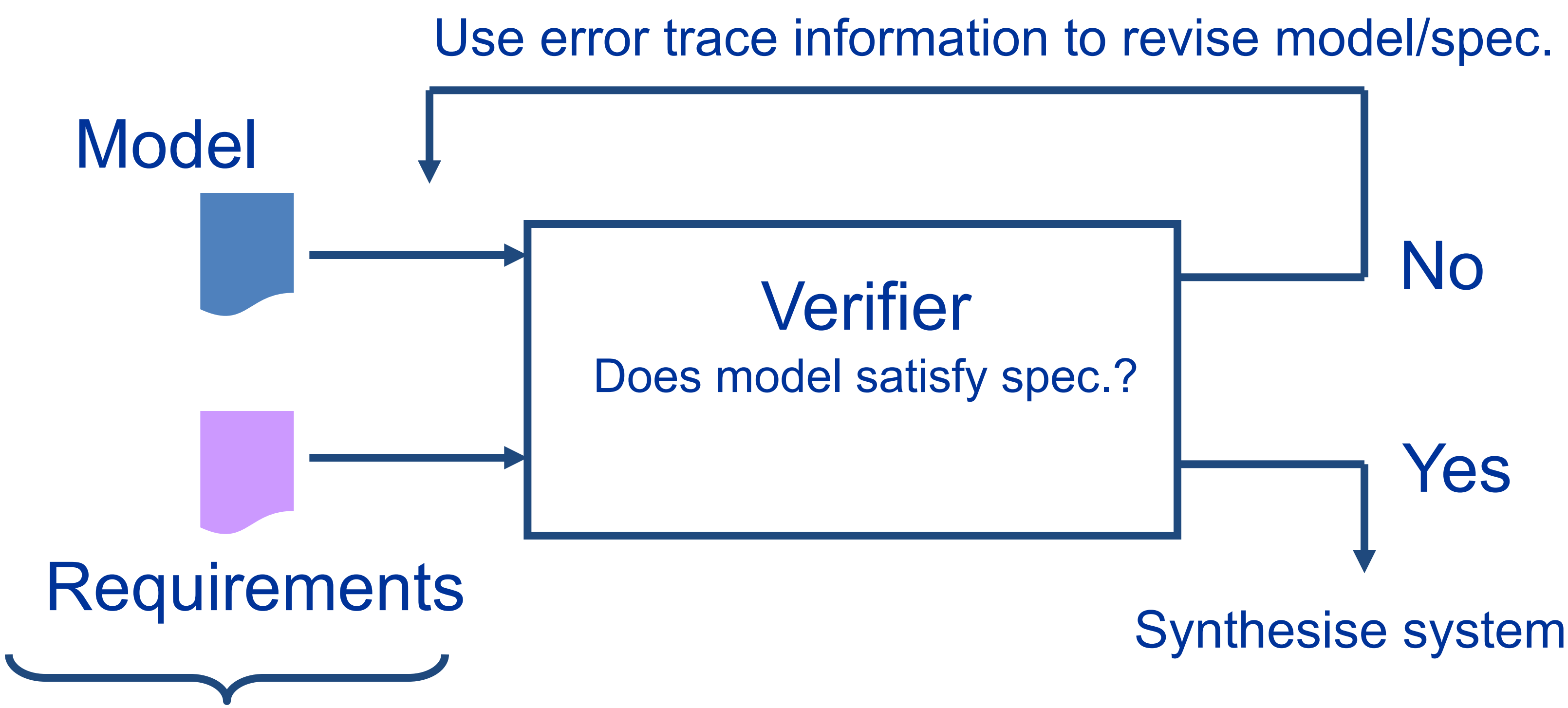
- **Verification**

  Does the designed system achieve its objective in the operating environment?

- **Controller Synthesis**

  Given an incomplete design, synthesise a strategy to complete the system so that it achieves its objective in the operating environment.

Model-Based Design: Verification & Synthesis



Need a formal way to write models and requirements (specification) so that an algorithm can process it

Temporal Logic

- A formal way to express properties of a system over time
  - e.g., behaviour of an FSM

- Many flavours of temporal logic
  - Propositional temporal logic (we will study this today)
  - Real-time temporal logic
  - Signal temporal logic
  - Interval temporal logic
  - …

Example: Specification of the SpaceWire Protocol (European Space Agency standard)

8.5.2.2 ErrorReset

a. The *ErrorReset* state shall be entered after a system reset, after link operation is terminated for any reason or if there is an error during link initialization.

b. In the *ErrorReset* state the Transmitter and Receiver shall all be reset.

c. When the reset signal is de-asserted the *ErrorReset* state shall be left unconditionally after a delay of 6,4 μs (nominal) and the state machine shall move to the *ErrorWait* state.

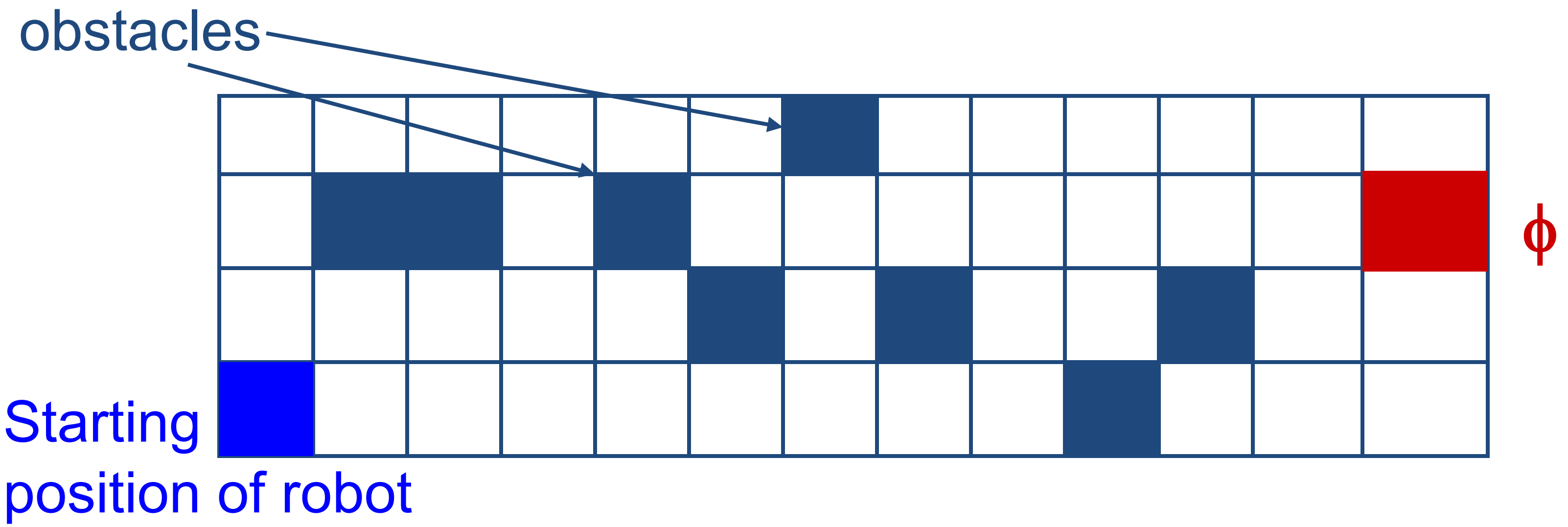
d. Whenever the reset signal is asserted the state machine shall move immediately to the *ErrorReset* state and remain there until the reset signal is de-asserted.

Example from Interrupts Lecture

```
    volatile uint timerCount = 0;
    void ISR(void) {
D →    … disable interrupts
       if(timerCount != 0) {
E →       timerCount--;
       }
       … enable interrupts
    }
    int main(void) {
       // initialization code
       SysTickIntRegister(&ISR);
       … // other init
A →    timerCount = 2000;
B →    while(timerCount != 0) {
        … code to run for 2 seconds
       }
C → … whatever comes next
    }
```

Property:
Assuming interrupts can occur infinitely often, it is always the case that position C is reached.

Robotic Navigation: Specifying Goals



ϕ = destination for robot

Specification:

The robot eventually reaches ϕ

Suppose there are n destinations $\phi_1, \phi_2, \ldots, \phi_n$

The new specification could be that

The robot visits $\phi_1, \phi_2, \ldots, \phi_n$ in that order

Propositional Logic

Atomic formulas: Statements about an input, output, or state of a state machine (at the current time).

Examples:

formula	meaning
x	x is *present*
$x = 1$	x is *present* and has value 1
s	machine is in state s

These are propositions (true or false statements) about a state machine with input or output x and state s.

Propositional Logic

- **Propositional logic formulas**: More elaborate statements about an input, output, or state of a state machine (at the current time). Examples:

formula	meaning
$p_1 \wedge p_2$	p_1 and p_2 are both true
$p_1 \vee p_2$	either p_1 or p_2 is true
$p_1 \implies p_2$	if p_1 is true, then so is p_2
$\neg p_1$	true if p_1 is false

- Here, p_1 and p_2 are either atomic formulae or propositional logic formulas.

Execution Trace of a State Machine

An **execution trace** is a sequence of the form

$$q_0,\ q_1,\ q_2,\ q_3,\ \ldots,$$

where $q_j = (x_j, s_j, y_j)$ where s_j is the state at step j, x_j is the input valuation at step j, and y_j is the output valuation at step j. Can also write as

$$s_0 \xrightarrow{x_0/y_0} s_1 \xrightarrow{x_1/y_1} s_2 \xrightarrow{x_2/y_2} \cdots$$

In the context of dynamical systems the trace is called system's trajectory or system's behaviour.

Propositional Logic on Traces

A propositional logic formula p **holds** for a trace

$$q_0,\ q_1,\ q_2,\ q_3,\ \ldots,$$

if and only if it holds for q_0.

This may seem odd, but we will provide temporal logic operators to reason about the trace.

Linear Temporal Logic (LTL)

- **LTL formulae**: Statements about an execution trace

$$q_0,\ q_1,\ q_2,\ q_3,\ \ldots,$$

formula	meaning
p	p holds in q_0
$\mathbf{G}\phi$	ϕ holds for every suffix of the trace
$\mathbf{F}\phi$	ϕ holds for some suffix of the trace
$\mathbf{X}\phi$	ϕ holds for the trace $q_1, q_2, \cdots$
$\phi_1 \mathbf{U} \phi_2$	ϕ_1 holds for all suffixes of the trace until a suffix for which ϕ_2 holds.

- Here, p is propositional logic formula and ϕ is either a propositional logic or an LTL formula.

Linear Temporal Logic (LTL)

- **LTL formulae**: Statements about an execution trace

$$q_0, q_1, q_2, q_3, \ldots,$$

formula	mnemonic
p	proposition
$\mathbf{G}\phi$	globally
$\mathbf{F}\phi$	finally, future, eventually
$\mathbf{X}\phi$	next state
$\phi_1 \mathbf{U} \phi_2$	until

- Here, p is propositional logic formula and ϕ is either a propositional logic or an LTL formula.

First LTL Operator: G (Globally)

The LTL formula $\mathbf{G}p$ **holds** for a trace

$$q_0, q_1, q_2, q_3, \ldots,$$

if and only if p holds for every suffix of the trace:

$$\begin{array}{r}
q_0, q_1, q_2, q_3, \ldots \\
q_1, q_2, q_3, \ldots \\
q_2, q_3, \ldots \\
q_3, \ldots
\end{array}$$

If p is a propositional logic formula, this means it holds for each q_i.

G p for propositional formula p, is also termed an invariant

Second LTL Operator: F (Eventually, Finally, Future)

The LTL formula $\mathbf{F}p$ **holds** for a trace

$$q_0,\ q_1,\ q_2,\ q_3,\ \ldots,$$

if and only if p holds for some suffix of the trace:

$$\begin{aligned} q_0,\ q_1,\ q_2,\ q_3,\ &\ldots \\ q_1,\ q_2,\ q_3,\ &\ldots \\ q_2,\ q_3,\ &\ldots \\ q_3,\ &\ldots \end{aligned}$$

If p is a propositional logic formula, this means it holds for some q_i.

Propositional Linear Temporal Logic

LTL operators can apply to LTL formulae as well as to propositional logic formulae.

e.g. Every input x is eventually followed by an output y

$$\mathbf{G}(x \implies \mathbf{F}y)$$

Globally
(at any point in time)

If x occurs

It is *eventually* followed by y

Every input x is eventually followed by an output y

The LTL formula $\mathbf{G}(x \implies \mathbf{F}y)$ holds for a trace

$$q_0, q_1, q_2, q_3, \ldots,$$

if and only if it holds for any suffix of the trace where x holds, there is a suffix of that suffix where y holds:

$$\begin{array}{rl} q_0, q_1, q_2, q_3, & \ldots \\ q_1, q_2, q_3, & \ldots \\ q_2, q_3, & \ldots \\ q_3, & \ldots \end{array}$$

x holds

y holds

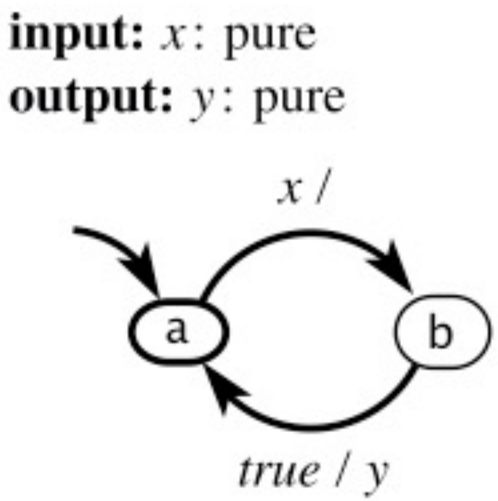
When is a Temporal Logic formula satisfied by a State Machine?

- A linear temporal logic (LTL) formula is satisfied by a state machine iff *every trace* of that state machine satisfies the LTL formula.

Example 1

Does the following temporal logic property hold for the state machine below?

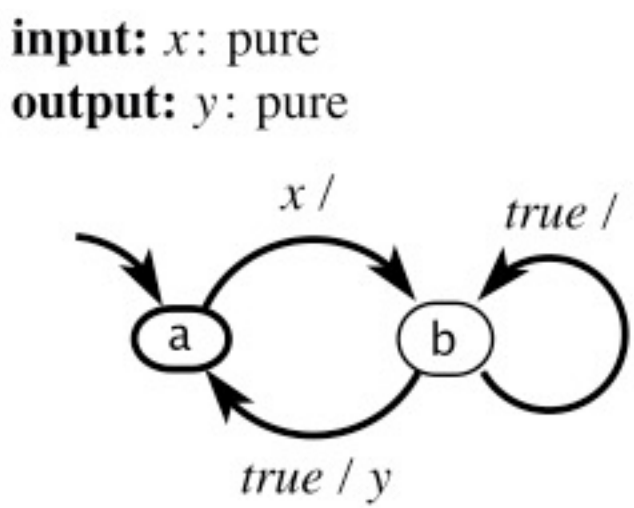
$$\mathbf{G}(x \implies \mathbf{F}y)$$



Example 2

Does the following hold?

$$\mathbf{G}(x \implies \mathbf{F}y)$$



Third LTL Operator: X (Next)

The LTL formula $\mathbf{X}p$ **holds** for a trace

$$q_0,\ q_1,\ q_2,\ q_3,\ \ldots,$$

if and only if it holds for the suffix $q_1, q_2, q_3, \ldots$

$$\begin{aligned} q_0,\ q_1,\ q_2,\ q_3,\ &\ldots \\ q_1,\ q_2,\ q_3,\ &\ldots \\ q_2,\ q_3,\ &\ldots \\ q_3,\ &\ldots \end{aligned}$$

Fourth LTL Operator: U (Until)

The LTL formula $p_1 \mathbf{U} p_2$ **holds** for a trace

$$q_0, q_1, q_2, q_3, \ldots,$$

if and only if p_2 holds for some suffix of the trace, and p_1 holds for all previous suffixes:

$$q_0, q_1, q_2, q_3, \ldots$$
$$q_1, q_2, q_3, \ldots$$
$$q_2, q_3, \ldots$$
$$q_3, \ldots$$

p_1 holds

p_2 holds (and maybe p_1 also)

Note: A variant, called "weak until," written W, does not require p_2 to eventually hold. The "U" version does.

Alternate Notation

- Sometimes you'll see alternative notation in the literature:

$\mathrm{G} \qquad \square$

$\mathrm{F} \qquad \Diamond$

$\mathrm{X} \qquad \bigcirc \qquad \mathbf{N}$

Examples: What do they mean?

<u>Remember:</u>

Gp p holds in all states
Fp p holds eventually
Xp p holds in the next state

- Infinitely many occurrences: **G F** *p*
  - *p holds infinitely often*
- Steady-state property: **F G** *p*
  - *Eventually, p holds henceforth*
- Request-response property: **G**(*p* => **F** *q*)
  - *Every p is eventually followed by a q*
- **F**(*p* => (**X X** *q*))
  - *If p occurs, then on some occurrence it is followed by a q two reactions later*

Temporal Operators & Relationships

- G, F, X, U: All express properties along system traces

- Can you express G p purely in terms of F, p, and Boolean operators?

$$\mathbf{G}\phi = \neg\mathbf{F}\neg\phi$$

- How about F in terms of U?

$$\mathbf{F}\phi = \mathit{true}\ \mathbf{U}\ \phi$$

- What about X in terms of G, F, or U?

Cannot be done

Some Points to Ponder

- A mathematical specification only includes properties that the system must or must not have
  - It requires human judgement to decide whether that specification constitutes "correctness"

- Getting the specification right is often as hard as getting the design right!

- Interesting research directions:
  - Inferring temporal logic from system traces
  - Translating natural language into (temporal) logic
  - Specification of temporal logic specifications for local subsystems so that the overall system meets a set of desired specifications.

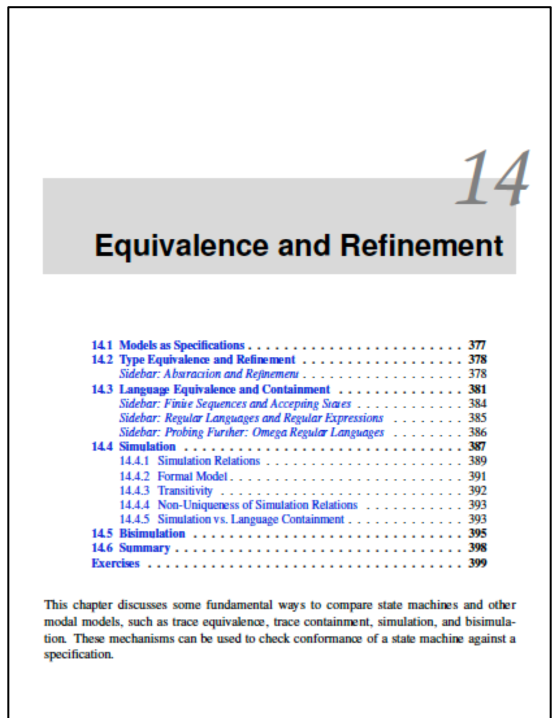
Exercises: Write in Temporal Logic

1. "Whenever the Kobuki is at the ramp-edge (cliff), eventually it moves 5 cm away from the cliff."
   - p – Kobuki is at the cliff
   - q – Kobuki is 5 cm away from the cliff

2. "Whenever the distance between cars is less than 2m, cruise control is deactivated"
   - p – distance between cars is less than 2 m
   - q – cruise control is active

Things to do …

- Next lecture: Equivalence and Refinement
- Read Chapter 14



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Equivalence and Refinement

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This chapter discusses some fundamental ways to compare state machines and other modal models, such as trace equivalence, trace containment, simulation, and bisimulation. These mechanisms can be used to check conformance of a state machine against a specification.