Lecture 10

Examples of Bode diagrams

Outline

- Recap of Bode diagrams
- Bode via Matlab
- Examples of Bode diagrams
- Conclusions

Bode via Matlab

- To get the magnitude in decibels, use

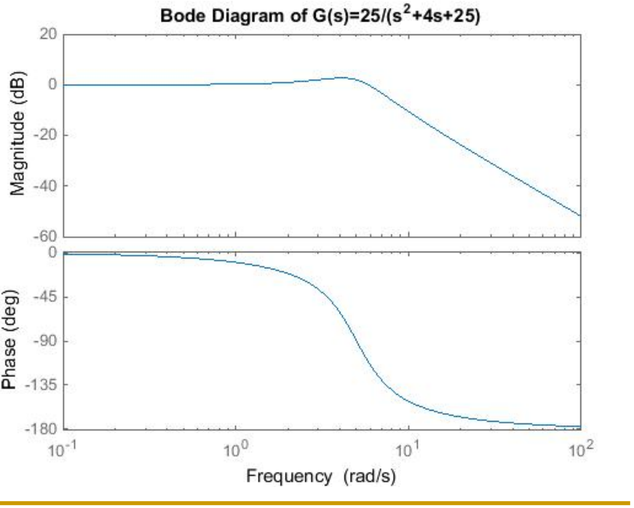
magdB=20*log10(mag)

- For this transfer function:

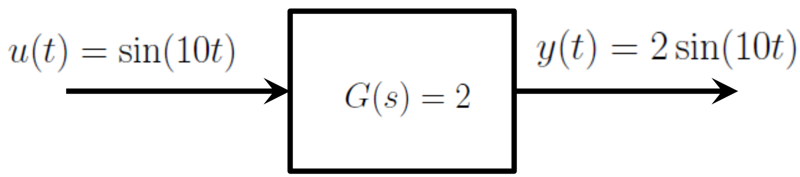
$$G(s) = \frac{25}{s^2+4s+25}$$

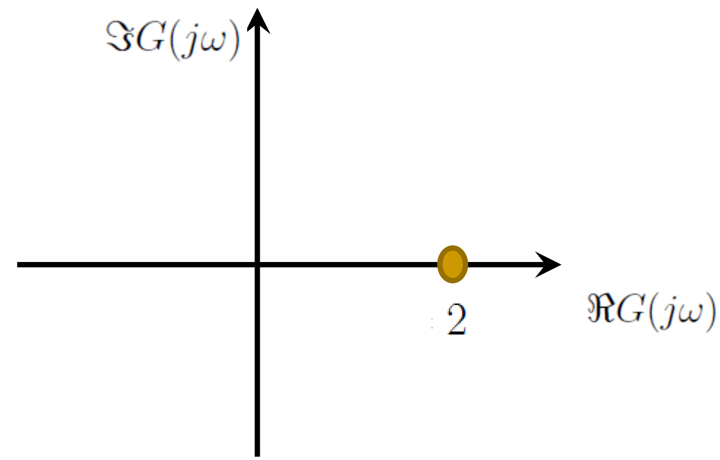
use

```
num=[0 0 25]
den=[1 4 25]
bode(num,den)
title('Bode Diagram of G(s)=25/(s^2+4s+25)')
```



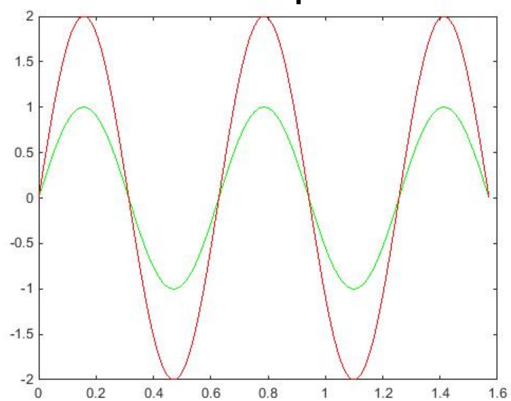
Bode diagram of a positive constant

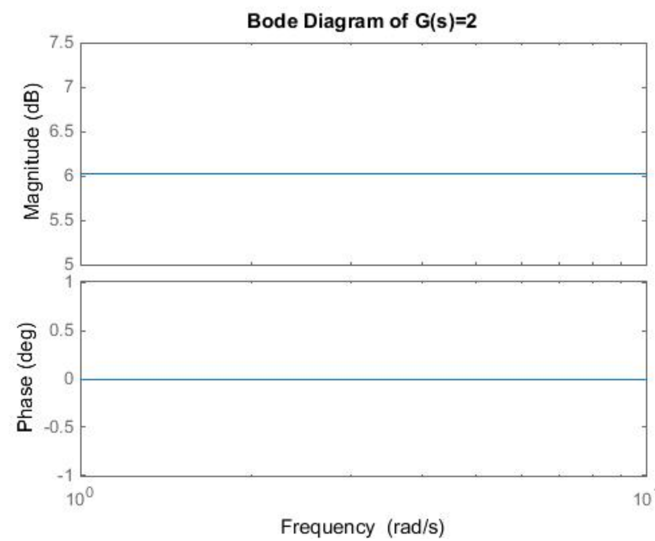




$$G(j\omega) = 2, \ \forall \omega \in (-\infty, +\infty)$$

Time response

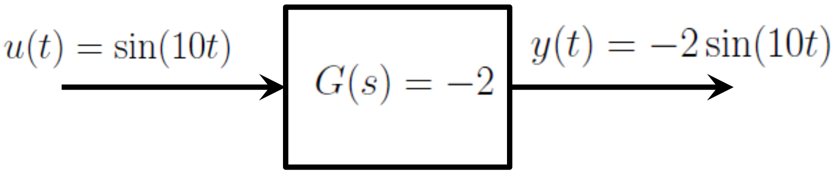


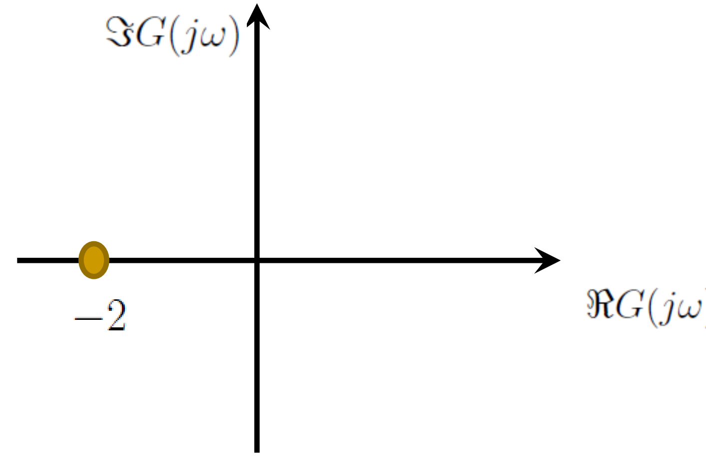


$$20 \log |G(j\omega)| = 20 \log 2 = 6.0206$$

$$\angle G(j\omega) = 0$$

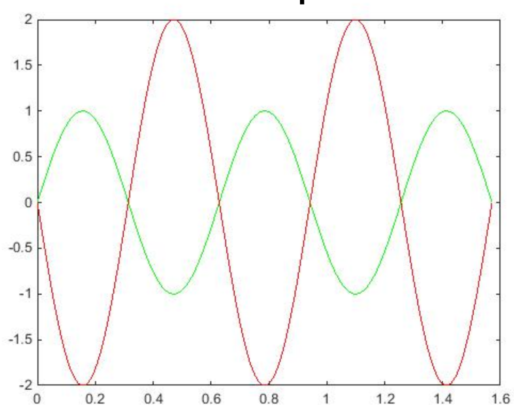
Bode diagram of a negative constant

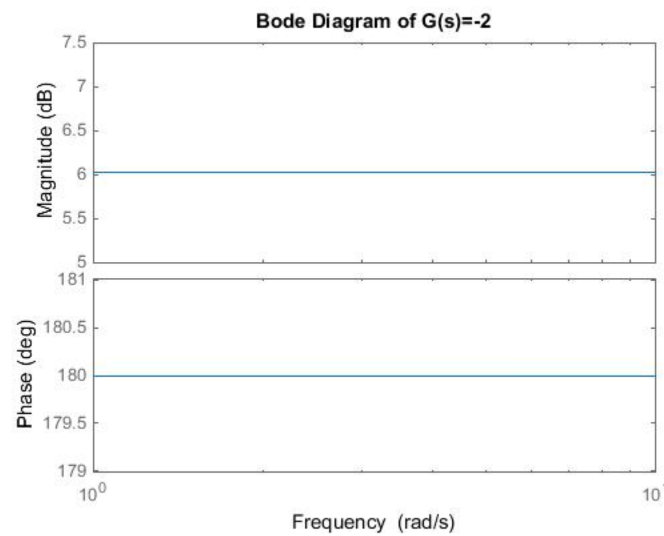




$$G(j\omega) = -2, \ \forall \omega \in (-\infty, +\infty)$$

Time response



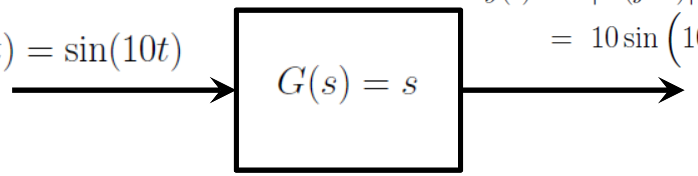


$$20 \log |G(j\omega)| = 20 \log 2 = 6.0206$$

$$\angle G(j\omega) = 180°$$

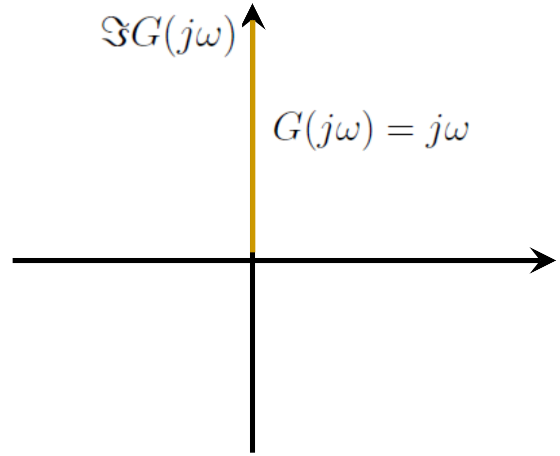
Bode diagram of a differentiator

$u(t) = \sin(10t)$

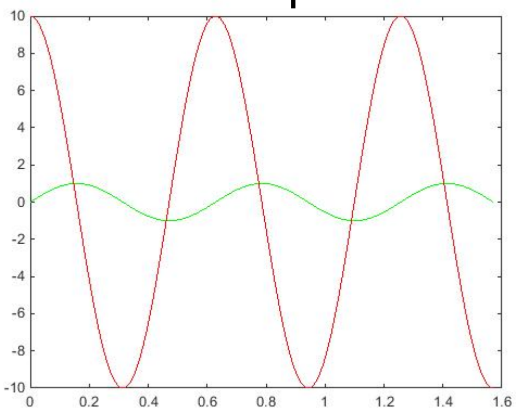


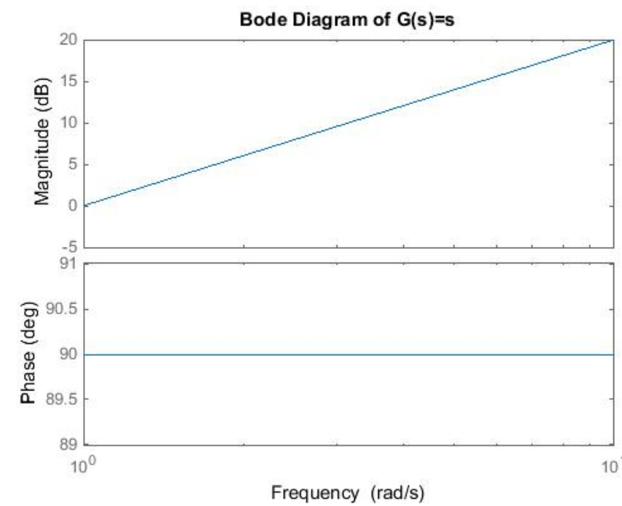
$$
\begin{aligned}
y(t) &= |G(j10)| \sin\left(10t + \angle G(j10)\right) \\
&= 10 \sin\left(10t + \frac{\pi}{2}\right)
\end{aligned}
$$

$G(j\omega) = j\omega$



Time response



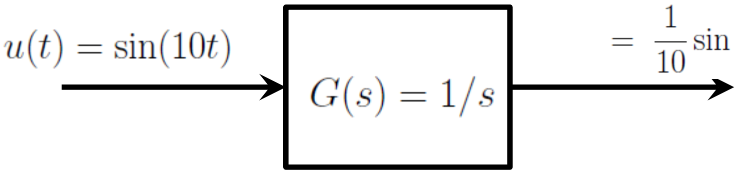


$|G(j\omega)| = |\omega|$

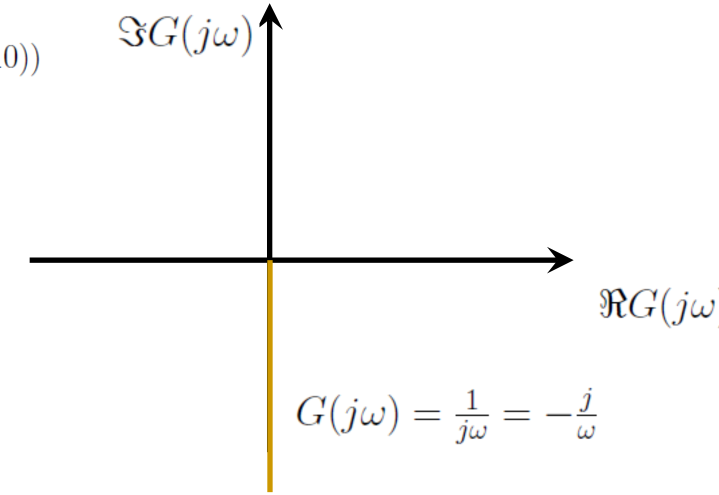
$\angle G(j\omega) = 90°$

Bode diagram of an integrator

$$
\begin{aligned}
y(t) &= |G(j10)| \sin\left(10t + \angle G(j10)\right) \\
&= \frac{1}{10} \sin\left(10t - \frac{\pi}{2}\right)
\end{aligned}
$$

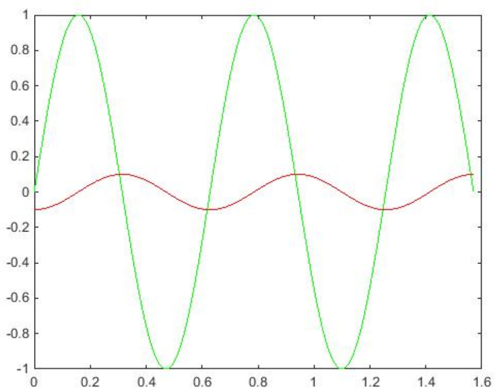


$$G(j\omega) = \frac{1}{j\omega} = -\frac{j}{\omega}$$



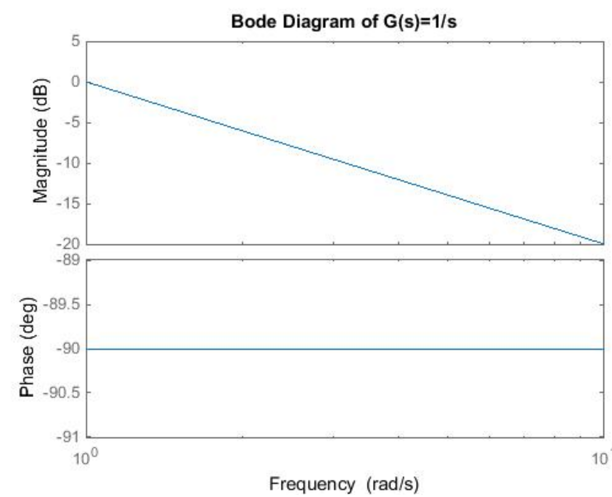
$$G(j\omega) = \frac{1}{j\omega} = -\frac{j}{\omega}$$

Time response

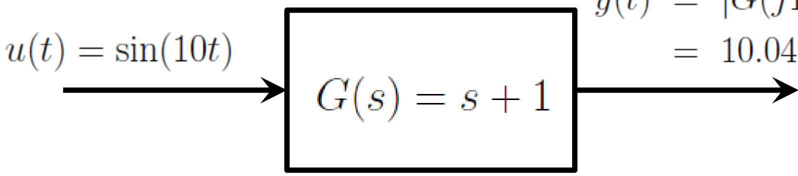


$$|G(j\omega)| = \frac{1}{|\omega|}$$

$$\angle G(j\omega) = -90^\circ$$

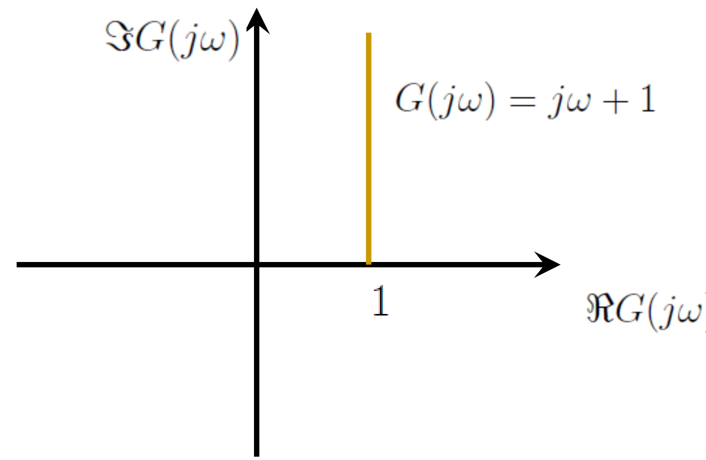


Bode diagram of a “ real zero” term

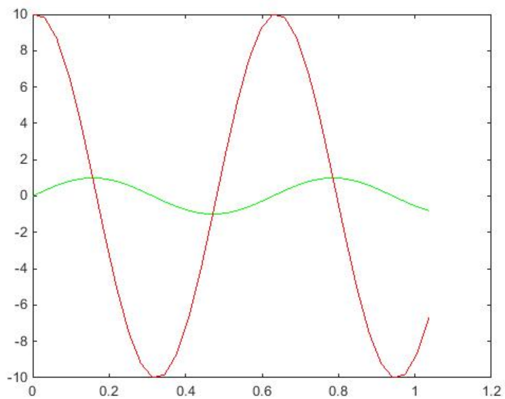


$$
\begin{aligned}
y(t) &= |G(j10)| \sin\left(10t + \angle G(j10)\right) \\
&= 10.0499 \sin\left(10t + 1.4711\right)
\end{aligned}
$$

$G(j\omega) = j\omega + 1$

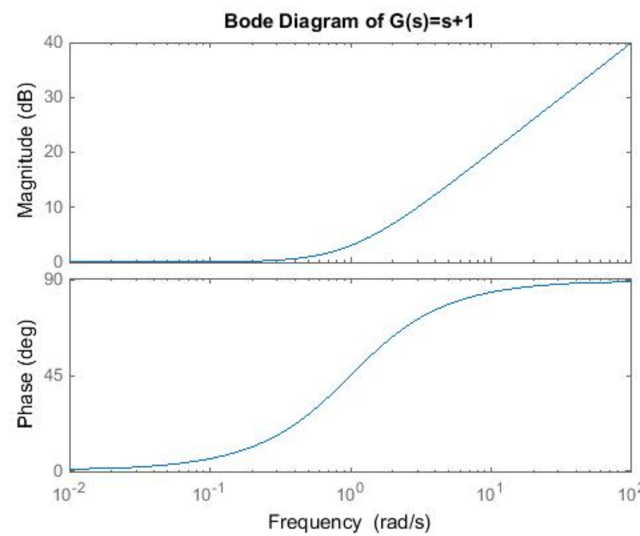


Time response



$|G(j\omega)| = \sqrt{1 + \omega^2}$

$\angle G(j\omega) = \tan^{-1} \frac{\Im G(j\omega)}{\Re G(j\omega)} = \tan^{-1} \omega$



Asymptotic behaviour

- For small frequencies the system behaves like positive constant gain

$$\omega << 1 \implies |G(j\omega)| = \sqrt{1+\omega^2} \approx 1$$

$$\omega << 1 \implies \angle G(j\omega) = \tan^{-1}\omega \approx \tan^{-1} 0 = 0$$

- For large frequencies the system behaves like a differentiator:

$$\omega >> 1 \implies |G(j\omega)| = \sqrt{1+\omega^2} \approx |\omega|$$

$$\omega >> 1 \implies \angle G(j\omega) = \tan^{-1}\omega \approx \tan^{-1} \infty = 90^\circ$$

A useful relationship

- For reciprocal factors, Bode diagrams just need to "change sign":

$$20\log\left|\frac{1}{G(j\omega)}\right| = -20\log|G(j\omega)|$$

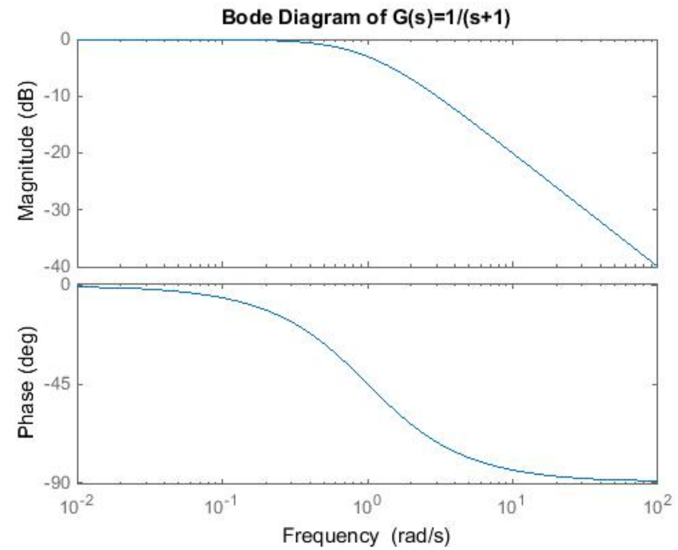
$$\angle\frac{1}{G(j\omega)} = -\angle G(j\omega)$$

Bode diagram of a "real pole" term

- For instance, we have for $G(s) = \frac{1}{s+1}$

$$20 \log \left| \frac{1}{j\omega+1} \right| = -20 \log |j\omega + 1| = -20 \log \sqrt{\omega^2 + 1}$$

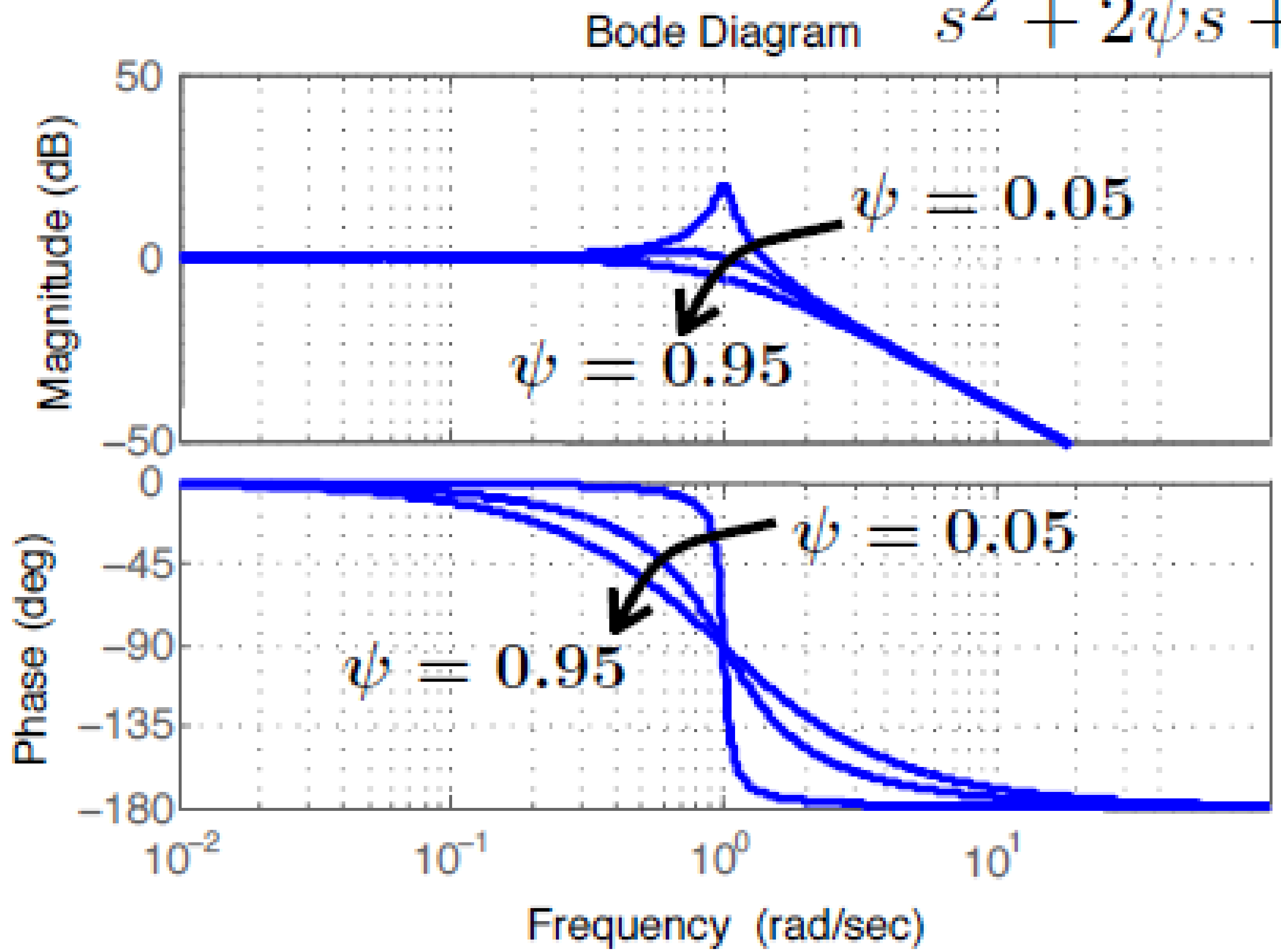
$$\angle \frac{1}{j\omega+1} = -\angle(j\omega + 1) = -\tan^{-1} \omega$$



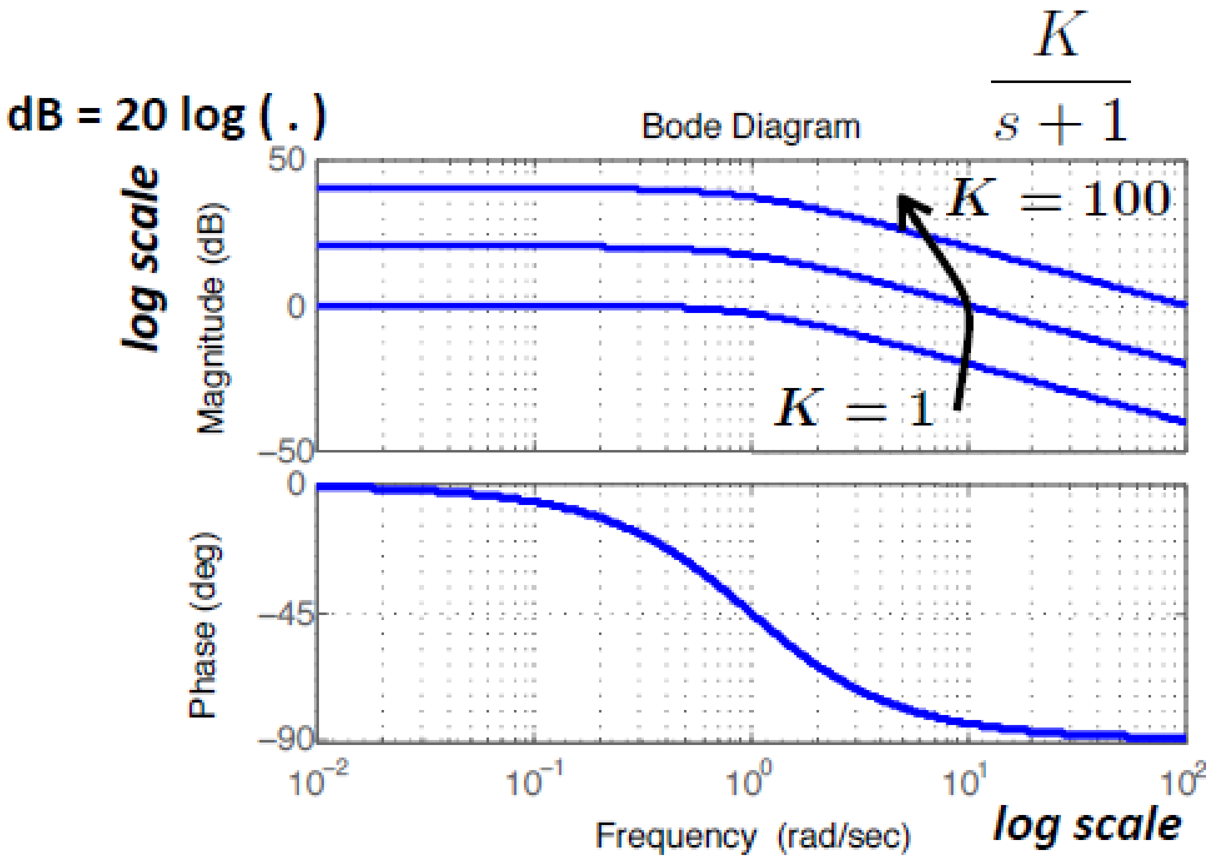
For exercise find the time response of the system as we did in previous examples.

A second order system

$$\frac{1}{s^2 + 2\psi s + 1}$$



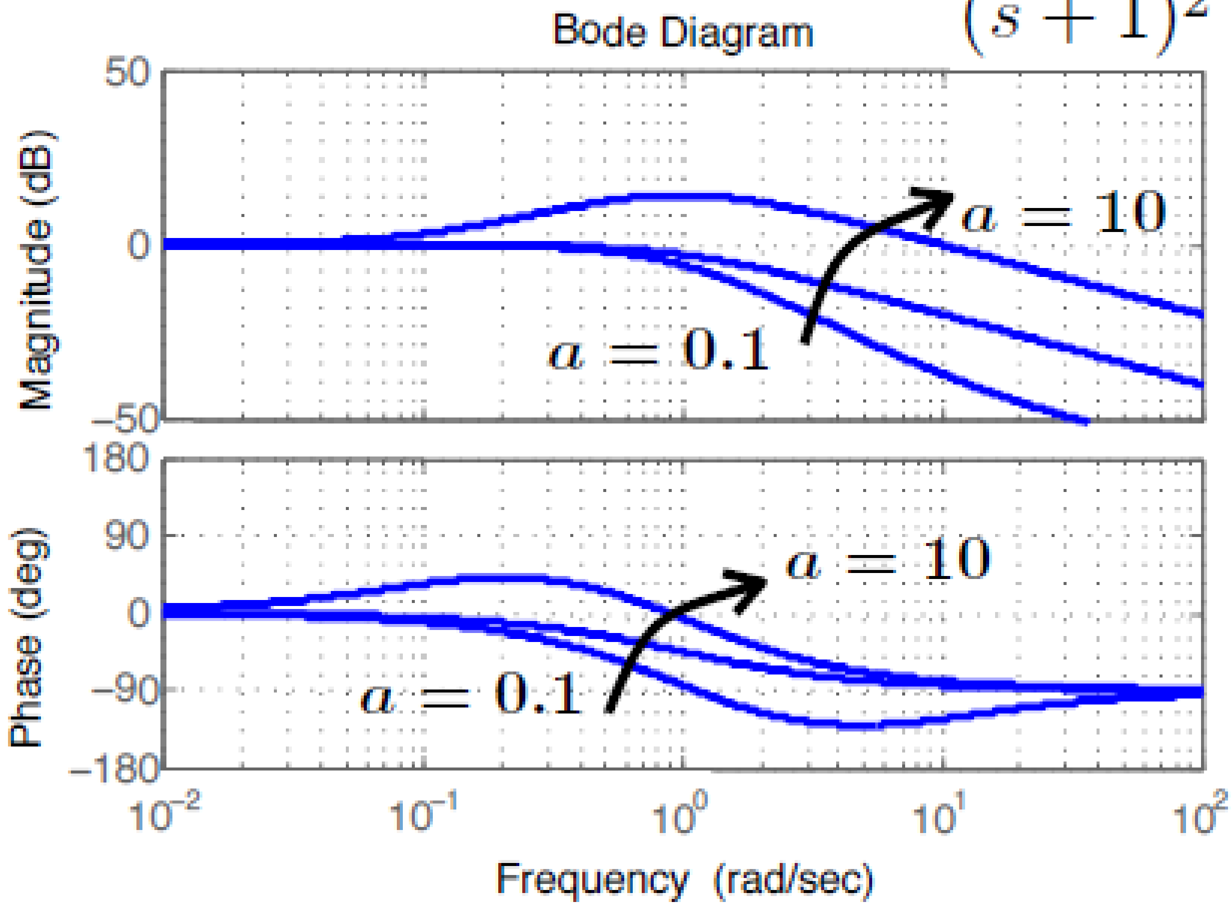
Example



dB = 20 log (.)
log scale
Bode Diagram
$\frac{K}{s+1}$
$K = 100$
$K = 1$
Magnitude (dB)
50
0
−50
Phase (deg)
0
−45
−90
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/sec)
log scale

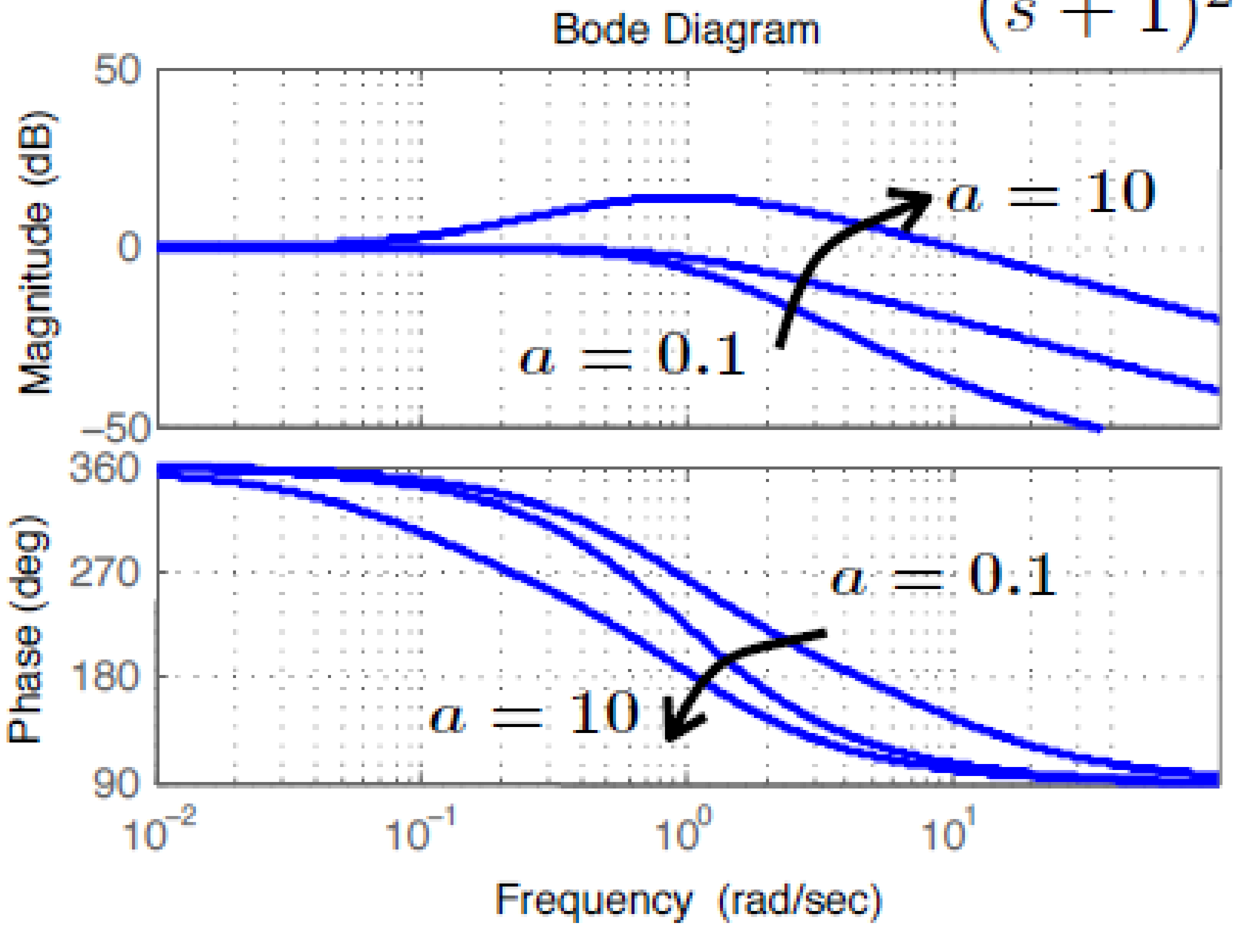
Example

$$\frac{as+1}{(s+1)^2}$$

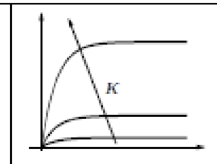
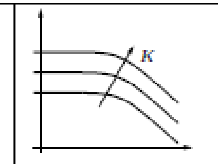
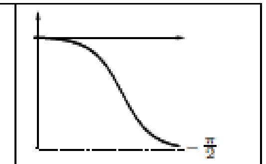
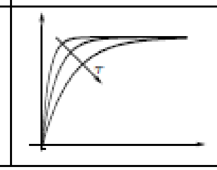
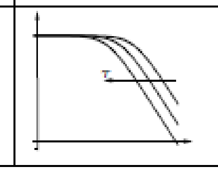
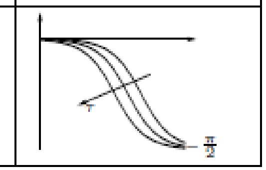
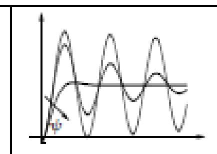
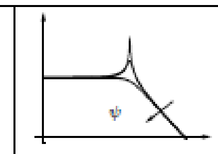
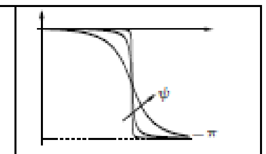
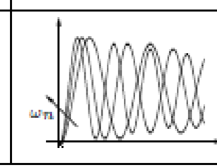
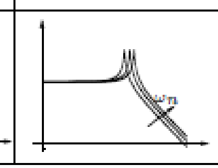
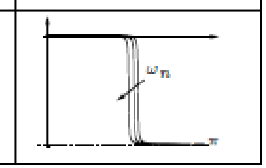
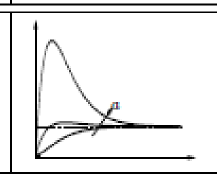
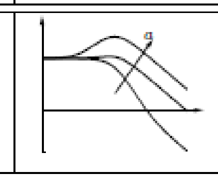
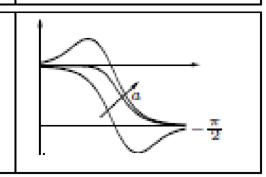
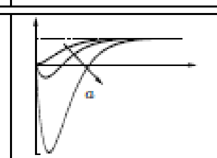
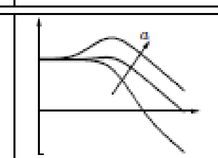
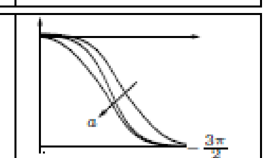


Example

$$\frac{-as+1}{(s+1)^2}$$



Summary of common cases

System	Parameter	Step response	Bode (gain)	Bode(phase)
$\dfrac{K}{\tau s+1}$	K			
	τ			
$\dfrac{\omega_n^2}{s^2+2\psi\omega_n s+\omega^2}$	ψ			
	ω_n			
$\dfrac{as+1}{(s+1)^2}$	a			
$\dfrac{-as+1}{(s+1)^2}$	a			

Taken from Goodwin, Graebe and Salgado, Control system design.

Example

$$\frac{(s+1)(s-50)}{s(s-0.1)(s+10)} = \frac{50(s+1)(0.02s-1)}{s(10s-1)(0.1s+1)}$$

$$\frac{1}{s} = \lim_{k\to\infty} \frac{k}{ks+1}$$