LESSON
1

A Picture Is Worth a Thousand Words

Understanding Quantities and Their Relationships

In this lesson, students identify the independent and dependent quantities in given contexts. They match graphs to the situations they model. Students then compare and contrast the graphs' characteristics, noticing that graphs are lines, connected line segments, or curves that increase, decrease, or both increase and decrease. They write a situation and sketch a graph for a given situation.

My Learning Goals

- I can use a reasonable scale for a graph modeling a situation.
- I can identify the independent and dependent quantities for a situation.
- I can identify key characteristics of graphs.

California High School

CALIFORNIA REPUBLIC

Big Ideas
Function Investigations
Features of Functions

Number and Quantity Standards
Quantities

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★

2. Define appropriate quantities for the purpose of descriptive modeling. ★

Functions Standard
Interpreting Functions

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★

Make a Connection

You have analyzed graphs of relationships and identified important features such as intercepts and slopes.

Learning Prompt

How can the key characteristics of a graph tell a story?

Setting the Stage

❯ Communicate the learning goals and key terms to look for.

❯ Tap into your students' prior learning by reading the *Make a Connection*.

❯ Provide a sense of direction by asking the *Learning Prompt*.

When one quantity depends on another, it is a dependent quantity. The quantity it depends upon is the independent quantity. The independent quantity is represented on the x-axis, and the dependent quantity is represented on the y-axis.

⫘MLL Language Goal

I can interpret and discuss real-world situations involving independent and dependent quantities using comparative language (e.g., *as x increases, y decreases*).

🪜 Content Connection

Exploring Changing Quantities

Lesson Structure and Pacing Guide

Topic 1 Quantities and Relationships

Session 1

Activate

~**5** minutes

Explaining Their Thinking

What Comes First?
Students identify the independent and dependent quantities in given situations.

Explore and Develop

~**25** minutes

Activity 1   **Real-World Problem Solving, Guided Practice**

Connecting Situations and Their Graphs
Students match graphs to situations and then compare and contrast the characteristics of the graphs.

Materials: Glue sticks or tape, *Graphs of Situations* Cutouts, scissors

Reflect

~**15** minutes

Exit Ticket Application

A Writer and a Mathematician
Students write a situation and sketch a graph for a possible trip to school.

Essential Ideas

- When one quantity depends on another in a situation, it is the dependent quantity. The quantity it depends on is the independent quantity.
- You can use a graph to model a situation.
- Label the x-axis with the independent quantity and the y-axis with the dependent quantity.
- Characteristics of a graph include whether the graph increases or decreases, is composed of line segments or a curve, or has a maximum or minimum point.

Math Language Routine ⋮ MLR

MLR 4: Information Gap

In Activity 1, *Connecting Situations and Their Graphs*, students determine the independent and dependent quantities for eight situations, match each of the situations to its graph, label the x- and y-axis with the appropriate quantity and a reasonable scale, and interpret the meaning of the origin for each situation. This is an opportunity to implement **Information Gap**.

1. Follow the guidance for facilitating the activity.
2. When students need additional support in matching a situation with its graph, explain how they need to determine the y-intercept of the situation by seeing what the value is when the independent variable is equal to 0 and explain how they need to determine how the graph changes over time, such as whether it changes at a constant rate over time or whether it decreases or increases or does some of both.
3. Have students complete the activity with a partner or small group.
4. Monitor students while they work to assess understanding.
5. Once students have finished the activity, have them discuss their experiences and share their strategies for matching each situation with the correct graph as a whole class.

Multilingual Learner Supports ⋛ MLL

This lesson provides scaffolded supports tailored for students at different English proficiency levels—Emerging, Expanding, and Bridging—to meet the Learning and Language Goals. These progressively challenging suggestions build language and mathematical understanding.

Go online to access the Multilingual Learner Support Handbook.

Visual Aids and Gestures

Use visual organizers, simplified graphs, hand motions, and cards to represent quantities, trends, and graph characteristics.

Sentence Frames

Provide sentence frames and starters for various proficiency levels to describe relationships and graph characteristics.

Word Banks

Include key mathematical terms for quantities, graph characteristics, relationships, and rate of change descriptions.

Partner/Group Work

Encourage peer discussions to compare graphs, explain situations, and engage in pair activities and peer reviews.

Hands-On Activities

Implement matching activities, situation acting, and physical objects to illustrate concepts and graph features.

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Activate

~5 minutes

Purpose: To identify the independent and dependent quantities in given situations

- 🖱 Go online to choose a variation of the activity.
- ❯ Display, read, and discuss the situation and quantity pairs.
- ❯ Use a Think-Pair-Share routine to discuss the activity.
- ❯ Share and summarize: When one quantity depends on another in a situation, it is the dependent quantity. The quantity it depends on is the independent quantity.
- 🖱 Consider using any embedded supports for this activity located in the Clear Lesson Player.

Explore and Develop   Activity 1

~25 minutes

Purpose: To match graphs to situations and then compare and contrast the characteristics of the graphs

Materials: Glue sticks or tape, *Graphs of Situations* Cutouts, scissors

⭐ **Driver of Investigation**
Make sense of the world.

- ❯ Read and discuss the introduction and directions.
- ❯ Group students to complete the activity.
- ❯ Share and summarize: To match a situation with its graph, you first need to identify the independent and dependent quantities to recognize the growth pattern. A graph's characteristics, such as whether it is a line or curve or increasing or decreasing, describe a relationship between two quantities.

Activate | **What Comes First?**

🖳 Consider the situation and quantity pairs shared by your teacher. Prepare to share your thinking with your classmates.

Explore and Develop | Activity 1

Connecting Situations and Their Graphs

Habits of Mind **SMP**
- Model with mathematics.
- Use appropriate tools strategically.

Graphs relay information about relationships in a visual way. You can use lines or smooth curves to represent relationships between points on a graph. In some problem situations, all the points on the line will make sense. In other problem situations, not all the points will make sense. So, when you model a relationship with a line or a curve, it is up to you to consider the situation and interpret the meaning of the data values.

Graphs of Situations — Cutouts

- ❯ **Cut out the graphs provided by your teacher. Then, read each of the six situations in this activity.**
  - Determine the independent and dependent quantities.
  - Match each situation to its corresponding graph. Glue the graph next to the situation.
  - For each graph, label the *x*- and *y*-axis with the appropriate quantity and a reasonable scale, and then interpret the meaning of the origin.

1 **Daredevil** **Graph E**

Jared completes a dive from a cliff 100-feet above a river. It takes him only 1.7 seconds to hit the water.

Think About . . .
Be sure to include the appropriate units of measure for each quantity.

- Independent quantity:
  Time (seconds)
- Dependent quantity:
  Jared's height above the water (feet)
  Origin: (0 seconds, 0 feet above the water)

Purposeful Questions   Activity 1

1 – 6 [Probing]   Did you select a graph that was increasing or decreasing? Why?

How did you determine the units and intervals for the axes?

What is the meaning of the *x*-intercept? The *y*-intercept?

1 [Probing]   Is the dependent quantity the distance Pablo traveled or his height above water? Explain your thinking.

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2 Something's Fishy Graph F

Parker is cleaning a 200-gallon office aquarium. She removes the fish and drains the water at a constant rate of 10 gallons per minute.

- Independent quantity:
  Time (minutes)

- Dependent quantity:
  Water (gallons)

 **Origin: (0 minutes, 0 gallons of water)**

Ask Yourself . . .
What strategies will you use to match each graph with one of the six situations?
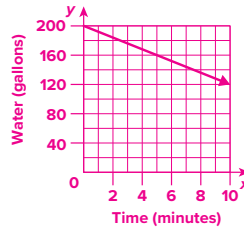

3 Smart Phone, But Is It a Smart Deal? Graph B

You want an upgraded smartphone but lack the funds. Your cousin offers to lend you the money with interest, starting at $1 and doubling weekly. You wonder if it's a good deal.

- Independent quantity:
  Time (weeks)

- Dependent quantity:
  Interest (dollars)

  Origin: (0 weeks, 0 dollars of interest)
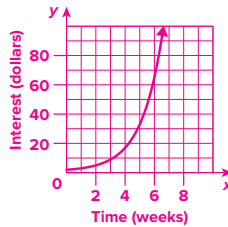
4 Baton Twirling Graph D

Juniper, the Altadena High drum major, tosses her baton 22 feet high during the halftime finale, giving her 2 seconds to twirl twice and catch it.

- Independent quantity:
  Time (seconds)

- Dependent quantity:
  Height of baton (feet)

  Origin: (0 seconds, height of 0 feet)

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Topic 1   Lesson 1   2 3 4 5 6   5
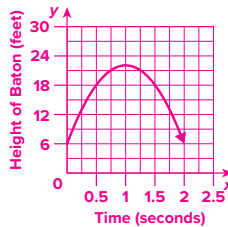
Differentiation Strategy

Just-in-Time Support
Engagement: Sustaining Effort & Persistence
To make the task more manageable, split it into two parts. Provide Graphs B, E, and F for the first three situations and Graphs A, C, and D for the last three situations.

Ongoing Assessment

Observe students as they match each situation to its graph.

- Are students using characteristics of the graphs to connect them to the situations?

- Are students using clues from the situations to determine the intervals?

NOTE: *This is the first pairing of these Habits of Mind:*

- *Model with mathematics.*

- *Use appropriate tools strategically.*

Have students refer to page xix for questions to ask themselves.

NOTE: *All situations provide opportunities for students to learn something new.*

Cultivate Connections

Considering an upgraded smartphone provides an opportunity to learn about classmates' experiences. Consider having students share examples of items they own that they have upgraded over time, or discuss the impact of mass tech upgrades on the environment.

Purposeful Questions Activity 1

2 | Probing | Is the dependent quantity the amount of water draining or the remaining water in the aquarium? Explain your thinking.

| Seeing Structure | How did you know the graph is a straight line?

3 | Probing | Explain how the graph shows the interest for the first several weeks.

| Seeing Structure | Why did you use a graph with a curve to represent this situation?

4 | Probing | What does each point on the graph represent?

| | What characteristic does this graph have that the others do not?

| Seeing Structure | When the baton goes straight up and comes straight back down, why isn't the graph a vertical segment?

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NOTE: *Once students have matched a graph with the appropriate situation, have them go back and examine all the graphs. Have students discuss the following:*

- *What similarities do you notice in the graphs?*
- *What differences do you notice in the graphs?*
- *How did you label the independent and dependent quantities in each graph?*
- *Analyze each graph from left to right. Describe any graphical characteristics you notice.*

Student Look-Fors

Social-Emotional Learning

Using self-motivation and self-discipline to persevere in problem solving

≧ MLL Multilingual Learner Support

Provide additional vocabulary support with illustrated examples of these terms: *increase*, *decrease*, *straight*, *curved*, *maximum*, and *minimum*.

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5 Cold Weather   Graph A

The number of guests at a ski resort depends on the day's high temperature. No one visits at −20°F or below. As temperatures rise, guests increase. At 0°F and above, attendance surges, reaching the 400-guest capacity at 10°F or higher.
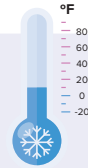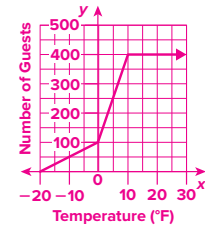
- Independent quantity:

- Dependent quantity:
  Number of guests

  Origin: (0 degrees Fahrenheit, 0 number of guests)

6 Jelly Bean Challenge   Graph C

Mr. Vasquez judges the Jelly Bean Challenge at the summer fair, recording all possible guesses and how far each is from the actual count.
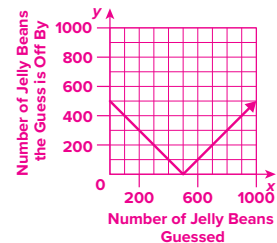
- Independent quantity:
  Number of jelly beans guessed
- Dependent quantity:
  Number of jelly beans the guess is off by

  Origin: (0 jelly beans guessed, 0 jelly beans the guess is off by)

7 Compare the graphs for each pair of situations given. Describe any similarities and differences you notice.

Think About . . .
Look closely when analyzing the graphs. What do you see?

a *Smart Phone, But Is It a Smart Deal?* and *Cold Weather*

Sample answers:
Both graphs increase from left to right.

The graph of the *Smart Phone, But Is It a Smart Deal*? situation is a smooth curve, but the graph of the *Cold Weather* situation is composed of two increasing and one constant line segments.

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Purposeful Questions   Activity 1

5 | Probing | Explain what the different segments of your graph represent.

How is "the number of guests soars" represented on the graph?

6 | Probing | Does the first part of the graph show guesses above or below the exact jelly bean count? Explain.

| Seeing Structure | What is the relationship between the symmetry of this graph and the situation?

7 | Seeing Structure | Why is one graph in part (a) composed of line segments while the other is a curve?

Why do both graphs in part (c) have symmetry?

Why is one graph in part (c) composed of line segments while the other is a curve?

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b *Something's Fishy* and *Daredevil*
Sample answers:
Both graphs decrease from left to right.

The graph of the *Something's Fishy* situation is a straight line, but the graph of the *Daredevil* situation is a smooth curve.

c *Baton Twirling* and *Jelly Bean Challenge*
Sample answers:
The graphs have either a minimum or a maximum value. Both graphs increase and decrease.

The graph of the *Baton Twirling* situation is a smooth curve, but the graph of the *Jelly Bean Challenge* situation is made up of two straight lines.

Reflect

~15 minutes

Purpose: To write a situation and sketch a graph for a possible trip to school

❯ Group students to complete **1** and **2**.
❯ Check in and share.
❯ Complete **3** as a class.
❯ Share and summarize: A graph's axes' labels, points, and whether the points form line segments or a curve provide insight into the situation it models.

Differentiation Strategy
Challenge Opportunity
Action & Expression: Expression & Communication
Have students create two graphs, one with the *y*-axis labeled as the distance from home and the other with the *y*-axis labeled distance from school. Then, have them compare the characteristics of the graphs.
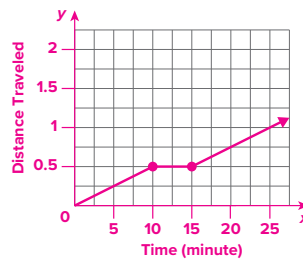
Reflect — A Writer and a Mathematician

1 Write a situation and sketch a graph to describe a possible trip to school.

Situation

Sample answer:
I walk half a mile to school in 10 minutes. Then, I stop to talk to a friend and tie my shoes for 5 minutes. I walk the remaining half-mile to school in 10 minutes.

Graph

2 Describe the meaning of the points, or smooth curve, represented by your graph.
Sample answer:
Each point on the graph represents possible times and the corresponding distances.

3 Compare your situation and sketch with your classmates' situations and sketches. What similarities do you notice? What differences do you notice?
Answers will vary.

👆 **Quick Check:** A Picture is Worth a Thousand Words

Purposeful Questions Reflect

1 Probing | How did you inform the reader about the pace?

Could more than one possible graph model your situation? Explain.

2 Probing | How did you use the axes' labels and scales to support your response? How did you know the graph is a straight line?

3 Seeing Structure | Is your graph increasing or decreasing? Why?

What type of situation would create a curved graph?

Session 1 | Close ~5 minutes

⚡ Go online to access the **Quick Check.**

Lesson 1

Practice and Apply

⊕ Assignment Suggestions

- Practice
- Journal
- Stretch (advanced learners)

⊕ Skills Practice Suggestions

❯ Assign this problem set, as needed, to support students' skill development.

Activity 1, *What Comes First?*

- Problem Set 1: *Identifying Independent and Dependent Quantities*

Practice builds new connections in your brain. You can use these practice activities to develop your mathematical knowledge.
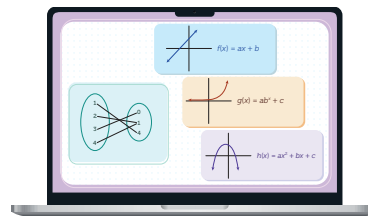
⊕ Interactive Assignment

A Picture is Worth a Thousand Words

with **LiveHint** 🤖

⊕ Interactive Skills Practice

Identifying Independent and Dependent Quantities

📄 Independent Practice

❯ **Determine the independent and dependent quantities in each situation. Be sure to include the appropriate units of measure for each quantity.**

1 Baila reads a 300-page book at a rate of 20 pages per hour.

2 A car wash business charges $15 per car for a basic wash.

3 Ashley drinks water at a steady rate of 0.5 liters per 15 minutes while hiking.

4 A library allows users to borrow books for 2 weeks per loan period.

5 A group of friends purchases tickets to a concert. Tickets cost $75 each.

6 A hose fills a swimming pool at a rate of 8 gallons per minute.

7 The temperature drops 2 degrees Fahrenheit per hour overnight.

8 A construction company rents a bulldozer for $150 per day.

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1. Independent quantity: Time (hours)
   Dependent quantity: Number of pages read

2. Independent quantity: Number of cars washes
   Dependent quantity: Cost (dollars)

3. Independent quantity: Time (minutes)
   Dependent quantity: Volume of water consumed (liters)

4. Independent quantity: Number of loans
   Dependent quantity: Time (weeks)

5. Independent quantity: Number of tickets purchased
   Dependent quantity: Cost (dollars)

6. Independent quantity: Time (minutes)
   Dependent quantity: Volume of water in the pool (gallons)

7. Independent quantity: Time (hours)
   Dependent quantity: Temperature change (degrees Fahrenheit)

8. Independent quantity: Number of rental days
   Dependent quantity: Cost (dollars)