
Decision making with the analytic hierarchy process

Thomas L. Saaty

Katz Graduate School of Business,
University of Pittsburgh,
Pittsburgh, PA 15260, USA
E-mail: saaty@katz.pitt.edu

Abstract: Decisions involve many intangibles that need to be traded off. To do that, they have to be measured along side tangibles whose measurements must also be evaluated as to, how well, they serve the objectives of the decision maker. The Analytic Hierarchy Process (AHP) is a theory of measurement through pairwise comparisons and relies on the judgements of experts to derive priority scales. It is these scales that measure intangibles in relative terms. The comparisons are made using a scale of absolute judgements that represents, how much more, one element dominates another with respect to a given attribute. The judgements may be inconsistent, and how to measure inconsistency and improve the judgements, when possible to obtain better consistency is a concern of the AHP. The derived priority scales are synthesised by multiplying them by the priority of their parent nodes and adding for all such nodes. An illustration is included.

Keywords: decision making; intangibles; judgements; priorities Analytic Hierarchy Process; AHP; comparisons; ratings; synthesis.

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Biographical notes: Thomas L. Saaty holds the Chair of University Professor at the University of Pittsburgh and is a Member of the National Academy of Engineering, USA. He is internationally recognised for his decision-making process, the Analytic Hierarchy Process (AHP) and its generalisation to network decisions, the Analytic Network Process (ANP). He won the Gold Medal from the International Society for Multicriteria Decision Making for his contributions to this field. His work is in decision making, planning, conflict resolution and in neural synthesis.

1 Introduction

We are all fundamentally decision makers. Everything we do consciously or unconsciously is the result of some decision. The information we gather is to help us understand occurrences, in order to develop good judgements to make decisions about these occurrences. Not all information is useful for improving our understanding and judgements. If we only make decisions intuitively, we are inclined to believe that all kinds of information are useful and the larger the quantity, the better. But that is not true. There are numerous examples, which show that too much information is as bad as little information. Knowing more does not guarantee that we understand better as illustrated by some author's writing "Expert after expert missed the revolutionary significance of

what Darwin had collected. Darwin, who knew less, somehow understood more". To make a decision we need to know the problem, the need and purpose of the decision, the criteria of the decision, their subcriteria, stakeholders and groups affected and the alternative actions to take. We then try to determine the best alternative, or in the case of resource allocation, we need priorities for the alternatives to allocate their appropriate share of the resources.

Decision making, for which we gather most of our information, has become a mathematical science today (Figuera et al., 2005). It formalises the thinking we use so that, what we have to do to make better decisions is transparent in all its aspects. We need to have some fundamental understanding of this most valuable process that nature endowed us with, to make it possible for us to make choices that help us survive. Decision making involves many criteria and subcriteria used to rank the alternatives of a decision. Not only does one need to create priorities for the alternatives with respect to the criteria or subcriteria in terms of which they need to be evaluated, but also for the criteria in terms of a higher goal, or if they depend on the alternatives, then in terms of the alternatives themselves. The criteria may be intangible, and have no measurements to serve as a guide to rank the alternatives, and creating priorities for the criteria themselves in order to weigh the priorities of the alternatives and add over all the criteria to obtain the desired overall ranks of the alternatives is a challenging task. How? In the limited space we have, we can only cover some of the essentials of multicriteria decision making, leaving it to the reader to learn more about it from the literature cited at the end of this paper.

The measurement of intangible factors in decisions has for a long time, defied human understanding. Number and measurement are the core of mathematics and mathematics is essential to science. So far, mathematics has assumed that all things can be assigned numbers from minus infinity to plus infinity in some way, and all mathematical modelling of reality has been described in this way by using axes and geometry. Naturally, all this is predicated on the assumption that one has the essential factors and all these factors are measurable. But there are many more important factors that we do not know how to measure than there are ones that we have measurements for. Knowing how to measure such factors could conceivably lead to new and important theories that rely on many more factors for their explanations. After all, in an interdependent universe everything depends on everything else. Is this just a platitude or is there some truth behind it? If we knew how to measure intangibles, much wider room would be open to interpret everything in terms of many more factors than we have been able to do so far scientifically. One thing is clear, numerical measurement must be interpreted for meaning and usefulness according to its priority to serve our values in a particular decision. It does not have the same priority for all problems. Its importance is relative. Therefore, we need to learn about how to derive relative priorities in decision making.

2 Background

There are two possible ways to learn about anything – an object, a feeling or an idea. The first is to examine and study it in itself to the extent that it has various properties, synthesize the findings and draw conclusions from such observations about it. The second is to study that entity relative to other similar entities and relate it to them by making comparisons.

The cognitive psychologist Blumenthal (1977) wrote that

“Absolute judgement is the identification of the magnitude of some simple stimulus...whereas comparative judgement is the identification of some relation between two stimuli both present to the observer. Absolute judgment involves the relation between a single stimulus and some information held in short-term memory, information about some former comparison stimuli or about some previously experienced measurement scale... To make the judgement, a person must compare an immediate impression with impression in memory of similar stimuli.”

Using judgements has been considered to be a questionable practice when objectivity is the norm. But a little reflection shows that even when numbers are obtained from a standard scale and they are considered objective, their interpretation is always, I repeat, always, subjective. We need to validate the idea that we can use judgements to derive tangible values to provide greater credence for using judgements when intangibles are involved.

3 The analytic hierarchy process

To make a decision in an organised way to generate priorities we need to decompose the decision into the following steps.

- 1 Define the problem and determine the kind of knowledge sought.
- 2 Structure the decision hierarchy from the top with the goal of the decision, then the objectives from a broad perspective, through the intermediate levels (criteria on which subsequent elements depend) to the lowest level (which usually is a set of the alternatives).
- 3 Construct a set of pairwise comparison matrices. Each element in an upper level is used to compare the elements in the level immediately below with respect to it.
- 4 Use the priorities obtained from the comparisons to weigh the priorities in the level immediately below. Do this for every element. Then for each element in the level below add its weighed values and obtain its overall or global priority. Continue this process of weighing and adding until the final priorities of the alternatives in the bottom most level are obtained.

To make comparisons, we need a scale of numbers that indicates how many times more important or dominant one element is over another element with respect to the criterion or property with respect to which they are compared. Table 1 exhibits the scale. Table 2 exhibits an example in which the scale is used to compare the relative consumption of drinks in the USA. One compares a drink indicated on the left with another indicated at the top and answers the question: How many times more, or how strongly more is that drink consumed in the US than the one at the top? One then enters the number from the scale that is appropriate for the judgement: for example enter 9 in the (coffee, wine) position meaning that coffee consumption is 9 times wine consumption. It is automatic that $1/9$ is what one needs to use in the (wine, coffee) position. Note that water is consumed more than coffee, so one enters 2 in the (water, coffee) position, and $1/2$ in the (coffee, water) position. One always enters the whole number in its appropriate position and automatically enters its reciprocal in the transpose position.

Table 1 The fundamental scale of absolute numbers

<i>Intensity of Importance</i>	<i>Definition</i>	<i>Explanation</i>
1	Equal Importance	Two activities contribute equally to the objective
2	Weak or slight	
3	Moderate importance	
4	Moderate plus	Experience and judgement slightly favour one activity over another
5	Strong importance	
6	Strong plus	
7	Very strong or demonstrated importance	An activity is favoured very strongly over another; its dominance demonstrated in practice
8	Very, very strong	
9	Extreme importance	The evidence favouring one activity over another is of the highest possible order of affirmation
Reciprocals of above	If activity <i>i</i> has one of the above non-zero numbers assigned to it when compared with activity <i>j</i> , then <i>j</i> has the reciprocal value when compared with <i>i</i>	
1.1–1.9	If the activities are very close	May be difficult to assign the best value but when compared with other contrasting activities the size of the small numbers would not be too noticeable, yet they can still indicate the relative importance of the activities.

Table 2 Relative consumption of drinks

<i>Which drink is consumed more in the USA?</i>							
<i>An example of examination using judgements</i>							
<i>Drink consumption in US</i>	<i>Coffee</i>	<i>Wine</i>	<i>Tea</i>	<i>Beer</i>	<i>Sodas</i>	<i>Milk</i>	<i>Water</i>
Coffee	1	9	5	2	1	1	1/2
Wine	1/9	1	1/3	1/9	1/9	1/9	1/9
Tea	1/5	2	1	1/3	1/4	1/3	1/9
Beer	1/2	9	3	1	1/2	1	1/3
Soda	1	9	4	2	1	2	1/2
Milk	1	9	3	1	1/2	1	1/3
Water	2	9	9	3	2	3	1

Note: The derived scale based on the judgements in the matrix is:

0.177 0.019 0.042 0.116 0.190 0.129 0.327

With a consistency ratio of 0.022.

the actual consumption (from statistical sources) is:

0.180 0.010 0.040 0.120 0.180 0.140 0.330

The priorities, (obtained in exact form by raising the matrix to large powers and summing each row and dividing each by the total sum of all the rows, or approximately by adding each row of the matrix and dividing by their total) are shown at the bottom of

the table along with the true values expressed in relative form by dividing the consumption of each drink (volume) by the sum of the consumption of all drinks. The information about actual consumption was obtained from the US Statistical Abstracts. We see the answers are very close and pair-wise comparison judgements of someone who knows can lead to very accurate results of drink consumption.

4 Validation

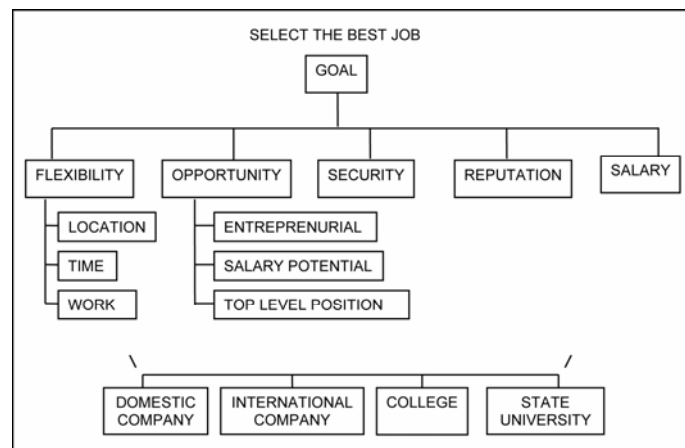
There are numerous examples to validate use of the 1–9 scale. We have already given one above.

Table 2 shows how an audience of about 30 people, using consensus to arrive at each judgement, provided judgements to estimate the dominance of the consumption of drinks in the USA (which drink is consumed more in the USA and how much more than another drink?). The derived vector of relative consumption and the actual vector, obtained by normalising the consumption given in official statistical data sources, are at the bottom of the table.

5 An example of a simple decision

The following is a simple decision examined by someone to determine what kind of job would be best for him/her after getting his/her PhD: either to work in two kinds of companies or to teach in two kinds of schools. The goal is to determine the kind of job for which he/she is best suited as spelled out by the criteria. Because of space limitations we will not define them in detail here (for more detail see Saaty, 1994; Saaty and Vargas, 2000) (Figure 1).

Figure 1 Best job decision



There are 12 pairwise comparison matrices in all: One for the criteria with respect to the goal, which is shown here in Table 3, two for the subcriteria, the first of which for the subcriteria under flexibility: location, time and work, that is given in Table 4 and one for the subcriteria under opportunity that is not shown here. Then, there are nine comparison

matrices for the four alternatives with respect to all the ‘covering criteria’, the lowest level criteria or subcriteria connected to the alternatives. The 9 covering criteria are: flexibility of location, time and work, entrepreneurial company, possibility for salary increases and a top-level position, job security, reputation and salary. The first six are subcriteria in the second level and the last three are criteria from the first level. We only show one of these 9 matrices comparing the alternatives with respect to potential increase in salary in Table 5.

Table 3 Pairwise comparison matrix of the main criteria with respect to the Goal

	<i>Flexibility</i>	<i>Opportunities</i>	<i>Security</i>	<i>Reputation</i>	<i>Salary</i>	<i>Priorities</i>
Flexibility	1	1/4	1/6	1/4	1/8	0.036
Opportunities	4	1	1/3	3	1/7	0.122
Security	6	3	1	4	1/2	0.262
Reputation	4	1/3	1/4	1	1/7	0.075
Salary	8	7	2	7	1	0.506

Table 4 Pairwise comparison matrix for the subcriteria with respect to flexibility

	<i>Location</i>	<i>Time</i>	<i>Work</i>	<i>Priorities</i>
Location	1	1/3	1/6	0.091
Time	3	1	1/4	0.218
Work	6	4	1	0.691

Table 5 Pairwise comparison matrix for the alternatives with respect to potential increase in salary

	<i>Domestic Co</i>	<i>Int’l Co</i>	<i>College</i>	<i>State Univ.</i>	<i>Priorities</i>
Domestic company	1	4	3	6	0.555
Int’l company	1/4	1	3	5	0.258
College	1/3	1/3	1	2	0.124
State University	1/6	1/5	1/2	1	0.064

In Table 1, the criteria listed on the left are one by one compared with each criterion listed on top as to which one is more important with respect to the goal of selecting a best job. In Table 2, the subcriteria on the left are compared with the subcriteria on top as to their importance with respect to flexibility. In Table 3, the alternatives on the left are compared with those on top with respect to relative preference for potential increase in salary. The subcriteria priorities in Table 2 are weighed by the priority of their parent criterion flexibility (0.036) to obtain their global priority.

The priorities for each matrix are obtained as they were from the matrix of comparisons for the drinks in the USA. In Table 6, the rankings of the alternatives are given against the nine covering criteria (only one of the matrices leading to the rankings was given, in Table 5). We need to multiply each ranking by the priority of its criterion or subcriterion and add the resulting weighs for each alternative to get its final priority. We call this part of the process, synthesis. It is given in Table 6. Because Table 6 is horizontally long, it is divided into two pieces where the lower piece follows to the right of the upper piece.

Table 6 Synthesising to obtain the final results

Criteria Subcriteria	Flexibility 0.036			Future opportunity 0.122			Security 0.262	Reputation 0.075	Salary 0.506	Overall Priority
	Location 0.091	Time 0.218	Work 0.691	Entrepreneurial 0.105	Salary increases 0.637	Top level position 0.258				
Domestic Company	0.003	0.008	0.025	0.013	0.078	0.032	0.225	0.064	0.124	0.193
Internatn'l Company	0.295	0.084	0.062	0.090	0.555	0.591	0.054	0.101	0.547	0.333
College	0.496	0.055	0.115	0.061	0.258	0.274	0.095	0.247	0.289	0.214
State University	0.131	0.285	0.249	0.239	0.124	0.083	0.626	0.588	0.039	0.262

The overall priorities for the alternative jobs, given on the far right of the lower piece of Table 6, are the sums across each row for the alternatives. Note that they sum to 1. These priorities may also be expressed in the ideal form by dividing each priority by the largest one, 0.333 for *International Company*, as given in Table 7. The effect is to make this alternative the ideal one with the others getting their proportionate value. One may then interpret the results to mean that a *State University* job is about 78% as good as one with an *International Company* and so on.

Table 7 Final results shown as normalised priorities and idealised priorities

<i>Name</i>	<i>Normalised priorities</i>	<i>Idealised priorities</i>
Domestic Company	0.193	0.579
Internatn'l Company	0.333	1.000
College	0.214	0.643
State University	0.262	0.785

6 The ratings mode

There is another method to obtain priorities for the alternatives. Here we establish rating categories for each covering criterion and prioritise the categories by pair-wise comparing them for preference. Alternatives are evaluated by selecting the appropriate rating category on each criterion.

The rating categories for the Job Security criterion are High, Medium and Low. We compare them for preference using a pair-wise comparison matrix in the usual way as given in Table 8. To obtain the idealised priorities, normalise by dividing by the largest of the priorities. The idealised priorities are always used for ratings.

Table 8 Deriving priorities for ratings on job security

<i>Job Security</i>	<i>High</i>	<i>Medium</i>	<i>Low</i>	<i>Priorities</i>	<i>Idealised Priorities</i>
High	1	3	7	0.6586	1.0000
Medium	1/3	1	4	0.2628	0.3989
Low	1/7	1/4	1	0.0786	0.1193

The rating categories for all the covering criteria and their priorities are established in a similar way and are given in Table 9.

Table 10 gives the verbal ratings of the four alternatives on each covering criterion and Table 11 gives their corresponding numerical ratings from Table 9 with their totals given in the first column on the left. The totals are converted to priorities by dividing by their sum in the second column on the left.

Comparing the results from the pair-wise comparison method called a relative model to these results from the ratings model as given in Table 12, we note that the first two alternatives' priorities are very close. The last two are a little different. This is to be expected. The two methods do not deliver the same priorities exactly. The relative model method where alternatives are compared with each other under the various criteria is more accurate. The ratings method has the advantage that one can rate large numbers of alternatives rather quickly, and the results are adequately close.

Table 9 The prioritised ratings categories for all criteria

<i>Job Security</i>	<i>Reputation</i>	<i>Salary</i>	<i>Location flexibility</i>	<i>Time flexibility</i>	<i>Work flexibility</i>	<i>Entrepreneurial</i>	<i>Salary increase</i>	<i>Top level</i>
High	Excellent	>3000	Very flexible	Flexible	Very flexible	Very probable	High	Likely
1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Med	Above Avg	2000–3000	Flexible	Avg	Flexible	Probable	Above Avg	Depends
0.399	0.487	0.551	0.431	0.405	0.665	0.373	0.299	0.399
Low	Average	1500–2000	Avg flexible	Not flexible	Avg flexible	Not probable	Average	Not likely
0.119	0.403	0.217	0.187	0.164	0.423	0.113	0.119	0.119
	Below Avg	1000–1500	Not flexible		Not flexible	Impossible		
	0.171	0.162	0.124		0.195	0.075		
	Poor	<1500	Stable		Stable			
	0.113	0.059	0.087		0.122			

Table 10 Ratings for the alternatives on each criterion

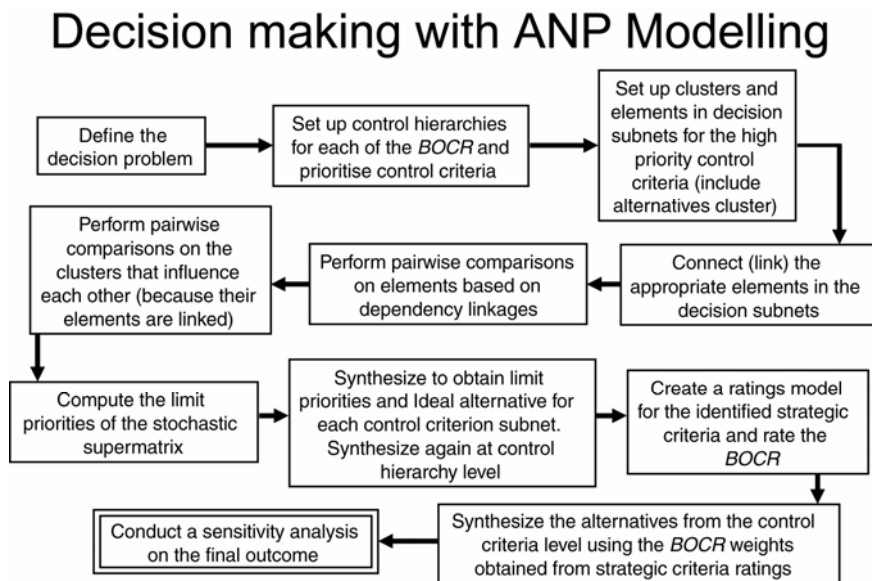
	<i>Total</i>	<i>Priorities</i>	<i>3 Jobsecurity</i> 0.261899	<i>4 Reputation</i> 0.074559	<i>5 Salary</i> 0.505585	<i>Location flexibility</i> 0.003296	<i>Time flexibility</i> 0.007846	<i>Work flexibility</i> 0.024909	<i>Entrepreneurial</i> 0.012766	<i>Salary increase</i> 0.077653	<i>Top level</i> 0.031487
International Company	0.619340	0.310067	Low	Above average	>3000	Very flexible	Not flexible	Not flexible	Impossible	Above avg.	Depends
Domestic Company	0.363962	0.182214	Medium	Average	1500–2000	Flexible	Avg.	Not flexible	Impossible	High	Likely
State University	0.505153	0.252900	High	Excellent	1000–1500	Stable	Flexible	Very flexible	Very probable	Avg.	Likely
College	0.508984	0.254818	Medium	Excellent	2000–3000	Not flexible	Flexible	Flexible	probable	Avg.	Depends

Table 12 Comparing relative and rating results

	<i>Relative model results</i>	<i>Ratings model results</i>
Domestic Company	0.192	0.182
Int. Company	0.333	0.310
College	0.214	0.255
State Univ.	0.261	0.253

The process of paired comparisons has far broader uses for making decisions. We can deal with a decision from four different standpoints: the benefits (*B*), that the decision brings, the opportunities (*O*) it creates, the costs (*C*) that it incurs and the risks (*R*) that it might have to face. We refer to these merits together as BOCR. Some people in the field of strategic planning use similar factors known as SWOT (Strengths, Weaknesses, Opportunities and Threats) having switched the order of weaknesses and opportunities in making the correspondence with BOCR. The alternatives must be ranked for each of the four merits. The four rankings are then combined into a single overall ranking by rating the best alternative in each of the BOCR on strategic criteria that an individual or a government uses to decide whether or not to implement one or the other of the numerous decisions that they face. The results of the four ratings determine the priorities each of which is used to weigh all the priorities of all the alternatives with respect to that merit.

There is in addition the possibility of the dependence of the criteria on the alternatives in addition to the mandatory dependence of the alternatives on the criteria or among themselves. In that case we have a decision with dependence and feedback. To determine the best course of action in such decisions needs a few days to do thoroughly. Its steps are carried out along the lines shown in Figure 2.

Figure 2 Steps followed in the analytic network decision process

7 Group decision making

Two important issues in group decision making are: how to aggregate individual judgements in a group into a single representative judgement for the entire group and how to construct a group choice from individual choices. The reciprocal property plays an important role in combining the judgements of several individuals to obtain a single judgement for the group. Judgements must be combined so that the reciprocal of the synthesised judgements is equal to the syntheses of the reciprocals of these judgements. It has been proved that the geometric mean, not the frequently used arithmetic mean, is the only way to do that. If the individuals are experts, they may not wish to combine their judgements but only their final outcomes obtained by each from their own hierarchy. In that case one takes the geometric mean of the final outcomes. If the individuals have different priorities of importance, their judgements (final outcomes) are raised to the power of their priorities and then the geometric mean is formed.

8 Future trends

There are two areas that need greater attention in decision-making. One is the integration and cataloguing of the structure of a variety of carefully studied decisions to create a dictionary to serve as a source of reference for others to consult, so they can benefit from the knowledge that went into making these decisions. Two successful attempts have already been made in this direction resulting in two books: *The Hierarchon* (Saaty and Forman, 1993) a dictionary of hierarchically structured decisions and the *Encyclicon* (Saaty and Ozdemir, 2005), a dictionary of more general network structured decisions.

Another important area of investigation is, how to factor psychological time into a decision in order to anticipate and deal with the future more successfully through prediction and planning. Many efforts are under way in this direction. Books and papers have been published that deal with the future and with planning using the prioritisation process described in this paper.

9 Conclusion

It appears inescapable that we need an organised way to make decisions and collect information relevant to them when a group must decide by laying out all the important factors and negotiating their understanding, beliefs and values. Here are a few examples where the process has been used in practice.

The Analytic Hierarchy Process (AHPs) has been used in various settings to make decisions.

- In public administration there have been many applications.
 - The state of North Carolina used it to develop evaluation criteria and assign ratings to vendors, leading to the selection of a best-value vendor acceptable to the decision makers.
 - The Nuclear Regulatory Commission (NRC) of the US with so many competing requirements for their information technology projects used it to allocate all of a \$100 million + portfolio. NRC's challenge to date has been

difficulty with prioritising so many competing requirements for IT work efforts as well as getting their 35 + members decision-making group to achieve consensus. Using the AHP not only helped allocate NRC's IT resources, but also reduced the amount of decision time from 15 to 20 meetings down to just a few.

- The Federal Financial Institutions Examination Council (FFIEC) is a governing body in the US composed of the Federal Reserve Board (FRB), the Federal Deposit Insurance Corporation (FDIC) and the Office of the Comptroller of Currency (OCC). They used AHP to prioritise strategic enhancements for an activity all the bodies needed to have: Call Data Reporting. They prioritised their objectives across competing requirements in a limited resource environment and were able to complete this in a one-day session.
 - The Boys and Girls Clubs of Greater Washington (BGCGW) is the largest affiliate of Boys and Girls Clubs of the USA. As part of their strategic planning process, the BGCGW needed to define 'at-risk' youth and used AHP to set relative priorities on the factors. The group reached consensus and has set the standards they now use in the BGCGW strategic plan.
 - The Department of Defence in the US uses it frequently and extensively to allocate their resources to their diverse activities.
 - The General Services Administration (GSA) of the USA used AHP to support their annual Information Technology Council (ITC) and Council of Controllers (COC) meeting to prioritise their major information technology initiatives. They used the process to refine their analytical framework, prioritise their criteria and then rate each IT initiative against them. The result was the first-ever GSA-wide prioritisation of major IT initiatives, which included a benefit-cost analysis and a benefit-risk analysis.
- In (2001), it was used to determine the best relocation site for the earthquake devastated Turkish city Adapazari.
 - British Airways used it in 1998 to choose the entertainment system vendor for its entire fleet of airplanes.
 - A company used it in 1987 to choose the best type of platform to build to drill for oil in the North Atlantic. A platform costs around 3 billion dollars to build, but the demolition cost was an even more significant factor in the decision.
 - A book was written in 1990 by Nagel and Mills: *Multicriteria Methods for Alternative Dispute Resolution* (NY: Quorum Books) applying the concepts of quantitative decision making in public administration. Examination of the book shows that the authors suggest using, ordinal scale numbers (as opposed to the absolute numbers used in making pairwise comparisons that can be added multiplied and divided) alongside actual numbers like money and other measurement. A mathematician may wonder, how one can add and multiply ordinal numbers and derive priorities from them. Nevertheless, the authors were thinking in the right direction by proposing use of multicriteria methods.
 - The process was applied to the US versus China conflict in the intellectual property rights battle of 1995 over Chinese individuals copying music, video and software tapes and CD's. An AHP analysis involving three hierarchies for

benefits, costs and risks showed that it was much better for the US not to sanction China. I was called by the office of Mr. Mickey Kantor, then President Clinton chief negotiator who read this paper, saying “This is Mickey Kantor’s office, congratulations, aren’t you glad we did not sanction China?” Shortly after that study another multicriteria analysis was completed (available on request) and sent to the US Congressional committee which showed that China should be admitted into the World Trade Organisation (WTO).

- Xerox Corporation has used the AHP to allocate close to a billion dollars to its research projects.
- In 1999, the Ford Motor Company used the AHP to establish priorities for criteria that improve customer satisfaction. Ford gave Expert Choice Inc, an Award for Excellence for helping them achieve greater success with its clients.
- In 1986 the Institute of Strategic Studies in Pretoria, a government-backed organisation, used the AHP to analyse the conflict in South Africa and recommended actions ranging from the release of Nelson Mandela to the removal of apartheid and the granting of full citizenship and equal rights to the black majority. All of these recommended actions were quickly implemented.
- The AHP has been used in student admissions, military personnel promotions and hiring decisions.
- In sports, it was used in 1995 to predict which football team would go to the Superbowl and win (correct outcome, Dallas won over my hometown, Pittsburgh). The AHP was applied in baseball to analyse which Padres players should be retained.
- IBM used the process in 1991 in designing its successful mid-range AS 400 computer. IBM won the prestigious Malcolm Baldrige award for Excellence for that effort. Bauer et al. (1992) devoted a paper on how AHP was used in benchmarking.
- Several military and political applications have been made. Of general interest was the analysis of the decision as to whether to build or not build the National Missile Defence (NMD) made two years prior to the time that decision was made in December 2002. The decision was the same as the study recommended in a briefing on the study to the military early in 2002: build it despite some scientists’ skepticism. We know an electrical engineer who is working on this project now (2006) and says great progress has been made to make the project feasible and successful.

In addition to the books referred above, a number of books have been written by different authors on the Analytic Hierarchy Process; examples are those by Bhushan and Ria (2004), Hummel (2001), Rabbani and Rabbani (1996), Saaty (1982), Saaty and Alexander (1989), Saaty and Kearns (1985), Saaty and Vargas, L.G. (1982, 1991, 2000, 2006) and Schmoldt et al. (2001).

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