Chapter 1 : Three physics principles

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0}$$ ⓐ

① mass conservation

② $F = ma$

③ Energy conservation

Euler Equs

ⓑ momentum cons.

ⓒ Energy cons.

Four fluid models.

1 FCV fixed in space (Integral, conservative)

2 FCV moving with fluids (Integral, non-cons)

3 IFE fixed in space (Differential, cons)

4 IFE moving with fluids (Differential, non-cons)

Two different approaches in Fluid dynamics:

1) macroscopic (fluid theory)

→ Euler Eqns, N–S, MHD, rad-hyd, rad-MHD, relativistic hyd, rel-MHD, etc.

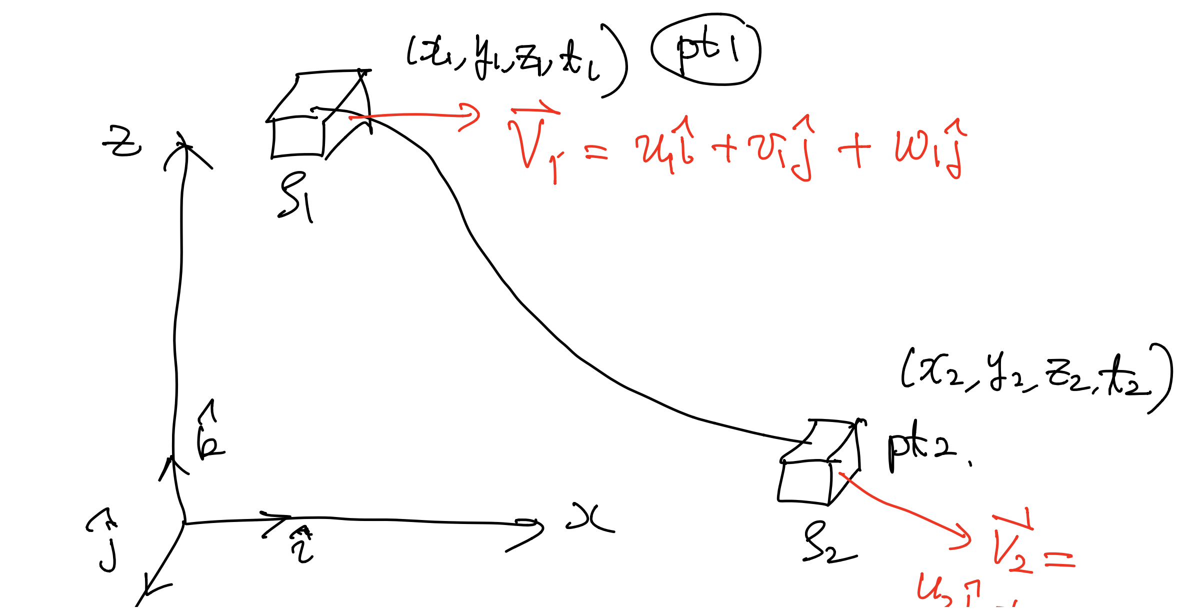
2) microscopic (kinetic theory)

→ Boltzmann Eqns.

1) $\frac{D}{Dt}$ (substantial derivative, or material derivative)

2) $\nabla \cdot \vec{V}$ (divergence of velocity fields)

→ Consider IFE moving with fluids.



$+ v_2 j + w_2 k$

y

$\rho = \rho(x, y, z, t)$ ⇐ density

$$\rho_2 = \rho_1 + \left(\frac{\partial \rho}{\partial x}\right)_1 (x_2 - x_1) + \left(\frac{\partial \rho}{\partial y}\right)_1 (y_2 - y_1) + \left(\frac{\partial \rho}{\partial z}\right)_1 (z_2 - z_1) + \left(\frac{\partial \rho}{\partial t}\right)_1 (t_2 - t_1) + \text{H.O.T}$$

→ Dividing both by $(t_2 - t_1)$:

$$\boxed{\lim_{t_2 \to t_1} \frac{\rho_2 - \rho_1}{t_2 - t_1}} \overset{\text{def}}{\equiv} \frac{D\rho}{Dt} \leftarrow \left(\frac{\partial \rho}{\partial t}\right)_1$$

total der.
subs. der.
material der.

$$= \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z}$$

$$= \frac{\partial \rho}{\partial t} + \underbrace{(\vec{V} \cdot \nabla)}_{\text{convective derivative}} \rho$$

total der.

$\frac{d\rho}{dt}$

$$d\rho = \frac{\partial \rho}{\partial t} dt + \frac{\partial \rho}{\partial x} dx + \frac{\partial \rho}{\partial y} dy + \frac{\partial \rho}{\partial z} dz$$

2) Divergence of velocity fields, $\nabla \cdot \vec{V}$

FCV