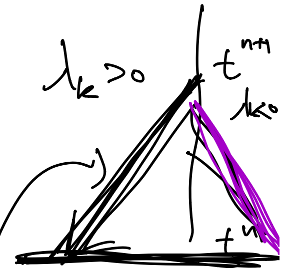
$$\underset{m\times 1}{\tilde U} = \underset{m\times m}{\tilde R}\,\underset{m\times 1}{\tilde W} = [r_1|\cdots|r_m]\begin{bmatrix}\omega_1\\ \omega_2\\ \vdots\\ \omega_m\end{bmatrix}$$



$$= \sum_{k=1}^{m} r_k\,\omega_k(x,t)$$

$$= \sum_{k=1}^{m} r_k\,\boxed{\omega_k(x-\lambda_k t, 0)}$$

known

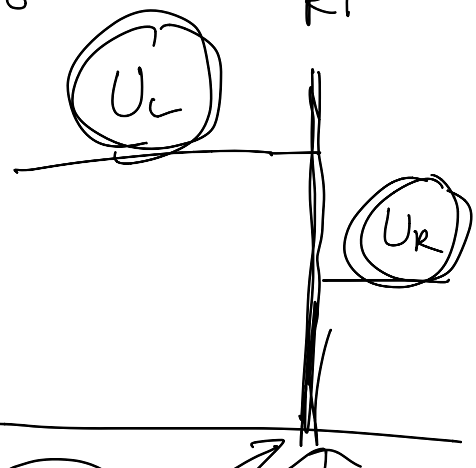
$$\omega_k(x-\lambda_k t, 0)$$

$$\boxed{U_t + (F(U))_x = 0}$$

$\tilde A = \cancel{A}(U_{avg})$

$$\Leftrightarrow U_t + \frac{\partial F}{\partial U} U_x = 0$$

RP



U_{avg} @ $x_{i+\frac{1}{2}}$

$$U = \begin{pmatrix}\rho\\ \rho u\\ \rho E\end{pmatrix}$$

$x_{i+\frac{1}{2}}$

an average state

① $U_{avg} = \frac{1}{2}(U_L + U_R) \Leftarrow \rho_{avg} = \frac{1}{2}(\rho_L + \rho_R)$

② U_{avg} based on the Roe average

$$\rho_{avg} = \sqrt{\rho_L \rho_R}$$

$$u_{avg} = \frac{u_L\sqrt{\rho_L} + u_R\sqrt{\rho_R}}{\sqrt{\rho_L} + \sqrt{\rho_R}}$$

$\vdots$

nonlinear system

$\downarrow \quad \uparrow$

linearized system

Δt

$n \longrightarrow n+1$

$U^n \longrightarrow U^{n+1}$

Reconstruction

RP

U_L & U_R $\rightarrow$ U_{avg}

$\downarrow$

Riemann

RP

$F^{*,\,\text{God}}_{i+\frac{1}{2}}$ ← 1st order

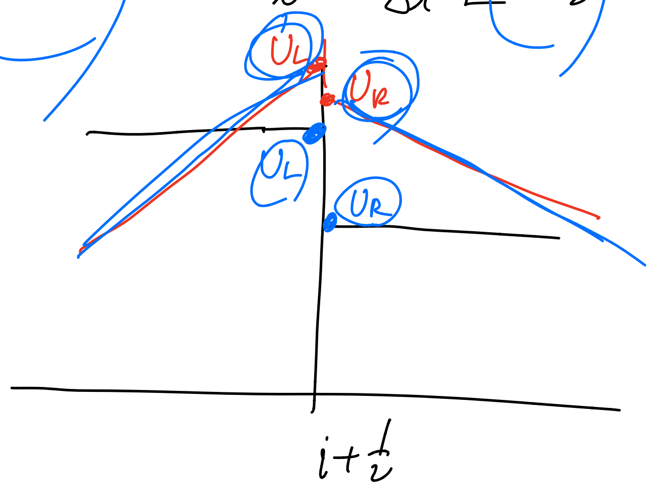
$F^{*,\,\text{PLM}}_{i+\frac{1}{2}}$ ← 2nd order

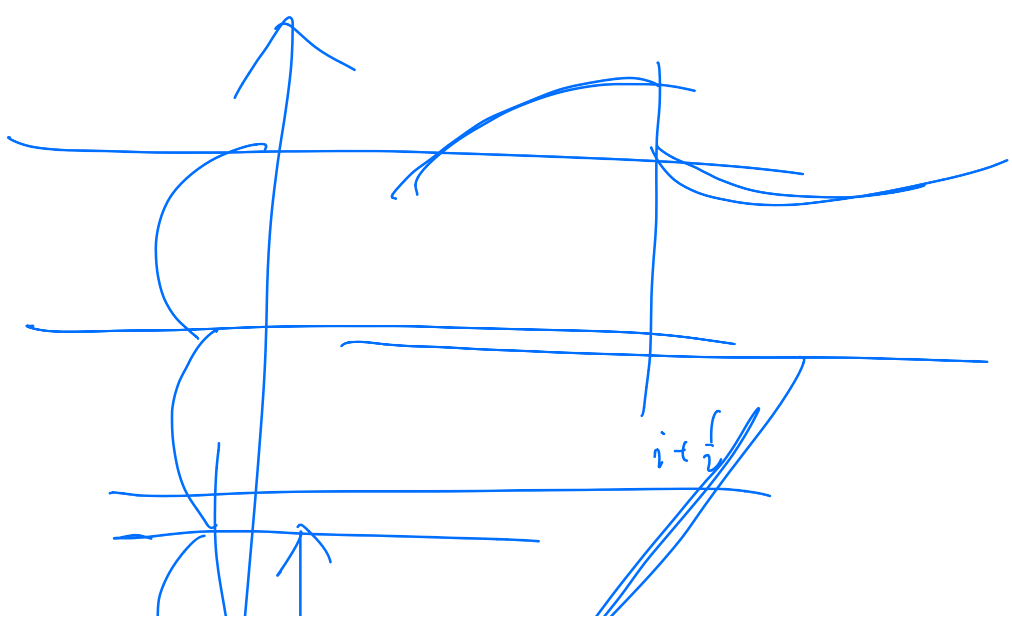
solves

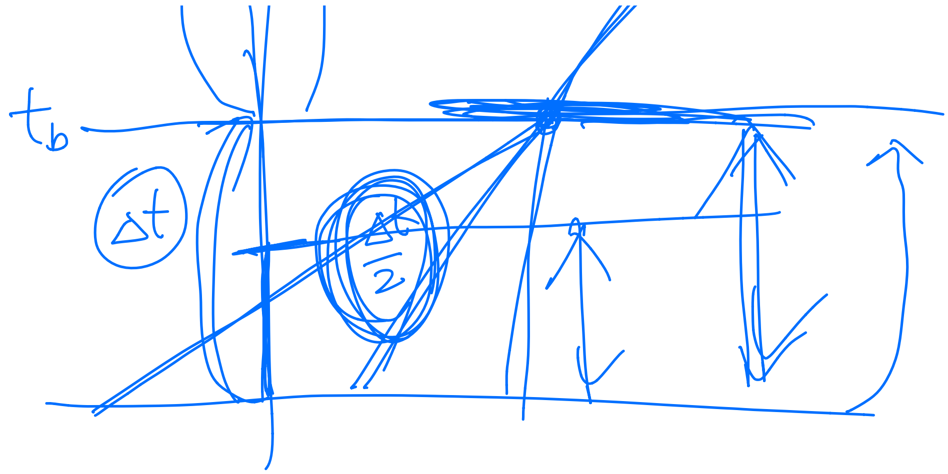
$F^{*\,n+\frac{1}{2}}_{i+\frac{1}{2}}$

PPM

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x}\left[F^*_{i+\frac{1}{2}} - F^*_{i-\frac{1}{2}}\right]$$







The Euler Eqns.

(i) U : conservative variable

(ii) W : characteristic variable

(iii) V : primitive variable

1D

$$U = \begin{bmatrix} \rho \\ \rho u \\ \rho E \end{bmatrix} \qquad V = \begin{bmatrix} \rho \\ u \\ p \end{bmatrix}$$

$$\rho E = \rho\left(\frac{u^2}{2} + e\right)$$

EoS

Caloric EoS

$$e = e(p, v)$$
$$p = p(e, v)$$
$$\left(\frac{1}{\rho} =\right) v = v(e, p)$$

Thermal EoS

$$T = T(p, v)$$
$$p = p(T, v)$$
$$v = v(T, p)$$

Ideal Gas law

Caloric EoS :

$$e = e(p, \rho) = \frac{p}{\rho(\gamma - 1)}$$

$\gamma = 1$

$\gamma = 1.00001$

Thermal EoS ;

$$p = \rho R T \iff pv = RT$$

$$\frac{\partial U}{\partial t} + F(U)_x = 0$$

$$U = U(V)$$
$$V = V(U)$$

$$\iff \frac{\partial U}{\partial t} + \frac{\partial F}{\partial U}\frac{\partial U}{\partial x} = 0 \quad \Leftarrow$$

$$\Leftrightarrow \frac{\partial U}{\partial V}\frac{\partial V}{\partial t} + \frac{\partial F}{\partial U}\frac{\partial U}{\partial V}\frac{\partial V}{\partial x} = 0$$

Q^{-1}

$$\Leftrightarrow Q\frac{\partial V}{\partial t} + \frac{\partial F}{\partial U} Q \frac{\partial V}{\partial x} = 0$$

$$\Leftrightarrow \frac{\partial V}{\partial t} + Q^{-1}\frac{\partial F}{\partial U} Q \frac{\partial V}{\partial x} = 0$$

$$\frac{\partial V}{\partial t} + \frac{\partial F}{\partial V}\frac{\partial V}{\partial x} = 0$$

$$\begin{pmatrix} \rho \\ u \\ p \end{pmatrix}$$

$$\because \quad Q^{-1}\frac{\partial F}{\partial U} Q = \frac{\partial F}{\partial V}.$$

$$\begin{cases} \frac{\partial F}{\partial V} = \boxed{A^{prim}(V)} \\ \frac{\partial F}{\partial U} = A^{cons}(U) \end{cases}$$

$$\begin{cases} A^c = R^c \wedge L^c \\ A^p = R^p \wedge L^p \end{cases}$$

$$W = \underbrace{L^c\, U}_{} = \underbrace{L^p\, V}_{}$$

$$(l_1^c, l_2^c, l_3^c)\begin{pmatrix} \rho \\ \rho u \\ \rho E \end{pmatrix}$$

$$\boxed{L^p U}$$

$$(l_1^p, l_2^p, l_3^p)\begin{pmatrix} \rho \\ u \\ p \end{pmatrix}$$

$$U_t + F(U)_x = 0 \quad \text{or}$$

$$U_t + A^c(U)\,U_x = 0 \qquad A^c = \frac{\partial F}{\partial U}$$

$$F(U) = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ u(\rho E + p) \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$$

$$\text{EoS} \Rightarrow e = \frac{p}{\rho(\gamma - 1)}, \qquad c_s\ (= a) = \sqrt{\frac{\gamma p}{\rho}}$$

$$A^c(U) = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \frac{\partial f_2}{\partial u_3} \\ \frac{\partial f_3}{\partial u_1} & \frac{\partial f_3}{\partial u_2} & \frac{\partial f_3}{\partial u_3} \end{bmatrix} \Leftarrow U = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} \rho \\ \rho u \\ \rho E \end{pmatrix}$$

$$A^c(U) = \left(\frac{\partial f_i}{\partial u_j}\right)$$

$$A^p(V) = \begin{bmatrix} \frac{\partial f_1}{\partial v_1} & \frac{\partial f_1}{\partial v_2} & \frac{\partial f_1}{\partial v_3} \\ & \ddots & \\ & & \ddots \end{bmatrix} \qquad V = \begin{pmatrix} \rho \\ u \\ p \end{pmatrix}$$

$$A^p(V) = \left(\frac{\partial f_i}{\partial v_j}\right)$$

$\gamma = 1.000 \ldots 1$

Hint: $p = \rho e(\gamma - 1)$

$$= (\gamma-1)\left(\rho E - \frac{(\rho u)^2}{2\rho}\right)$$

$$= (\gamma-1)\left(u_3 - \frac{u_2^2}{2u_1}\right) \cdots$$

$$U_t + (F(U))_x = 0$$

$$V_t + (F(V))_x = 0$$

$$A^P = Q^{-1} A^C Q$$

$$\Downarrow$$

$$W_t + \Lambda W_x = 0$$

$$\frac{\partial w_k}{\partial t} + \lambda_k \frac{\partial w_k}{\partial x} = 0, \quad k = 1, \dots, m$$

3

RP