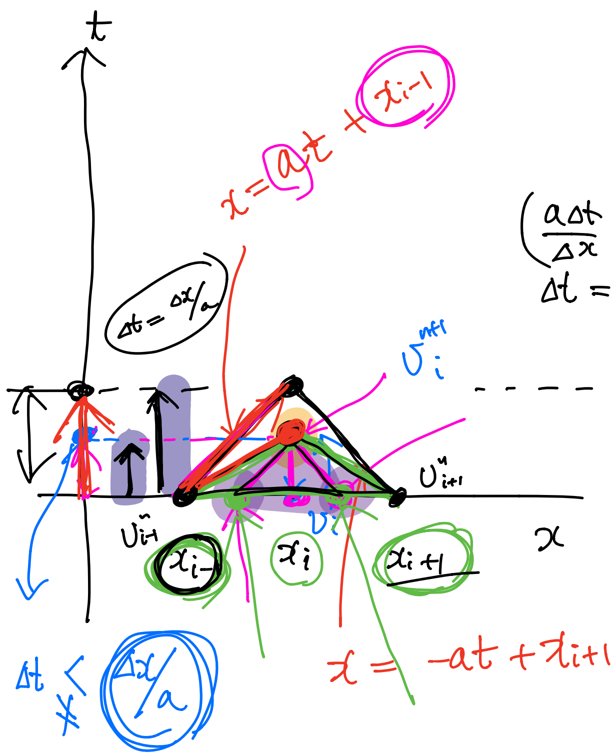
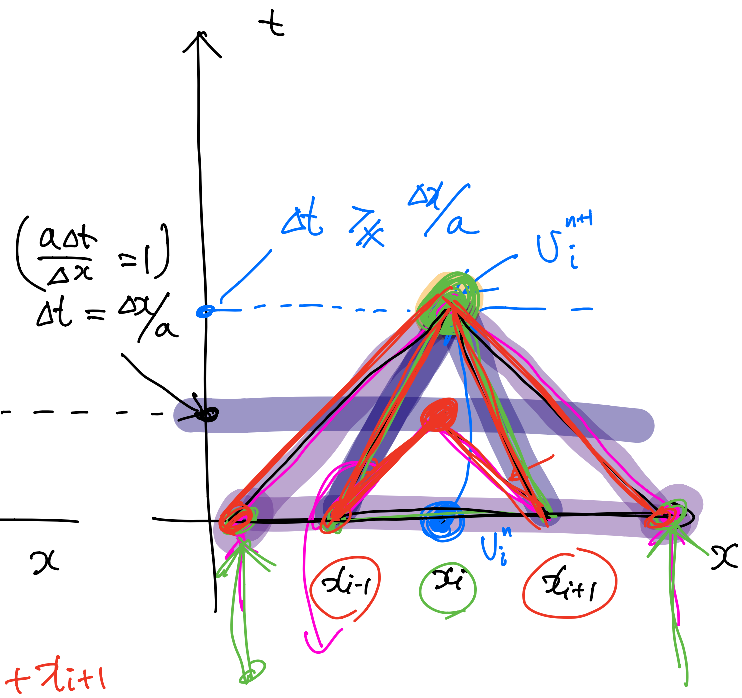
Case 1: $\frac{a\Delta t}{\Delta x} < 1$ Case 2: $\frac{a\Delta t}{\Delta x} > 1$





Model equ: $u_t + a u_x = 0$, $\frac{a<0}{a>0}$

$$\rightarrow U_i^{n+1} = U_i^n - \frac{a\Delta t}{\Delta x}\left[U_i^n - U_{i-1}^n\right]$$

U_{i+1}^n

⇒ { purple △ are the analytical DoD

△ : x_{i-1}, x_i, x_{i+1}

↳ the numerical DoD.

$\Rightarrow$ num DoD $\supset$ analytical DoD

$\Rightarrow$ CFL condition is satisfied

Conclusion

We want : ① $\frac{a\Delta t}{\Delta x} \leq 1$ for stability Ca

② $\frac{a\Delta t}{\Delta x} \approx 1$ for accuracy

code A $\frac{a\Delta t}{\Delta x} = 0.9$

code B $\frac{a\Delta t}{\Delta x} = 0.1$

Ex) My method is stable for

$0 \le C_a \le 1$

Ex) For advection, $(u_t + a u_x = 0)$ $\underline{0 < C_a \le 1}$ for CFL condition

$C_a = \dfrac{a\Delta t}{\Delta x}$

Ex) For diffusion, $(u_t = k u_{xx})$ $(k > 0)$ $0 < C_k \le 1$ for CFL condition

$C_k = \dfrac{k\Delta t}{\Delta x^2}$

$\underline{\text{Rmk}}$. $\Delta t =$

$$\begin{cases} \underbrace{C_a \dfrac{\Delta x}{|a|}}_{\equiv \Delta t_{adv}} & \text{advection } (a>0 \text{ or } a<0) \\ \underbrace{C_k \dfrac{\Delta x^2}{k}}_{\equiv \Delta t_{diff}} & (k>0) \text{ for diffusion} \end{cases}$$

0.8

Rmk. $u_t + a u_x = k u_{xx}$ centered diff

$\longrightarrow \Delta t = \min \{ \Delta t_{adv} , \Delta t_{diff} \}$

Rmk $\Delta t_{adv} \gg \Delta t_{diff}$

~ 0.1 $\quad$ 0.001

(1) operator splitting

explicit for advection $\quad$ implicit for diffusion

$$\begin{cases} \text{step 1: } u_t + a u_x = 0 \xrightarrow{\text{(explicit)}} \tilde{u} \\ \text{step 2: } \tilde{u}_t = k \tilde{u}_{xx} \text{ (implicit)} \longrightarrow u^{n+1} \end{cases}$$

Ex $\quad u_t + a u_x + b u_y = 0 \quad$ 2D

$C_a = \frac{|a|\Delta t}{\Delta x}, \quad C_b = \frac{|b|\Delta t}{\Delta y} \quad C$

(i) $\Delta t \le C \sqrt{\left\{ \frac{\Delta x}{|a|} + \frac{\Delta y}{|b|} \right\}}$

$\longrightarrow$ simple, easy $\quad \Delta x = \Delta y$

$a = b$

$b = 0 \quad \Delta t_{2D} \to 1$

$0 \neq b = a \quad \Delta t_{2D} \to \frac{1}{2}$

(ii) $\Delta t \le C \min \left\{ \frac{\Delta x}{|a|}, \frac{\Delta y}{|b|} \right\} \Leftarrow$

$\Delta t \approx 1$