Big-Oh

Def. We write $f(h) = O(g(h))$ as $h \to 0$ if $\exists\, C$ constant s.t.

$$\boxed{\left|\frac{f(h)}{g(h)}\right| < C} \quad \forall h \text{ sufficiently small.}$$

Rmk. $f(h)$ decays to zero "at least" as fast as $g(h) \to 0$

Def. Little-oh

$f(h) \to o(g(h))$ as $h \to 0$ if

$$\left|\frac{f(h)}{g(h)}\right| \to 0 \quad \text{as } h \to 0.$$

Rmk. $f(h) \to 0$ as $h \to 0$ faster than $g(h) \to 0$ as $h \to 0$.

Rmk. Little oh $\subset$ Big Oh

Ex $\quad u_{x,i} \left(= \frac{\partial u}{\partial x}\bigg|_{x=x_i} \right) \approx \frac{u_{i+1} - u_i}{\Delta x}$

(Forward in space)

$$u_{x,i} = \frac{u_{i+1} - u_i}{\Delta x} - \frac{\Delta x}{2} u_{xx,i}(x_\xi)$$

$$x_i \le x_\xi \le x_{i+1}$$

$$= \frac{u_{i+1} - u_i}{\Delta x} + O(\Delta x)$$

Q Compute the truncation error of Forward in space ?

A
$$\begin{cases} u_{i+1} = u_i + u_{x_i} \Delta x + u_{xx_i} \frac{\Delta x^2}{2} + H.O.T. \\ u_{i-1} = u_i - u_{x_i} \Delta x + u_{xx_i} \frac{\Delta x^2}{2} + \cdots \end{cases}$$

$\rightarrow \frac{u_{i+1} - u_i}{\Delta x} = u_{x_i}$

Δx

$2 \longrightarrow O(\Delta x)$

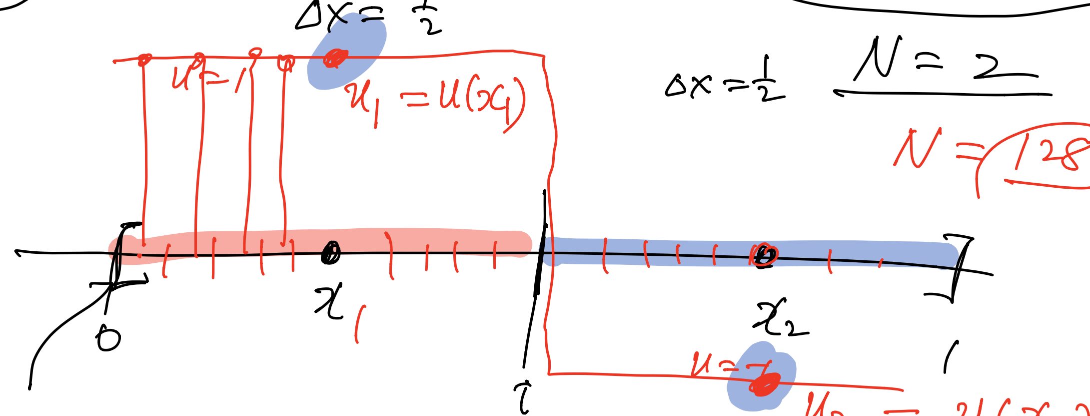
Ex $\quad u_{x,i} = \dfrac{u_i - u_{i-1}}{\Delta x}$ $\quad$ IC: u

Ex

$$u_t + a u_x = 0$$

$\Delta x = \frac{1}{2}$ $\quad N = 2$

$N = 128$



$x_{i-\frac{1}{2}}$

$x_{i+\frac{1}{2}} \longrightarrow$ half-integer

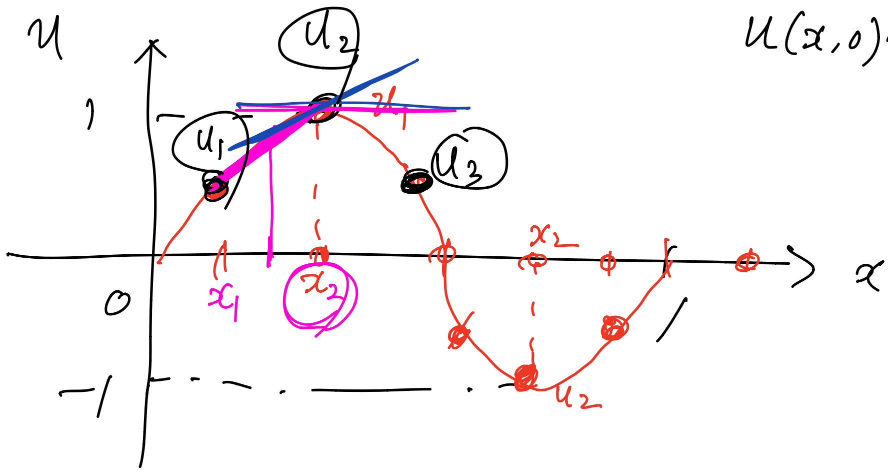
$$u = \begin{cases} 1 & x < 0.5 \\ -1 & x > 0.5 \end{cases}$$

- x_i : cell-center location
- ith cell: $[x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}] = I_i$
- $x_{i-\frac{1}{2}} = x_i - \dfrac{\Delta x}{2}$
- $\Delta x = \dfrac{1-0}{N} = \dfrac{1}{2} = 0.5$

u_i

$$u_{x,2} \approx \frac{u_2 - u_1}{\Delta x}$$

$u(x,0) = \sin(2\pi x)$



Ex **Centered diff.** or (centered in space)

$$u_{x,i} = \frac{u_{i+1} - u_{i-1}}{2\Delta x} + O(\Delta x^2)$$

Δx^2

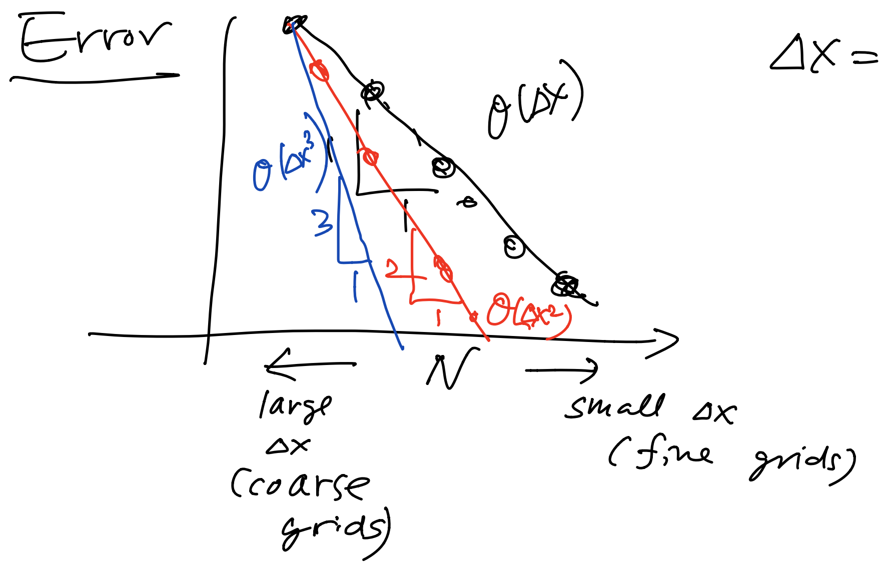
$\Delta x \ll 1$

Rmk

CS $\rightarrow$ $O(\Delta x^2)$

BS $\rightarrow$ $O(\Delta x)$

FS



$$\Delta x = \frac{1}{N}$$