Homework 3 on Chapter 2 and 3

January 24, 2022

1 Wave packet approximation

Prove equation (2.48) of the lecture notes starting from (2.44), filling in the steps that are omitted in the lecture notes (i.e. providing a few extra steps and explanations).

2 Application : A sound mirror

Read Section 2.7.3 (on ray tubes and spherical loudspeakers) and do the following question.

At the San Francisco Exploratorium, there is a rather amazing example of a sound mirror. The display is composed of two walls facing each other at two ends of a (usually very crowded) room, and each wall has a 2m-size half-sphere carved into it (see Figure 1 on the next page). When one person stands at the edge of the half-sphere and speaks into it, even very gently, another person standing in the same position but near the other half-sphere can hear the first person's voice very clearly. If, on the other hand, the first person turns around and speaks directly towards the second, it's usually impossible for the two people to hear each other across the noisy room.

Explain semi-quantitatively (i.e. no need for detailed mathematics, but at least some quantitative reasoning) how the sound mirror works, and why speaking directly to one another wouldn't. Hint: use ray theory and assume that a half-sphere is not too different from a parabola for simplicity. A parabola has the property that parallel rays coming in from infinity all focus in the same point (called the focal point). Similarly, rays emitted from the focus end up going to infinity on parallel lines, after reflection on the mirror. See Wikipedia article on Parabolic Reflectors for instance.

3 Diffraction of sound waves

Consider a 2D uniform medium (with constant sound speed). Then, consider a small region where the sound speed is slightly larger, i.e. by writing that

$$c_s(X,Y) = c_0 + (\Delta c)\exp\left[-\frac{(X^2+Y^2)}{2}\right] \quad (1)$$

where $\Delta c > 0$.

What will happen to the paths of waves coming from $Y \to +\infty$, along wavevectors that are initially along $\boldsymbol{e}_y$, as they approach the sound speed anomaly? A qualitative answer is sufficient. Sketch some

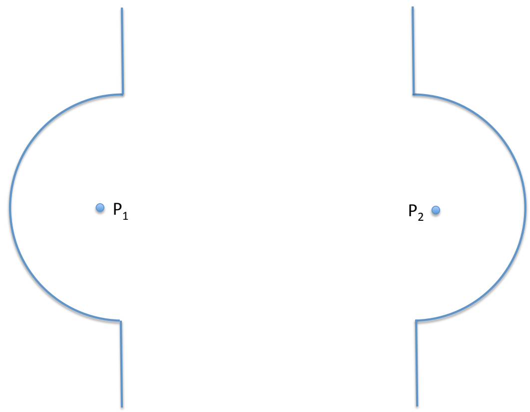

Figure 1: Schematic of the sound mirror. The two people are sitting at the two foci of the mirrors at P_1 and P_2, talking to each other while facing the mirrors, can hear each other across a crowded, noisy room.

representative ray paths. What happens if the sound speed anomaly Δc is negative? Sketch the ray paths in that case.

4 Gravity waves wave packet equations

Consider the governing equation for internal gravity waves (equation 4.7), in the case where N is a slowly varying function of $\boldsymbol{X}$ and the slow time τ. Assume a wave packet solution of the form

$$\phi = A(\boldsymbol{X}, \tau)e^{i\theta(\boldsymbol{x},t)} \tag{2}$$

with $\boldsymbol{k} = \nabla\theta$ and $\omega = -\partial\theta/\partial t$.

Starting from these assumptions only, and using the wave-packet approximation, directly prove equations (4.23), (4.28), (4.29) of the lecture notes.

5 Gravity waves in a linearly stratified medium

What are the ray paths of gravity wave packets in an ocean basin of uniform depth H that has $N = aZ$, where $Z = 0$ is the surface of the water, and Z increases downward? Note that you may assume that the basin is infinite in the horizontal direction. Plot an upward moving ray path starting from $X = 0$ and $Z = H$.