Computational $\frac{29}{30}$

Numerical Linear Algebra

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Contents

4: Assignment 4

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1 Part 1

1.1 Symmetric to tridiagonal

Using Householder matrices

$$A = \begin{pmatrix} 5 & 4 & 1 & 1 \\ 4 & 5 & 1 & 1 \\ 1 & 1 & 4 & 2 \\ 1 & 1 & 2 & 4 \end{pmatrix}.$$

the matrix reduces to the similar (tridiagonal) matrix ✓ $\frac{10}{10}$

$$\begin{pmatrix} 5.0 & -4.246 & 0.0 & 0.0 \\ -4.246 & 6.0 & 1.4142 & 0.0 \\ 0.0 & 1.4142 & 5.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 2.0 \end{pmatrix}.$$

1.2 QR to calculate eigenvalues

$$A = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}.$$

1.2.1 Without shifts

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The algorithm takes 45 iterations and produces the following eigenvalues along the diagonal

$$\lambda_1 \approx 3.7321 \qquad \lambda_2 \approx 2.0 \qquad \lambda_3 \approx 0.2679.$$

using a tolerance of 10^{-12} on the norm of the subdiagonal of the input matrix

1.2.2 With shifts

This algorithm, using the same tolerance as above on the on the last subdiagonal entry of the input matrix, calculates the eigenvalues of A in just 7 iterations

$$\lambda_1 \approx 3.7321 \qquad \lambda_2 \approx 2.0 \qquad \lambda_3 \approx 0.2679.$$

1.3 Eigenvectors

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$$A = \begin{pmatrix} 2 & 1 & 3 & 4 \\ 1 & -3 & 1 & 5 \\ 3 & 1 & 6 & -2 \\ 4 & 5 & -2 & -1 \end{pmatrix}.$$

which has eigenvalues $\lambda_1 = -8.0286$, $\lambda_2 = 7.9329$, $\lambda_3 = 5.6689$, and $\lambda_4 = -1.5732$

Using the inverse iteration algorithm and the initial guess of $\lambda_1 = -8$ and the same tolerance of 10^{-12} on the two norm difference between (absolute value of) eigenvectors of successive iterations, we converge to

$$v_1 \approx \begin{pmatrix} -0.2635 \\ -0.6590 \\ 0.1996 \\ 0.6756 \end{pmatrix}.$$

for eigenvalue λ_1, in 7 iterations. Then for $lambda_2$, the associated eigenvector converges in 20 iterations to, with guess of $\lambda_2 = 7$

$$v_2 \approx \begin{pmatrix} 0.3787 \\ 0.3624 \\ -0.5379 \\ 0.6602 \end{pmatrix}.$$

Need to tweak μ values to get distinct e-vecs here.

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Then for guess $\lambda_3 = 5$, the associated eigenvector converges in 12 iterations to

$$v_3 \approx \begin{pmatrix} 0.3787 \\ 0.3624 \\ -0.5379 \\ 0.6602 \end{pmatrix}.$$

For guess $\lambda_4 = -1.5$, the associated eigenvector converges in 8 iterations to

$$v_4 \approx \begin{pmatrix} -0.6880 \\ 0.6241 \\ 0.2598 \\ 0.2638 \end{pmatrix}.$$