To Be Appended

Ycr

Seishun Academy

1 Common Distributions and Their Characteristic Functions

With this section, we will introduce characteristic functions of some common distributions. And we may not derive them during the presentation.

1.1 Common Distributions and Their Characteristic Functions

Const If $X \equiv a$ where $a =$const, then $\phi_X(t) = e^{iat}$.

Binomial Random Variable

$$X \sim Binomial(m, p),\ m = 1, 2, \cdots, 0 \le p \le 1;$$
$$p_X(n) = \binom{n}{m} p^n (1-p)^{m-n}, n = 0, 1, \cdots, m;$$
$$\phi_X(t) = (pe^{it} + (1-p))^m.$$

Poisson Random Variable

$$X \sim Poisson(\lambda), \lambda > 0;$$
$$p_X(n) = \frac{\lambda^n}{n!} e^{-\lambda}, n = 0, 1. \cdots;$$
$$\phi_X(t) = e^{\lambda(e^{it}-1)}.$$

Normal Random Variable

$$X \sim N(0, 1), \mu \in \mathbb{R};$$
$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, x \in \mathbb{R};$$
$$\phi_X(t) = e^{-\frac{t^2}{2}}.$$

$$Y \sim N(\mu, \sigma^2), \mu \in \mathbb{R};$$
$$f_Y(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu)^2}{2\sigma^2}}, y \in \mathbb{R};$$
$$\phi_Y(t) = e^{iat - \frac{\sigma^2 t^2}{2}}.$$

1.2 To be continued

The Inversion and Parseval identity
Some Theorems
Proofs of CLT or something else

References