Introduction to LaTeX
Part II: Writing a Technical Paper

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Getting Started with your LaTeX Technical Paper

First Step: Set up the preamble, which is the area between `\documentclass` and `\begin{document}`.

The preamble includes the definition of the document class with options,

```
\documentclass[journal,onecolumn]{IEEEtran}
```

Global style commands.

```
\setlength{\parindent}{0pt}
```

Packages that you want to include,

```
\usepackage[pdftex]{graphicx}
```

Your own special features and definitions

```
\def\pr{{\rm P}}
```


Paper Preamble

For the sample paper, the preamble is quite simple:

```
%\documentclass[journal,onecolumn,twoside]{IEEEtran}
\documentclass[10pt,twocolumn,twoside]{IEEEtran}
\usepackage[pdftex]{graphicx}
\usepackage{amsmath,amssymb}
\usepackage{setspace}
\usepackage{subfigure}
\def\pr{{\rm P}}

\begin{document}
```

Paper Header

Random Variables: An Overview

Monson Hayes, *Fellow, IEEE*
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Seoul, Korea
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(Invited Paper)

LaTeX

For the sample paper, the header is straightforward:

```
\title{Random Variables: An Overview}
\author{Monson Hayes,~\IEEEmembership{Fellow,~IEEE} \\
        Chung-Ang University \\
        Seoul, Korea \\
        \textit{mhh3@gatech.edu}}

\markboth{IEEE Transactions on \LaTeX\ }
         {Hayes}

\IEEEspecialpapernotice{(Invited Paper)}
\maketitle
```


Paper Abstract

***Abstract*—This paper introduces the concept of a random variable, which is nothing more than a variable whose numeric value is determined by the outcome of an experiment. To describe the probabilities that are associated with these numeric values in a concise and conceptually useful manner, the probability distribution and probability density function are introduced. Then, the moment generating function is defined, and several examples are given. Finally, the concept of a correlation function and correlation matrices is introduced.**

LaTeX

```
\begin{abstract}
This paper introduces the concept of a random variable,
which is nothing more than a variable whose numeric
value is determined by the outcome of an experiment.
To describe the probabilities that are associated with
these numeric values in a concise and conceptually useful
manner, the probability distribution and probability
density function are introduced.
Then, the moment generating function is defined, and
several examples are given.
Finally, the concept of a correlation function and
correlation matrices is introduced.
\end{abstract}
%\doublespace
```

Paper Introduction

I. Introduction

The concept of a random variable is a simple one, and one that is important. Although perhaps sounding at first like something difficult, random variables are conceptually quite simple. Given a sample space Ω corresponding to some random experiment, this sample space contains elementary events, $\omega \in \Omega$, and when an experiment is performed, a specific elementary event (experimental outcome) is observed.

LaTeX

```
\section{Introduction}
The concept of a random variable is a simple one, and
one that is important.
Although perhaps sounding at first like something difficult,
random variables are conceptually quite simple.
Given a sample space $\Omega $ corresponding to some random
experiment, this sample space contains elementary events,
$\omega \in \Omega $, and when an experiment is performed,
a specific elementary event (experimental outcome) is
observed.
```


Section II with Numbered Equation and Footnote

II. Probability Assignments

Let N be a variable that represents the number of α particles that are counted over a given period of time. The ensemble for N is the set of non-negative integers

$$\mathcal{E}_N = \{0, 1, 2, \ldots\}$$

Since the number of outcomes is unknown until we actually make a count, then N is a random variable. In many cases, it is appropriate to model N as a *Poisson random variable* where[1]

$$\mathrm{P}\{N = n\} = \frac{\lambda^n}{n!} e^{-\lambda} \qquad n \geq 0 \tag{1}$$

for some $\lambda > 0$.

Given this probability assignment for N, it is then easy to find the probability of any event that is defined in terms of

[1] Note that with this probability assignment it is assumed that the number of particles may be arbitrarily large and, in fact, approach infinity.

LaTeX

```
\section{Probability Assignments}
Let $N$ be a variable that represents the number
of $\alpha $ particles that are counted over a given period of
time.
The ensemble for $N$ is the set of non-negative integers
    \[ {\cal E}_N = \{ 0, 1, 2, \ldots \} \]
Since the number of outcomes is unknown until we actually make a
count, then $N$ is a random variable.
In many cases, it is appropriate to model $N$ as a
\emph{Poisson random variable} where\footnote{Note that with this
probability assignment it is assumed that the number of particles
may be arbitrarily large and, in fact, approach infinity.}
\begin{equation}
    \pr \{ N = n \}
        = \frac {\lambda ^n}{n!} e^{-\lambda } \qquad n\geq 0
    \label{eq:prob_assgn}
    \end{equation}
for some $\lambda > 0$.
```

Union

values of N. For example, the probability that the number of α particles is less than some number, N_0, may be found as follows. Since the event $\{N < N_0\}$ is the union of the events $\{N = k\}$ for $k = 0, 1, \ldots, N_0 - 1$,

$$\{N < N_0\} = \bigcup_{n=0}^{N_0-1} \{N = n\}$$

and since these events are mutually exclusive, then

$$\mathrm{P}\{N < N_0\} = \sum_{n=0}^{N_0-1} \mathrm{P}\{N = n\} = \sum_{n=0}^{N_0-1} \frac{\lambda^n}{n!} e^{-\lambda}$$

LaTeX

```
For example, the probability that the number of $\alpha $
particles is less than some number, $N_0$, may be found as
follows.
Since the event $\{ N < N_0 \}$ is the union of the events
$\{ N=k \}$ for $k = 0, 1, \ldots , N_0-1$,
    \[ \{ N < N_0 \} = \bigcup _{n=0} ^{N_0-1} \{N = n\} \]
and since these events are mutually exclusive, then
\begin{equation*}
    \pr \{N < N_0 \}
       = \sum _{n=0} ^{N_0-1} \pr \{ N=n \}
       =  \sum _{n=0} ^{N_0-1}
            \frac {\lambda ^n}{n!} e^{-\lambda}
    \end{equation*}
```


An Integral and a Sum

This last sum may be evaluated using the following

$$\sum_{n=0}^{k} \frac{\lambda^n}{n!} e^{-\lambda} = \frac{\Gamma(k+1, \lambda)}{k!}$$

where

$$\Gamma(k, \lambda) = \int_{\lambda}^{\infty} x^{k-1} e^{-x} dx$$

LaTeX

```
This last sum may be evaluated using the following
    \[ \sum _{n=0} ^{k} \frac {\lambda ^n}{n!} e^{-\lambda }
            = \frac {\Gamma(k+1,\lambda)}{k!} \]
where
    \[ \Gamma (k,\lambda )
            = \int _{\lambda }^{\infty} x^{k-1}e^{-x}dx \]
```

Arrays with Invisible Delimiters

In order to express probability mass functions mathematically, we introduce the *delta function*,[2] which is defined as follows:

$$\delta[n] = \left\{ \begin{array}{cl} 1 & \ ; \ n = 0 \\ 0 & \ ; \ n \neq 0 \end{array} \right.$$

Shifted delta functions may be used to represent functions that have a value of one at other values of n. For example, $\delta[n-1]$ is equal to one when $n = 1$ and equal to zero for all other values of n. Therefore, for an integer-valued discrete random variable X with

$$\mathrm{P}\{ X = n \} = p_X[n] \ ; \ -\infty < n < \infty$$

[2] In digital signal processing, $\delta[n]$ is referred to as the unit sample function.

LaTeX

```
In order to express probability mass functions mathematically,
we introduce the \emph{delta function},\footnote{In digital
signal processing, $\delta [n]$ is referred to as the unit
sample function.} which is defined as follows:
    \[ \delta [n] = \left\{ \begin{array}{cl}
                    1 & \ ; \ n = 0 \\
                    0 & \ ; \ n \neq 0 \end{array} \right. \]
Shifted delta functions may be used to represent functions
that have a value of one at other values of $n$.  For example,
$\delta [n-1]$ is equal to one when $n=1$ and equal to zero for
all other values of $n$.
Therefore, for an integer-valued discrete random variable $X$
with
    \[ \pr \{ X = n \} = p_X[n] \ ; \ -\infty < n < \infty \]
```


Fractions in Equations

Another example is the *The Geometric Random Variable* that has an ensemble equal to the set of all positive integers

$$\mathcal{E}_X = \{ 1, 2, 3, \ldots \}$$

with a probability law given by

$$\mathrm{P}\{ N = k \} = (\tfrac{1}{2})^k \ ; \ k > 0$$

The probability mass function for this random variable is

$$p_N(n) = \sum_{k=1}^{\infty} (\tfrac{1}{2})^k \delta[n-k]$$

LaTeX

```
Another example is the \emph{The Geometric Random Variable}
that has an ensemble equal to the set of all positive
integers
    \[ {\cal E}_X = \{ 1, 2, 3, \ldots \} \]
with a probability law given by
    \[ \pr \{ N = k \} = (\tfrac{1}{2})^k \ ; \ k > 0 \]
The probability mass function for this random variable is
    \[ p_N(n) = \sum _{k=1} ^{\infty }
                (\tfrac {1}{2})^k \delta [n - k] \]
```

Combinatorics and Phantoms

Another random variable that occurs frequently in applications is one that corresponds to the number of successes, N, in n Bernoulli trials, with the probability of a success being equal to p. In this case, N has a *Binomial Distribution* with

$$p_N(k) = \mathrm{P}\{N = k\} = \dbinom{n}{k} p^k(1-p)^{n-k} \ ; \ 0 \leq k \leq n$$

where $\tbinom{n}{k}$ is the number of combinations of n objects that are taken k at a time, and is defined by

$$\dbinom{n}{k} = \frac{n!}{k!(n-k)!}$$

Alternative notations include $C(n,k)$, ${}_nC_k$, ${}^nC_k$, and C^n_k.

LaTeX

```
Another random variable that occurs frequently in
applications is one that corresponds to the number of
successes, $N$, in $n$ Bernoulli trials, with the
probability of a success being equal to $p$.
In this case, $N$ has a \emph{Binomial Distribution}
with
    \[ p_N(k) = \pr \{ N = k \}
              = \dbinom{n}{k} p^k(1-p)^{n-k} \ ; \
                0 \leq k \leq n \]
where $\tbinom{n}{k}$ is the number of combinations of
$n$ objects that are taken $k$ at a time, and is defined
by
    \[ \dbinom{n}{k} = \frac {n!}{k!(n-k)!} \]
Alternative notations include $C(n,k)$,
${}_nC_k$, ${}^nC_k$, and $C^n_k$.
```


Substack Command

Interesting problems that are sometimes challenging to solve, are those such as

$$\mathrm{P}\{N \text{ is odd}\} = \sum_{\substack{0 \leq n \leq \infty \\ n \text{ odd}}} \mathrm{P}\{N = n\}$$

LaTeX

```
Interesting problems that are sometimes challenging
to solve, are those such as
    \[ \pr \{ N \text{ is odd}\}
        = \sum _{\substack{0 \leq n \leq \infty \\[0.5ex]
                           n \text{ odd}}}
            \pr \{N = n\} \]
```

Tables

Name	Density Function
Exponential	$f_X(x) = \lambda e^{-\lambda x}$
Laplace	$f_X(x) = \frac{1}{2}\alpha e^{-\alpha \|x - m\|}$
Rayleigh	$f_X(x) = \alpha^2 x e^{-\alpha^2 x^2/2}, \ x \geq 0$
Uniform	$f_X(x) = 1/(b-a), \ b \leq x \leq a$

TABLE I
A TABLE OF COMMON AND IMPORTANT RANDOM VARIABLES.

LaTeX

```
\begin{table}[t]
\begin{center}
\begin{tabular}[t]{|ll|}
\hline\hline
Name  & Density Function \\
\hline\hline
    & \\
Exponential & $f_X(x) = \lambda e^{-\lambda x}$ \\
Laplace     & $f_X(x) = \frac{1}{2}\alpha e^{-\alpha |x - m|}$ \\
Rayleigh    & $f_X(x) = \alpha ^2 x e^{-\alpha ^2x^2/2},
                        \ x \geq 0$ \\
Uniform     & $f_X(x) = 1/(b-a), \ b \leq x \leq a$ \\
& \\
 \hline
\end{tabular}
\end{center}
    \caption{A table of common and important random variables.}
    \label{table:RandomVariables}
\end{table}
```


Enumeration with Item Separation

Two properties of the density function are:

1) $f_X(x) \geq 0$ for all x.
2) $\displaystyle\int_{-\infty}^{\infty} f_X(x)dx = 1$

LaTeX

```
Two properties of the density function are:
\begin{enumerate}\setlength{\itemsep}{4pt}
    \item $f_X(x) \geq 0$ for all $x$.
    \item ${\displaystyle \int _{-\infty } ^{\infty }
                f_X(x) dx } = 1$
    \end{enumerate}
```

Left and Right Brackets

$$\begin{array}{rcl} |M_X(j\omega)| & = & \left| \int_{-\infty}^{\infty} e^{j\omega x} f_X(x) dx \right| \leq \int_{-\infty}^{\infty} \left| e^{j\omega x} f_X(x) \right| dx \\ & = & \int_{-\infty}^{\infty} \left| e^{j\omega x} \right| \ \left| f_X(x) \right| dx = \int_{-\infty}^{\infty} f_X(x) dx = 1 \end{array}$$

then the characteristic function is well-defined and will always exist for any probability density function.

LaTeX

```
\begin{array*}
    |M_X(j\omega )|
       &= \left| \int _{-\infty } ^{\infty }
                          e^{j\omega x} f_X(x) dx \right|
          \leq   \int _{-\infty } ^{\infty } \left|
                          e^{j\omega x} f_X(x) \right| dx \\
       &= \int _{-\infty } ^{\infty }
              \left| e^{j\omega x} \right| \
              \left| f_X(x) \right| dx
        =     \int _{-\infty } ^{\infty } f_X(x) dx = 1
    \end{eqnarray*}
```


Figures

B. Gaussian Random Variable

A zero-mean Gaussian random variable X has a density function of the form

$$f_X(x) = \frac{1}{\sigma_x\sqrt{2\pi}} e^{-x^2/2\sigma_x^2}$$

where σ_x^2 is the variance of X. A plot of the density function of a Gaussian for several different values of σ_x is shown in Fig. 1.

LaTeX

```
\subsection{Gaussian Random Variable}

A zero-mean Gaussian random variable $X$ has a density
function of the form
    \[ f_X(x) = \frac {1}{\sigma _x\sqrt {2\pi }}
                e^{-x^2/2\sigma _x^2} \]
where $\sigma _x^2$ is the variance of $X$.
A plot of the density function of a Gaussian for several
different values of $\sigma _x$ is shown in
Fig.~\ref{fig:Gaussian}.
\begin{figure}
\begin{center}
  \includegraphics[width=\hsize]{images/Gaussian.png}\\
  \end{center}
  \caption{A Gaussian Density Function}
  \label{fig:Gaussian}
\end{figure}
```

Underbraces

The characteristic function is

$$
\begin{aligned}
M_X(j\omega) &= \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{j\omega x} e^{-x^2/2\sigma_x^2} \\
&= e^{-\omega^2\sigma_x^2/2} \underbrace{\frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(x-j\omega\sigma_x)^2/2\sigma_x^2} dx}_{=1}
\end{aligned}
$$

LATEX

```
The characteristic function is
\begin{align*}
   M_X(j\omega )
      &= \frac {1}{\sigma _x\sqrt {2\pi }}
         \int _{-\infty } ^{\infty } e^{j\omega x}
                e^{-x^2/2\sigma _x^2} \\
      &= e^{-\omega ^2 \sigma _x^2/2}
           \underbrace{\frac {1}{\sigma _x\sqrt {2\pi }}
           \int _{-\infty } ^{\infty }
           e^{-(x - j\omega \sigma _x)^2/2\sigma _x^2} dx}_{=1}
    \end{align*}
```


Double Integrals

$$
\mathrm{P}\{x_1 \leq X \leq x_2,\ y_1 \leq X \leq y_2\} = \int_{x_1}^{x_2} \int_{y_1}^{y_2} f_{XY}(x,y)dxdy \tag{6}
$$

LATEX

```
\begin{equation}
    \pr \{ x_1 \leq X \leq x_2, \  y_1 \leq X \leq y_2 \}
        = \int _{x_1}^{x_2} \int _{y_1}^{y_2} f_{XY}(x, y) dx dy
    \label{eq:IntegrateJointDensity}
    \end{equation}
```

Integrals with Limits at Bottom of Integral Sign

$$\mathrm{P}\{(X,Y) \in R\} = \iint\limits_{R} f_{XY}(x,y)dxdy \tag{7}$$

LaTeX

```
\begin{equation}
    \pr \{ (X,Y) \in R \}
        = \iint\limits _{R} f_{XY}(x,y) dx dy
    \label{eq:IntegrateOverR}
    \end{equation}
```


Matrices

$$\mathbf{R}_X = E\{\mathbf{X}\mathbf{X}^T\} = \left[\begin{array}{cc} E\{X_1^2\} & E\{X_1X_2\} \\[1ex] E\{X_2X_1\} & E\{X_2^2\} \end{array} \right]$$

For n random variables, the correlation matrix has the form

$$\mathbf{R}_X = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nn} \end{bmatrix}$$

LaTeX

```
${\bf X}{\bf X}^T$,
     \[ {\bf R}_X
        = E\{ {\bf X} {\bf X}^T \}
        = \left[ \begin{array}{cc}
                E\{ X_1^2 \} & E\{ X_1X_2 \} \\[1ex]
                E\{ X_2X_1\}  & E\{ X_2^2 \}
                \end{array} \right] \]
For $n$ random variables, the correlation matrix has the
form
    \[ {\bf R}_X
            = \begin{bmatrix}
                r_{11} & r_{12} & \cdots & r_{1n} \\
                r_{21} & r_{22} & \cdots & r_{2n} \\
                \vdots & \vdots & \ddots & \vdots \\
                r_{n1} & r_{n2} & \cdots & r_{nn} \\
                \end{bmatrix}
                \]
```

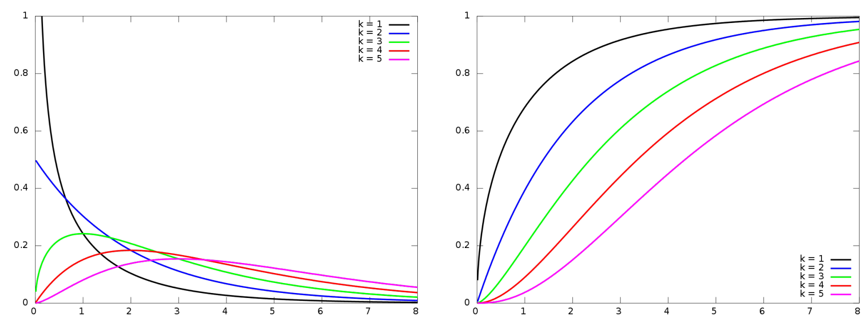
Sums and Integrals in Fractions

$$\hat{r}_{xy} = \frac{\displaystyle\sum_{i=1}^{n}(x_i - x)(y_i - y)}{\displaystyle\left[\sum_{i=1}^{n}(x_i - x)^2 \sum_{i=1}^{n}(y_i - y)^2\right]}$$

LaTeX

```
\[ \hat{r}_{xy}
    = \frac {\displaystyle \sum _{i=1}^n (x_i - x)(y_i - y)}
            {\displaystyle \left[ \sum _{i=1}^n (x_i - x)^2
            \sum _{i=1}^n (y_i - y)^2 \right]} \]
```


The Subpicture Environment



(a) Density Function (b) Distribution Function

Fig. 2. The Chi-square Random Variable.

LaTeX

```
\begin{figure}
\begin{center}
\subfigure[Density Function]{
    \includegraphics[width=0.45\hsize]
        {images/Chi-Square_distributionPDF.png}}
\subfigure[Distribution Function]{
    \includegraphics[width=0.45\hsize]
        {images/Chi-Square_distributioncDF.png}}
  \end{center}
  \caption{The Chi-square Random Variable.}
  \label{fig:ChiSquare}
\end{figure}
```

References Using BibTEX

VIII. Conclusion

There are many excellent textbooks where the reader may find advanced developments of the results presented in this paper. The classic work in the field is the text by Papoulis [1]. Another recommended text is [2]. An introduction to Monte Carlo simulations may be found in [3].

References

[1] A. Papoulis and S. Pillai, *Probability, Random Variables, and Stochasic Processes*. New York: McGraw-Hill, 2002.
[2] H. Larsen and B. Shubert, *Probabilistic Models in Engineering Sciences, Vol. 1*. New York: John Wiley and Sons, 1979.
[3] S. Raychaudhuri, "Introduction to monte carlo simulation," in *Simulation Conference, 2008. WSC 2008. Winter*, pp. 91 –100, Dec. 2008.

LATEX

```
\section{Conclusion}

There are many excellent textbooks where the reader may
find advanced developments of the results presented in
this paper.
The classic work in the field is the text by
Papoulis~\cite{Pap2002}.
Another recommended text is~\cite{Larsen}.
An introduction to Monte Carlo simulations may be found
in~\cite{MonteCarlo}.

\bibliography{mybibliography}
\bibliographystyle{ieeetr}

\end{document}
```


BibTEX

- BibTEX makes it easy to cite sources in a consistent manner, by separating bibliographic information from the presentation of this information.
- BibTEX takes, as input
  1. An .aux file produced by LATEX on an earlier run;
  2. A .bst file (the style file), that specifies the general reference-list style and specifies how to format individual entries,
  3. A .bib file(s) constituting a database of all reference-list entries the user might ever hope to use.
- BibTEX chooses from the .bib file(s) only those entries specified by the .aux file (that is, those given by LATEX's `\cite` or `\nocite` commands), and creates as output a .bbl file containing these entries together with the formatting commands specified by the .bst file.
- LATEX uses the .bbl file, perhaps edited by the user, to produce the reference list.

BibTEX File

```
@book{Pap2002,
author = {A. Papoulis and S. Pillai},
title = {Probability, Random Variables, and Stochastic Processes},
publisher = {McGraw-Hill},
Address = {New York},
year = {2002}
}

@INPROCEEDINGS{MonteCarlo,
author={Raychaudhuri, S.},
booktitle={Simulation Conference, 2008. WSC 2008. Winter},
title={Introduction to Monte Carlo Simulation},
year={2008},
month={Dec.},
volume={},
number={},
pages={91 -100},
keywords={Monte Carlo simulation;repeated random sampling;
         statistical analysis;Monte Carlo methods;random processes},
doi={10.1109/WSC.2008.4736059},
ISSN={},}
```

Manual Specification of References

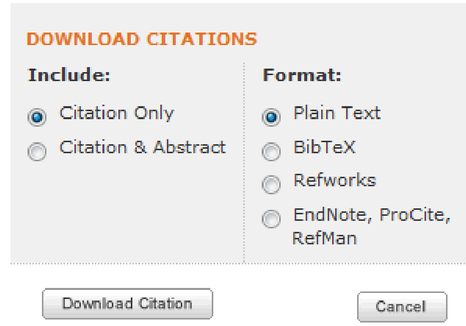
```
\begin{thebibliography}{9}

\bibitem{Pap2002} A. Papoulis and S. Pillai,
    \emph{Probability, Random Variables, and Stochastic Processes},
    Mc-Graw Hill, 2002.
\bibitem{Larsen}
    H Larsen and B. Shubert,
    \emph{Probabilistic Models in Engineering Sciences, Vol. 1},
    John Wiley and Sons, New York, 1979.
\bibitem{MonteCarlo}
    S. Raychaudhuri,
    "Introduction to Monte Carlo Simulation,"
    \emph{Simulation Conference}, pp. 91-100, Dec. 2008

\end{thebibliography}
```


BibTEX and IEEExplore

- IEEExplore and other databases will export citations into BibTEX format, as well as others.



Ifthen Package

```
%---------Ignore all text from here until \fi ---------
%---Replace \iffalse with \iftrue to include text -----
\iffalse
It is clear that this probability assignment satisfies
the first probability axiom since all probabilities in
Eq.~\ref{eq:prob_assgn} are positive.
\fi
%------------end ----------------
```


Verbatim Text

- To include computer listings or other similar text, we would like to have unformatted text to produce something like:

```
\begin{align*}
  M_X(j\omega )
    &= \frac {1}{\sigma _x\sqrt {2\pi }}
       \int _{-\infty } ^{\infty } e^{j\omega x}
             e^{-x^2/2\sigma _x^2} \\
    &= e^{-\omega ^2 \sigma _x^2/2}
         \underbrace{\frac {1}{\sigma _x\sqrt {2\pi }}
         \int _{-\infty } ^{\infty }
         e^{-(x - j\omega \sigma _x)^2/2\sigma _x^2} dx}_{=1}
   \end{align*}
```

- There are several ways to introduce text that won't be interpreted by the compiler.
  - With the verbatim environment, everything input between a `\begin{verbatim}` and an `\end{verbatim}` command will be processed as if by a typewriter.
  - Also see the `\verb` command for short in-line verbatim text.

Graphics

- Now that you have your beautifully typeset journal paper or article, you want your figures, block diagrams, plots and other graphics to be beautifully typset.
- There are a number of very powerful packages that allow you to create graphics in postscript or PDF file format. Some of these are:
  - xfig,
  - TikZ and PGF,
  - XY-Pic,
  - PSTricks and PDFTricks,
  - Metapost
  - Adobe Illustrator
- See the web for a description of these packages and for documentation.

Next Time

- How to prepare and deliver an effective presentation.
- Presentations with PowerPoint Using Aurora
- LaTeX Presentations using the Beamer and Prosper Classes