Statistics for Data Science
Unit 4 Part 2 Homework: Continuous Random Variables

May 21, 2019

1. **Processing Pasta**

   A certain manufacturing process creates pieces of pasta that vary by length. Suppose that the length of a particular piece, L, is a continuous random variable with the following probability density function.

$$f(l) = \begin{cases} 0, & l \leq 0 \\ l/2, & 0 < l \leq 2 \\ 0, & 2 < l \end{cases}$$

   (a) Write down a complete expression for the cumulative probability function of L.

   (b) Using the definition of expectation for a continuous random variable, compute the expected length of the pasta, $E(L)$.

2. **The Warranty is Worth It**

   Suppose the life span of a particular (shoddy) server is a continuous random variable, T, with a uniform probability distribution between 0 and 1 year. The server comes with a contract that guarantees you money if the server lasts less than 1 year. In particular, if the server lasts t years, the manufacturer will pay you $g(t) = \$100(1-t)^{1/2}$. Let $X = g(T)$ be the random variable representing the payout from the contract.

   Compute the expected payout from the contract, $E(X) = E(g(T))$.

3. **(Lecture)#Fail**

   Suppose the length of Paul Laskowski's lecture in minutes is a continuous random variable C, with pmf $f(t) = e^{-t}$ for $t > 0$. This is an example of an exponential random variable, and it has some special properties. For example, suppose you have already sat through t minutes of the lecture, and are interested in whether the lecture is about to end immediately. In statistics, this can be represented by something called the *hazard rate*:

$$h(t) = \frac{f(t)}{1 - F(t)}$$

   To understand the hazard rate, think of the numerator as the probability the lecture ends between time t and time $t + dt$. The denominator is just the probability the lecture does not end before time t. So you can think of the fraction as the conditional probability that the lecture ends between t and $t + dt$ given that it did not end before t.

Compute the hazard rate for C.

4. **Optional Advanced Exercise: Characterizing a Function of a Random Variable**

   Let X be a continuous random variable with probability density function $f(x)$, and let h be an invertible function where h^{-1} is differentiable. Recall that $Y = h(X)$ is itself a continuous random variable. Prove that the probability density function of Y is

$$g(y) = f(h^{-1}(y)) \cdot \left| \frac{d}{dy} h^{-1}(y) \right|$$