Problem
Let X = the outcome when a fair die is rolled once. If before the die is rolled you are offered either (1/3.5) dollars or *h*(X) = 1/X dollars, would you accept the guaranteed amount or would you gamble? [NOTE: It is not generally true that 1/E(X) = E(1/X).]

Solution
The big question here is, which bet offers the best possibility to win, and to come up with the answer we have to analyze the possibilities, and ask ourselves the following: $E(1/X) \geq 1/E(X) = 1/3.5$, or if $E(1/X)$ < 1/E(X) = 1/3.5. In other words, is the expected value of the function *h*(X)=1/X greater than or equal to the expected value of 1/E(X) which is equivalent to 1/3.5, or is the expected value of the function *h*(X) is less than the expected value of 1/E(X). So before we gamble lets calculate the two expected values using probabilities techniques.

Given:
1. Offered bet = (1/3.5) = 1/E(X)
2. Gamble bet = *h*(X) = (1/X) = Y

Find:
- Which of the two bet options gives the best odds of winning by comparing the expected amount one would win in each game.

Solution approach
- First find the values of X along with their probabilities collectively to specify the probability distribution (pmf) for both bet options.
- Find the expected values for each bet [e.g. E(X) = 1/3.5 and E(Y) = E[*h(X)] = E(1/X).]*
- Finally compare the two expected values, and decide which one gives the best probability of winning.
- I will also provide the pmf and cdf graph for further understanding.

Calculations:
First lets define our random variables.
- X = The outcome when the fair die is rolled once.
- Y = 1/X in dollars

We are now ready to create our table of outcomes like table 1 below.

Table 1: Outcomes and their Probabilities for Offered Bet

x	**1**	**2**	**3**	**4**	**5**	**6**
P(x)	1/6	1/6	1/6	1/6	1/6	1/6

From the table above, we can calculate the **expected values** or **mean value** by using equation 1 below which is in the class text in page 104.

$$E(X) = \mu_x = \sum_{x \in D} x \cdot p(x) \qquad \textbf{equation 1}$$

E(X) = 1(1/6)+2(1/6)+3(1/6)+4(1/6)+5(1/6)+6(1/6) = 3.5, so
(1/E(X)) = 1/3.5 = .285714 which was already given. If we take the offered bet then we win approximately .286 cents for every dollar we bet, so do we go with the safe bet or risked?. Figure one below is a graph of this pmf.

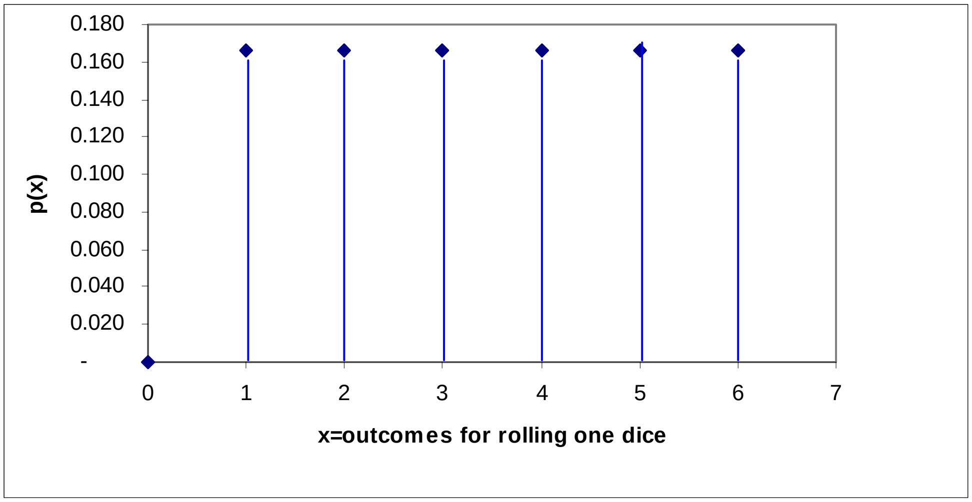

figure 1: pmf for the outcomes of rolling one dice

Then we do the same procedure for the function *h*(X), gamble bet. Create table 2, and plot its values on the pmf graph. E(Y) = E[*h*(X)] = 1/X, feeling lucky!!!!

Table 2: Outcomes and Probabilities for the Gamble Bet.

X	1	2	3	4	5	6
Y	1	1/2	1/3	1/4	1/5	1/6
P(Y)	1/6	1/6	1/6	1/6	1/6	1/6

We can calculate E(Y) = 1(1/6)+(½)(1/6)+(1/3)(1/6)+(¼)(1/6)+(1/5)(1/6)+(1/6)(1/6)
E(Y) = .4083333

So the chances of winning with the gamble bet are approximately .408 cents for every dollar gamble, therefore if one selects the gamble bet one can enhance its probabilities of winning more money. Figure two shows the probability distribution for this set of data.

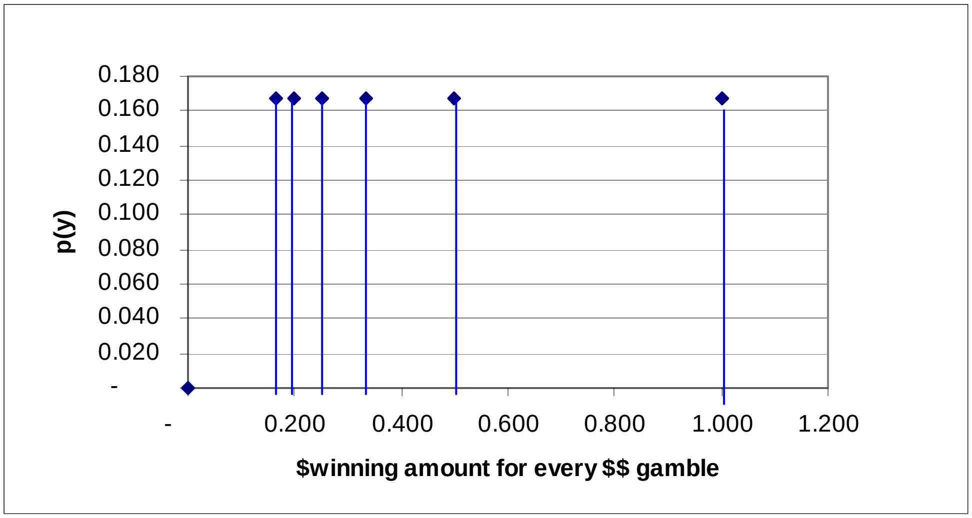

figure 2: pmf for the chances of winning for the gamble bet $$

For further understanding of this problem let look into the cumulative distribution function (cdf) of both options the offer and gamble bets, the graphs are shown in figure 3 and 4 respectively, and the expected values E(X) are represented with an arrow along the x axis.

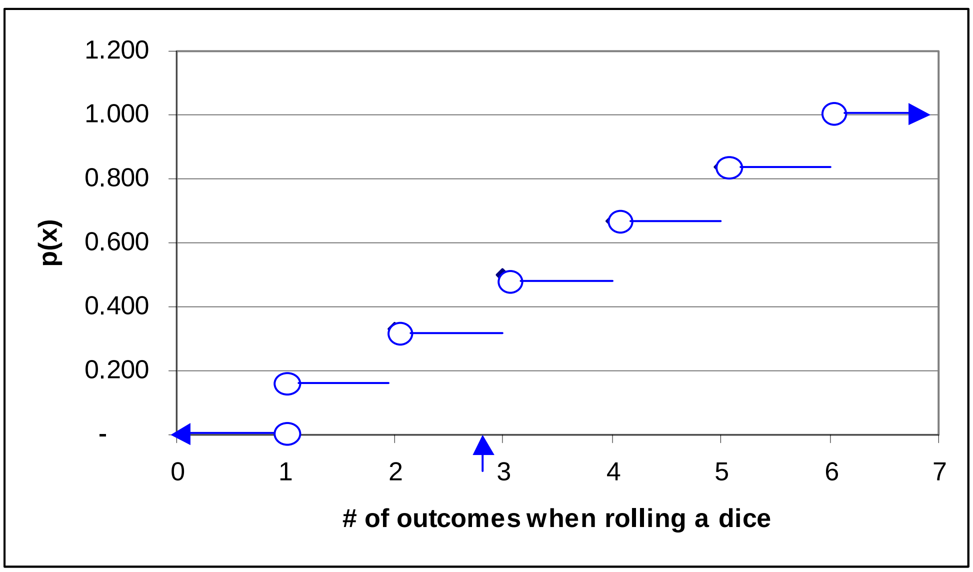

figure 3: cdf for outcomes when dice is roll

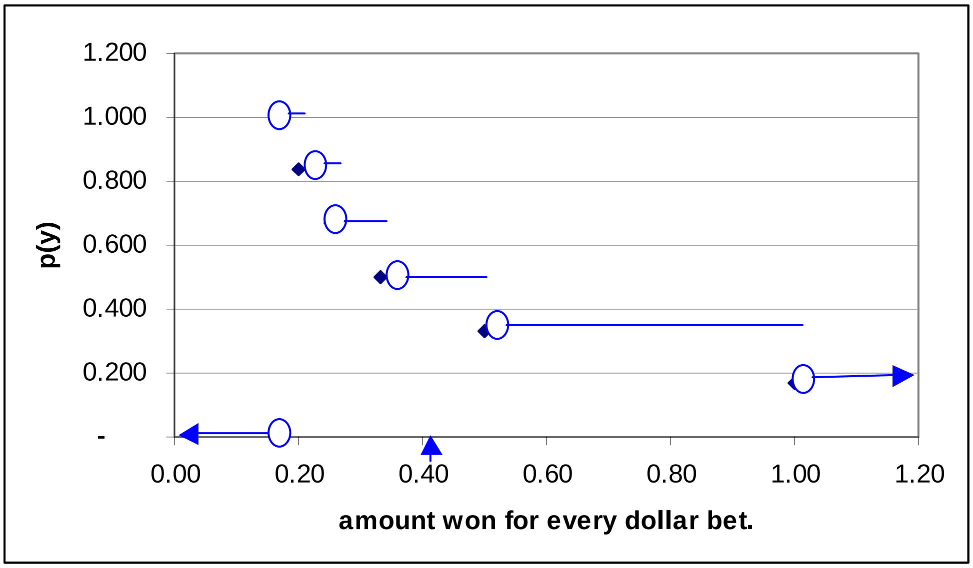

figure 4: cdf for amount won for every dollar bet

Notice how figure four is different in the sense that it does not follow a linear path like figure three, or other examples in the text that may be do the function Y = *h*(X) = 1/X as X increases Y decreases. Finally, after looking at all the results, it is safe to conclude that the gamble option will win you more $$.