Symmetries in 3D Shapes: Analysis and Applications

Course at the USTC Summer School 2012 on
"Advances in Computer Graphics"

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This course serves as …

An introduction of the interesting problem

An overview of the state-of-the-art

- The key ideas
- The main approaches
- Other related works as further readings

A showcase of the main applications

A discussion of future trends

Two excellent survey papers

Yanxi Liu, Hagit Hel-Or, Craig S. Kaplan and Luc Van Gool. "**Computational Symmetry in Computer Vision and Computer Graphics**," *Foundations and Trends in Computer Graphics and Vision*, 2009

Niloy J. Mitra, Mark Pauly, Michael Wand, and Duygu Ceylan. "**Symmetry in 3D Geometry: Extraction and Applications**," *Eurographics STAR*, 2012

Outline

Concept and background

- Symmetry and symmetry groups

Problem definition

- Classification of problems

Symmetry detection in 3D

- Extrinsic symmetry detection
- Intrinsic symmetry detection

Applications

- Structured organization of symmetry
- Applications of symmetry in shape analysis and processing

Summary

- Discussion: future

Concept and background

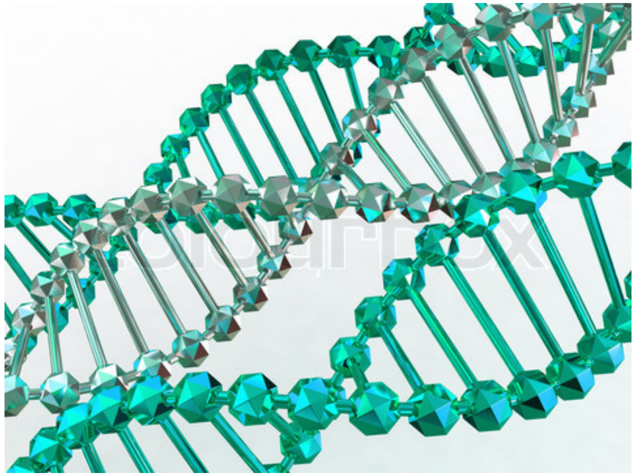
Symmetry is ubiquitous: in nature

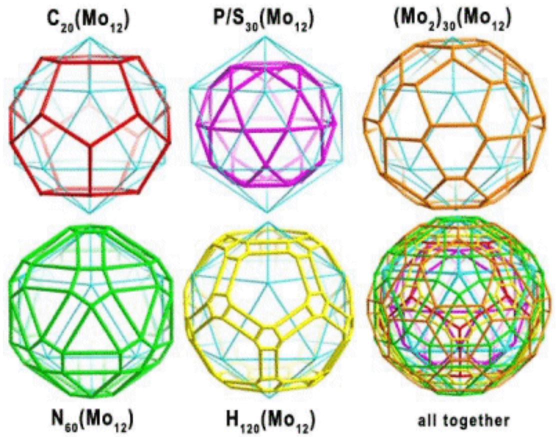
The most ***general law in nature*** *is equity-the principle of balance and symmetry which guides the growth of forms along the lines of the* ***greatest structural efficiency****. -- Herbert Read*

Symmetry is ubiquitous: in art

Beauty is our weapon against nature; by it ***we make objects, giving them limit, symmetry, proportion****. -- Camille Paglia*

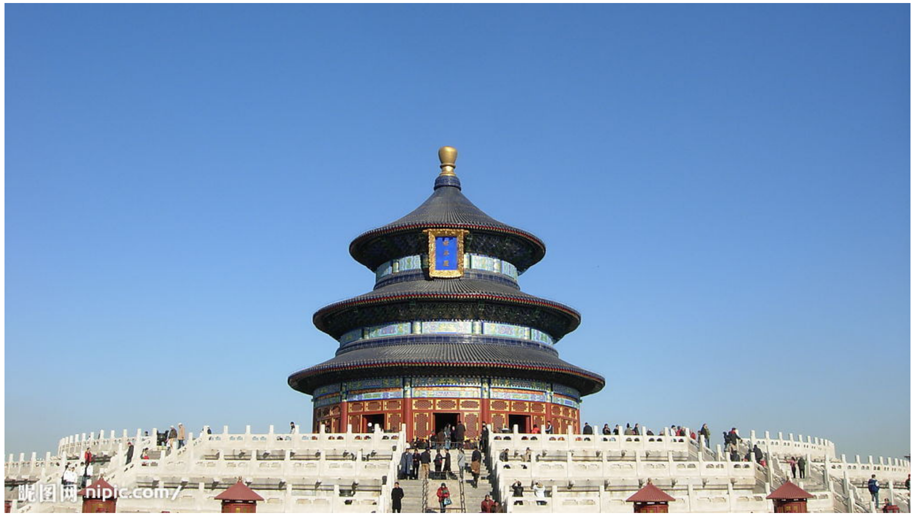
Symmetry is ubiquitous: in science





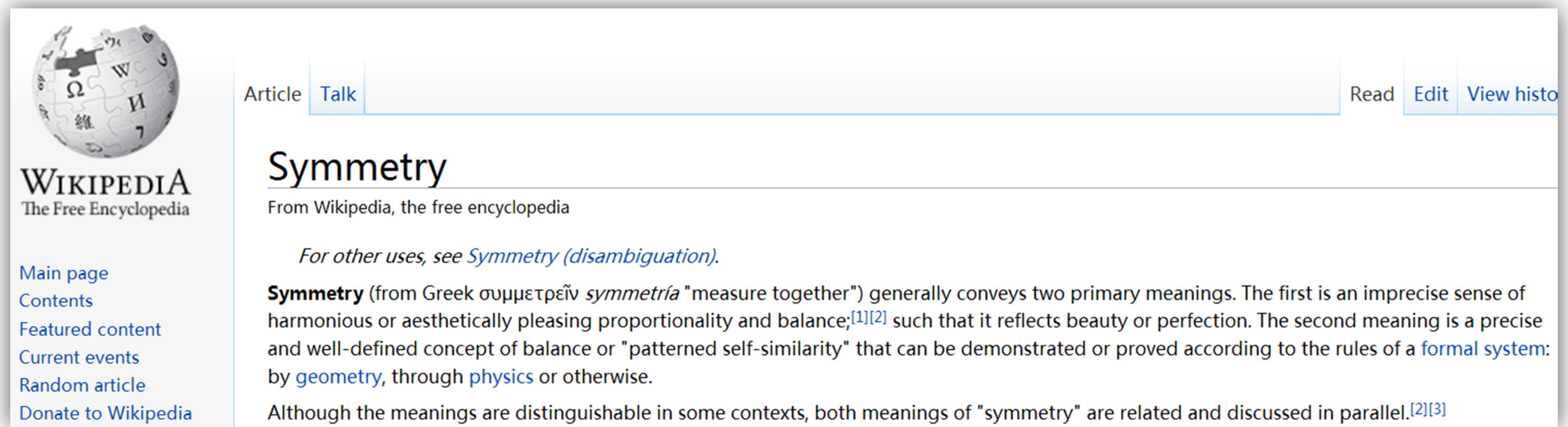
*Symmetry considerations have provided us with an extremely powerful and useful tool in our effort to understand nature … they have become **the backbone of our theoretical formulation of physical laws**. -- Tsung-Dao Lee*

Symmetry is ubiquitous: in architecture



Symmetry finds its ways into architecture at every scale. -- Wikipedia

Symmetry in Wikipedia



a precise and well-defined concept of balance or ***"patterned self-similarity"*** *that* ***can be demonstrated or proved*** *according to* ***the rules of a formal system****: by geometry, through physics or otherwise. -- Wikipedia*

Geometric symmetry

*"Geometry is the study of a space that is **invariant under a given transformation group**." – Felix Klein's Erlangen program*

In a metric space M, a *symmetry* $g \in G$ of a set $M' \subseteq M$ is an ***isometry*** (a distance preserving transformation) that maps M' to itself (an ***automorphism***) $g(M') = M'$. → ***"Self-isometry"***

Symmetry is geometrically measurable!

Geometric symmetry is detectable!

Why interested in symmetry detection?

Some obvious reasons:

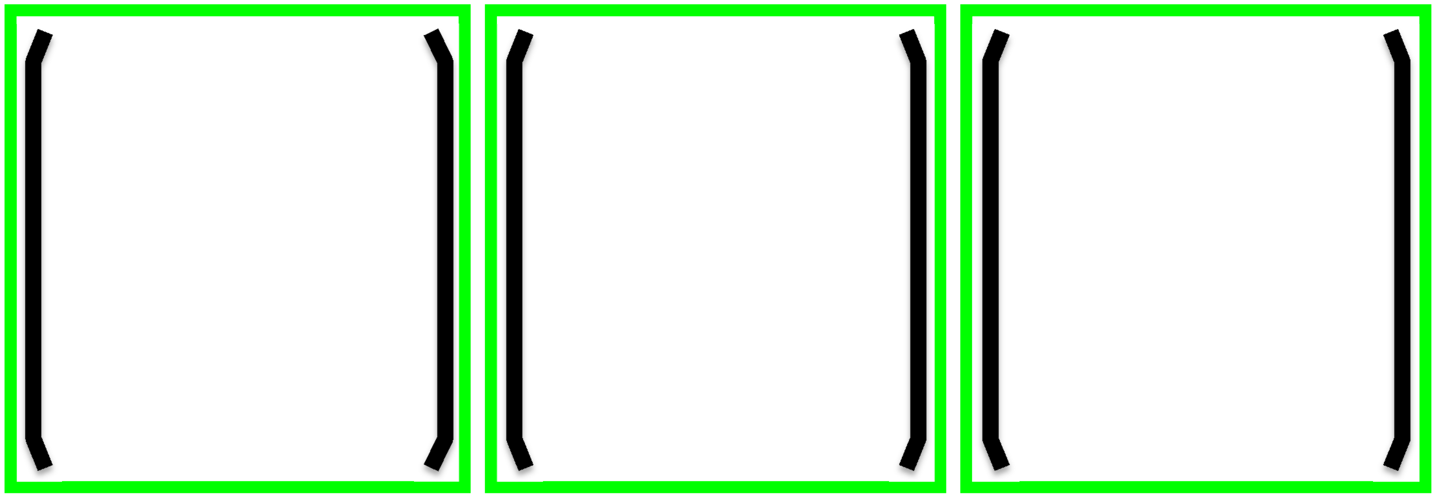
- Symmetry is ubiquitous
- Symmetry is detectable

"Symmetry is a complexity-reducing concept ...; seek it everywhere." -- Alan J. Perlis

Why interested in symmetry detection?

Some obvious reasons:

- Symmetry is ubiquitous
- Symmetry is detectable
- Aesthetic/Human perception → Gestalt Law of symmetry



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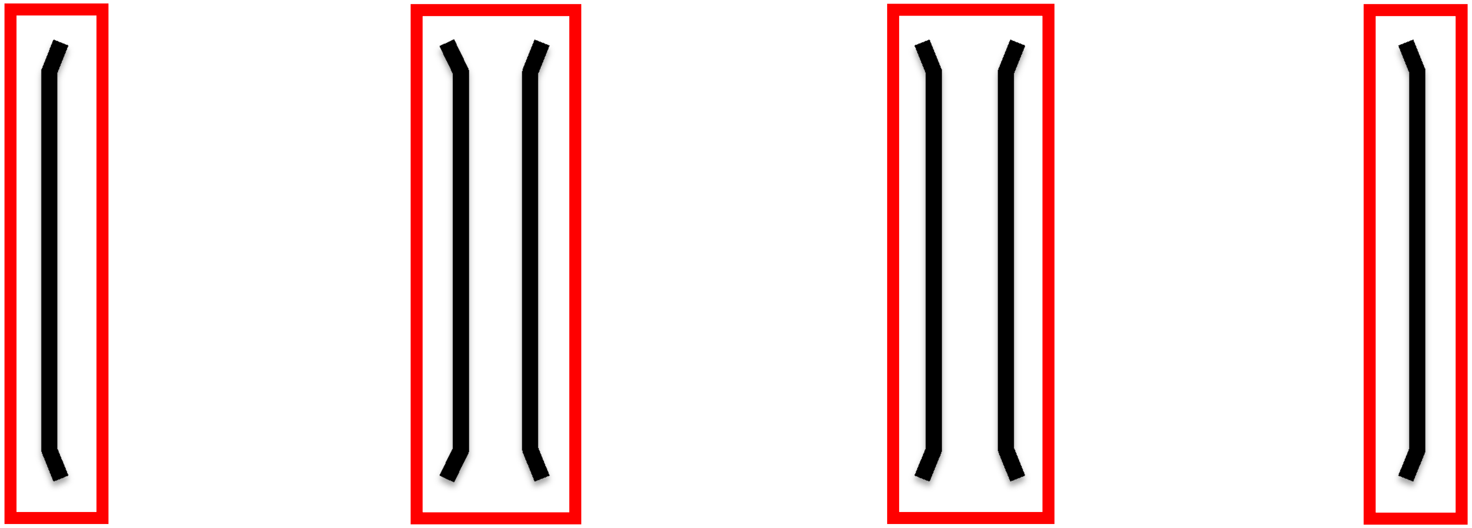
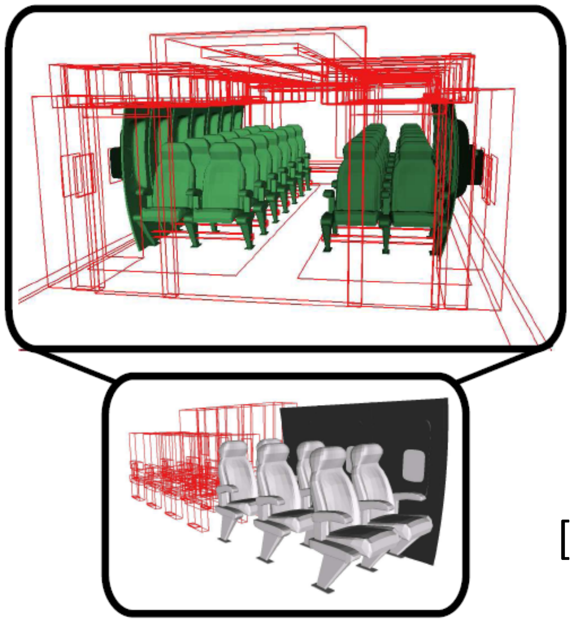


Why interested in symmetry detection?

Some obvious reasons:

- Symmetry is ubiquitous
- Symmetry is detectable
- Aesthetic/Human perception → Gestalt Law of symmetry
- Efficiency → complexity reduction



[Martinet 2007]

Why interested in symmetry detection?

Symmetry implies high-level structural information

- bridges the gap between low-level geometry properties and high-level structural information

Self-symmetric region serves as a meaningful part

Symmetric parts together serve a particular function

Structural analysis is to the center of shape understanding!

Symmetry and symmetry group

*“**invariance** of a configuration of elements under a group of automorphic transformations.” -- Hermann Weyl*

The notion of invariance ⇔ *group theory*

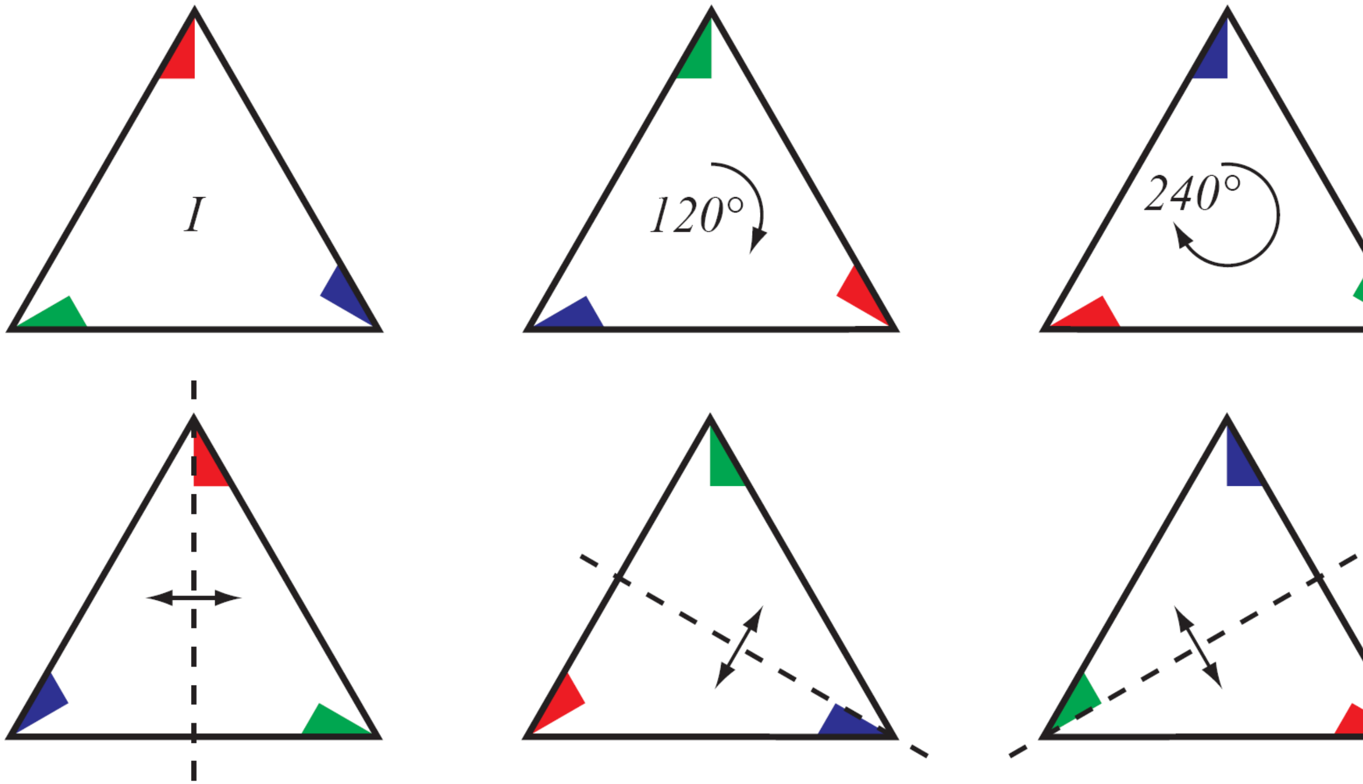
Symmetry and symmetry group

The set G of all symmetry transformations of an geometric object M form a *group*:

- **Closure**: if $g_1, g_2 \in G$, it follows that $(g_1 g_2) \in G$.
- **Identity element**: $Id(M) = M$.
- **Inverse element**: For each $g \in G$, there exist an inverse element $g^{-1} \in G$, such that $g^{-1} g = g g^{-1} = Id$.
- **Associativity**: $(g_1 g_2) g_3 = g_1 (g_2 g_3) \ \forall g_1, g_2, g_3 \in G$.

Symmetry and symmetry group

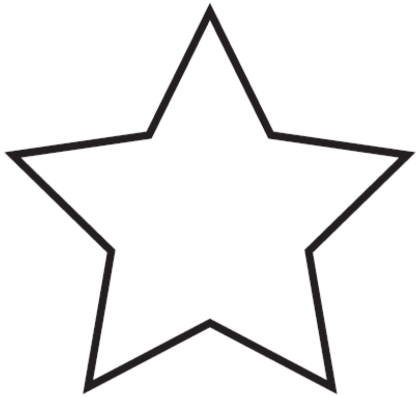
Classifying geometric objects based on symmetry groups:



dihedral group D3

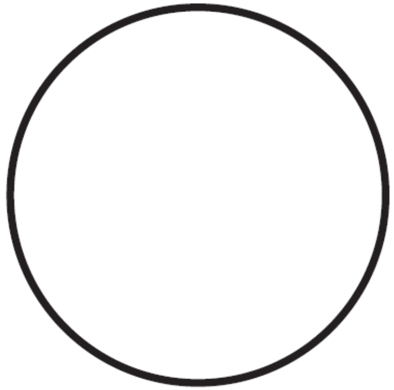
Symmetry and symmetry group

Classifying geometric objects based on symmetry groups:



dihedral group D_5

cyclic group C_3



infinite group $O(2)$

Symmetry and symmetry group

Classifying geometric objects based on symmetry groups [Liu et al. Survey]:

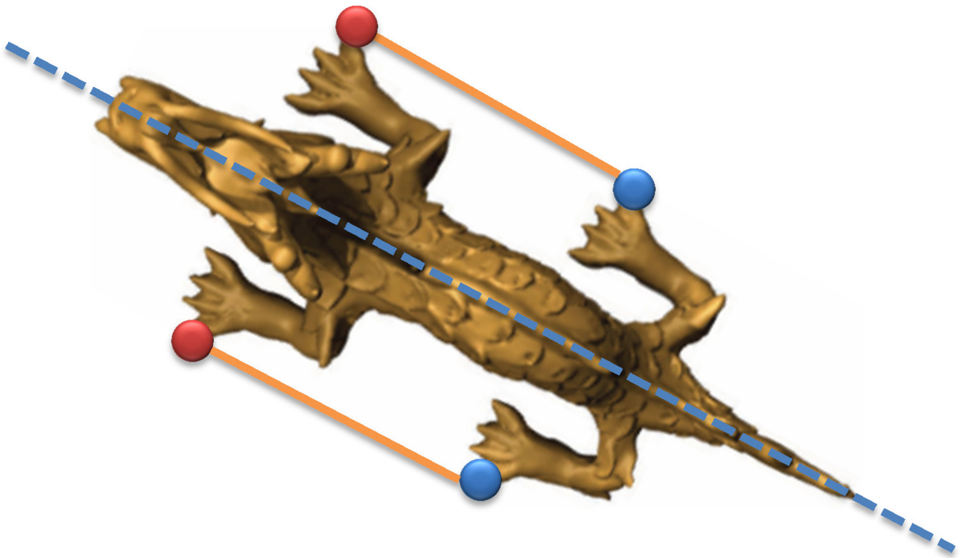
Canonical groups	Representative group element
Identity group	
G_{id}	$\{1\}$
Rotation subgroups	
$\text{SO}(3)$	$\{\mathbf{rot}(\mathbf{i},\theta)\mathbf{rot}(\mathbf{j},\sigma)\mathbf{rot}(\mathbf{k},\phi) \mid \theta,\sigma,\phi \in R\}$
$O(2)$	$\{\mathbf{rot}(\mathbf{k},\theta)\mathbf{rot}(\mathbf{i},n\pi) \mid \theta \in R, n \in \mathcal{N}\}$
$\text{SO}(2)$	$\{\mathbf{rot}(\mathbf{k},\theta) \mid \theta \in R\}$
D_n	$\{\mathbf{rot}(\mathbf{k},2\pi/n)\mathbf{rot}(\mathbf{i},m\pi) \mid m \in \mathcal{N}\}, n \in \mathcal{N}$
C_n	$\{\mathbf{rot}(\mathbf{k},2\pi/n)\}, n \in \mathcal{N}$
Translation subgroups	
$\mathcal{T}^1$	$\{\mathbf{trans}(0,0,z) \mid z \in R\}$
$\mathcal{T}^1_{\text{dis}}(t_0)$	$\{\mathbf{trans}(0,0,t_0)\}, t_0 \in R$
$\mathcal{T}^2$	$\{\mathbf{trans}(x,y,0) \mid x,y \in R\}$
$\mathcal{T}^3$	$\{\mathbf{trans}(x,y,z) \mid x,y,z \in R\}$

Problem definition

The problem of symmetry detection

Problem statement:

[Mitra et al. Survey]: Given a shape, identify and extract *pairs of regions* such that each pair of regions, under an appropriate *distance measure*, is similar when the respective regions are aligned using an *allowable transformation*.

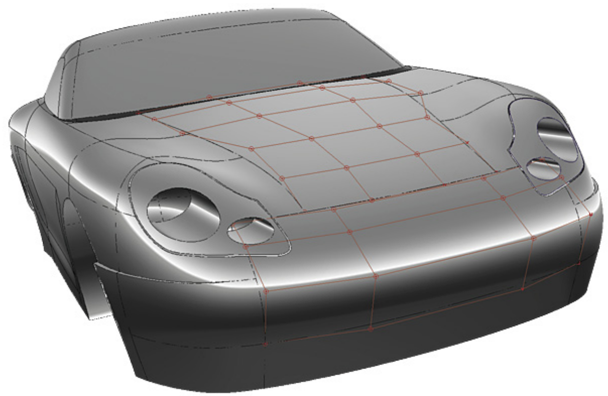


Symmetry detection in 3D

Input: shapes represented as surfaces.

triangle mesh

point cloud



NURBS surfaces

Symmetry detection in 3D

Output:

- Pairs of symmetric regions
- Symmetry correspondence/map
- For partial symmetries: segmentation

Classification

Classification of symmetry detection problems:

- Exact vs. approximate
- Global vs. partial
- Extrinsic vs. intrinsic
- Symmetry groups assumed/detected

Classification: Exact vs. approximate

Symmetry in real world is never perfect

Approximate symmetry:

distance function: $d(M, T(M))$

ϵ-*symmetry*: $d\big(M, T(M)\big) < \epsilon$

Distance measure:

- Squared distance to the closet point (used in Iterative Closest Point):

$$d\big(M, T(M)\big) = \int_{\mathbf{x} \in M} ||T\mathbf{x} - \Theta_M(T\mathbf{x})||^2 d\mathbf{x}$$

- Hausdorff distance:

$$d\big(M, T(M)\big) = \max\{\sup_{\mathbf{x} \in M} \inf_{\mathbf{y} \in T(M)} \| \mathbf{x} - \mathbf{y} \|, \sup_{\mathbf{y} \in T(M)} \inf_{\mathbf{x} \in M} \| \mathbf{x} - \mathbf{y} \|\}$$

Classification: Global vs. partial

Self-similarities often occur on parts of a shape:

Symmetric parts, but
no global symmetry

Global symmetry is a special case of partial symmetry

Classification: Global vs. partial

Partial symmetry is harder to detect:

- Solving two problems simultaneously: **segmentation** and **shape matching**

Larger search space!

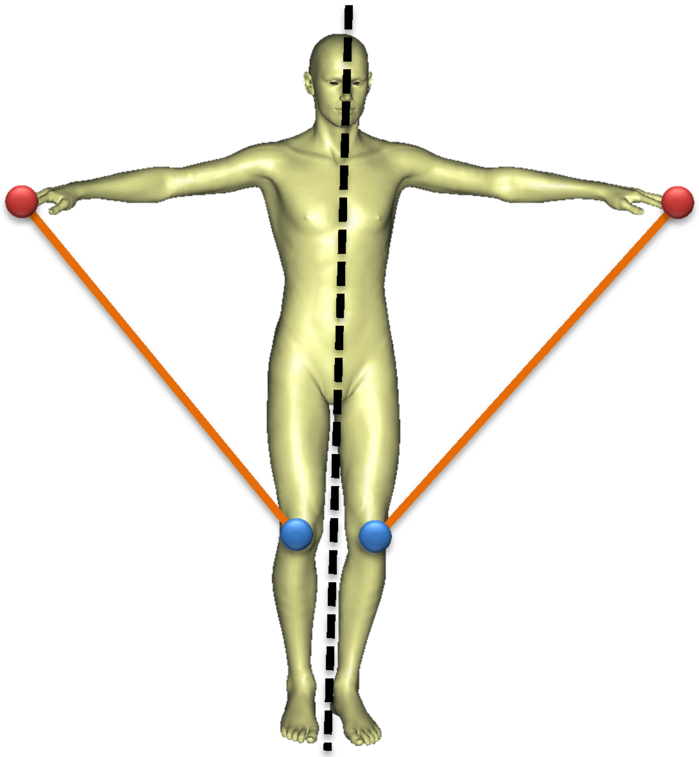
- To be meaningful: "prominent" partial symmetry

(ϵ, δ)-*symmetry*: (M', T) with $M' \subseteq M$, if $d\big(M', T(M')\big) < \epsilon$ and $\delta = |M'|/|M|$

Classification: Extrinsic vs. intrinsic

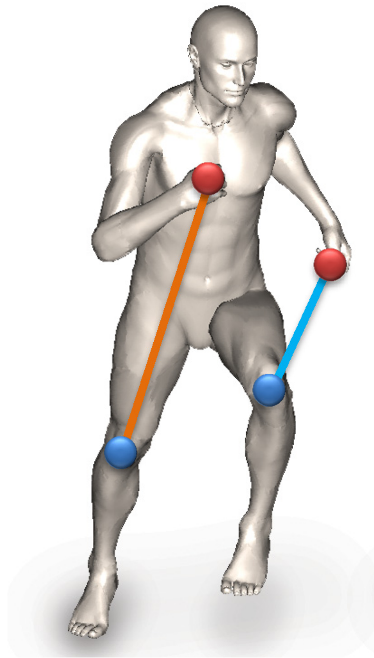
Symmetry: self-isometry (distance preserving)

- How the distance is measured?



Extrinsic

Preserving Euclidean distance

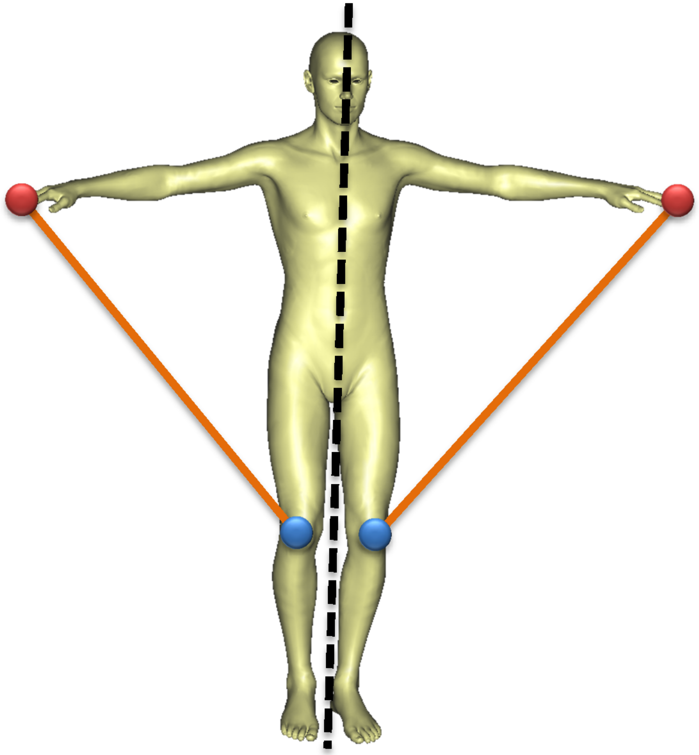


?

Classification: Extrinsic vs. intrinsic

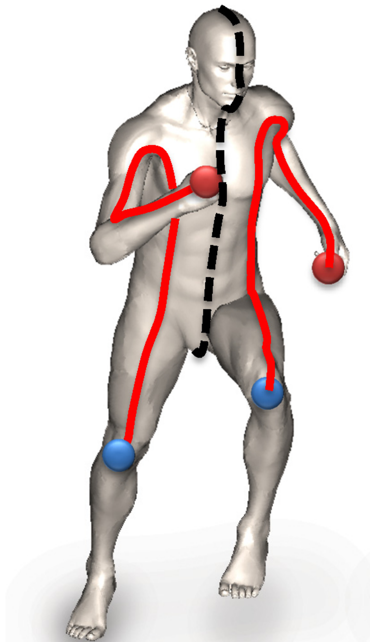
Symmetry: self-isometry (distance preserving)

- How the distance is measured?



Extrinsic

Preserving Euclidean distance



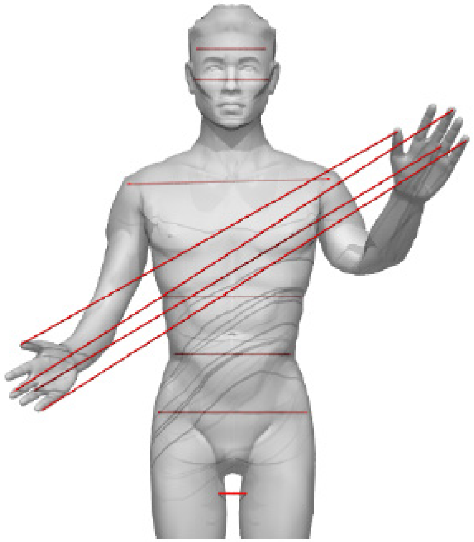
Intrinsic

Preserving geodesic distance

Classification: Symmetry groups detected

What symmetry group is assumed?

Reflectional symmetry
(extrinsic)



Reflectional symmetry
(intrinsic)

Not specified
(correspondence-space)
* But difficult for intrinsic

Symmetry detection in 3D

How to detect geometric symmetry in 3D?

Mainly focus on three aspects:

- Extrinsic symmetry detection: [Mitra et al. SIGGRPAH 2006]
- Global intrinsic symmetry detection: [Ovsjaniskov et al. SGP 2008]
- Partial intrinsic symmetry detection: [Xu et al. SIGGRAPH Asia 2009]

	Global	Partial
Extrinsic	[Mitra et al. 2006]	
Intrinsic	[Ovsjaniskov et al. 2008]	[Xu et al. 2009]

How to detect geometric symmetry in 3D?

Mainly focus on three aspects:

- Extrinsic symmetry detection: [Mitra et al. SIGGRPAH 2006]
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- Partial intrinsic symmetry detection: [Xu et al. SIGGRAPH Asia 2009]

Other state-of-the-art works covered briefly

Extrinsic symmetry detection

Main approaches:

- Transformation-space voting
- Direct (partial) matching
- Symmetry transform computation (voting)
- Spectral analysis in correspondence space

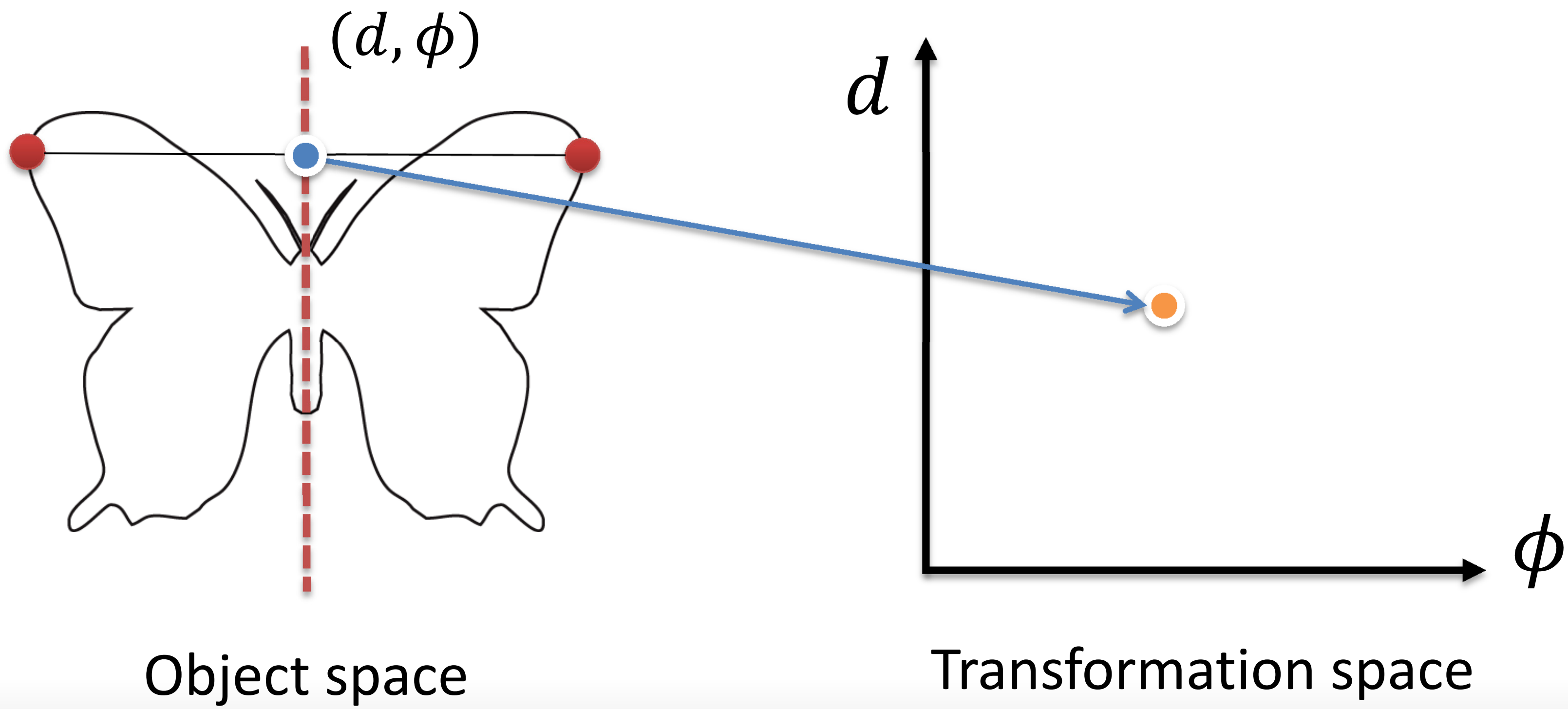
Extrinsic symmetry detection: Transformation-space voting

Niloy J. Mitra, Leonidas Guibas, and Mark Pauly. "**Partial and Approximate Symmetry Detection for 3D Geometry**," ACM SIGGRAPH 2006.

Extrinsic symmetry detection: Transformation-space voting

Key idea:

Perform analysis in the **parameter space** of symmetry transformations



Extrinsic symmetry detection: Transformation-space voting

Key idea:

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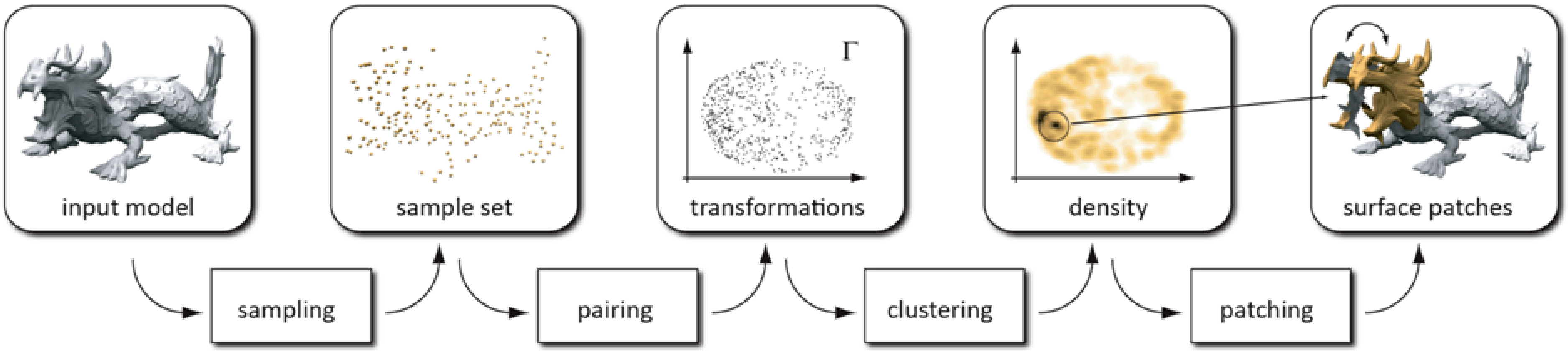
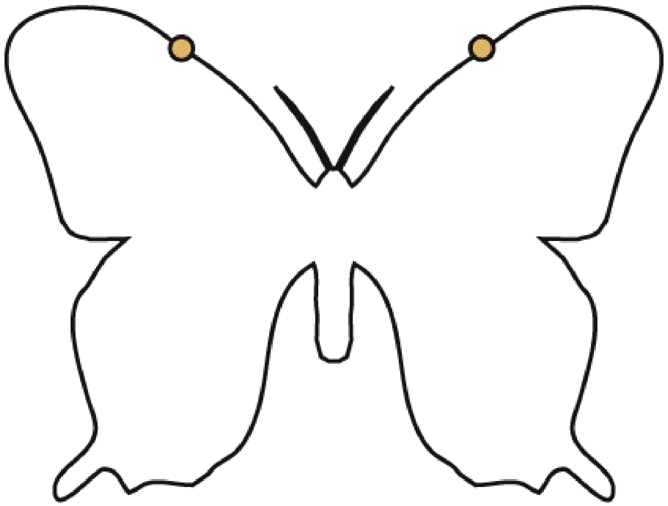


Extrinsic symmetry detection: Transformation-space voting

Voting in reflective symmetry space:



Extrinsic symmetry detection: Transformation-space voting

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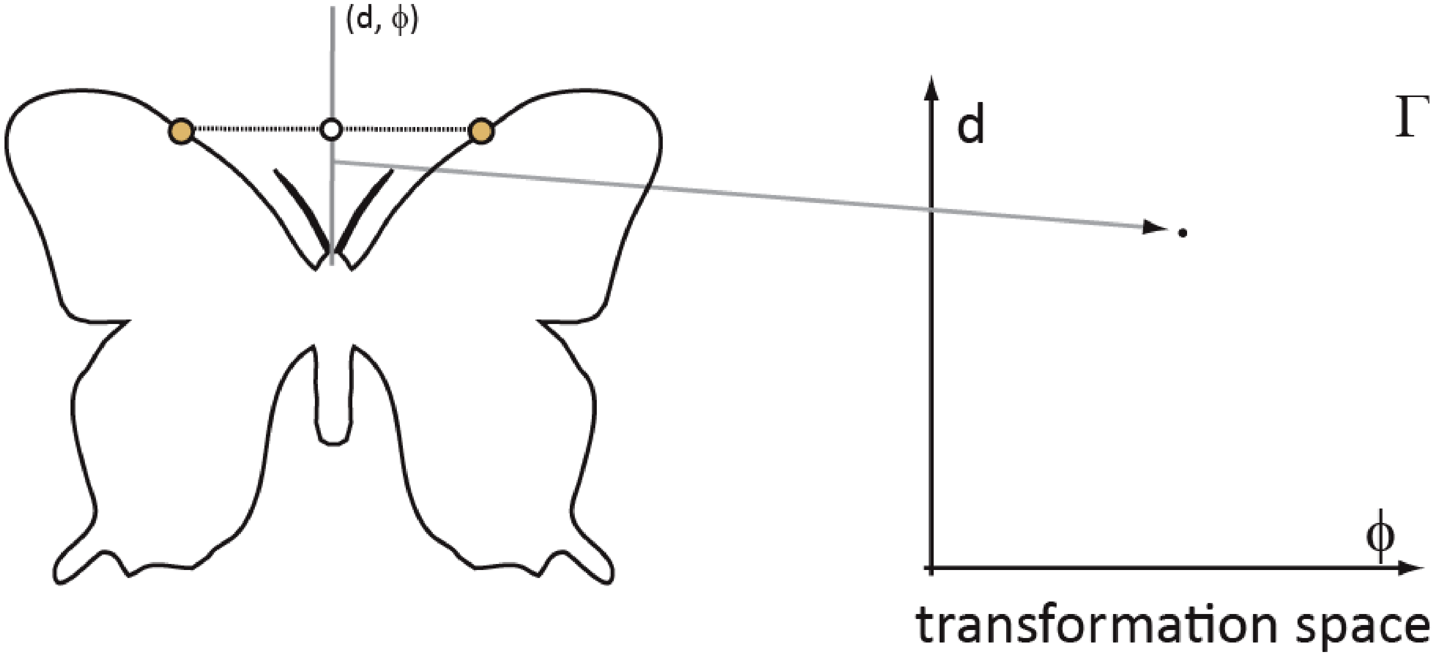
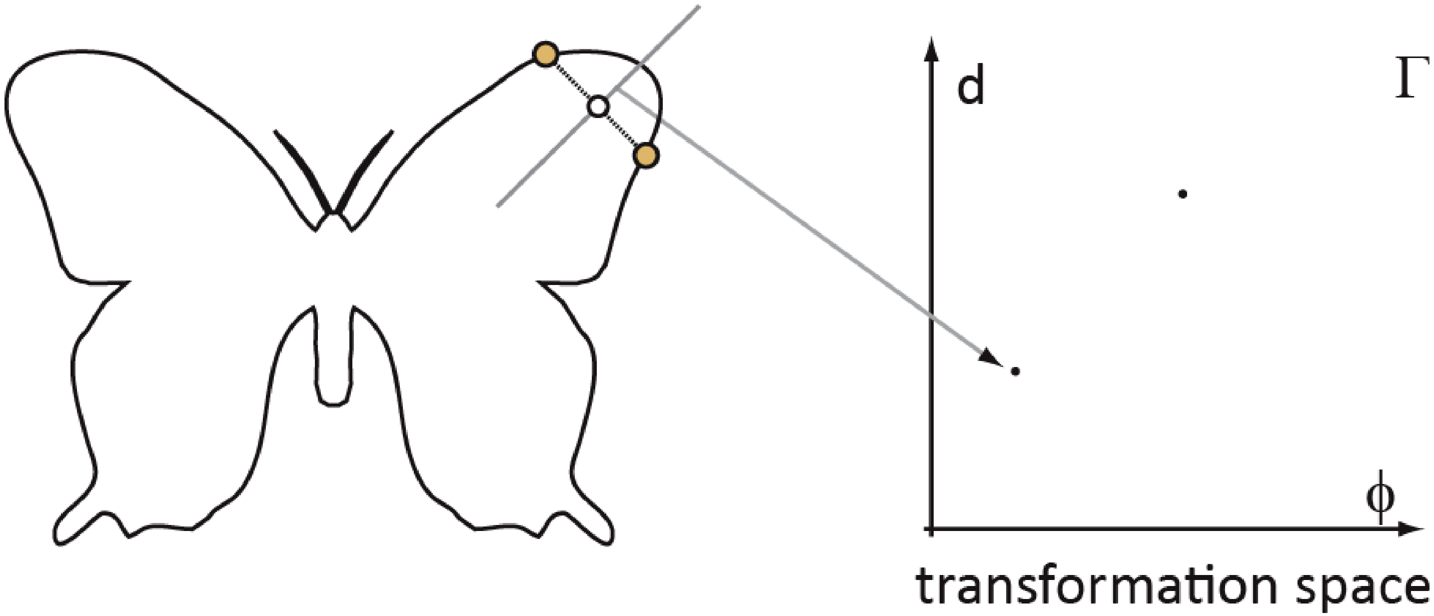


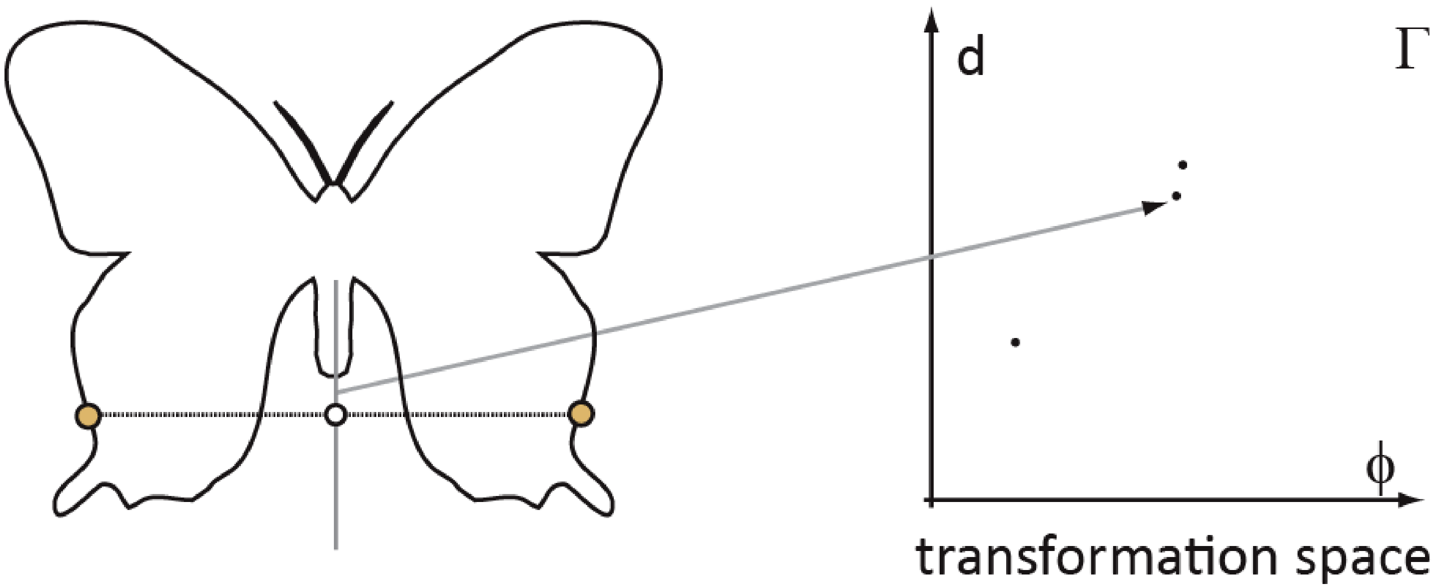
Extrinsic symmetry detection: Transformation-space voting

Voting in reflective symmetry space:



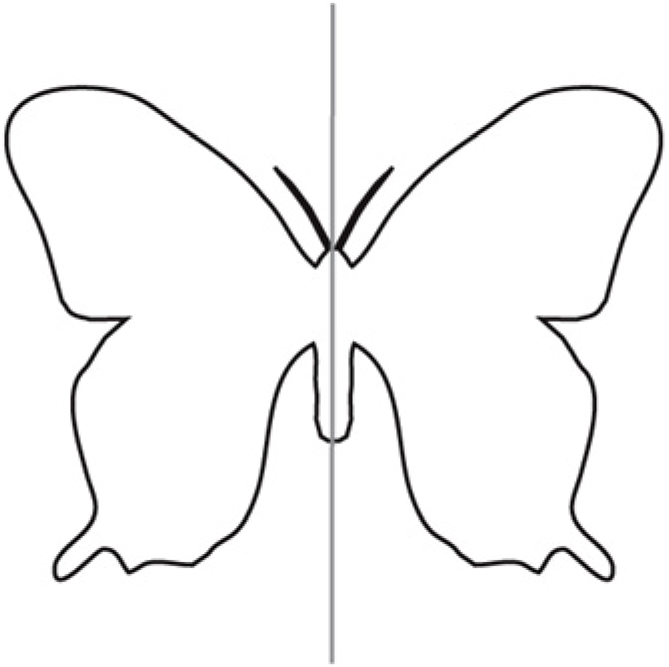
Extrinsic symmetry detection: Transformation-space voting

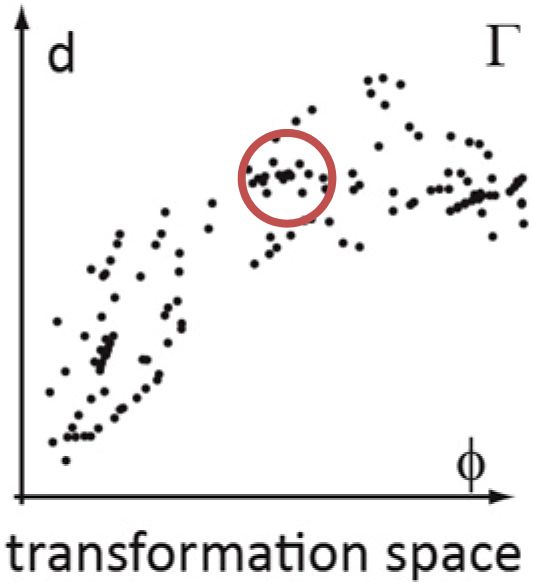
Voting in reflective symmetry space:



Extrinsic symmetry detection: Transformation-space voting

Voting in reflective symmetry space:



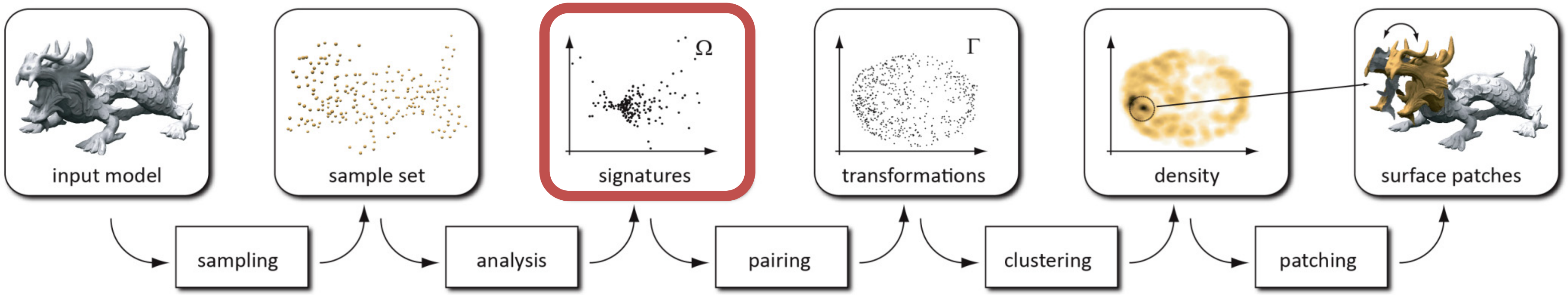


Clusters not quite prominent!

Height of cluster → size of patch
Spread of cluster → level of approximation

Extrinsic symmetry detection: Transformation-space voting

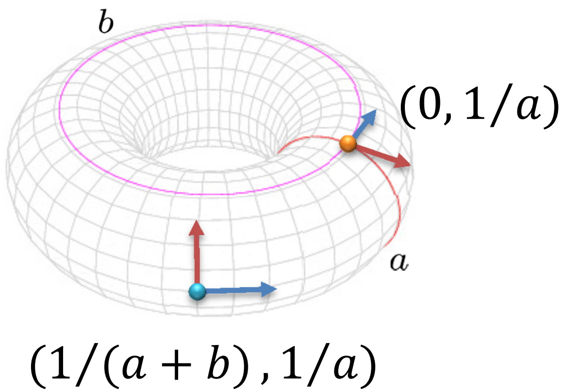
Pruning point pairs with local signatures



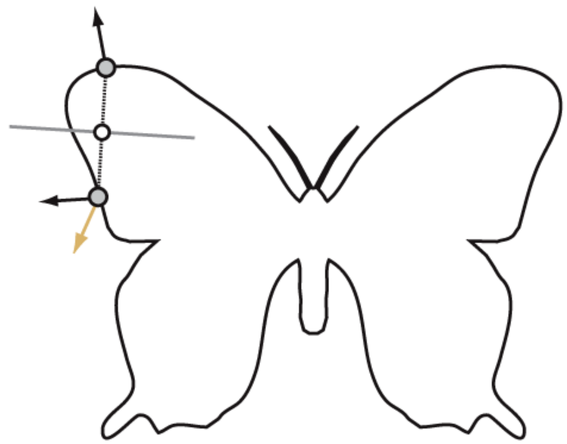
Local signature → invariant under transforms
Signature disagree → points don't correspond

Extrinsic symmetry detection: Transformation-space voting

Pruning point pairs with local signatures



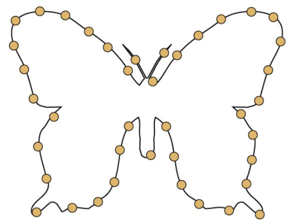
Pruning based on principal curvature (κ_1, κ_2)

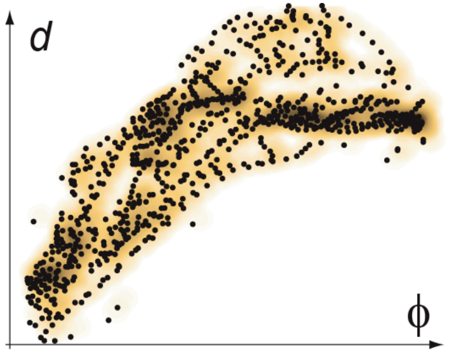


Normal based pruning

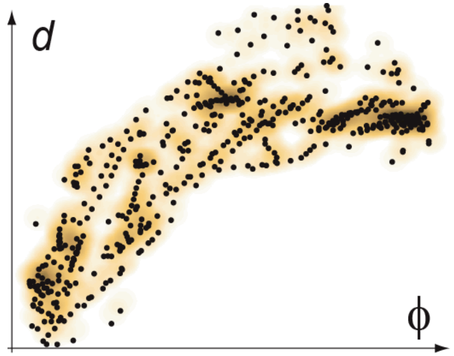
Extrinsic symmetry detection: Transformation-space voting

Pruning point pairs with local signatures

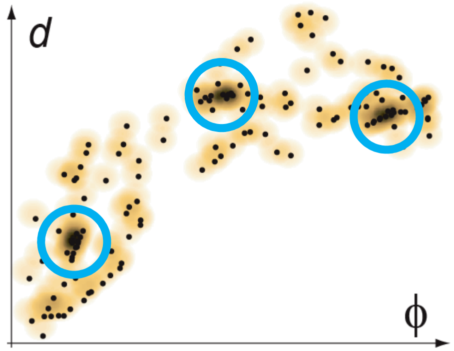




all pairs



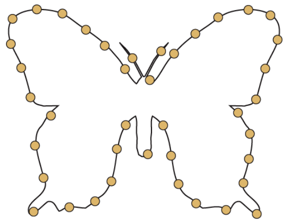
curvature based

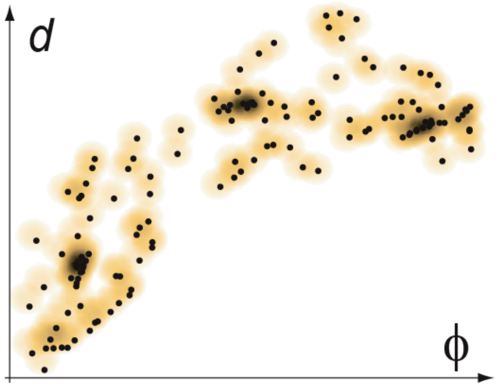


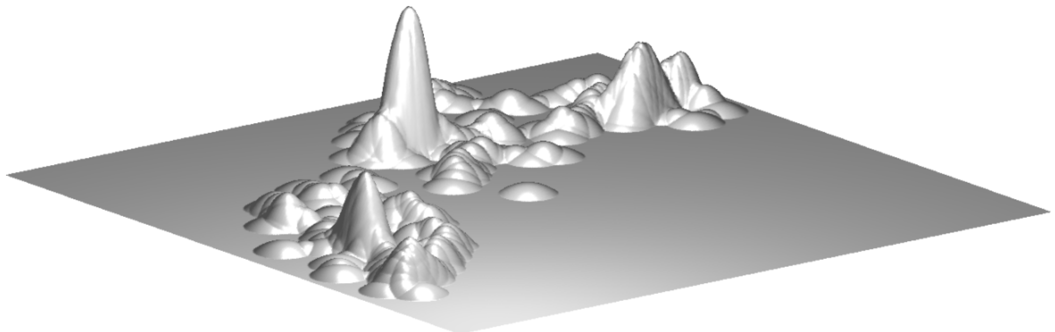
curvature + normal based

Extrinsic symmetry detection: Transformation-space voting

Mean-Shift Clustering



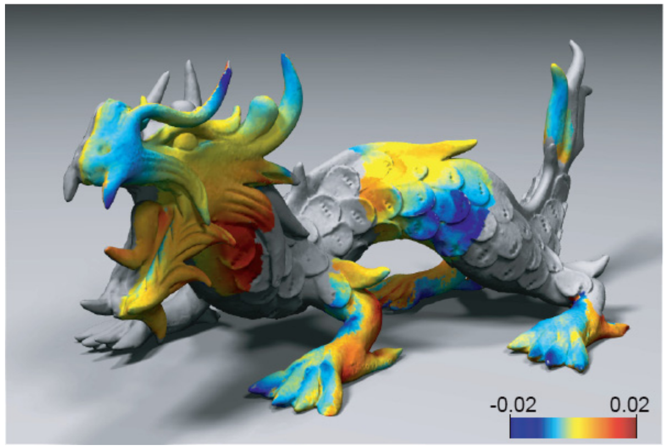




Extrinsic symmetry detection: Transformation-space voting

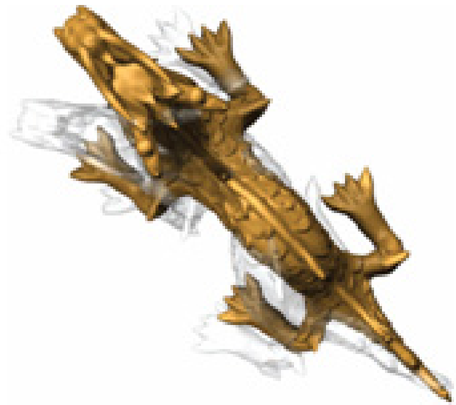
Result

detected symmetries



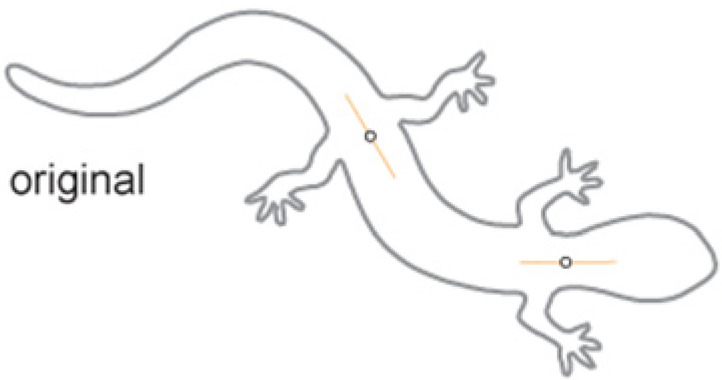
correction field

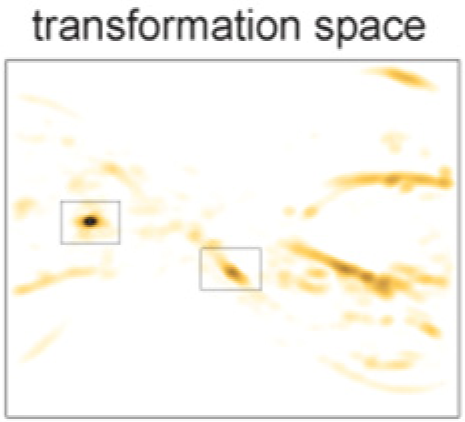
Transformation-space voting: Follow-up I



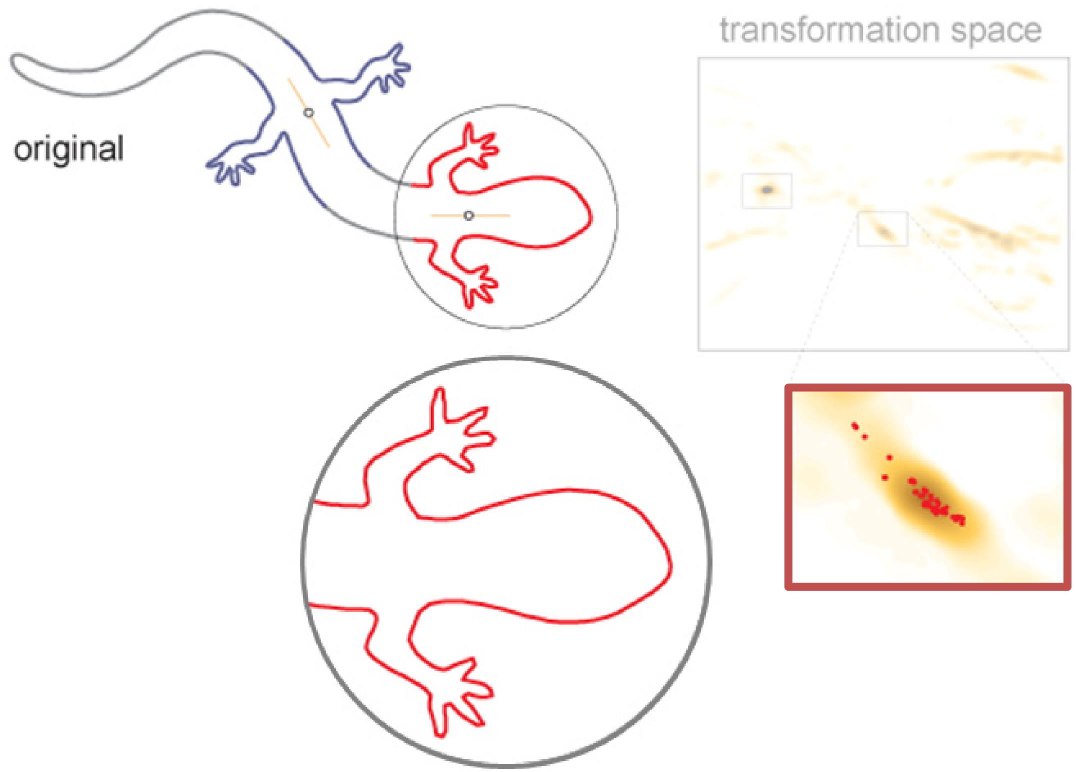
Niloy J. Mitra, Leonidas Guibas, and Mark Pauly.
"**Symmetrization**," *SIGGRAPH*, 2007

Symmetrization with transformation space





Symmetrization with transformation space

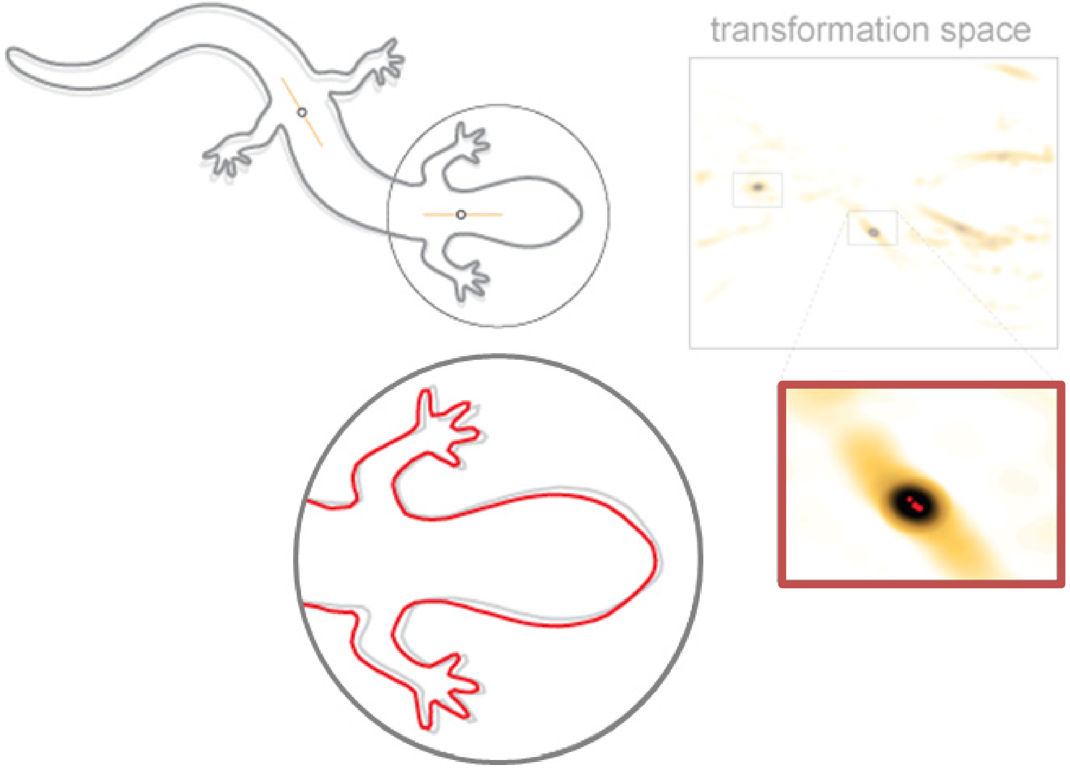


Spread of cluster → deviation from perfect symmetry

Cluster contraction → Local symmetrization

Transformation-space voting: Follow-up I

Symmetrization with transformation space



Cluster contraction **in transform space** → Constrained deformation **in object space**

Transformation-space voting: Follow-up I

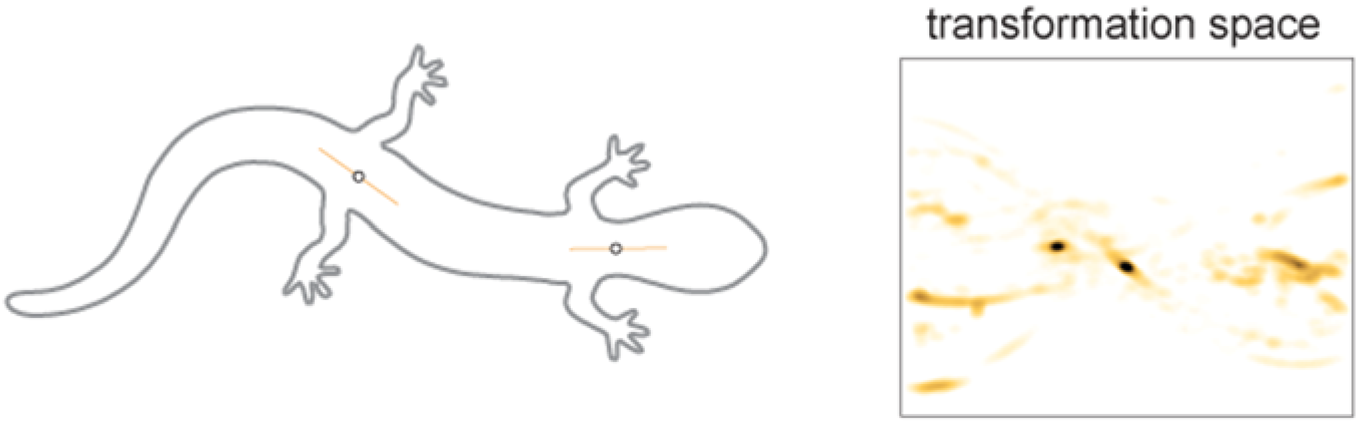
Symmetrization with transformation space

Different clusters → different partial symmetries

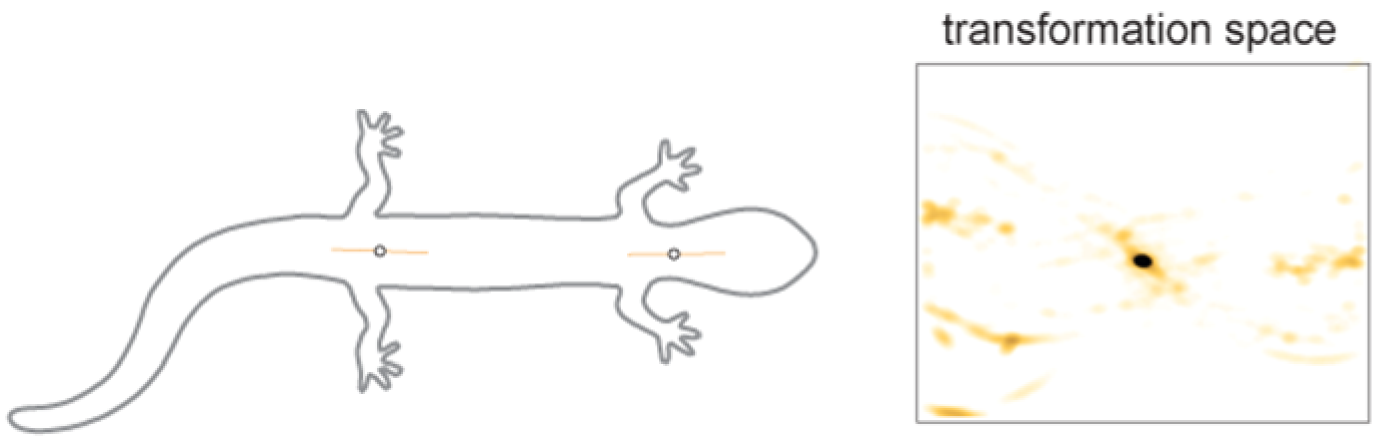
Cluster merging → Global symmetrization

Transformation-space voting: Follow-up I

Symmetrization with transformation space



Symmetrization with transformation space



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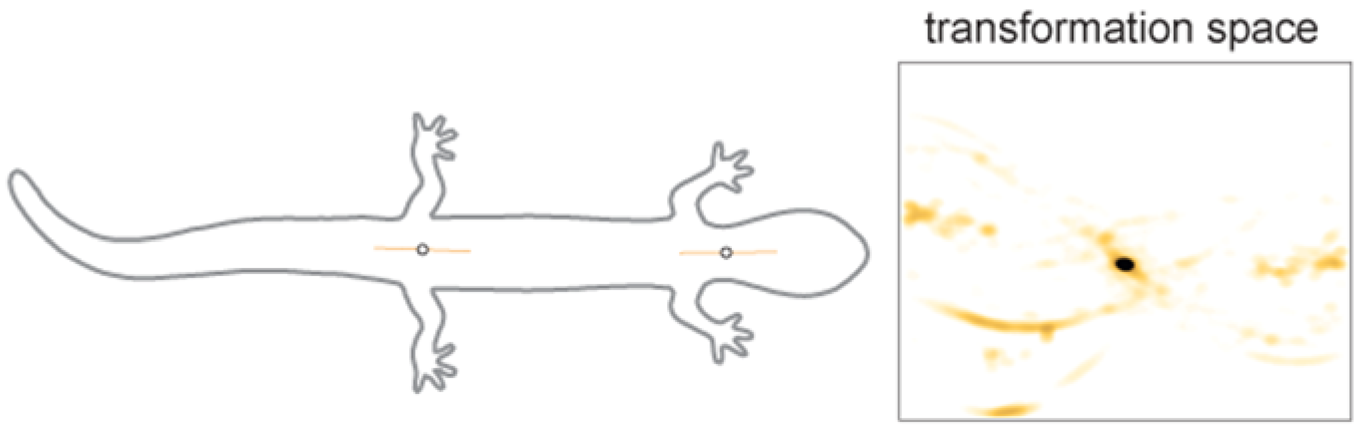
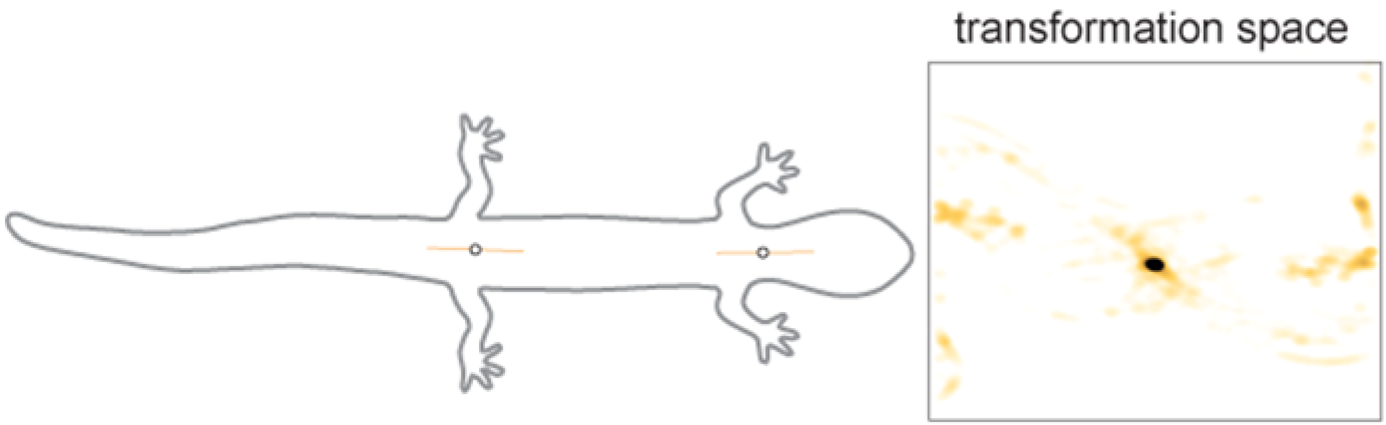
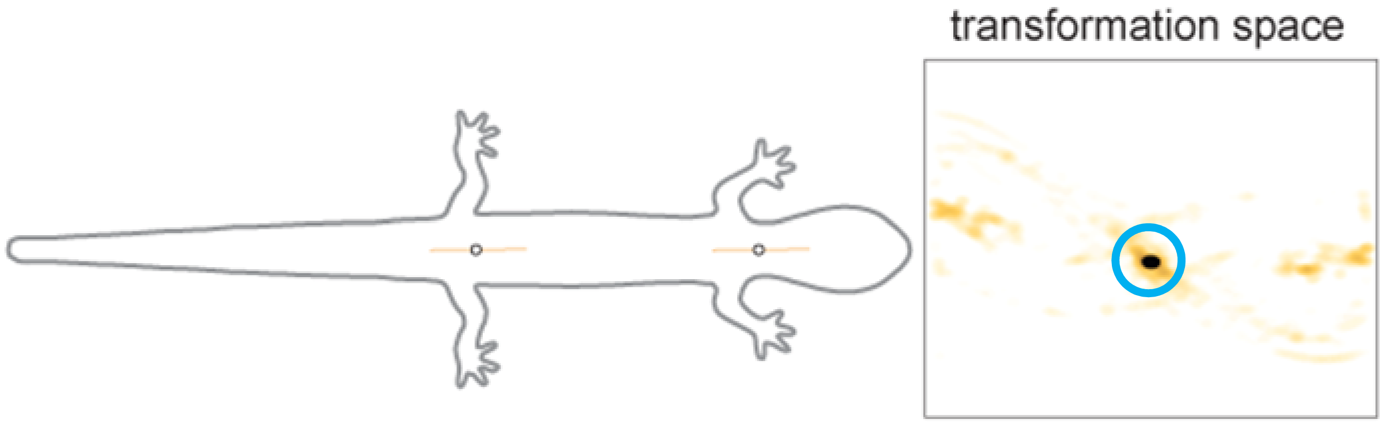


Transformation-space voting: Follow-up I

Symmetrization with transformation space

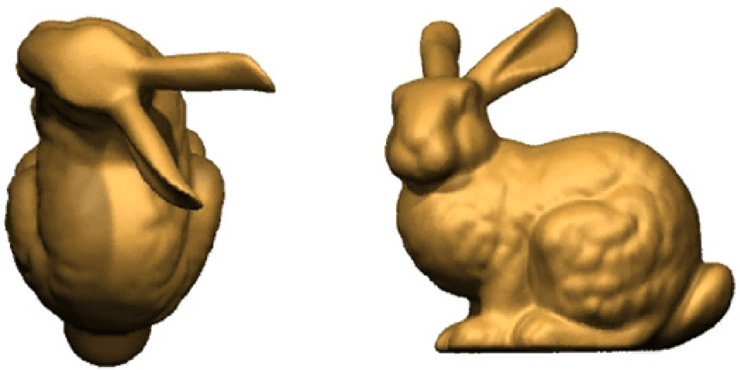


Symmetrization with transformation space

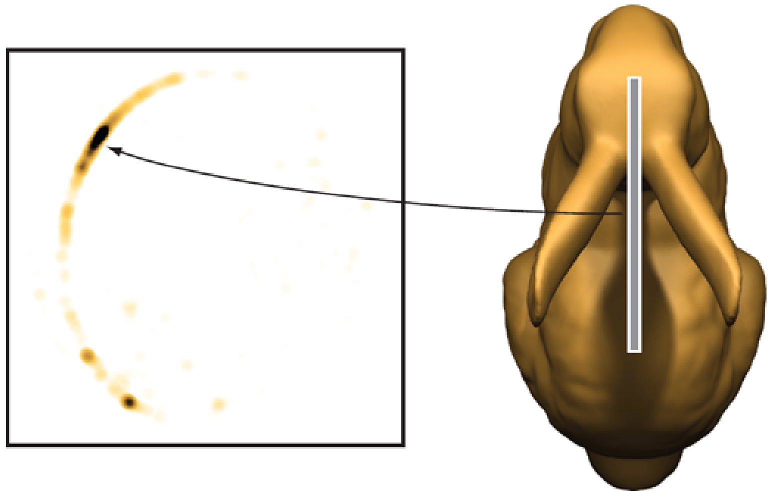


Transformation-space voting: Follow-up I

Symmetrization in 3D



Input



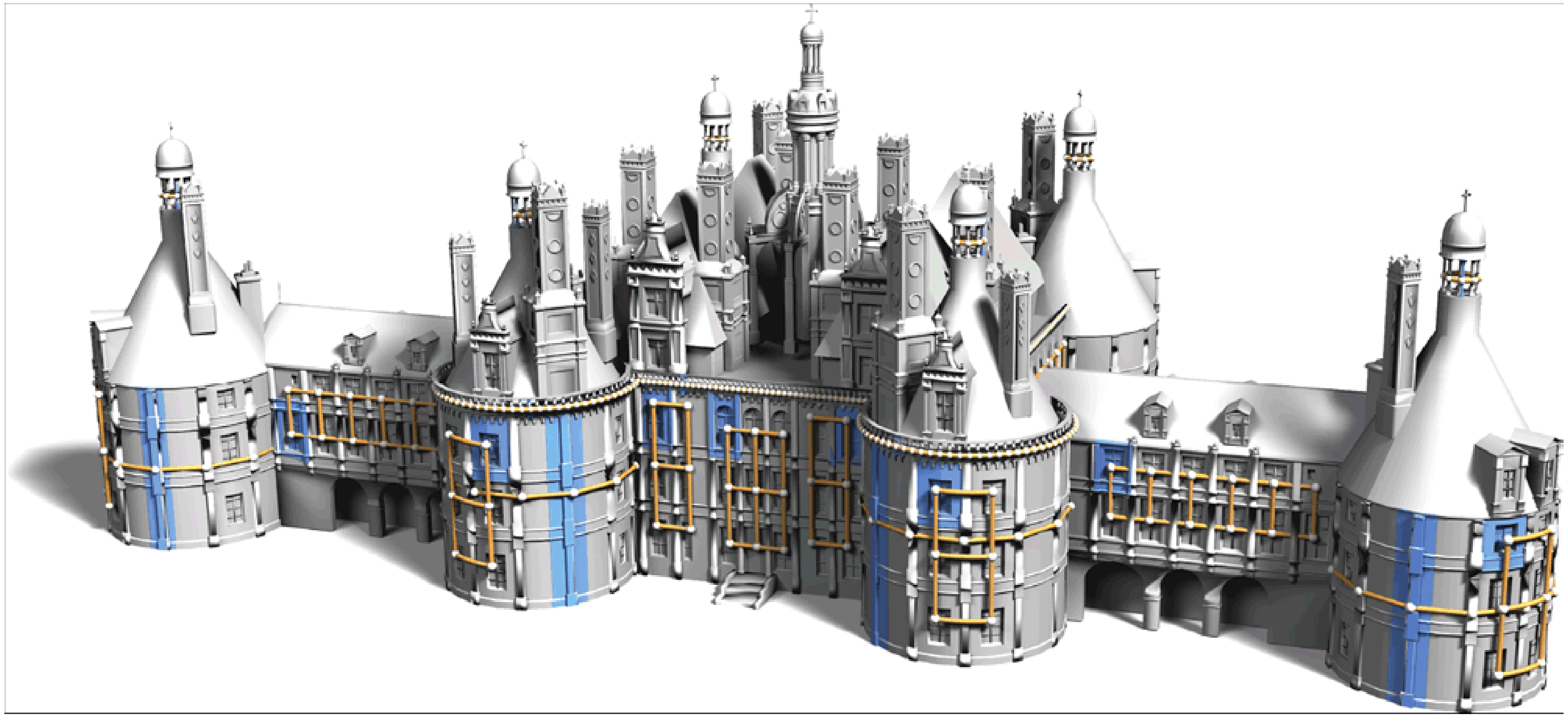
Symmetrized

Transformation-space voting: Follow-up II

Mark Pauly, Niloy J. Mitra, Johannes Wallner, Helmut Pottmann, and Leonidas Guibas. "**Discovering Structural Regularity in 3D Geometry**," *SIGGRAPH*, 2008

Transformation-space voting: Follow-up II

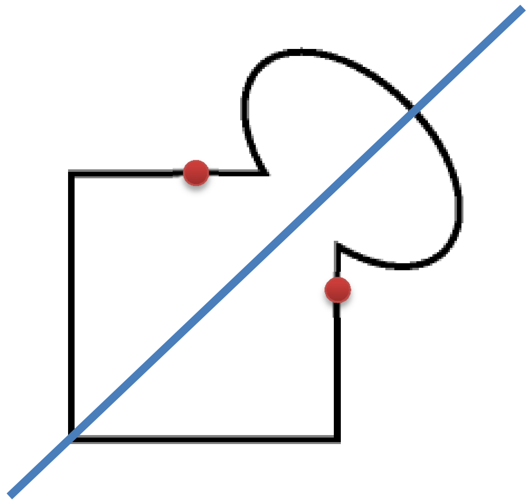
Discovering Structural Regularity



Extrinsic symmetry detection: Other state-of-the-arts

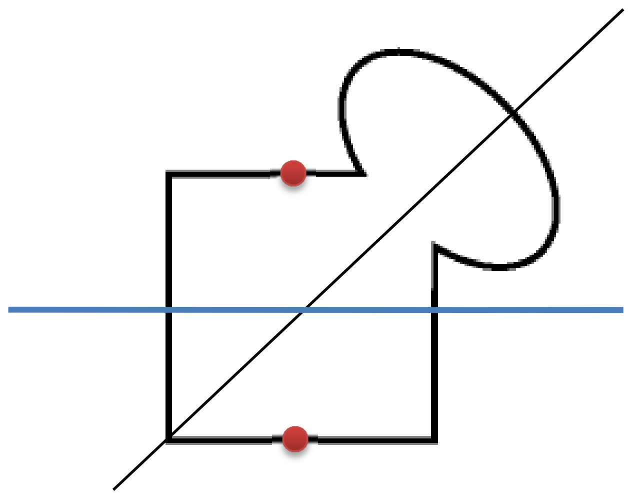
Symmetry transform computation:

J. Podolak, P. Shilane, A. Golovinskiy, S. Rusinkiewicz, and T. Funkhouser. "**A Planar-Reflective Symmetry Transform for 3D Shapes**," *SIGGRAPH 2006*



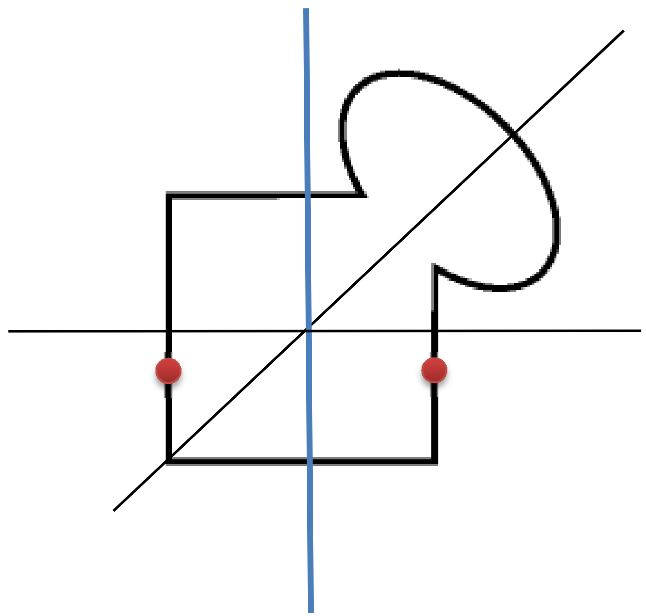
Extrinsic symmetry detection: Other state-of-the-arts

Symmetry transform computation:



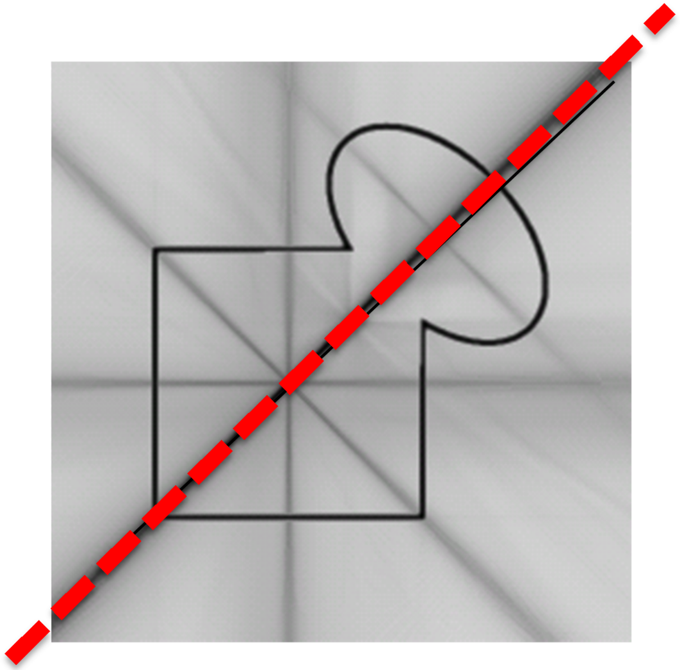
Extrinsic symmetry detection: Other state-of-the-arts

Symmetry transform computation:



Extrinsic symmetry detection: other state-of-the-arts

Symmetry transform computation:

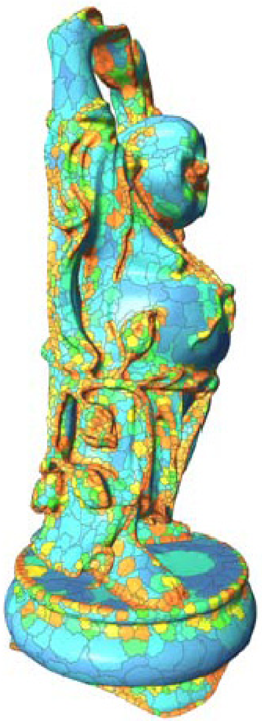


$$\mathrm{PRST}^2(f,\gamma) = 1 - \frac{d\big(f,\gamma(f)\big)^2}{\|f\|^2}$$

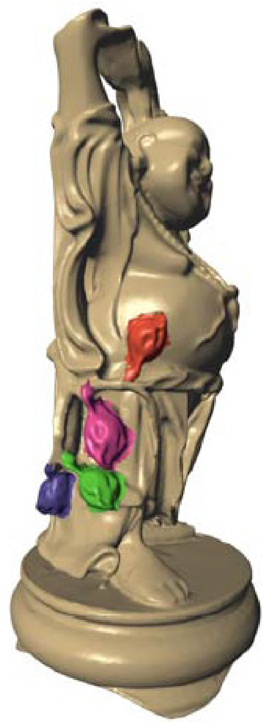
Extrinsic symmetry detection: other state-of-the-arts

Direct (partial) matching: Salient features

Ran Gal and Daniel Cohen-Or. "**Salient Geometric Features for Partial Shape Matching and Similarity**," *ACM Transactions on Graphics*, 2006



Quadratic patches with saliency



Partial matching by geometric hashing

Extrinsic symmetry detection: other state-of-the-arts

Direct (partial) matching: Feature curves

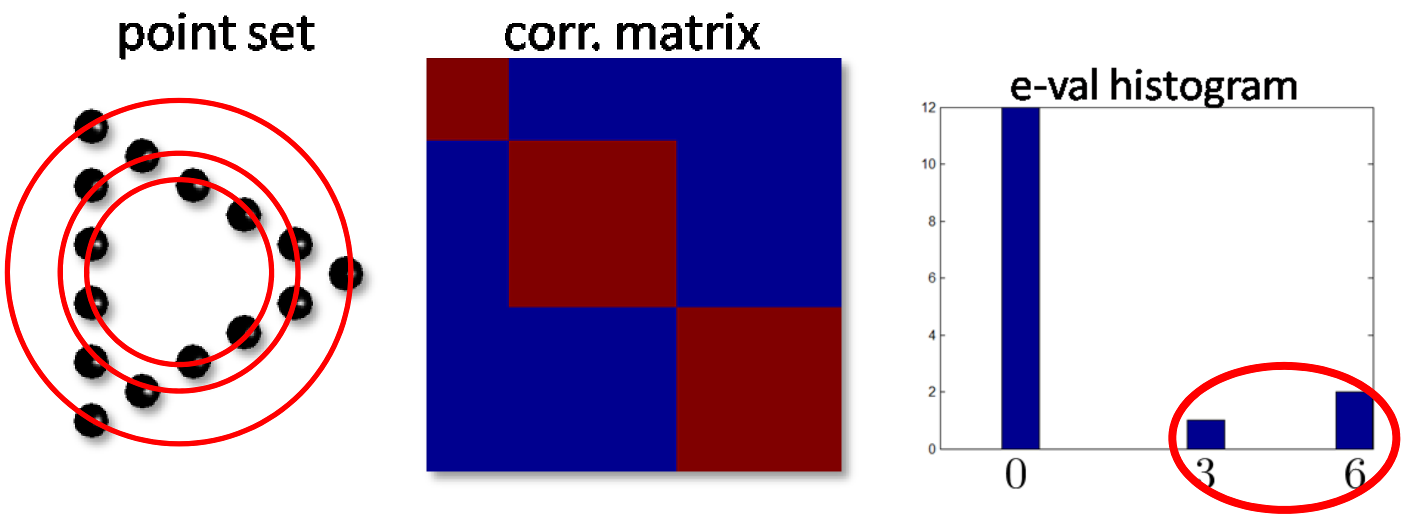
M. Bokeloh, A. Berner, M. Wand, H.-P. Seidel, and A. Schilling .
"**Symmetry Detection Using Line Features**," *Eurographics*, 2009

Graph matching of the networks of feature curves

Extrinsic symmetry detection: other state-of-the-arts

Correspondence space analysis

Yaron Lipman, Xiaobai Chen, Ingrid Daubechies, and Thomas Funkhouser. "**Symmetry Factored Embedding and Distance**," *SIGGRAPH 2010*.



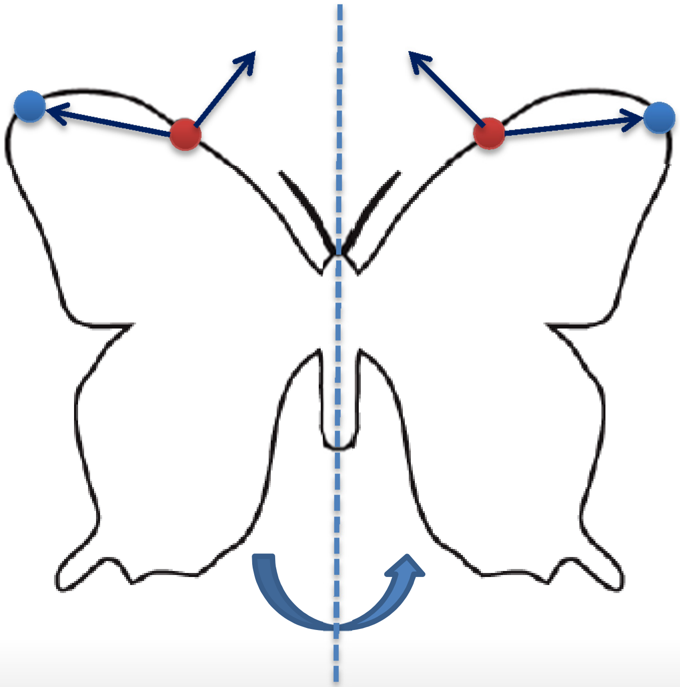
Top non-constant eigenvalues characterize symmetry orbits

Extrinsic symmetry detection: other state-of-the-arts

Correspondence space analysis

Yaron Lipman, Xiaobai Chen, Ingrid Daubechies, and Thomas Funkhouser.
"**Symmetry Factored Embedding and Distance**," *SIGGRAPH 2010*.

The key is defining the Symmetry Correspondence Matrix:



1. Select two pairs of points defining two local frames.
2. Align the two local frames
3. Compute Root Mean-Square Deviation

Extrinsic symmetry detection: other state-of-the-arts

Correspondence space analysis

Yaron Lipman, Xiaobai Chen, Ingrid Daubechies, and Thomas Funkhouser. "**Symmetry Factored Embedding and Distance**," *SIGGRAPH 2010*.

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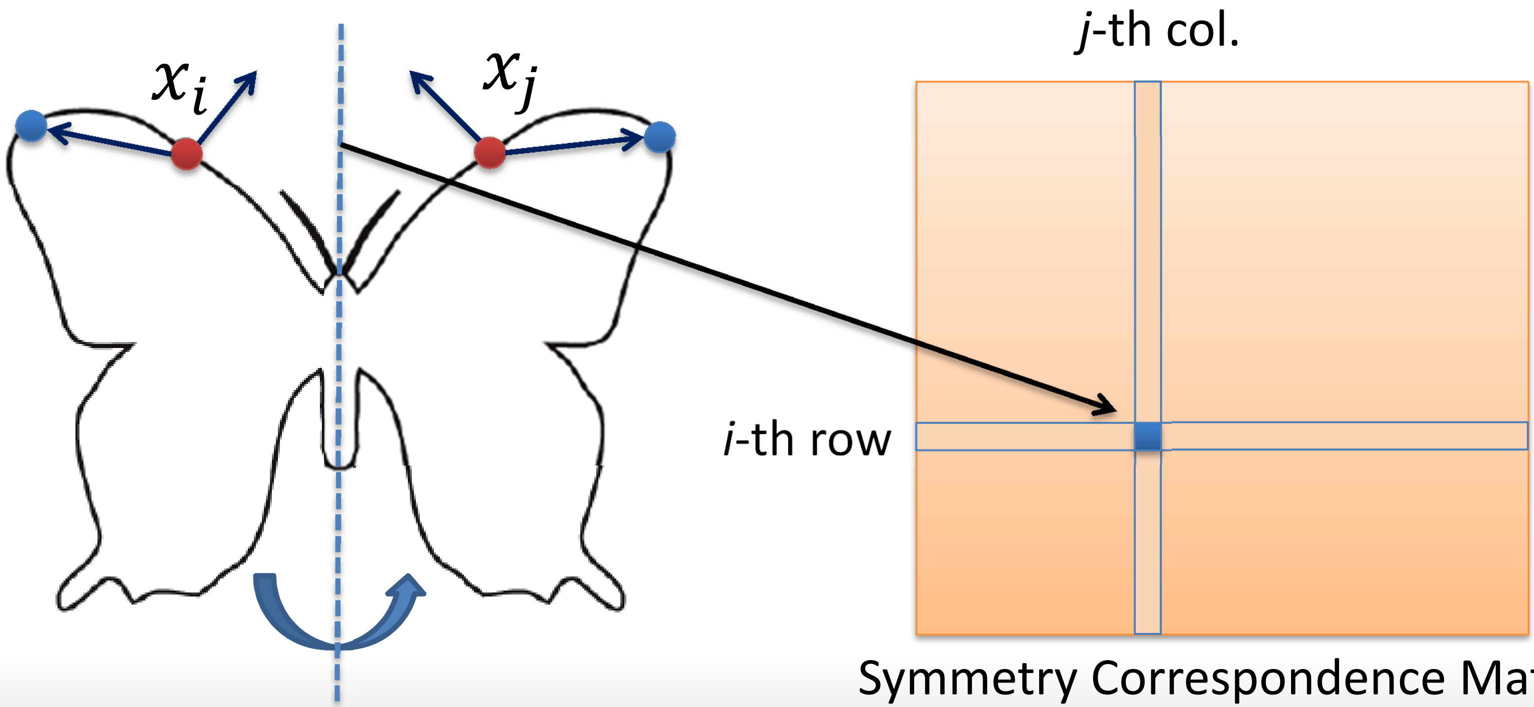

Symmetry Correspondence Matrix

Extrinsic symmetry detection: other state-of-the-arts

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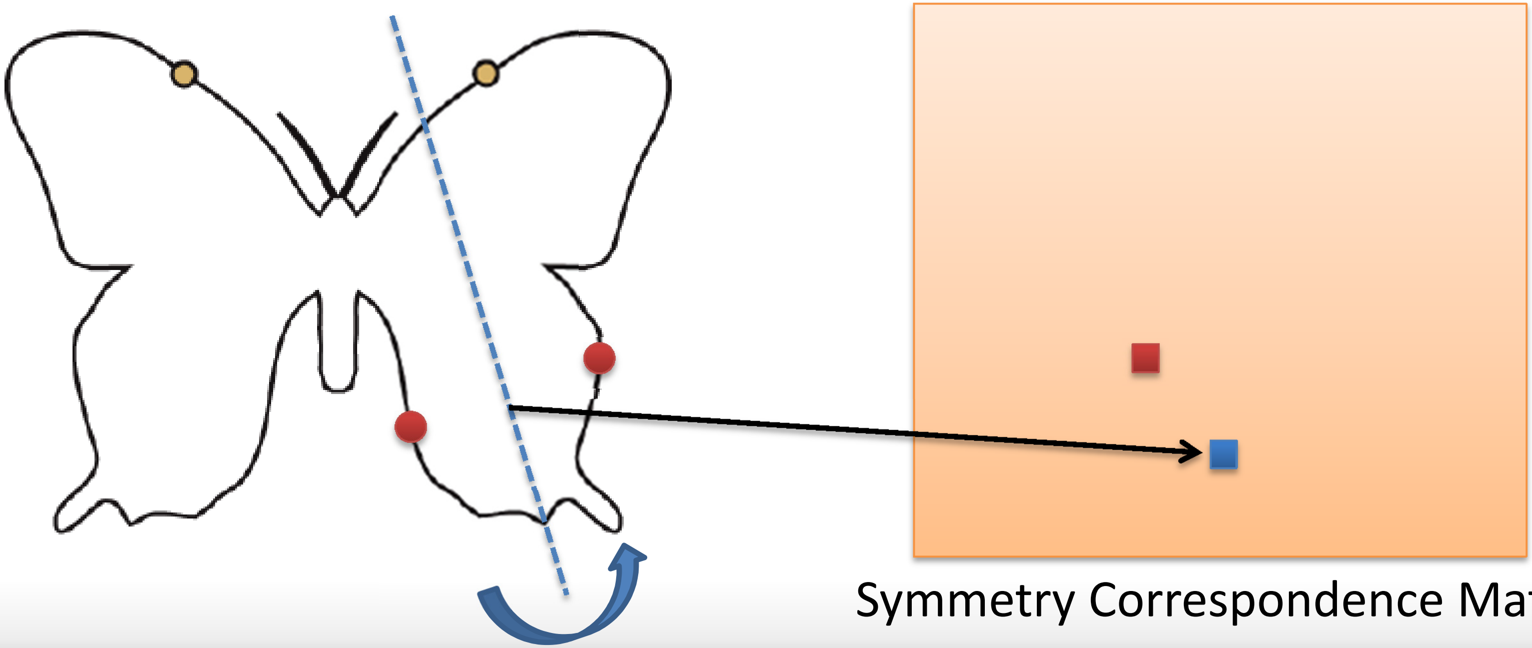


Symmetry Correspondence Matrix

Extrinsic symmetry detection: other state-of-the-arts

Correspondence space analysis

Yaron Lipman, Xiaobai Chen, Ingrid Daubechies, and Thomas Funkhouser. "**Symmetry Factored Embedding and Distance**," *SIGGRAPH 2010*.

Extrinsic symmetry detection: Summary

Characteristics of the problem:

- Transformation in Euclidean space:
  - Symmetry transformation easy to parameterize
  - Relatively small search space
  - Often handle partial and global in the same framework

Extrinsic symmetry detection: Summary

In a nutshell:

- Symmetry is global property
- Detection is relegated to local similarity
- Key approach: aggregation ← randomized voting

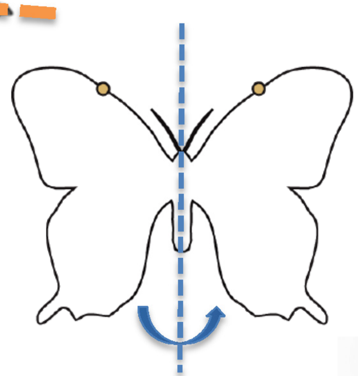
The three "typical stages" summarized in [Mitra et al. Survery]

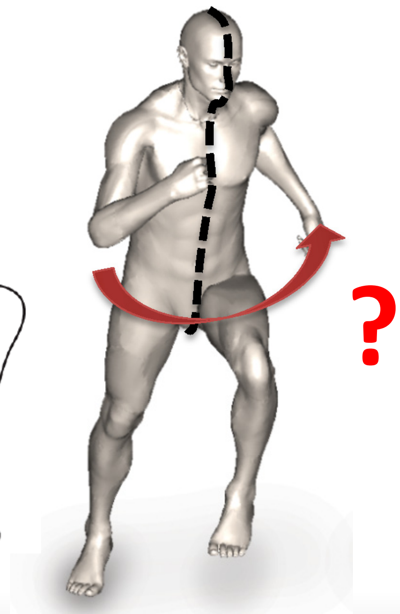
- Feature selection
- Aggregation
- Extraction

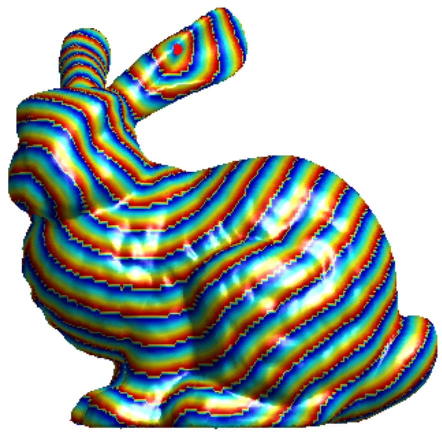
Global intrinsic symmetry detection

The main challenges of *intrinsic*

- Hard to parameterize
- Image under intrinsic map is hard to compute
- Measures in the intrinsic setting are expensive to compute







Global intrinsic symmetry detection

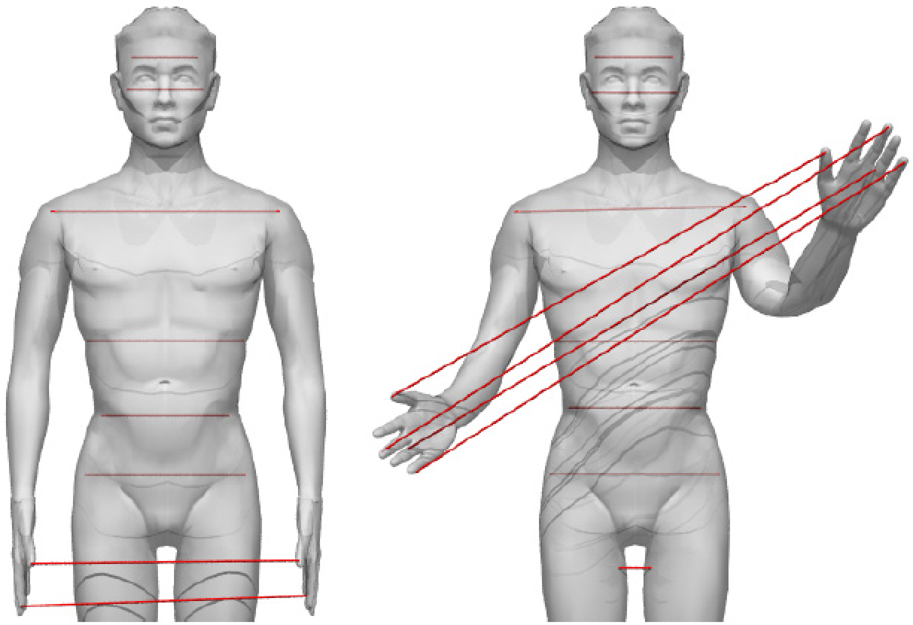
The main challenges of *intrinsic*

- Hard to parameterize
- Image under intrinsic map is hard to compute
- Measures in the intrinsic setting are expensive to obtain

Main approaches

- Directly minimize distance distortion [Raviv et al. 2007]
- Transform into spectral domain [Ovsjanikov et al. 2008]
- Transform into a more general space [Kim et al. 2010]
- Continuous definition [Ben-Chen et al. 2010]
- Approximate criteria of intrinsic isometry [Xu et al. 2009]

Intrinsic symmetry detection: Pose-normalizing spectral embedding



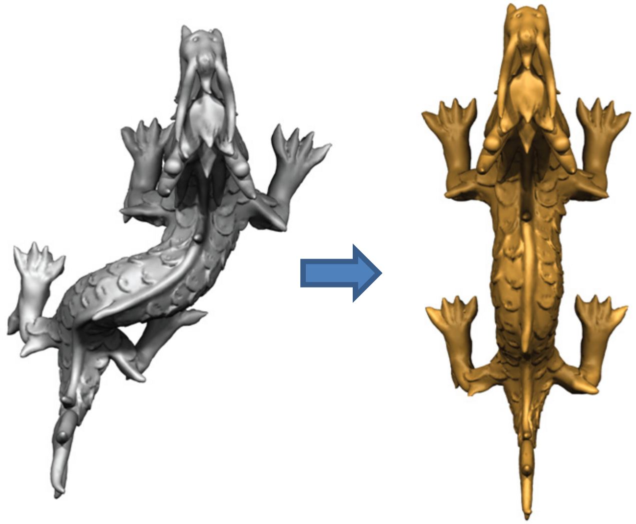
Maks Ovsjanikov, Jian Sun, and Leonidas Guibas. “**Global Intrinsic Symmetries of Shapes**,” SGP 2008.

Intrinsic symmetry detection: Pose-normalizing spectral embedding

Key observation:

The intrinsic symmetries of a shape are **transformed into the Euclidean symmetries** in the *signature space* defined by the *eigenfunctions of the Laplace-Beltrami operator*.

Spectral embedding normalizes poses



Symmetrization?

Costly!

Intrinsic symmetry detection: Pose-normalizing spectral embedding

Spectral embedding:

Eigendecomposition of Laplace-Beltrami operator:

$$\Delta\phi_i = \lambda_i \phi_i$$

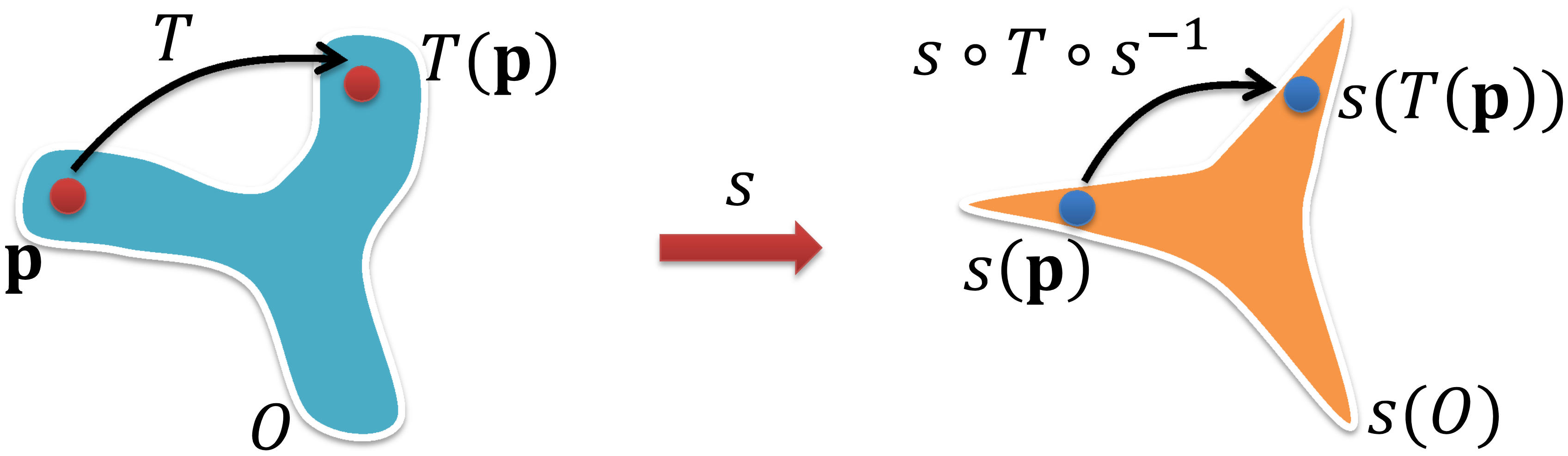
Consider Global Point Signature (GPS) embedding:

$$s(\mathbf{p}) = \left(\frac{\phi_1(\mathbf{p})}{\sqrt{\lambda_1}}, \frac{\phi_2(\mathbf{p})}{\sqrt{\lambda_2}}, \ldots, \frac{\phi_i(\mathbf{p})}{\sqrt{\lambda_i}}, \ldots \right)$$

Intrinsic symmetry detection: Pose-normalizing spectral embedding

Theorem Suppose O has an intrinsic symmetry with an associated isometric self-mapping T. Then $s \circ T \circ s^{-1}$ is an extrinsic symmetry of $s(O)$. In other words, if $\mathbf{p}_1, \mathbf{p}_2 \in O$:

$$\|s(\mathbf{p}_1) - s(\mathbf{p}_2)\|_2 = \|s(T(\mathbf{p}_1)) - s(T(\mathbf{p}_2))\|_2$$

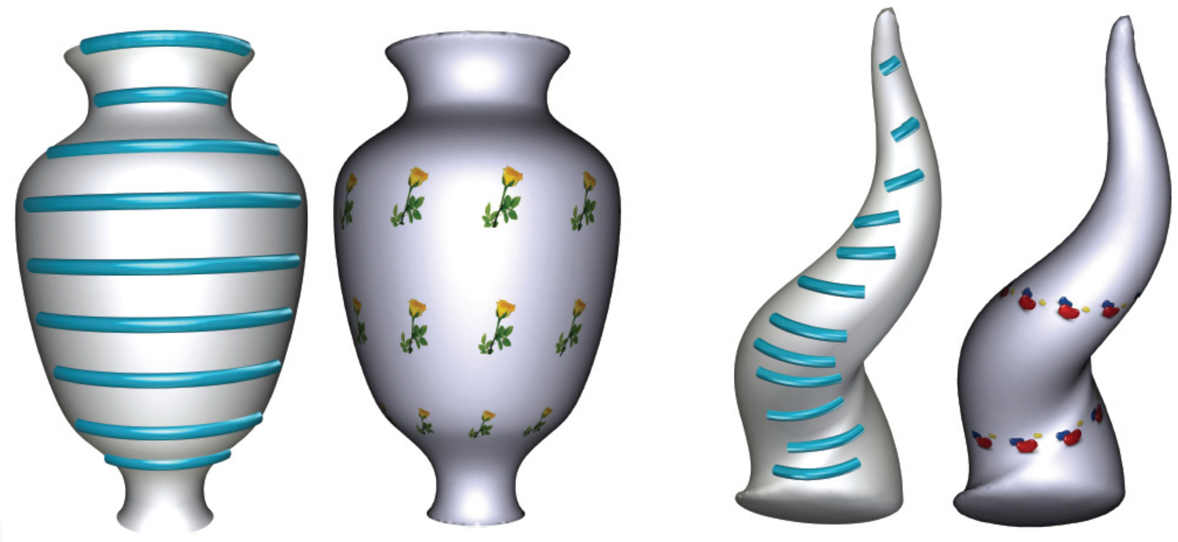


Intrinsic symmetry detection: Other state-of-the-arts

Continuous tangent vector field

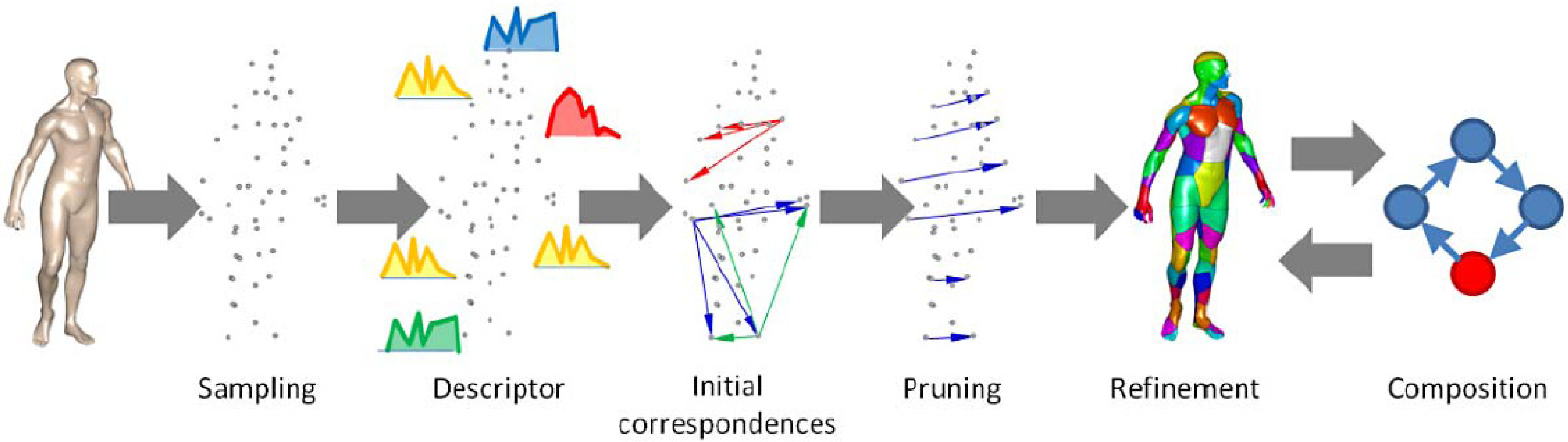
Mirela Ben-Chen, Adrian Butscher, Justin Solomon, and Leonidas Guibas. "**On Discrete Killing Vector Fields and Patterns on Surfaces**," *SGP 2010*.

Observation: Continuous intrinsic symmetries can be represented using *infinitesimal rigid transformations*, which are given as tangent vector fields on the surface: *Killing Vector Fields*



Directly minimize distance distortion

D. Raviv, A. M. Bronstein, M. M. Bronstein, and Ron Kimmel. "**Full and Partial Symmetries of Non-rigid Shapes**," *IJCV*, 2010.



Key: Compute correspondence through directly minimizing intrinsic distortion using Generalized Multi-Dimensional Scaling (GMDS)

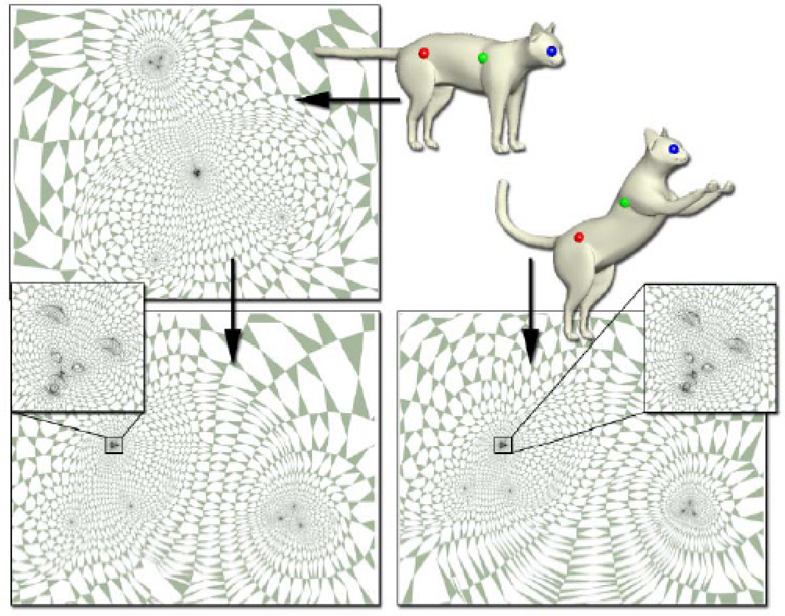
Intrinsic symmetry detection: Other state-of-the-arts

Möbius transformations

Vladimir G. Kim, Yaron Lipman, Xiaobai Chen, and Thomas Funkhouser.
"**Möbius Transformations for Global Intrinsic Symmetry Analysis**," *SGP 2010*.

Observations [Lipman and Funkhouser, 2009]:

1. Isometry is a subset of Möbius group
2. Möbius transformation has low degree of freedom



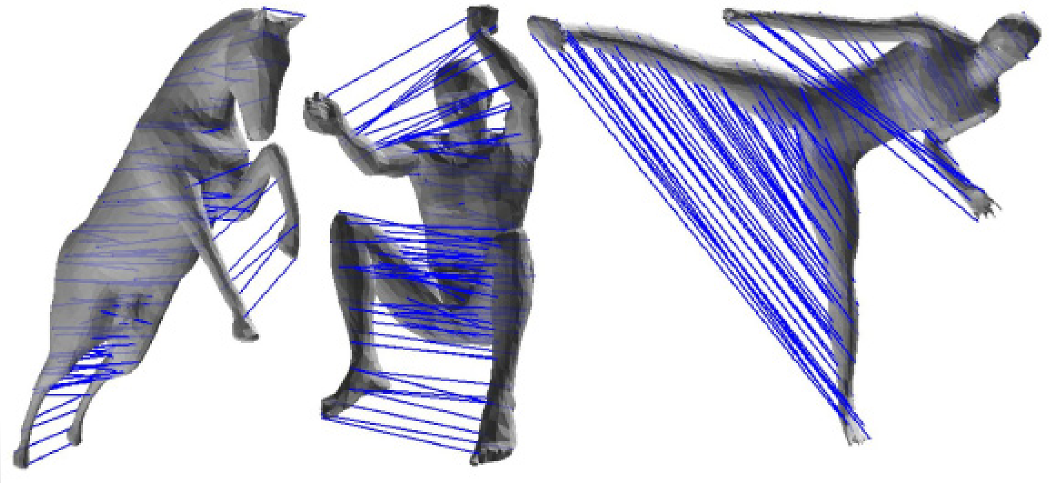
Intrinsic symmetry detection: Other state-of-the-arts

Möbius transformations

Vladimir G. Kim, Yaron Lipman, Xiaobai Chen, and Thomas Funkhouser.
"**Möbius Transformations for Global Intrinsic Symmetry Analysis**," *SGP 2010*.

Steps:

1. Generates a set of symmetric points
2. Generate *candidate Möbius transformations* via enumerating small subsets of those feature points
3. Selects among those which best map the surface onto itself



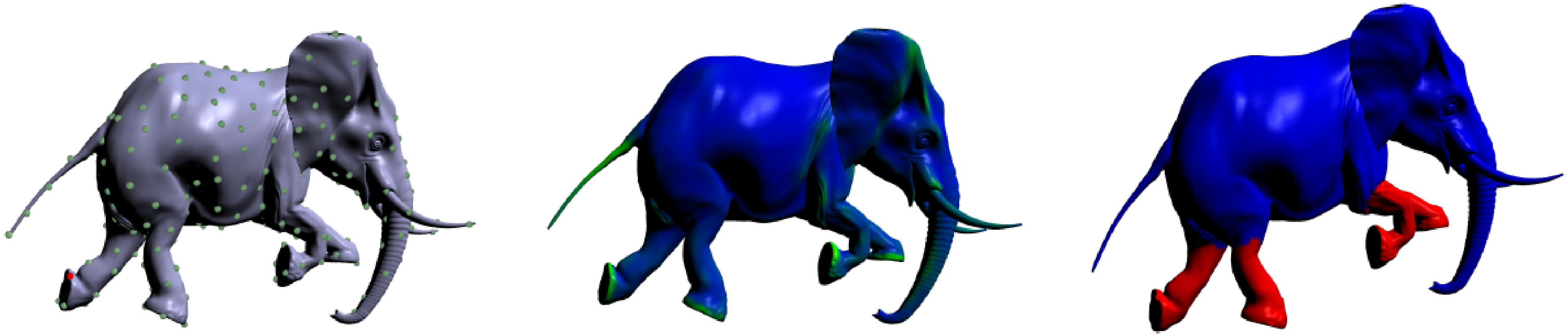
Intrinsic symmetry detection

Probabilistic framework

Ruxandra Lasowski, and Art Tevs, and Hans-Peter Seidel, and Michael Wand, "**A Probabilistic Framework for Partial Intrinsic Symmetries in Geometric Data**," *ICCV 2009*.

Key components:

1. Markov random field model → Probability distribution over all possible intrinsic self-mappings
2. Approximate marginals of this distribution using sum-product loopy belief propagation



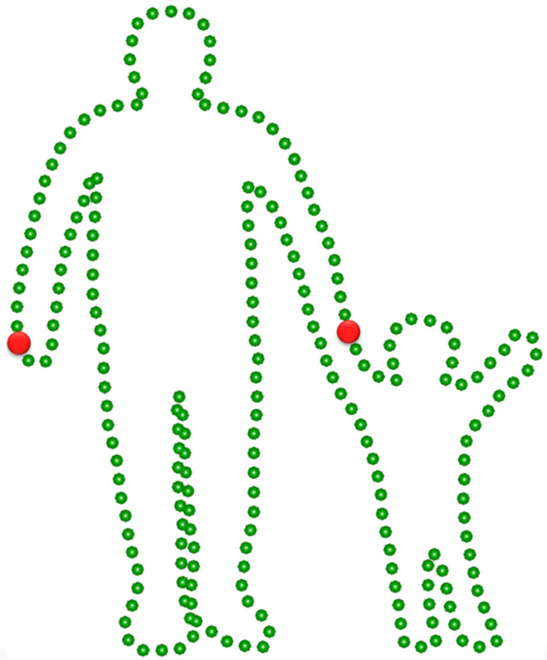
Marginals

Symmetry

Intrinsic symmetry detection: Summary

In a nutshell:

- Symmetry is global property; its detection relies on local similarity
- Isometry-invariant similarity measure?
- Not working for partial symmetries!



Local similarity fails!

Partial intrinsic symmetry detection

Partial symmetry is more natural!

Partial intrinsic symmetry detection

Challenges of *partial intrinsic* isometry

- Intrinsic
  - Hard to parameterize
  - Expensive to compute
- Partial
  - larger search space

Do previous methods for global intrinsic symmetry work for *partial intrinsic*?

Partial intrinsic symmetry detection

Spectral embedding: [Ovsjanikov et al. 2008]

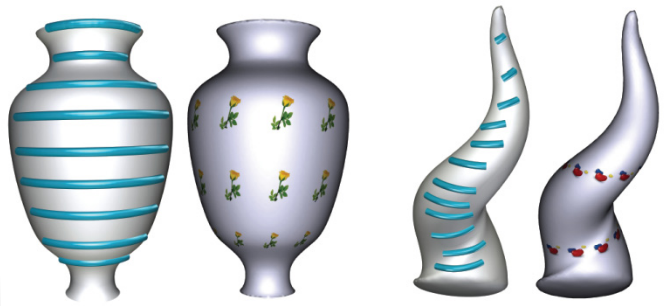
Search in high-D space for partial symmetries costly!

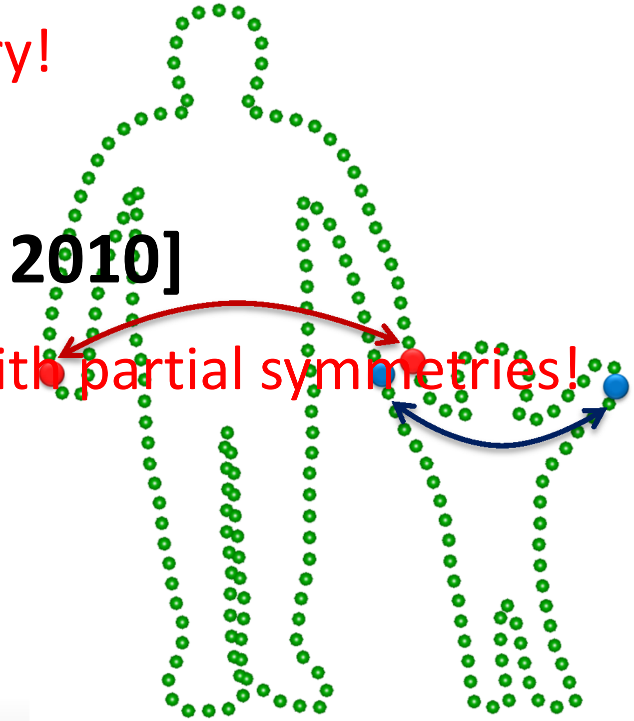
Distortion-minimizing self-mapping [Raviv et al. 2007]

Discontinuous map for partial symmetry!

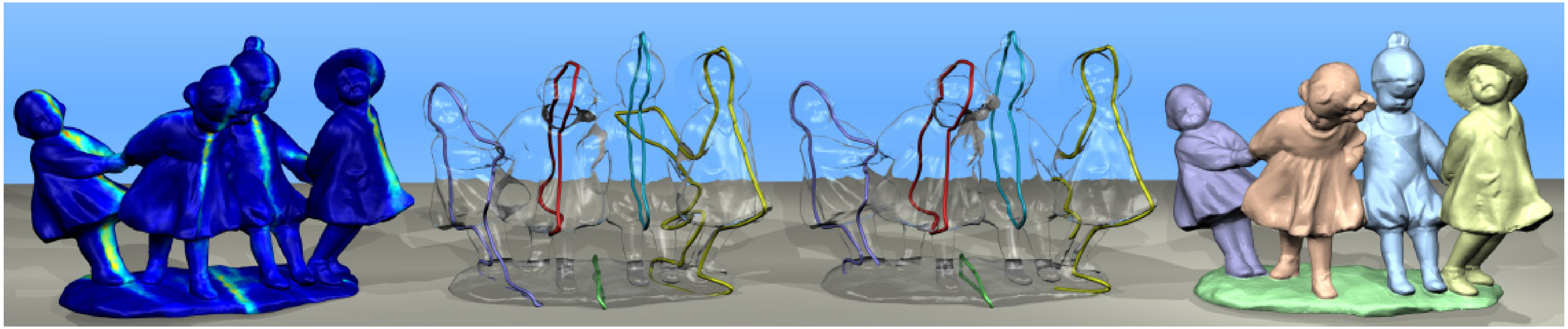
Killing vector field: [Ben-Chen et al. 2010]

No continuous vector field for shape with partial symmetries!





Partial intrinsic symmetry detection

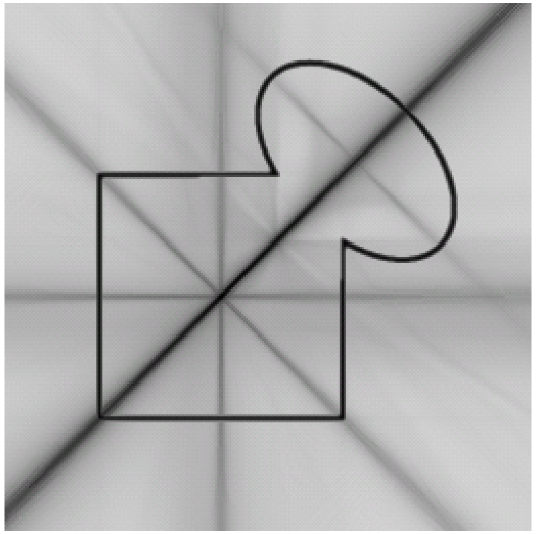


Kai Xu, Hao Zhang, Andrea Tagliasacchi, Ligang Liu, Guo Li, Min Meng, and Yueshan Xiong. "**Partial Intrinsic Reflectional Symmetry of 3D Shapes**," *SIGGRAPH Asia 2009*.

Partial intrinsic symmetry detection

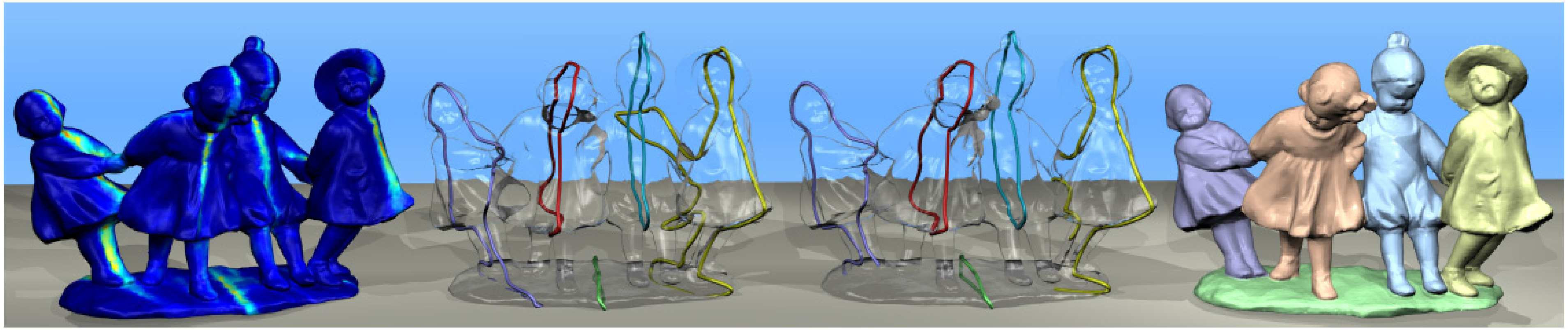
Voting for Intrinsic Reflectional Symmetry Axis (IRSA) transform

Intrinsic ref-sym axis (IRSA) transform



Planar ref-sym axis transform (PRST)

Partial intrinsic symmetry detection



IRSA transform

Partial reflectional symmetry axes

Pruned symmetry axes

Symmetry induced segmentation

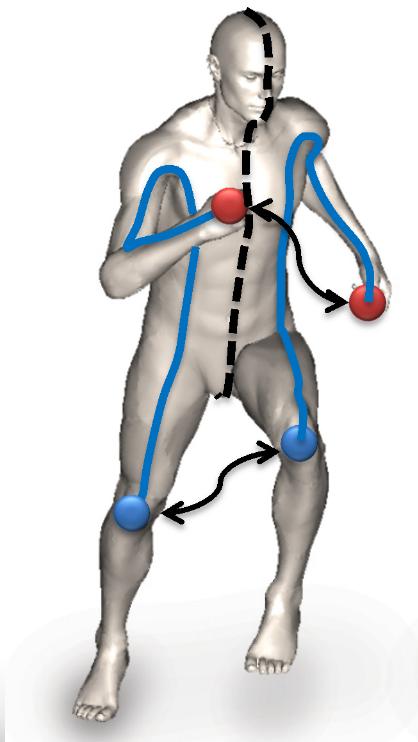
Partial intrinsic symmetry detection

Work on closed 2-manifold meshes M

M has a global symmetry if there is a homeomorphism:

$$T: M \to M$$

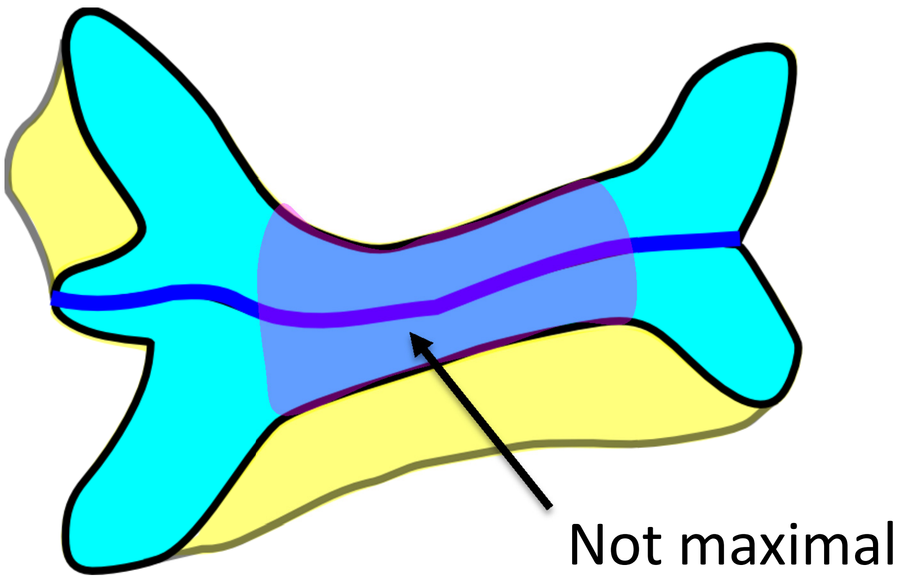
that is an isometry (geodesic distance preserving)



Partial intrinsic symmetry detection

Preliminaries: What we seek

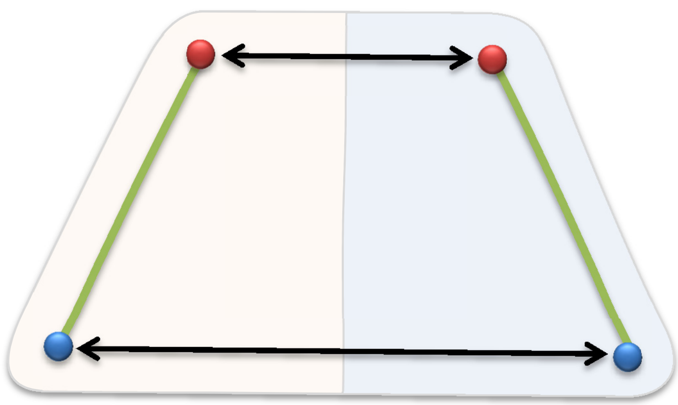
- Partial symmetry: isometry on connected subset $M' \subset M$
- Prominent symmetries: maximal partial symmetries

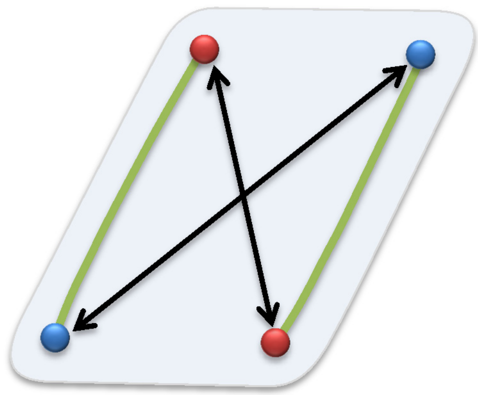


Partial intrinsic symmetry detection

Preliminaries: Reflectional symmetries

- *Involute* isometry over manifold on geodesic distances
- Set of *fixed points form a curve* (not a point)



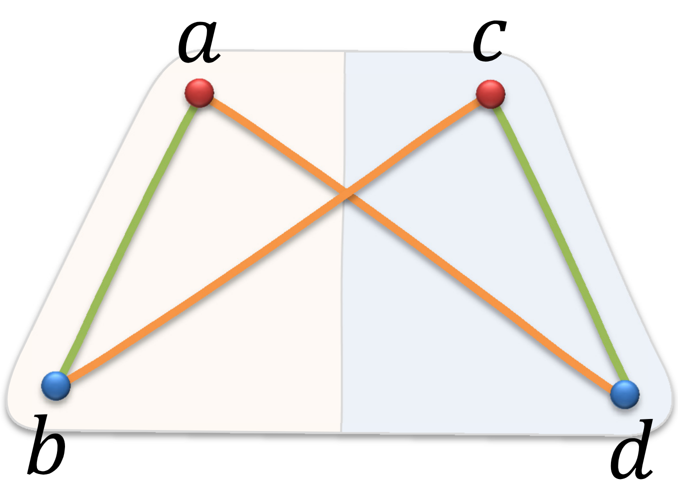


~~A point symmetry~~

Partial intrinsic symmetry detection

Preliminaries: Reflectional symmetries

- *Involute* isometry over manifold on geodesic distances
- Set of *fixed points form a curve* (not a point)



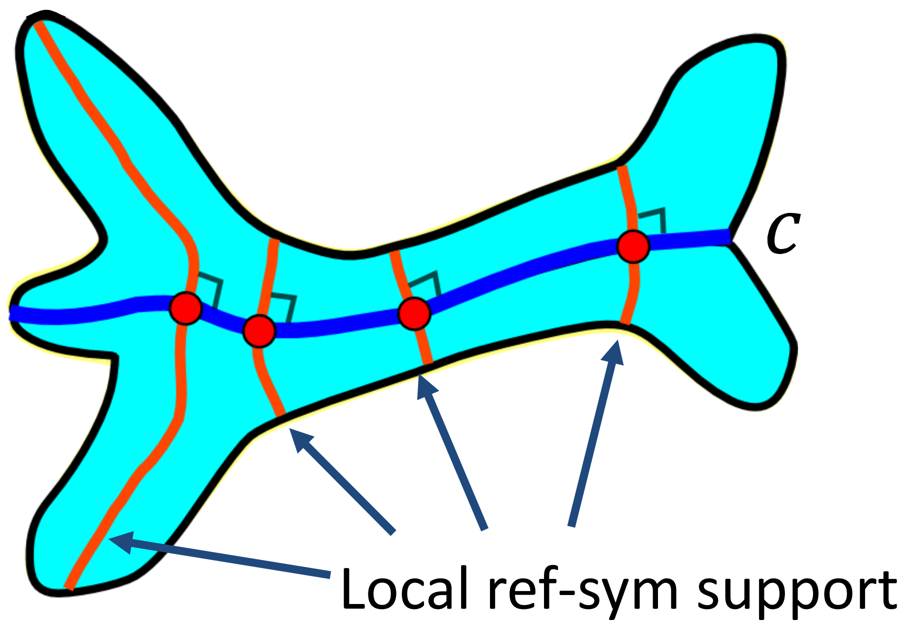
$$d_M(a,b) = d_M(c,d)$$
$$d_M(a,d) = d_M(b,c)$$

Necessary conditions of Involute isometry

Partial intrinsic symmetry detection

Preliminaries: Sym-Generating Set

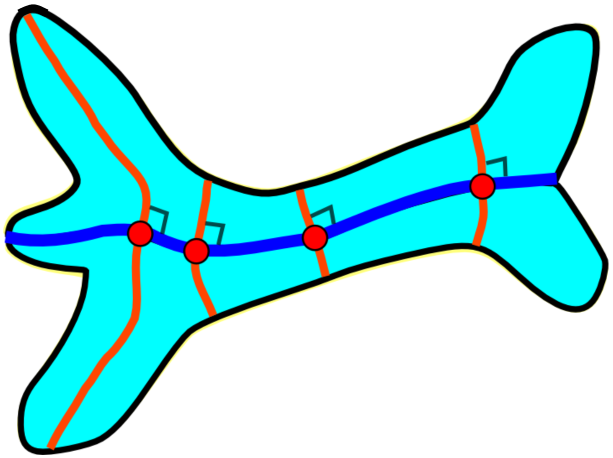
- Region that is reflectionally symmetric about a curve c
- Symmetry-Generating Set (SGS) of a curve c



Partial intrinsic symmetry detection

IRSA transform:

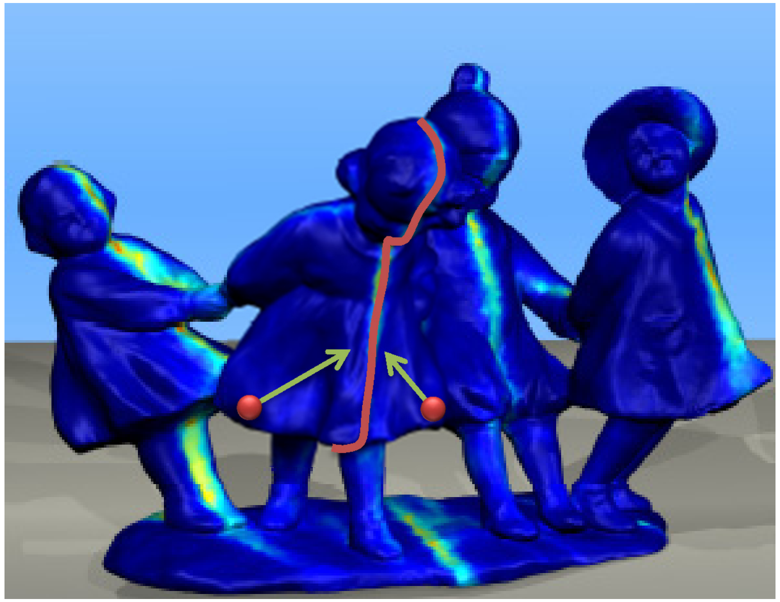
- Scalar field reflecting *prominence* of IRSA curves
  - Prominence = Area of maximal SGS of an IRSA curve



Partial intrinsic symmetry detection

IRSA transform:

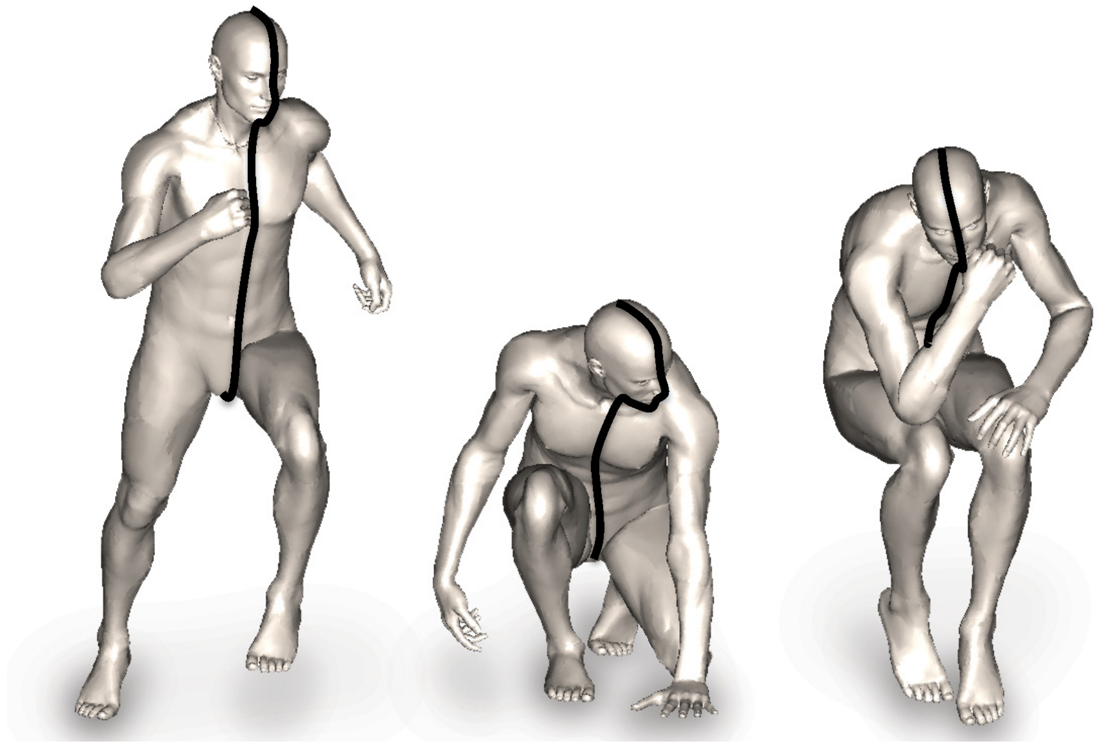
- Accumulated curve prominence
- *Randomized voting*: point pairs vote for their IRSA curves



Partial intrinsic symmetry detection

Technical challenges:

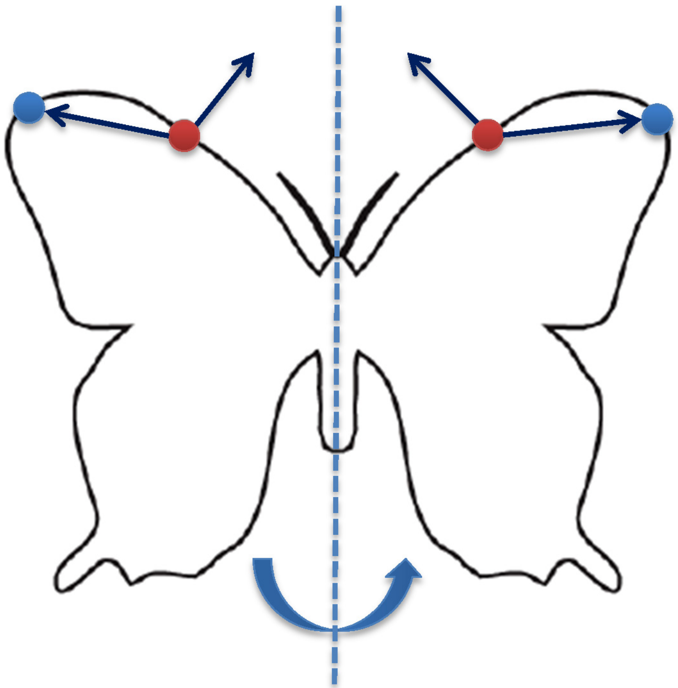
- Reflection axes can be arbitrary curves on manifold
  - Difficult to parameterize space of axes

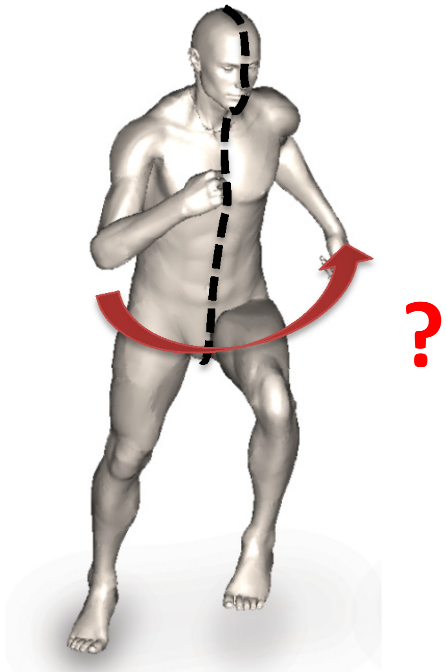


Partial intrinsic symmetry detection

Technical challenges:

- Reflection axes can be arbitrary curves on manifold
  - Difficult to parameterize space of axes
- Difficult to reflect over an intrinsic manifold

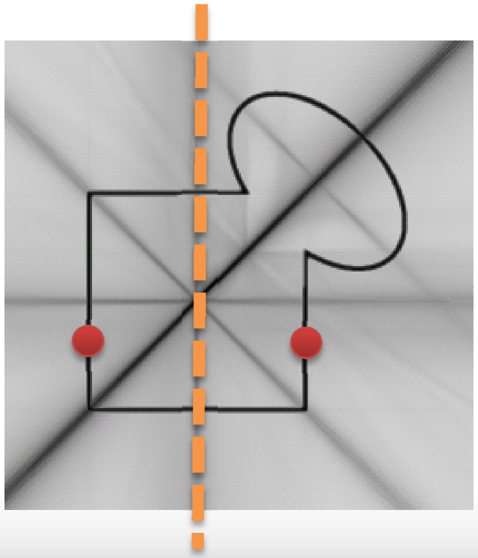


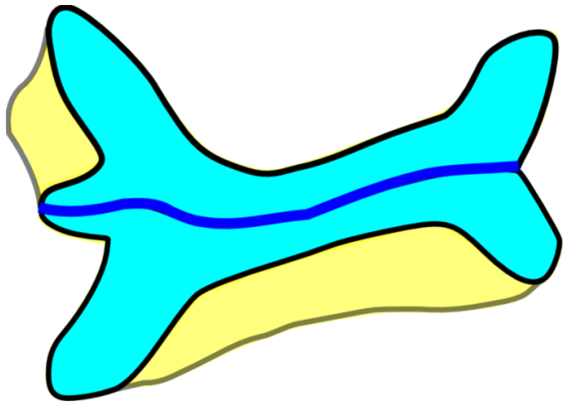


Partial intrinsic symmetry detection

Technical challenges:

- Reflection axes can be arbitrary curves on manifold
  - Difficult to parameterize space of axes
- Difficult to reflect over an intrinsic manifold
- Boundary bias does not help
  - Intrinsic problem has much higher dimension
  - Need new set of filters to select eligible voters





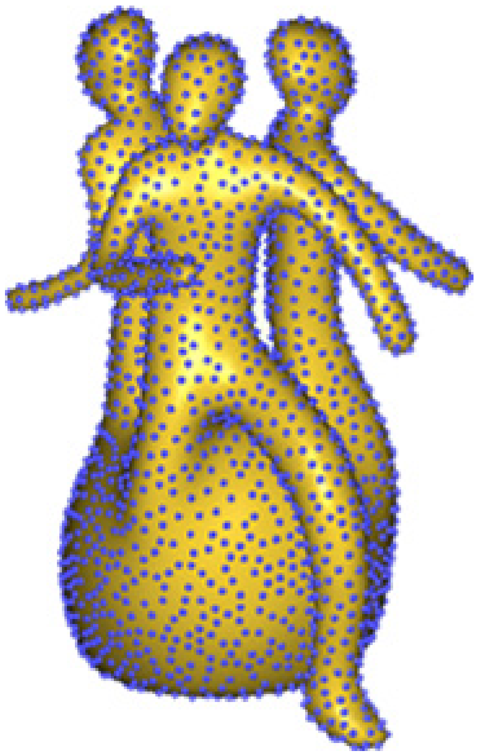
Boundary is unknown!

Partial intrinsic symmetry detection

Voting algorithm

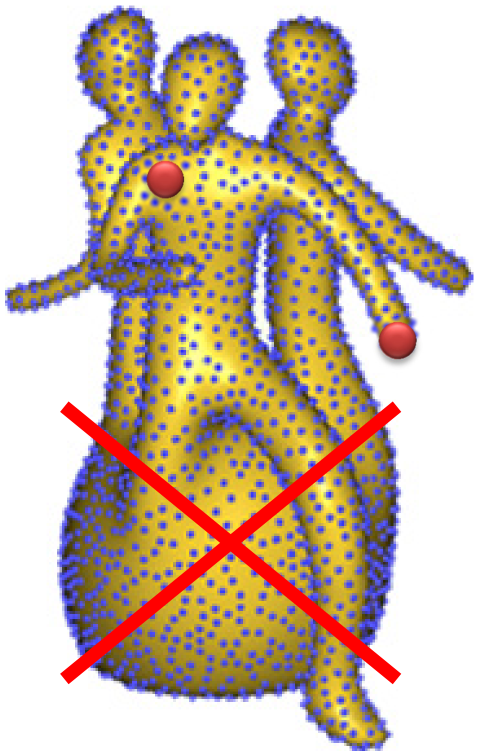
Partial intrinsic symmetry detection

Voting algorithm



Partial intrinsic symmetry detection

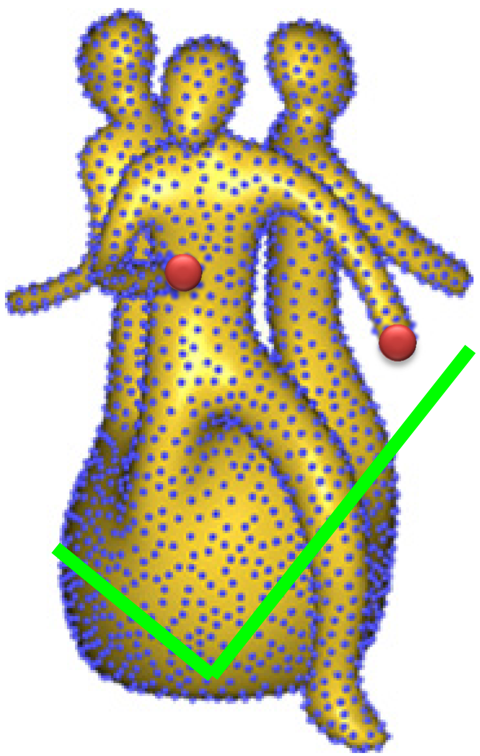
Voting algorithm



Only samples with sufficient local similarity can vote

Partial intrinsic symmetry detection

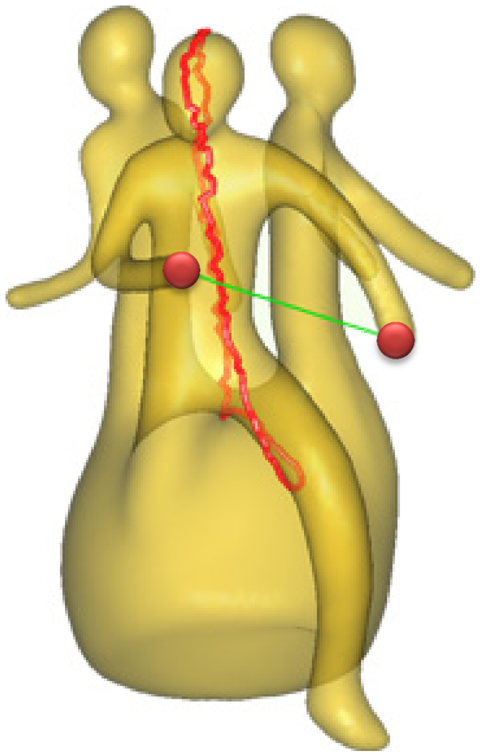
Voting algorithm



Only samples with sufficient local similarity can vote

Partial intrinsic symmetry detection

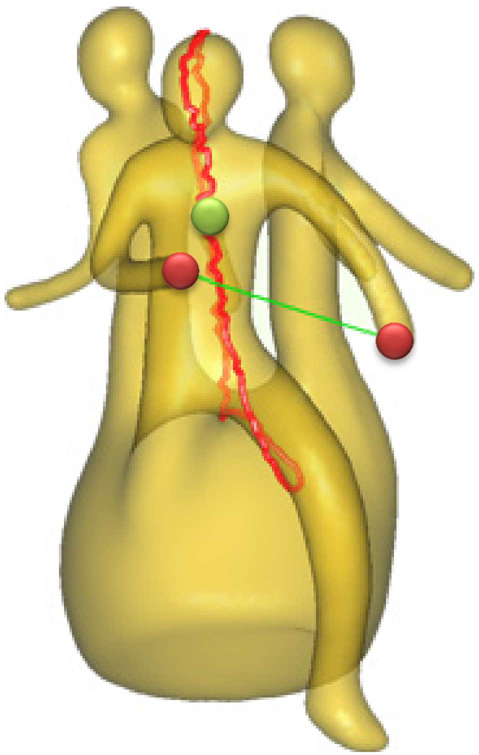
Voting algorithm

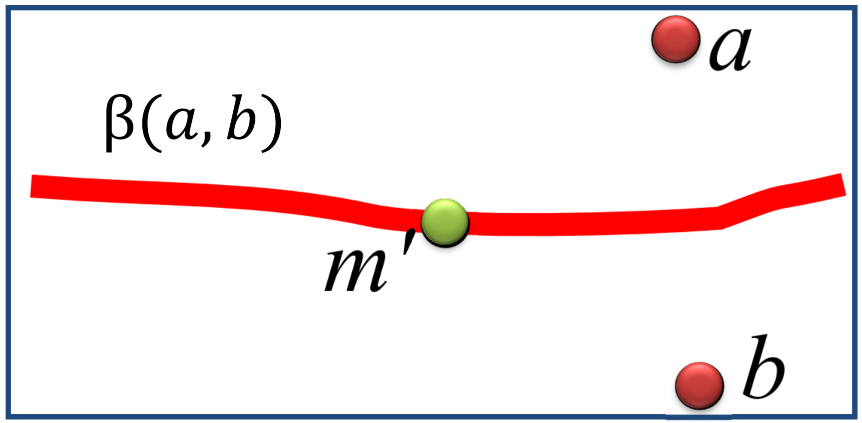


Votes are cast on intrinsic *Voronoi boundary*
of the two voters

Partial intrinsic symmetry detection

Vote on Voronoi boundary



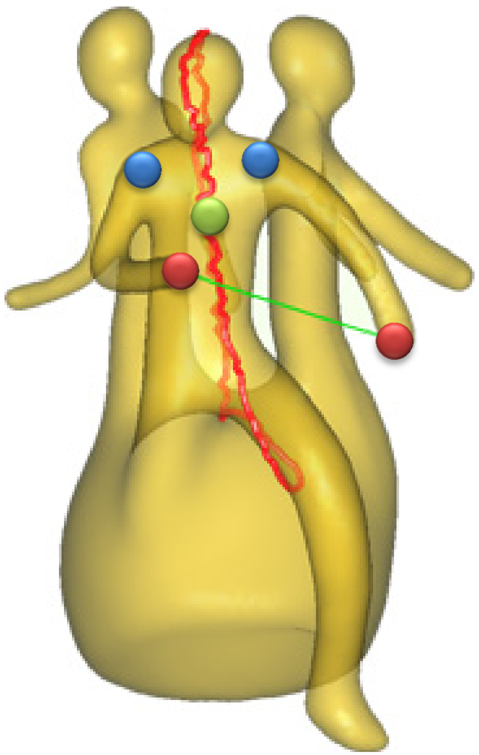


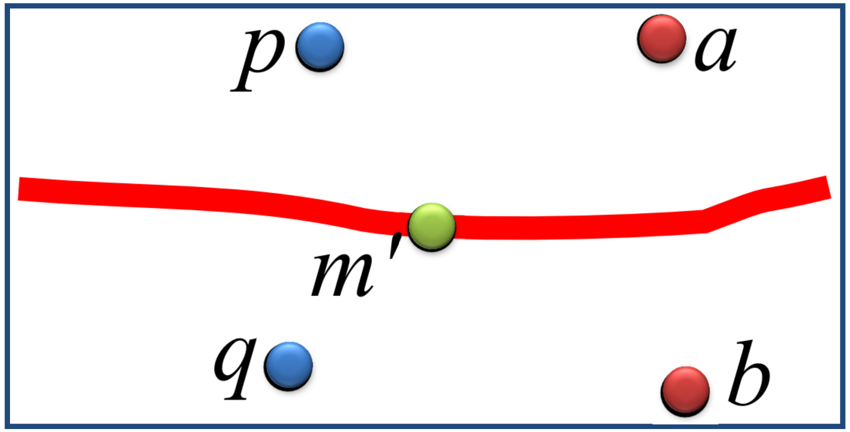
vote at m'

local symmetry support at m' as part of the IRSA for a and b

Partial intrinsic symmetry detection

Vote on Voronoi boundary

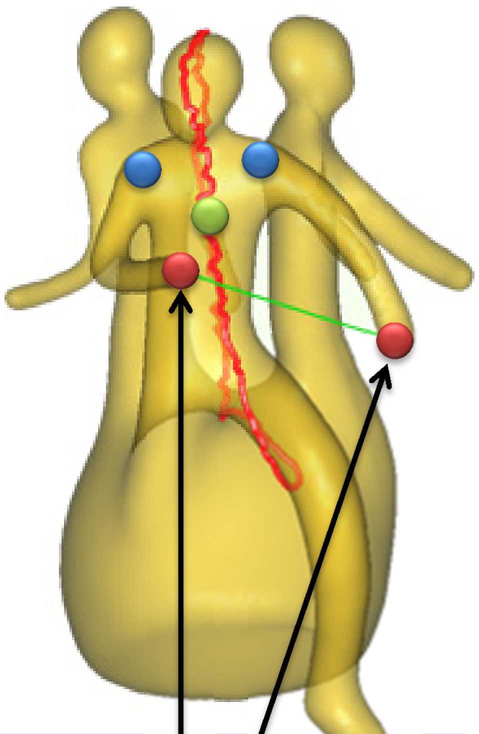




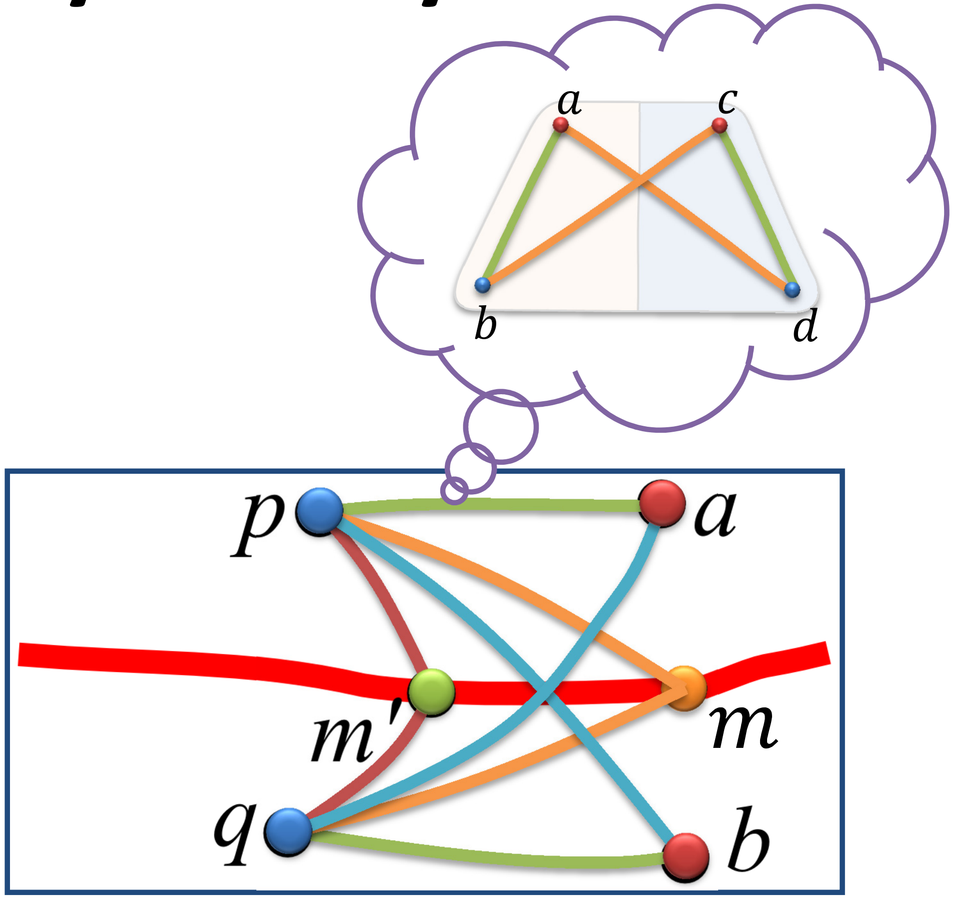
symmetry support at m' ⟺ accumulated over point pairs sharing the same PIRS as a, b

Partial intrinsic symmetry detection

Filters on voters



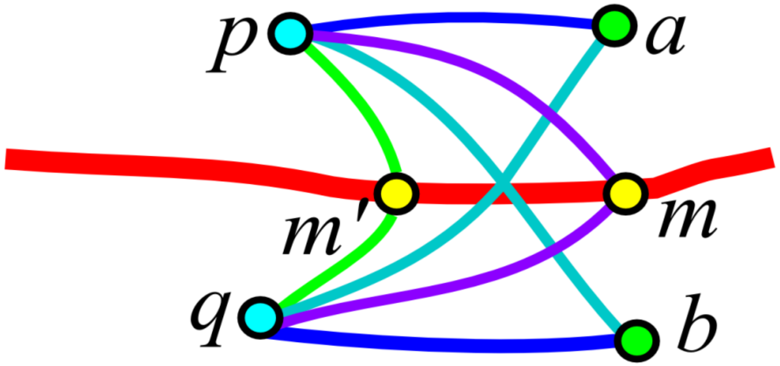
Similarity filter:
Voters p, q need sufficient similarity

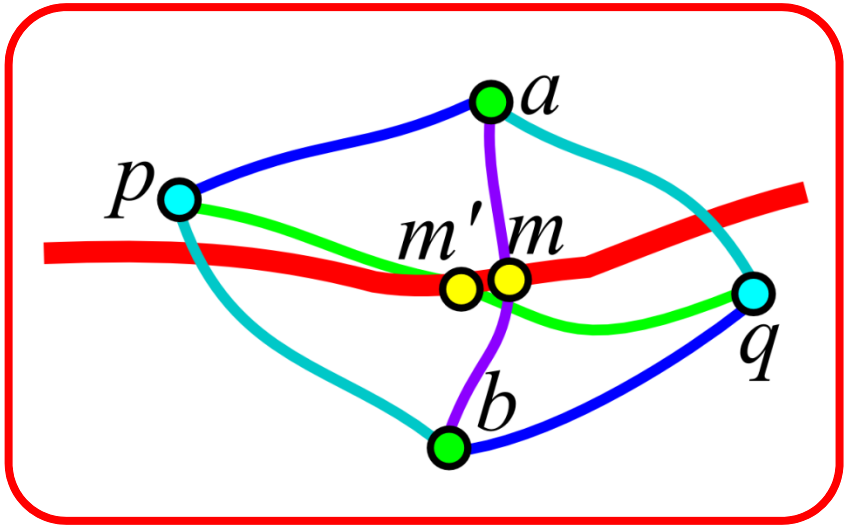


Distance filter:
Geodesic distances to reflect isometry

Partial intrinsic symmetry detection

Filters on voters





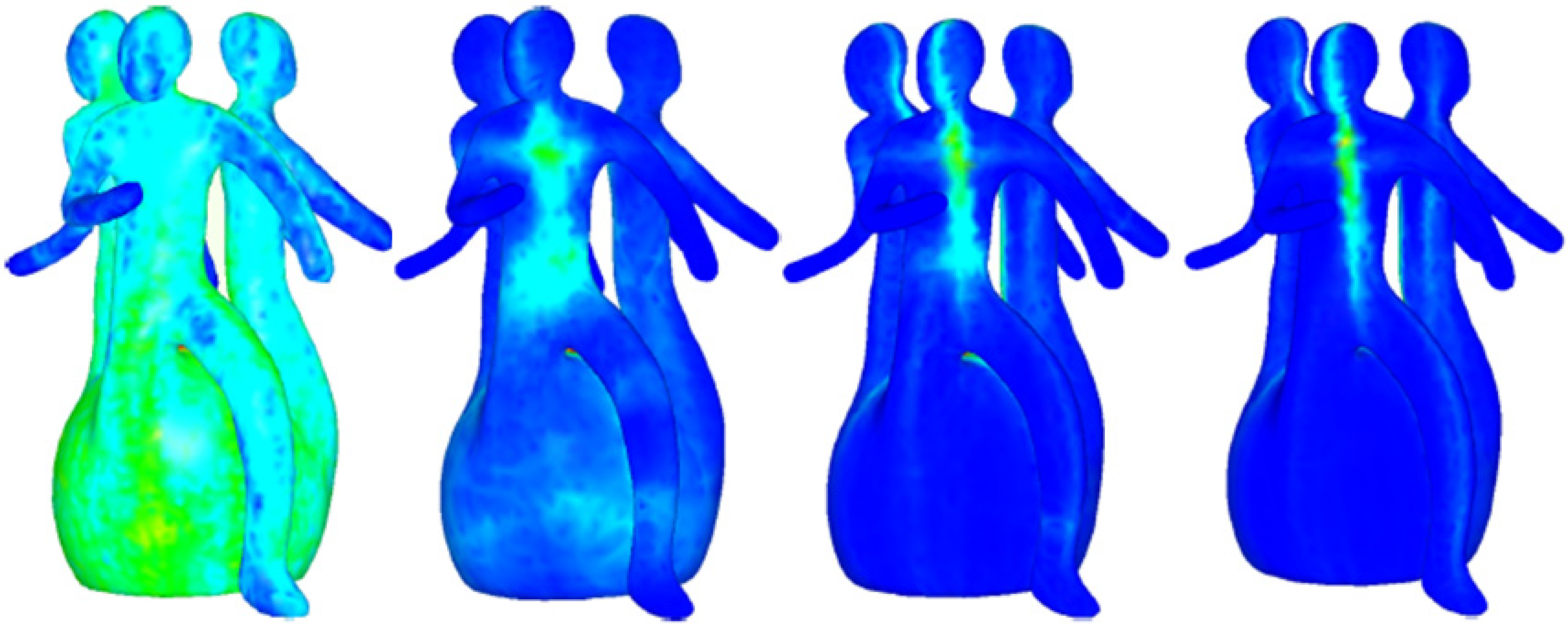
Axial-Sym filter:
Preclude point symmetry

Partial intrinsic symmetry detection

Filters on voters

Partial intrinsic symmetry detection

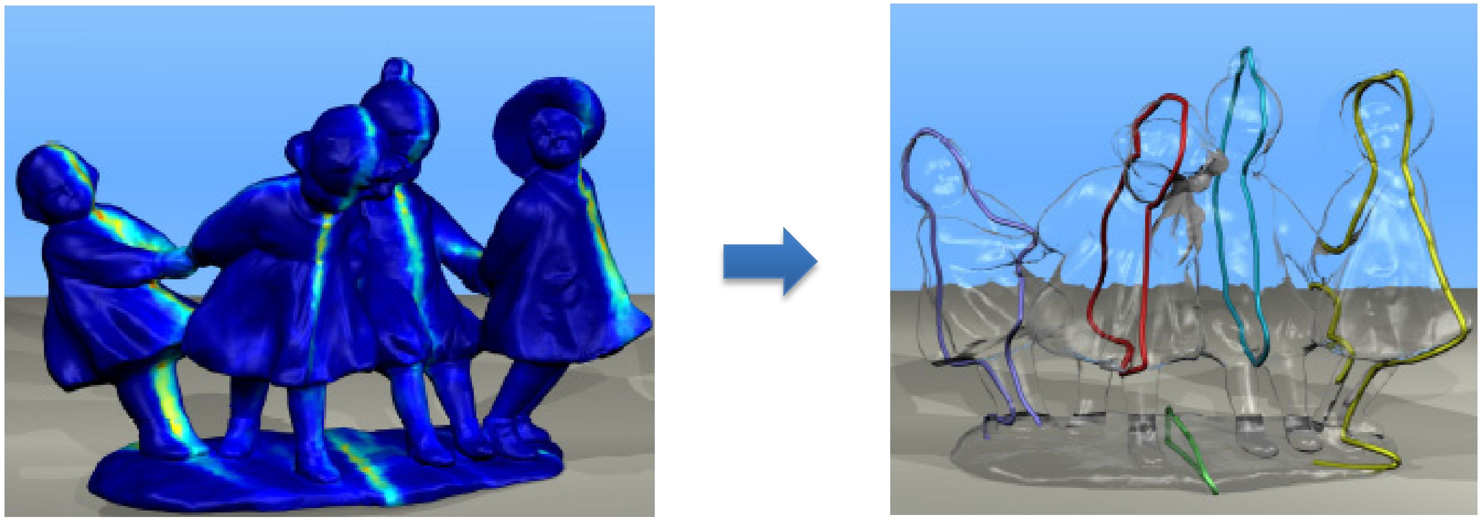
Filters at work



Similarity Distance Locality Axil-sym

Partial intrinsic symmetry detection

Explicit IRSA curves



Partial intrinsic symmetry detection

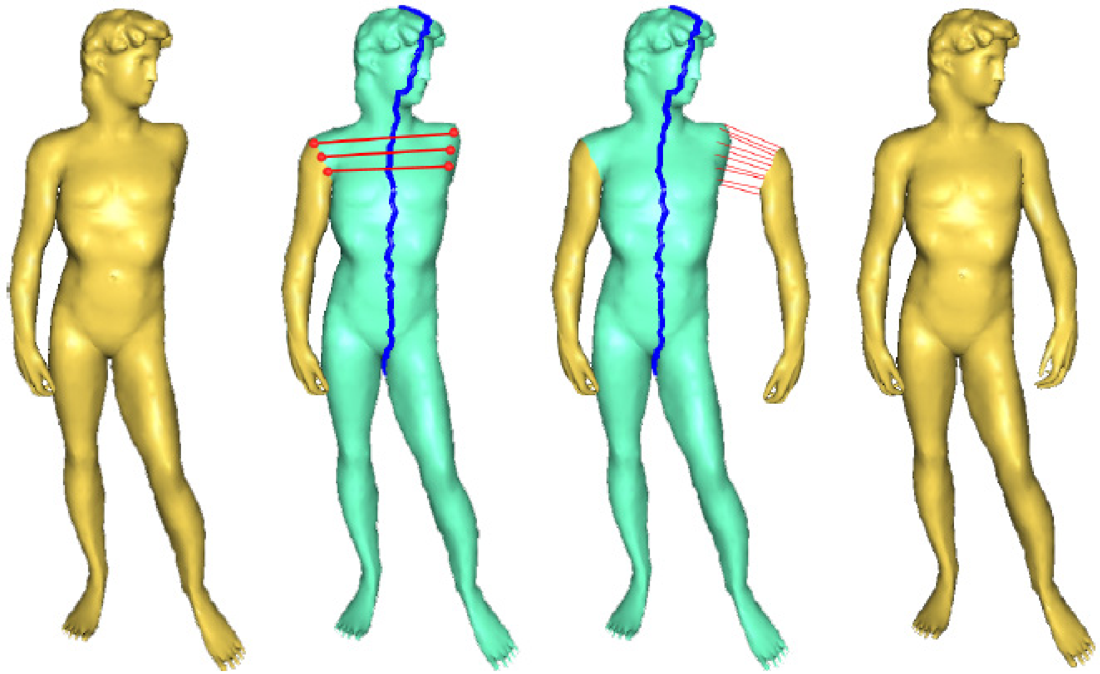
Application I: Symmetry-driven mesh segmentation

Conventional approaches:
Minima rule

Symmetry induced :
A part has maximal intrinsic ref-sym

Partial intrinsic symmetry detection

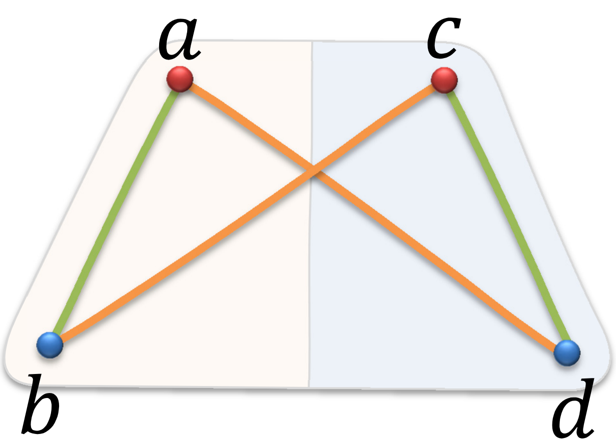
Application II: Symmetry-driven part repair



Intrinsic symmetry detection: Summary

In a nutshell:

- Partial intrinsic symmetry detection is hard
- Rely on the aggregation of approximate involute intrinsic
- Not assuming symmetry group? Hard



Necessary cond. → aggregation of voting

Applications

Applications

What to do with symmetry?

- Symmetry-guided X
  - Use of symmetry in geometry processing
- Structuring symmetries
  - High-level shape understanding
- Structure-aware shape editing

Symmetry-guided X

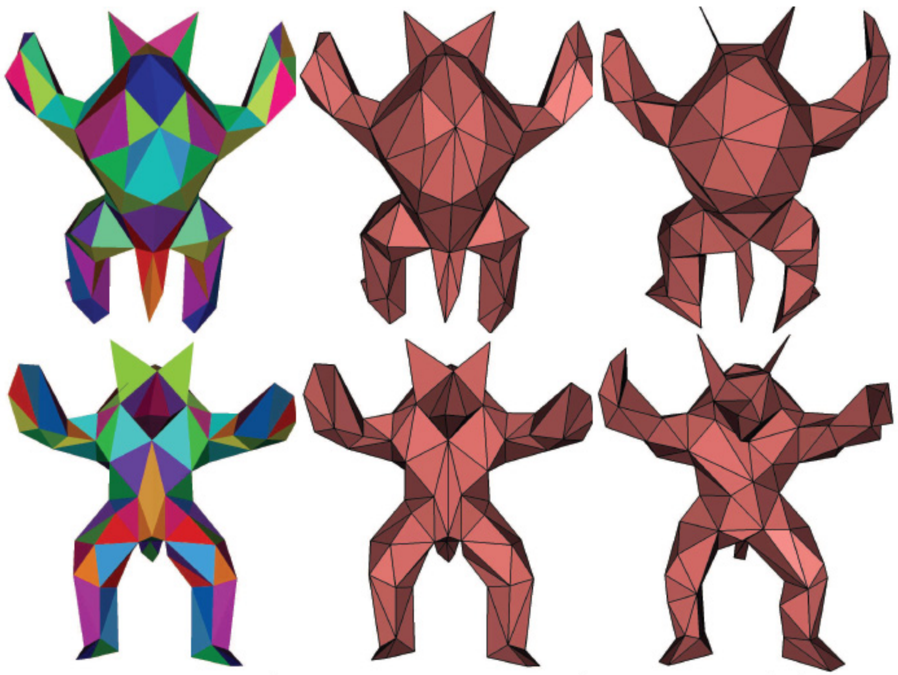
Symmetry guided mesh segmentation

[Podolak et al. 2006]

[Xu et al. 2009]

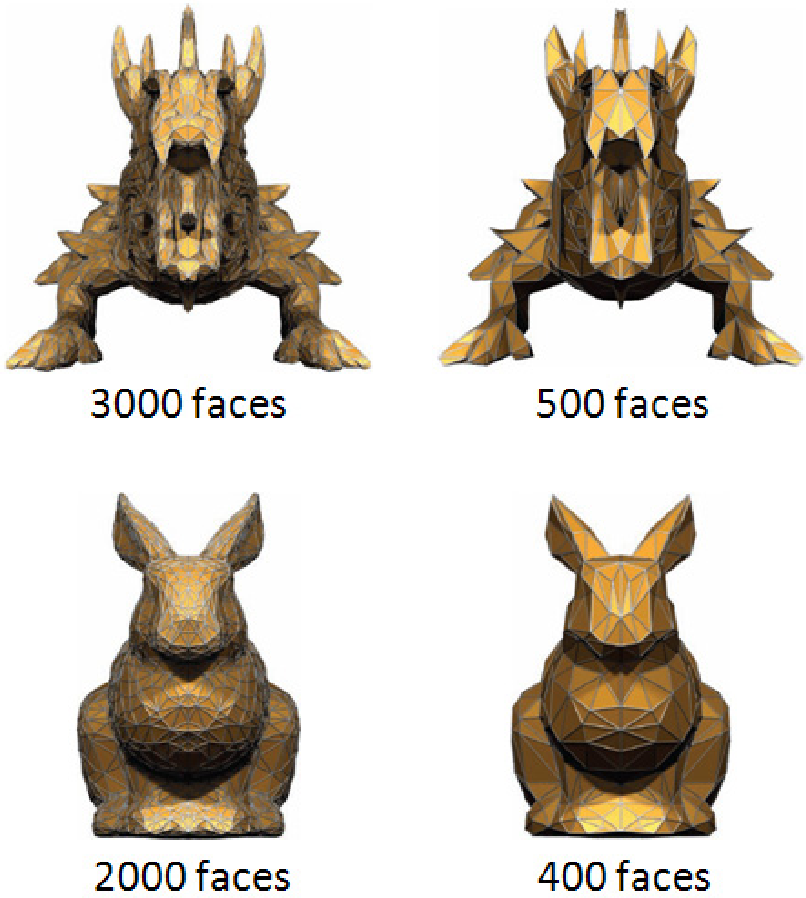
Symmetry-guided X

Symmetry guided mesh simplification



(a) Symmetry-preserving QSlim (face correspondences)   (a) Symmetry-preserving QSlim (edges)   (a) Original QSlim (edges)

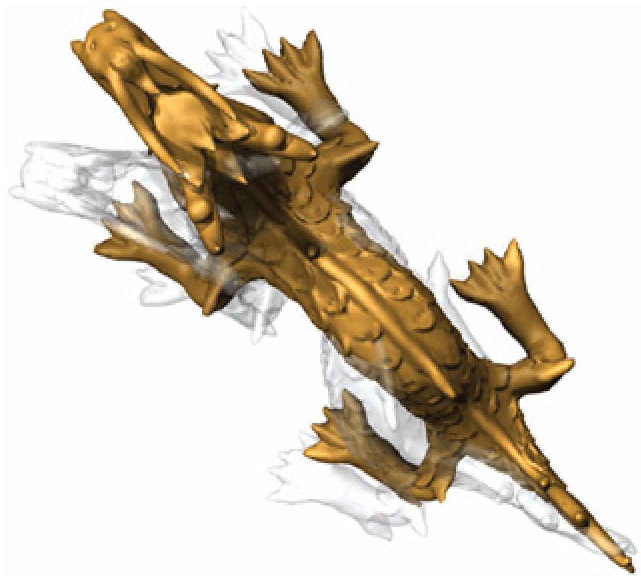
[Golovinskiy et al. 2009]



[Mitra et al. 2007]

Symmetry-guided X

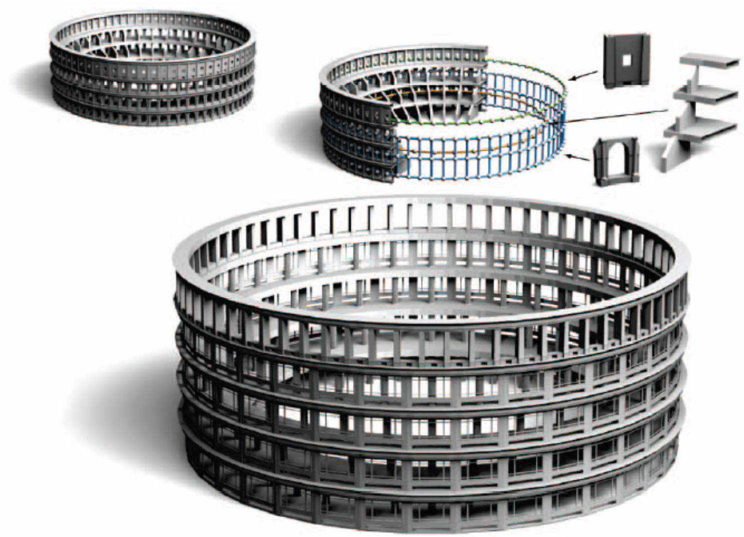
Symmetrization

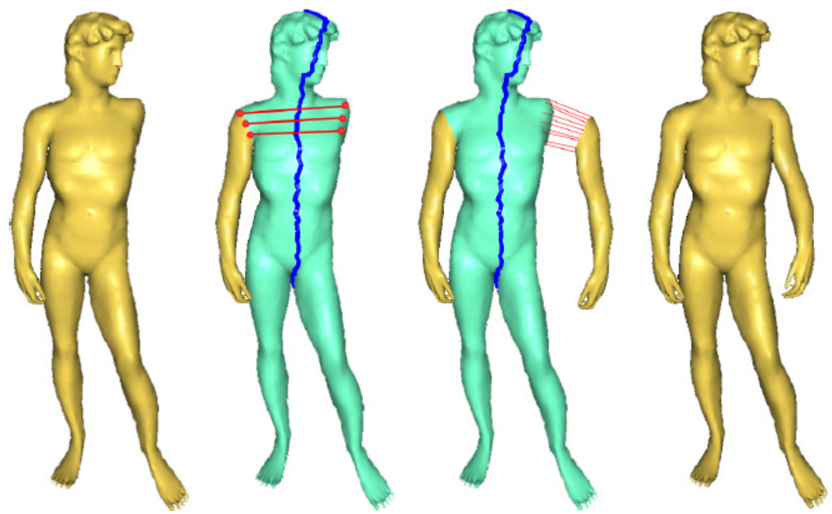
[Mitra et al. 2007]

Symmetry-guided X

Model completion/repair



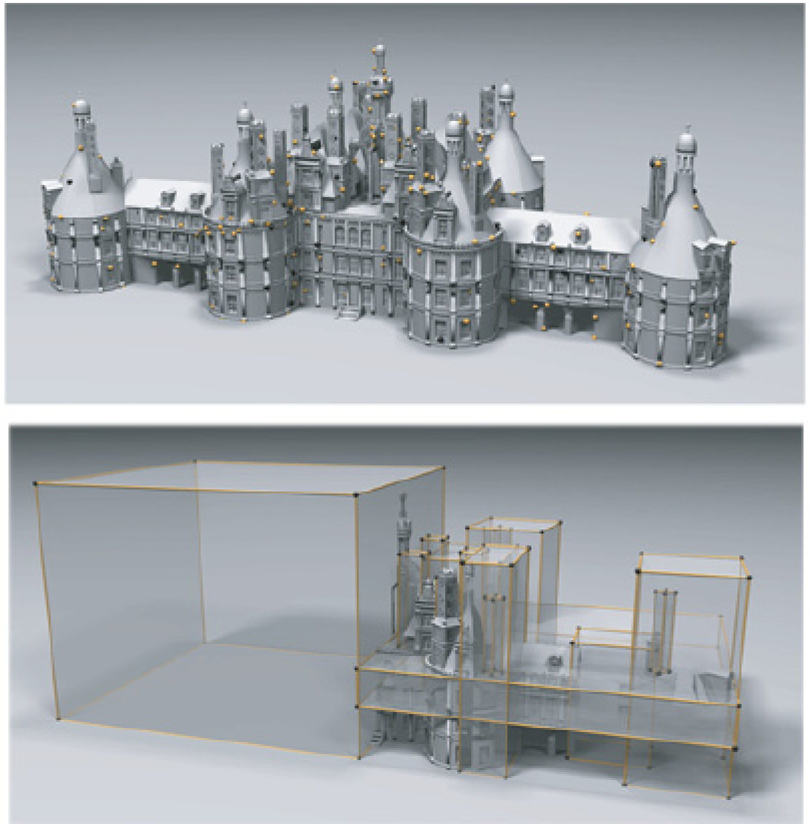
[Pauly et al. 2008]



[Xu et al. 2009]

Symmetry-guided X

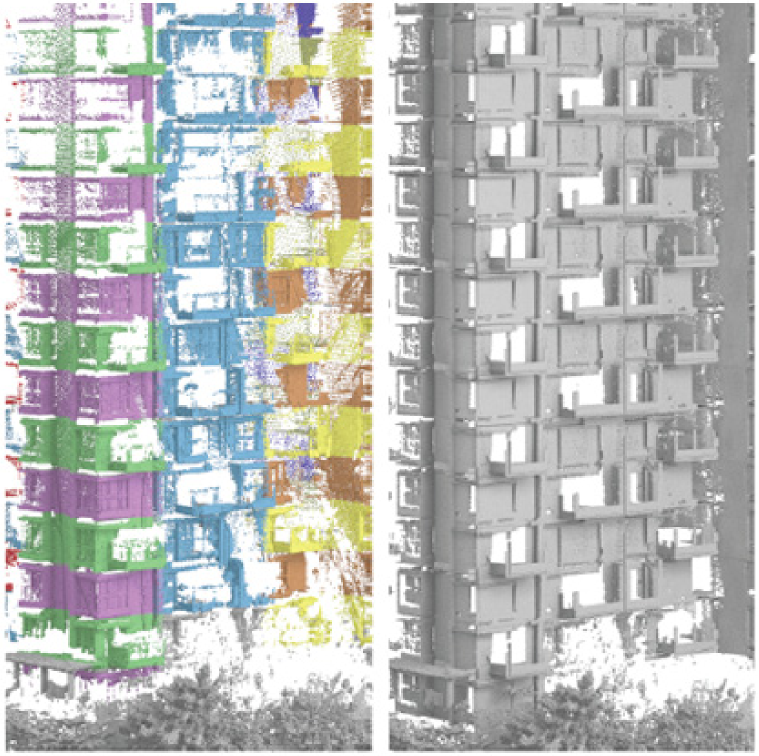
Model compression



[Mitra et al. 2006]

Symmetry-guided X

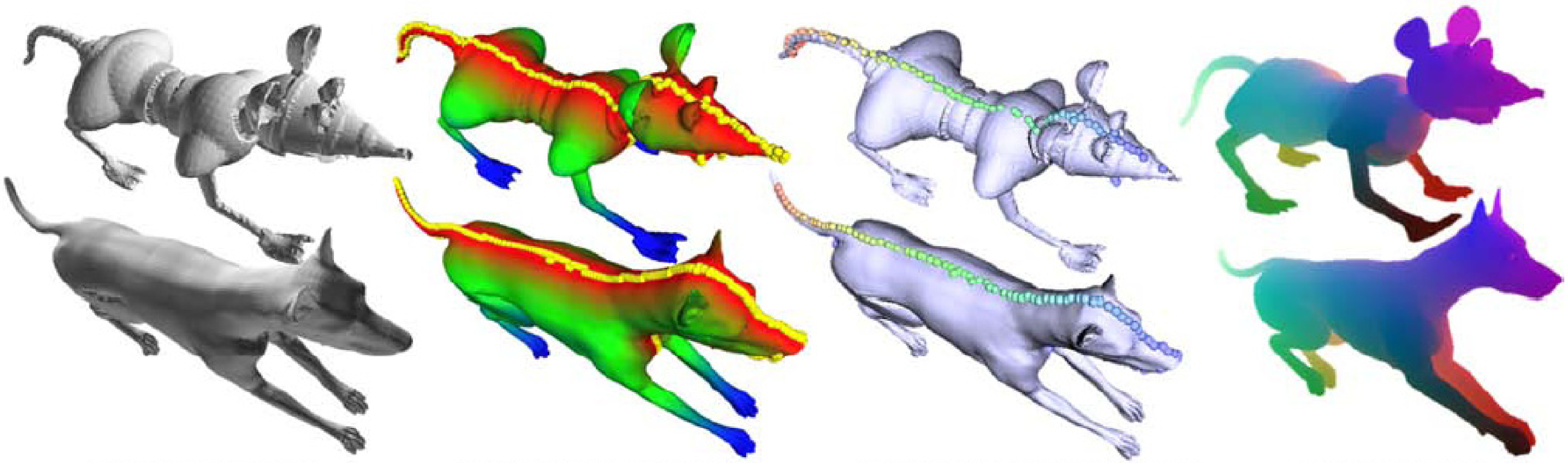
Scan consolidation



[Zheng et al. 2010]

Symmetry-guided X

Correspondence

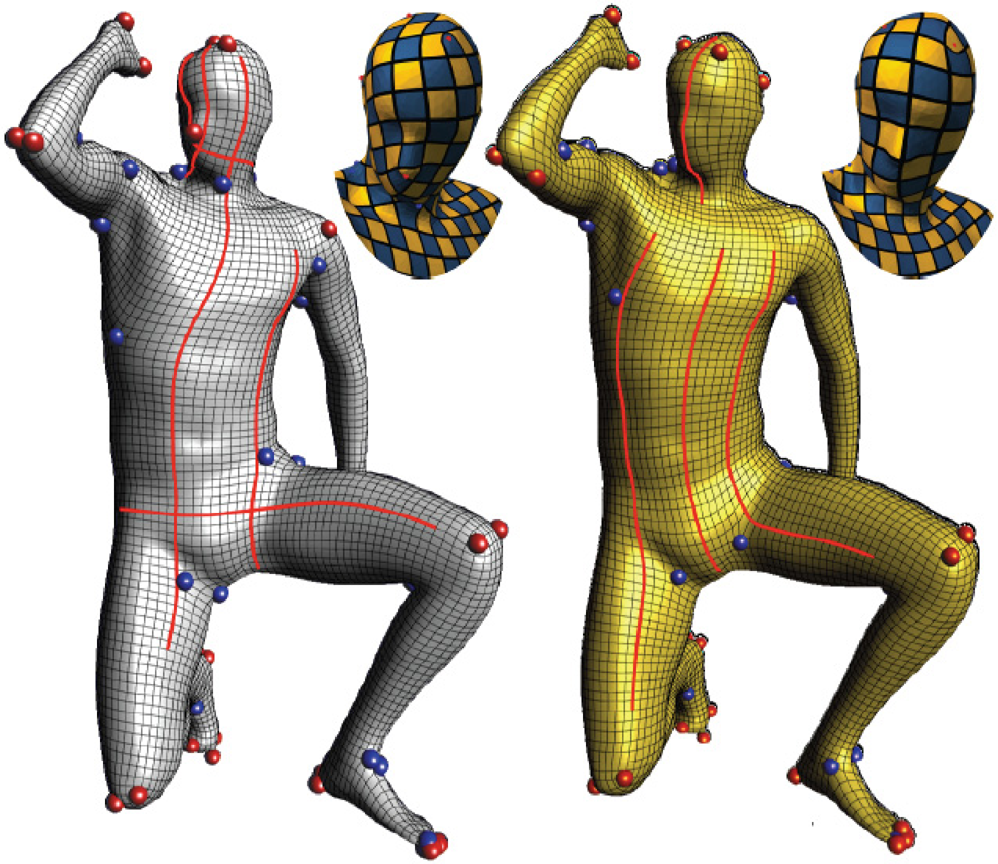


(a) Genus-0 meshes (b) Symmetry axis curves (c) Symmetry axis alignment (d) Surface map

Tianqiang Liu, Vladimir G. Kim, and Thomas Funkhouser. "**Finding Surface Correspondences Using Symmetry Axis Curves**," *SGP 2012*.

Symmetry-guided X

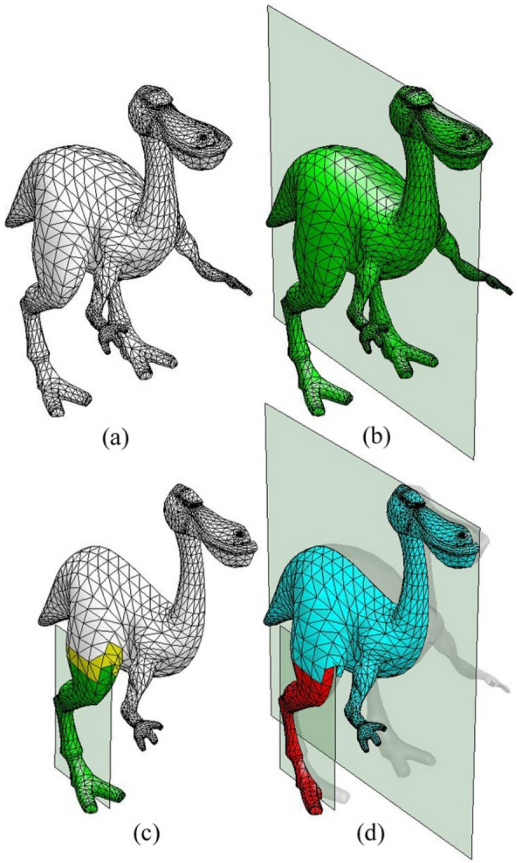
Remeshing

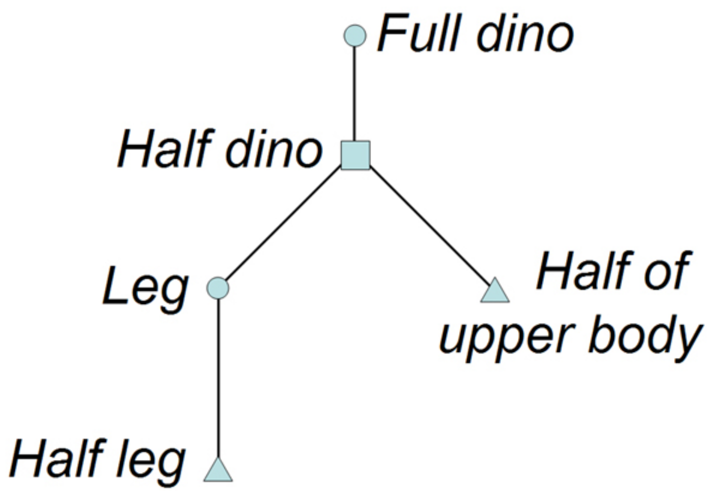


Daniele Panozzo, Yaron Lipman, Enrico Puppo, Denis Zorin. "**Fields On Symmetric Surfaces**," *SIGGRAPH 2012*.

Structuring symmetries

Folding mesh

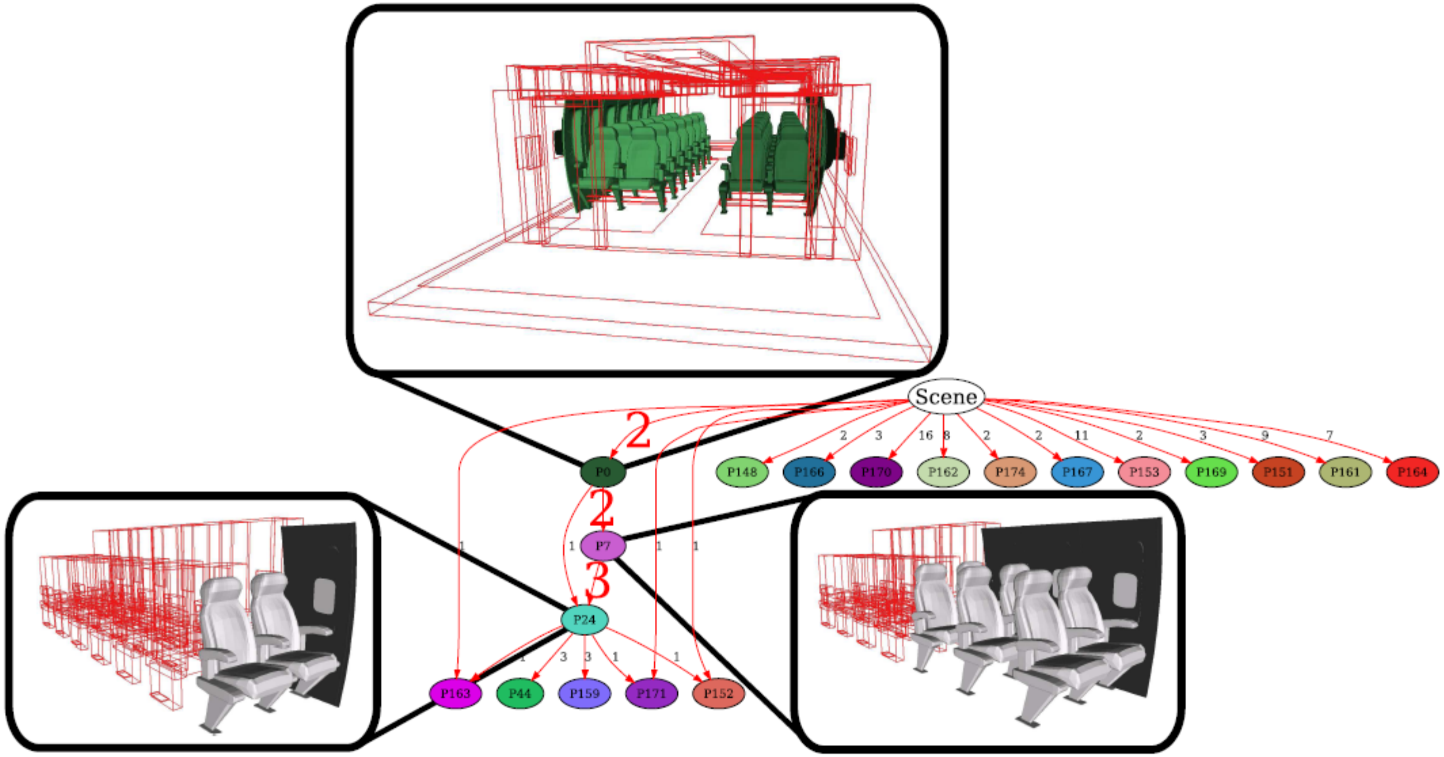




[Simari et al. 06]

Structuring symmetries

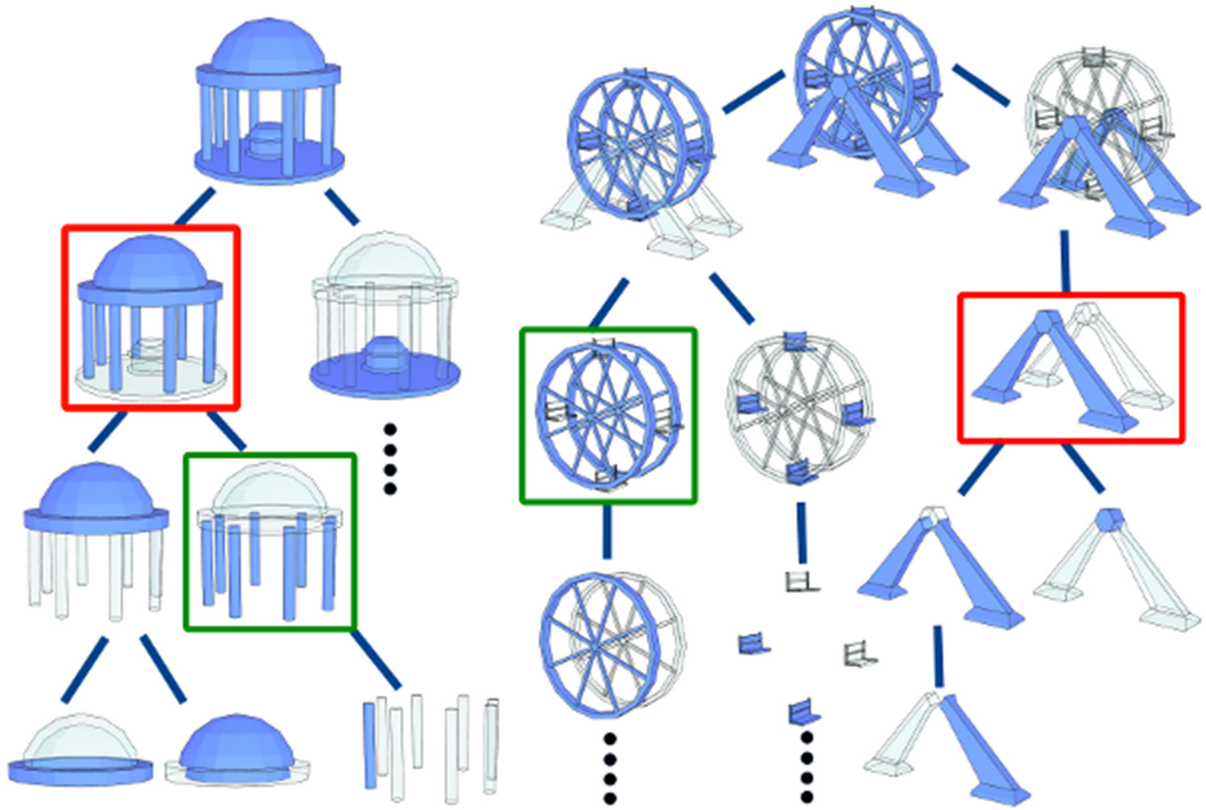
Structuring 3D Geometry based on Symmetry and Instancing Information



[Martinet 2007]

Structuring symmetries

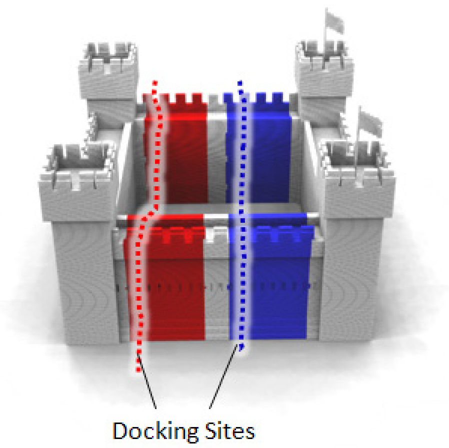
Symmetry Hierarchy of Man-Made Objects

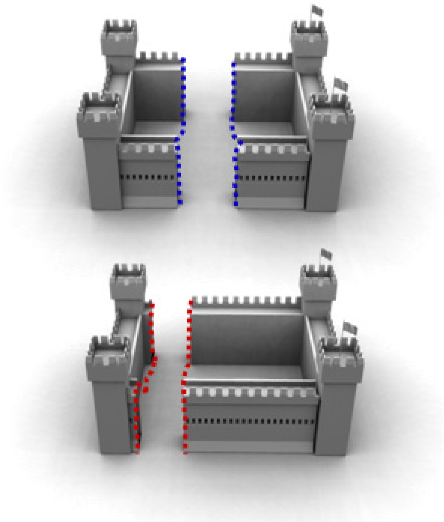


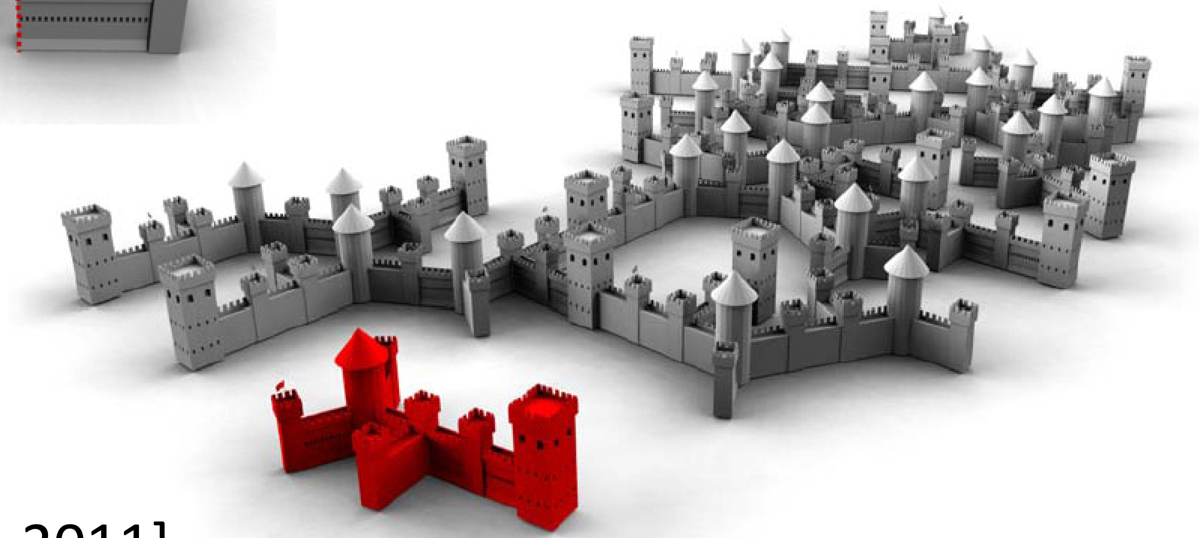
[Wang et al. 2011]

High-level shape understanding

Inverse procedural modeling



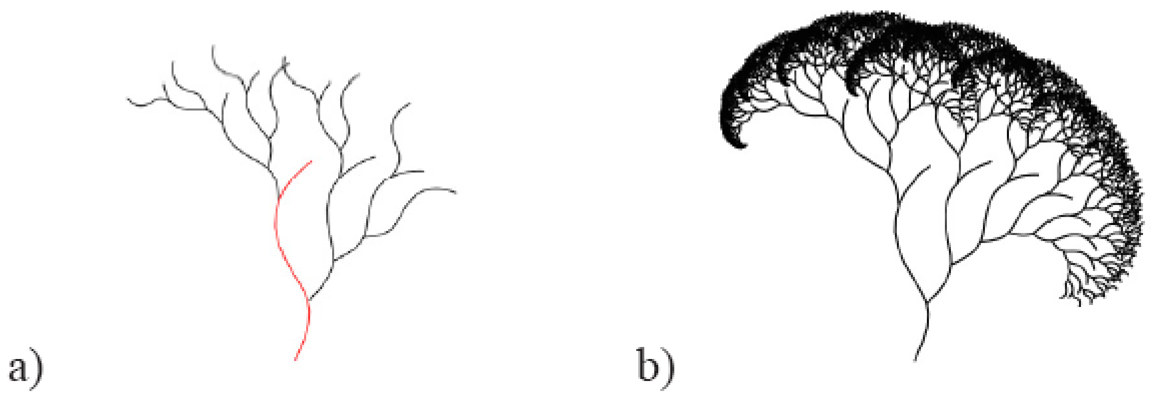


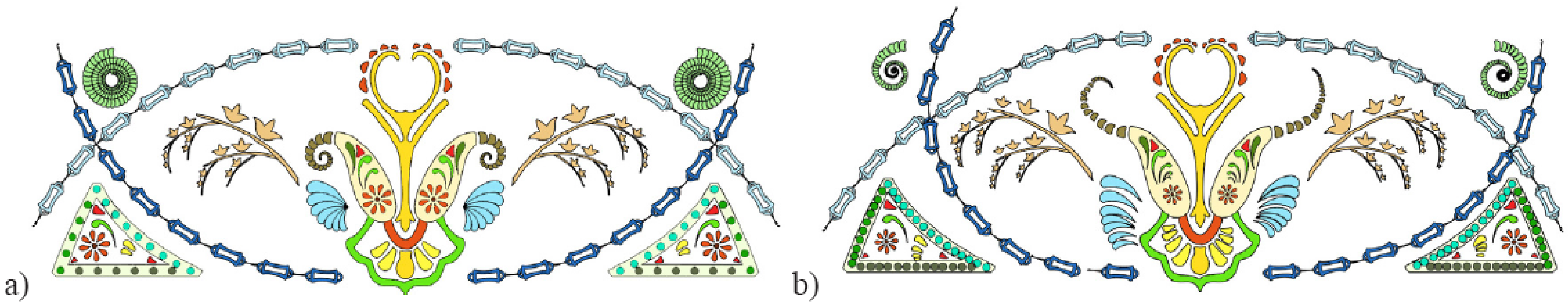


[Bokeloh et al. 2011]

High-level shape understanding

Inverse L-system

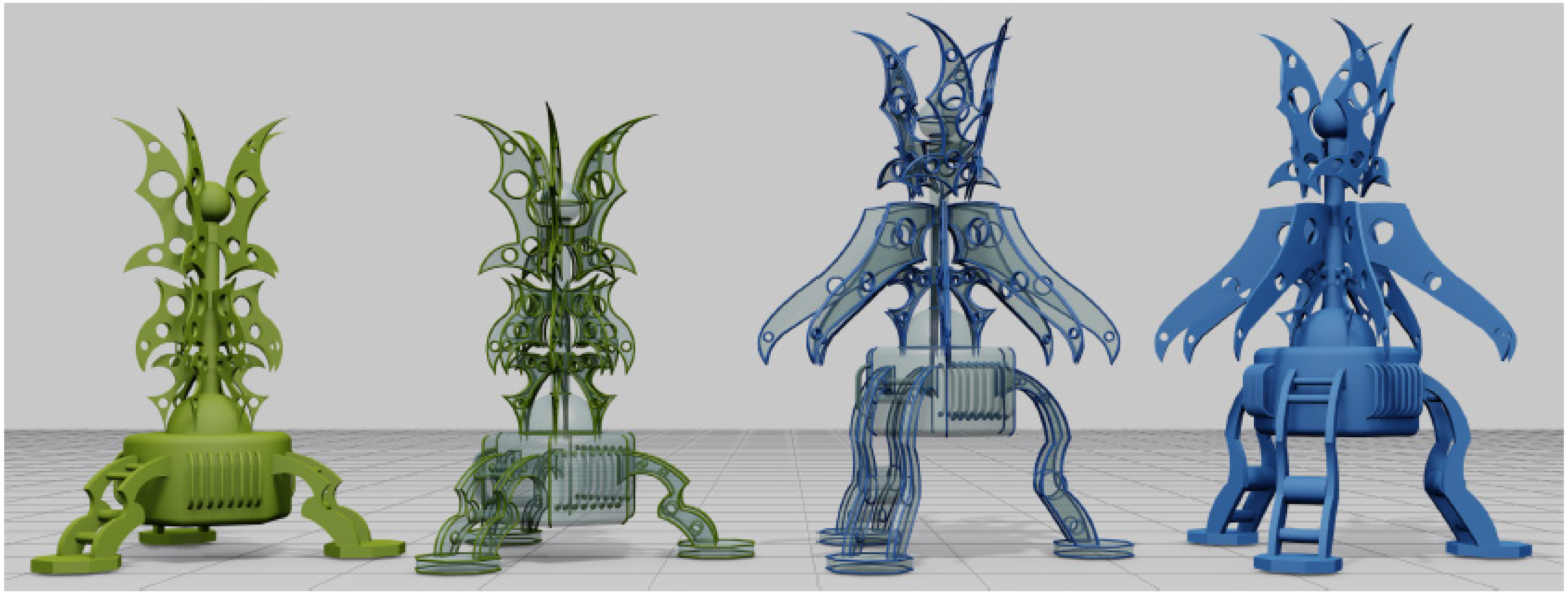




[Stava et al. 2010]

Structure-aware shape editing

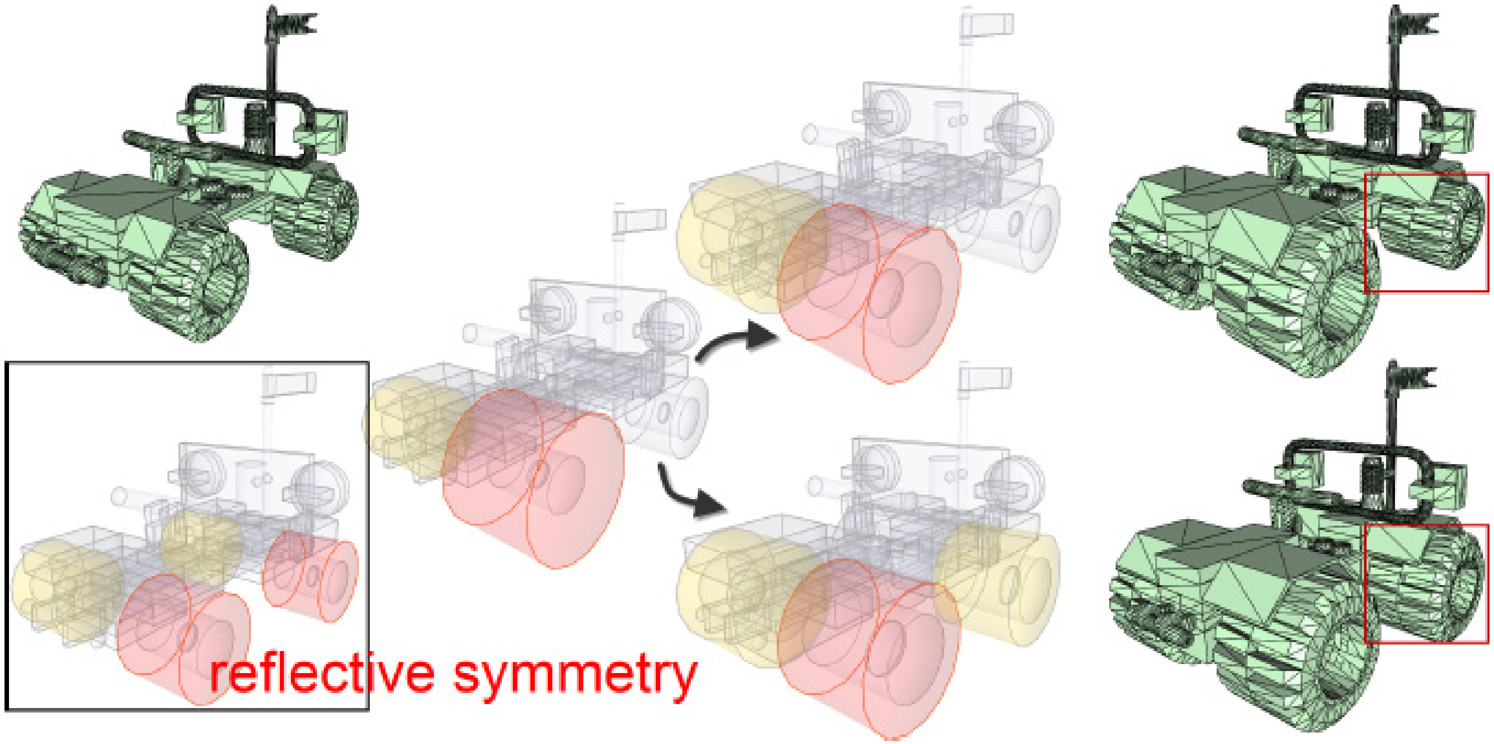
iWIRES



[Gal et al. 2009]

Structure-aware shape editing

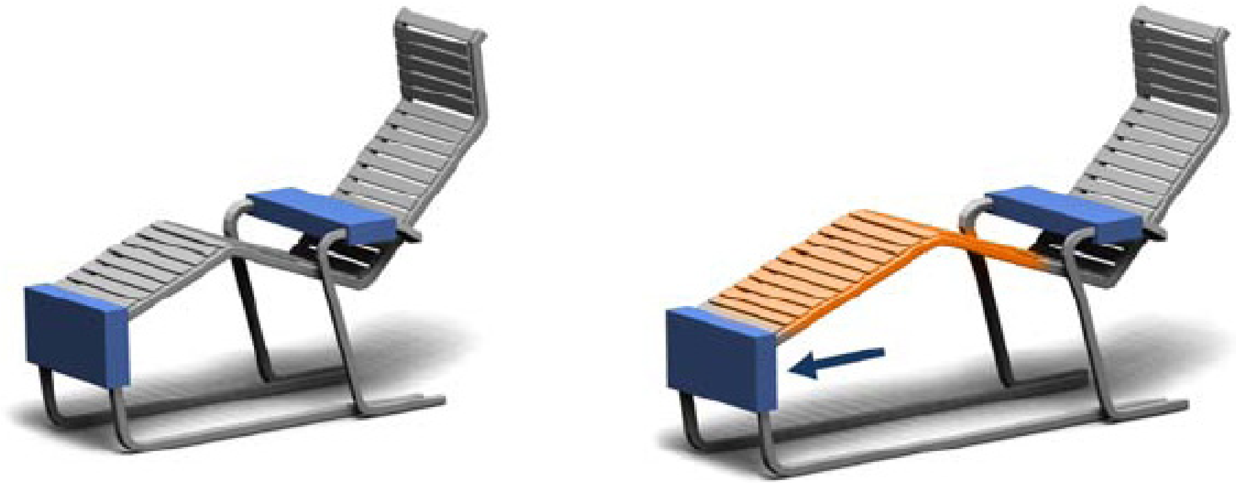
Component-wise controllers



[Zheng et al. 2010]

Structure-aware shape editing

Pattern-aware shape deformation

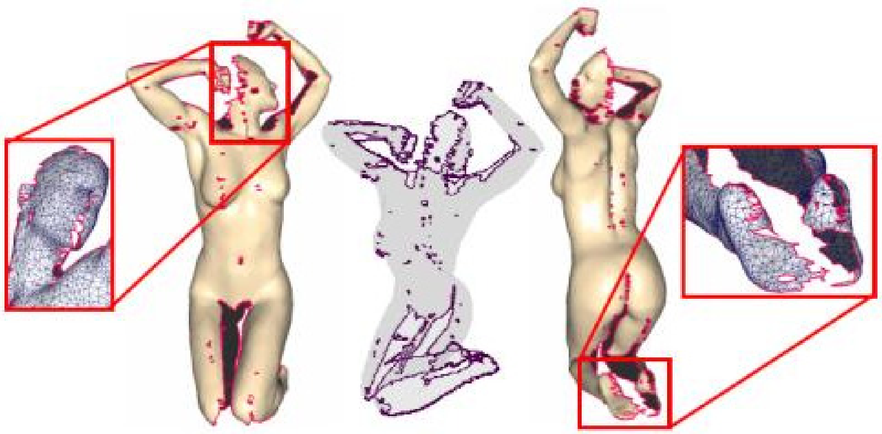


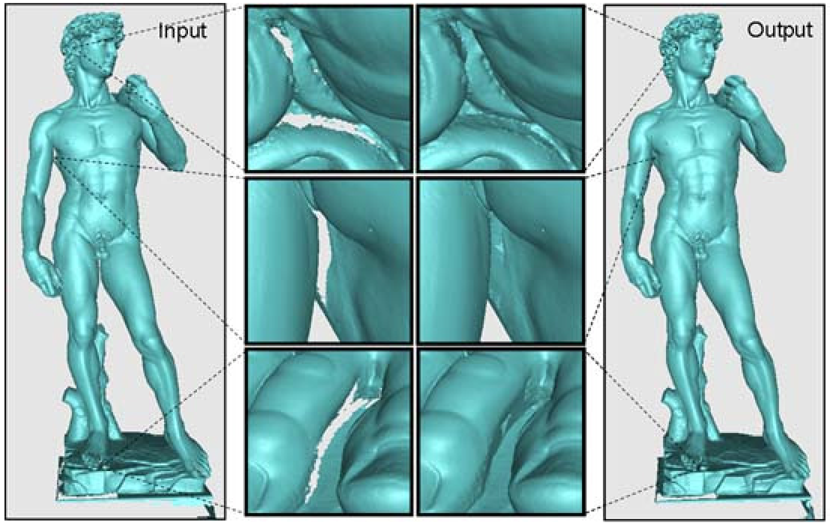
[Bokeloh et al. 2011]

Discussion

Future directions: Symmetry detection

- Handling of poor input data
  - Incomplete scan
  - Topological noise





Discussion

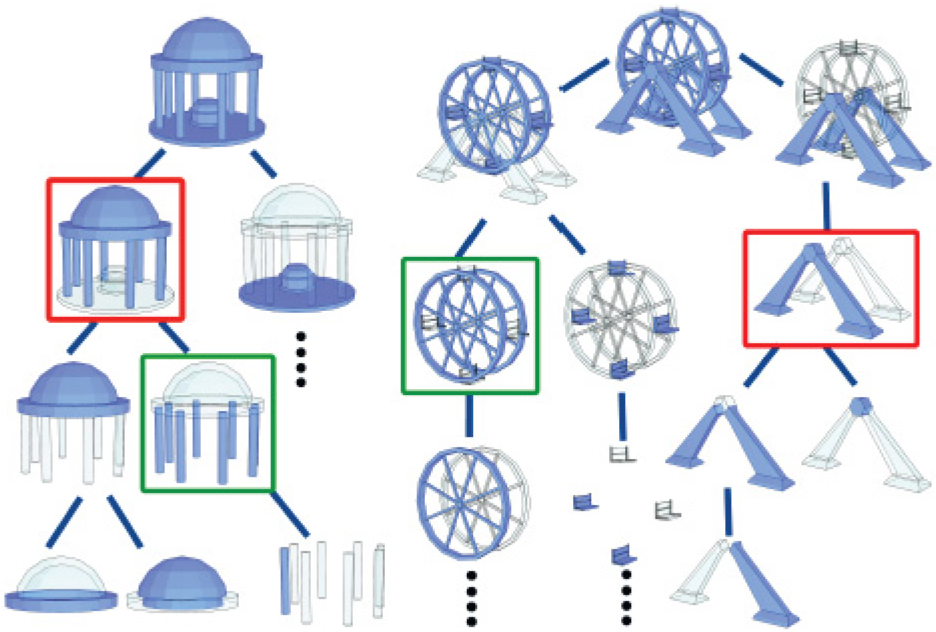
Future directions: Symmetry detection

- Handling of poor input data
  - Incomplete scan
  - Topological noise
- Uncertain symmetry detection
  - Statistical representation of potential symmetry
- Partial intrinsic symmetry
  - More efficient method
  - Break the symmetry group assumption

Discussion

Future directions: Symmetry for shape understanding and processing

- Structural/hierarchical shape representation
  - Supervised/unsupervised learning
  - Human perception: crowd-sourcing; user study



Discussion

Future directions: Symmetry for shape understanding and processing

- Symmetry related shape styles
  - Understanding *shape styles* → symmetry related?

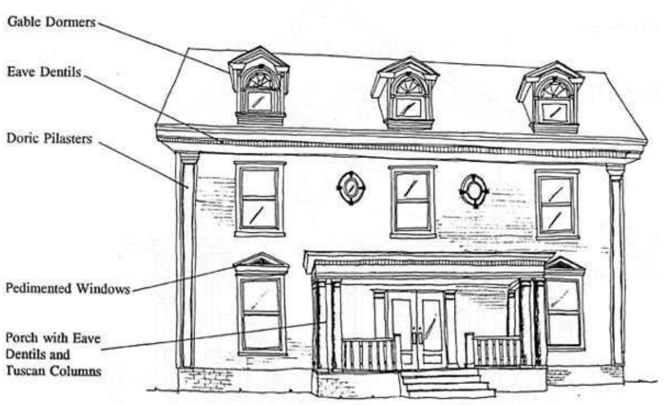
Style-content separation
[Xu et al. 2010]

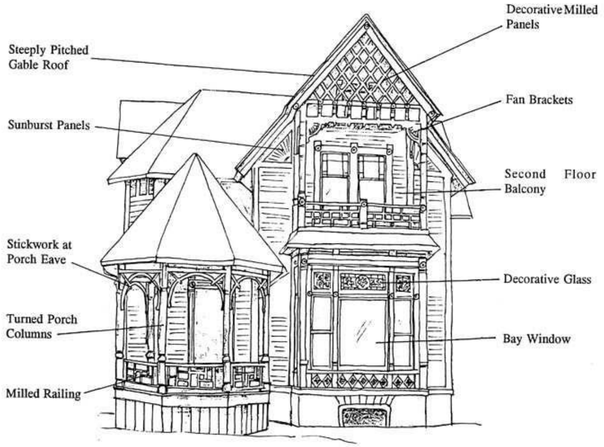
Curve style
[Li et al. 2012]

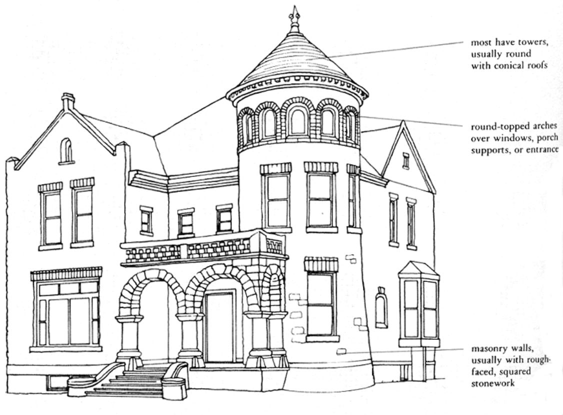
Discussion

Future directions: Symmetry for shape understanding and processing

- Symmetry related shape styles
  - Understanding *architecture styles* based on the rich symmetries in architectures



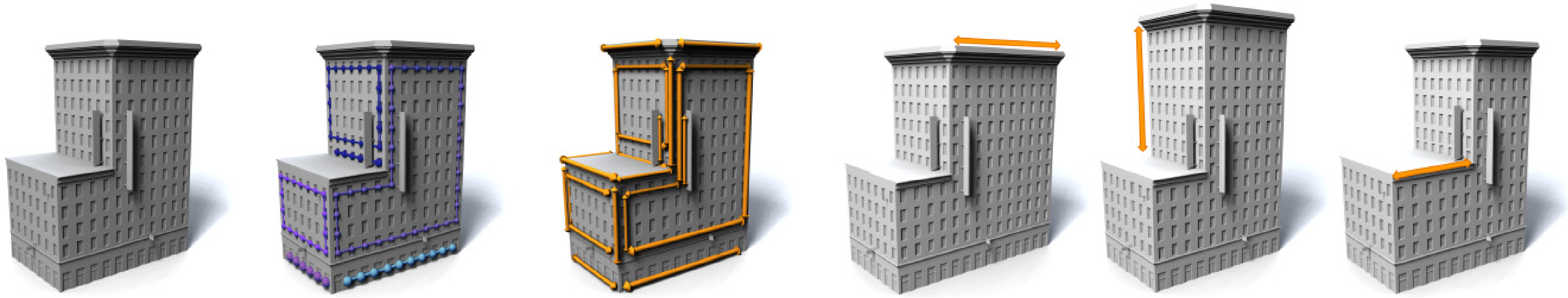




Discussion

Future directions: Symmetry for shape understanding and processing

- Structure-aware shape editing
  - So far symmetry is handled in a discrete manner
  - Integrate continuous symmetry measure into structure aware editing



Martin Bokeloh, Michael Wand, Hans-Peter Seidel, Vladlen Koltun. "**An Algebraic Model for Parameterized Shape Editing**," *SIGGRAPH 2012*.

THANK YOU

Q/A

Homepage: www.kevinkaixu.net