

CS 106B

Lecture 21: Graphs

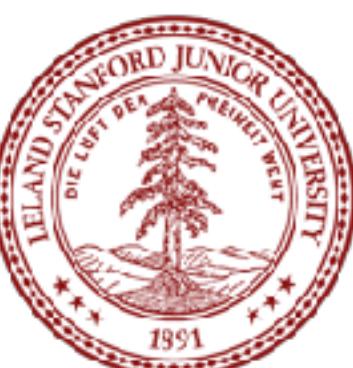
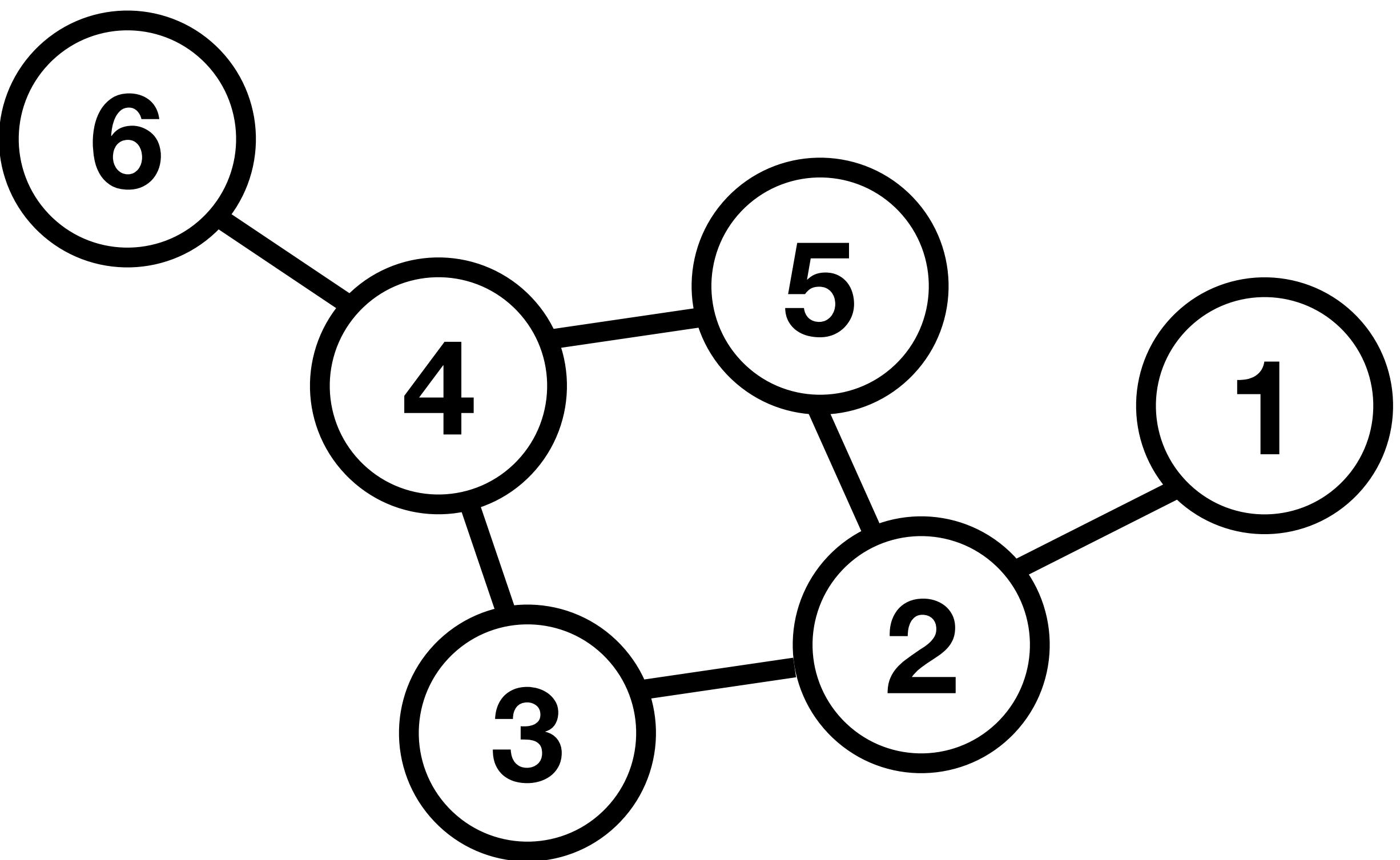
Friday, May 19, 2017

Programming Abstractions
Spring 2017
Stanford University
Computer Science Department

Lecturer: Chris Gregg

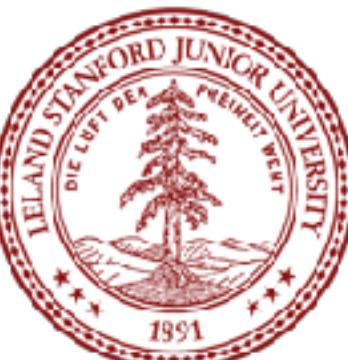
reading:

Programming Abstractions in C++, Chapter 18



Today's Topics

- Logistics
 - YEAH hours video: posted.
 - You must use your remaining late credits before the last assignment.
- Introduction to Graphs
 - The wild west of the node world
 - First Graph
 - Who is your lover?



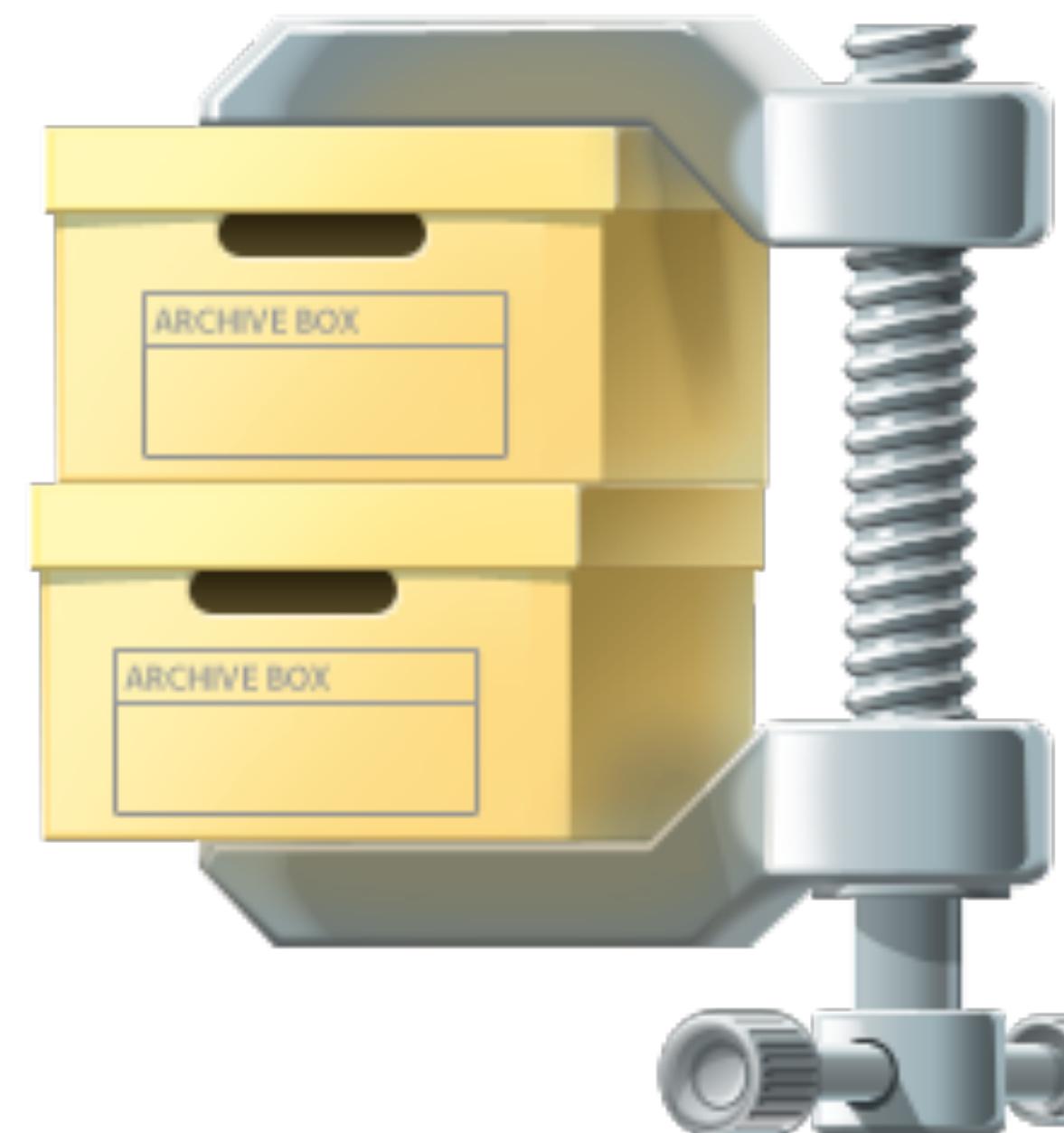
Assignment 6b: Huffman Encoding

Beautiful
mathematically

Used in everyday
life (both JPEG and
MP3)

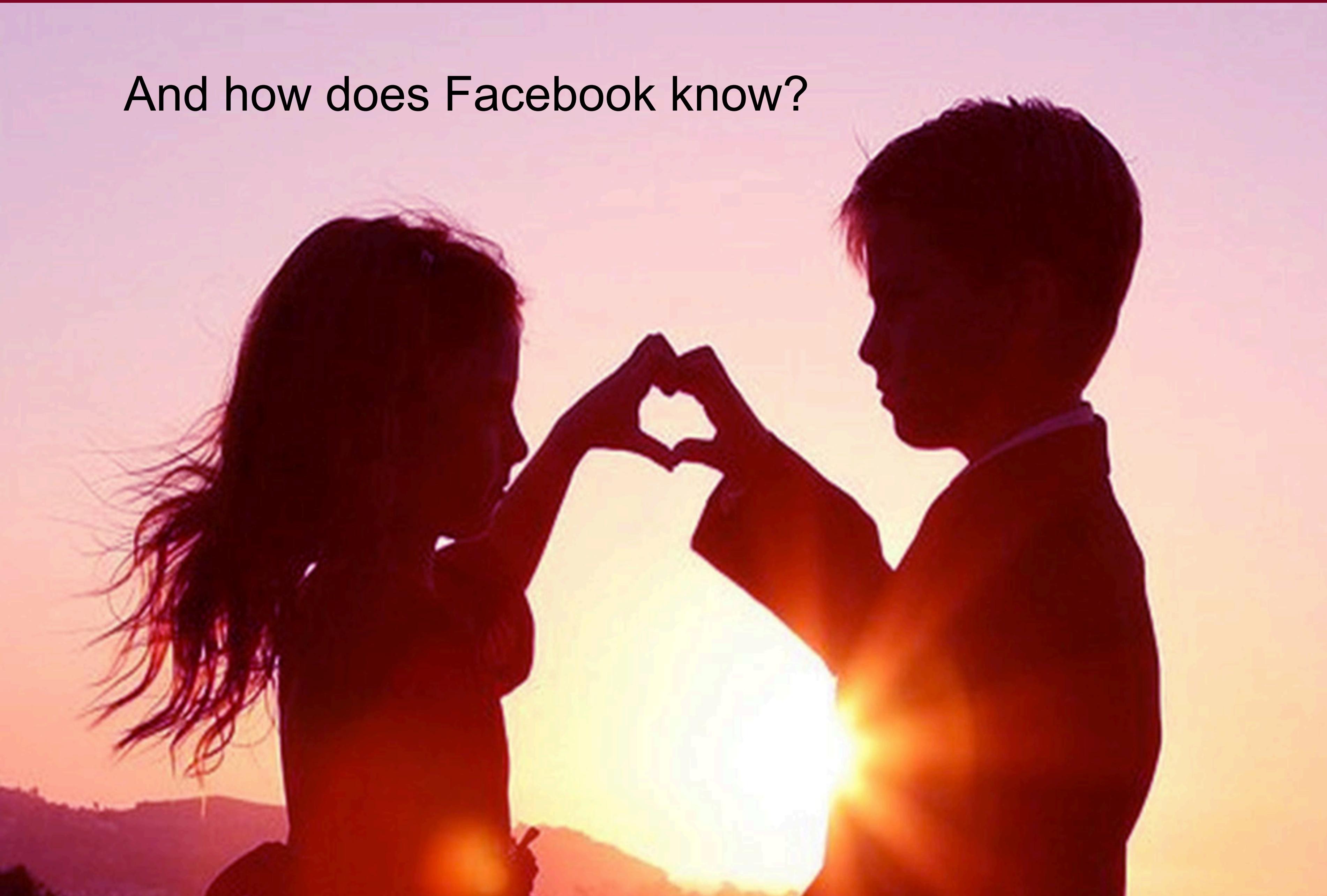
Sweet history

Great
practice with
trees



Intro to Graphs: Who do You Love?

And how does Facebook know?



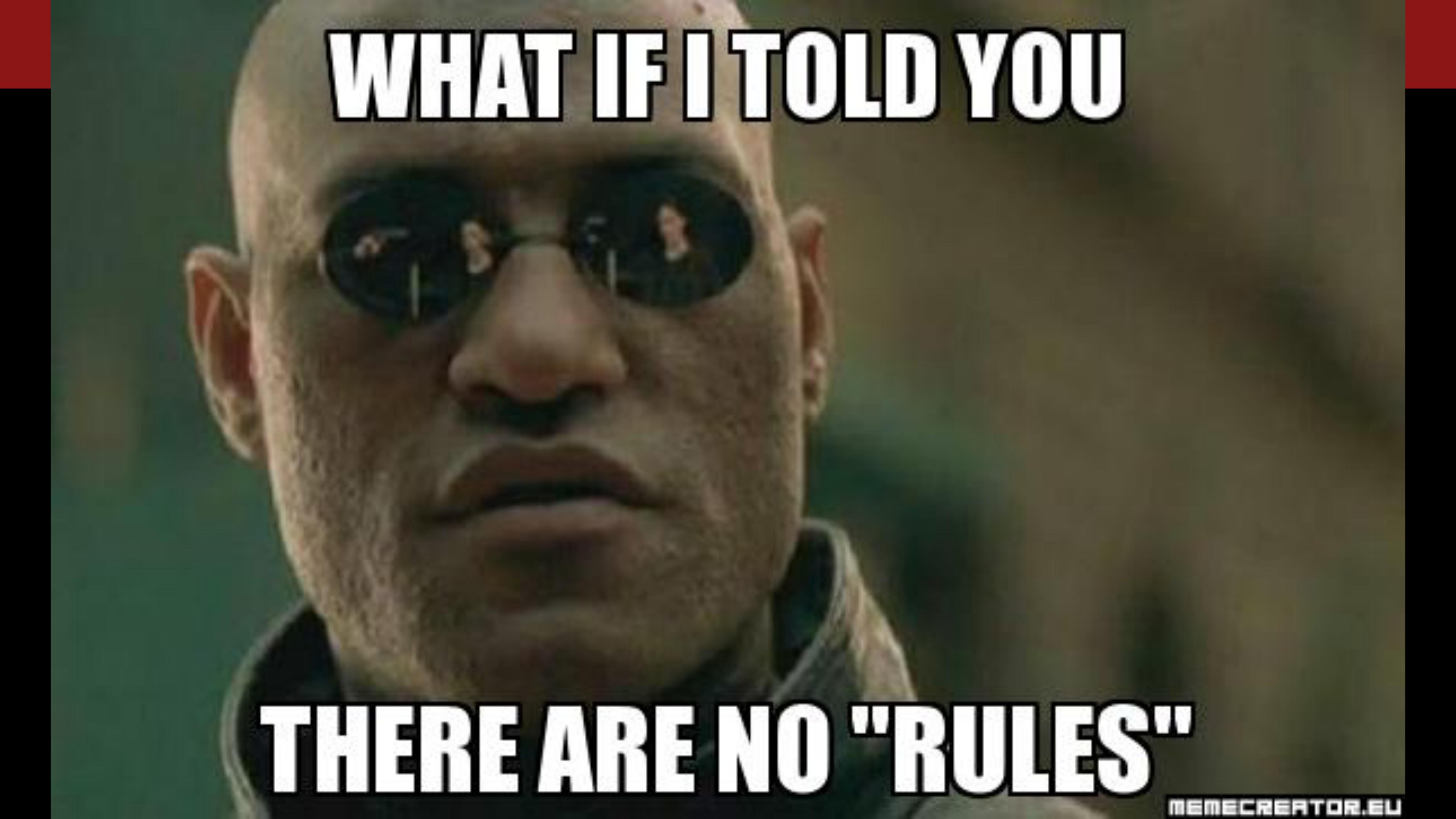
Tree Definition



Only One Parent



No Cycles



WHAT IF I TOLD YOU

THERE ARE NO "RULES"

Graph Definition

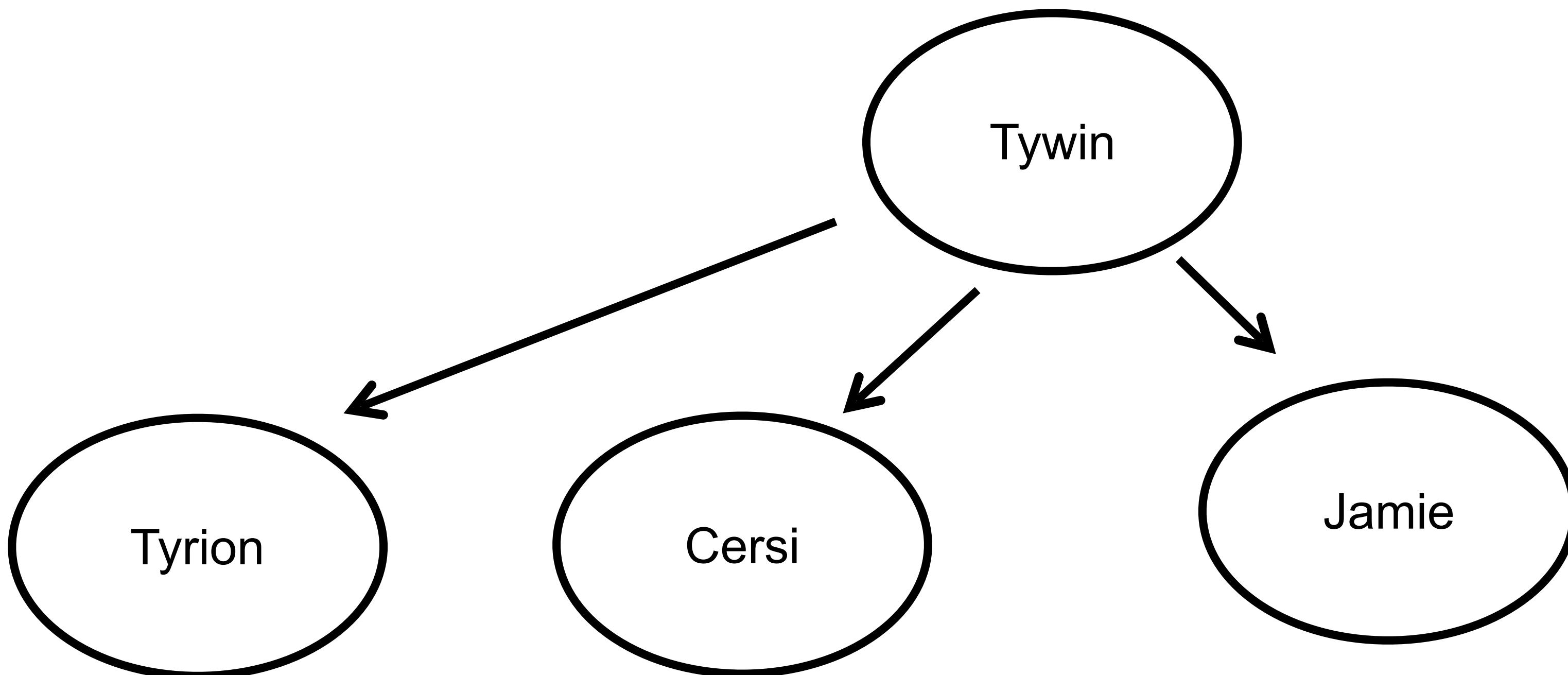
A **graph** is a mathematical structure for representing relationships using nodes and edges.

*Just like a tree without the rules

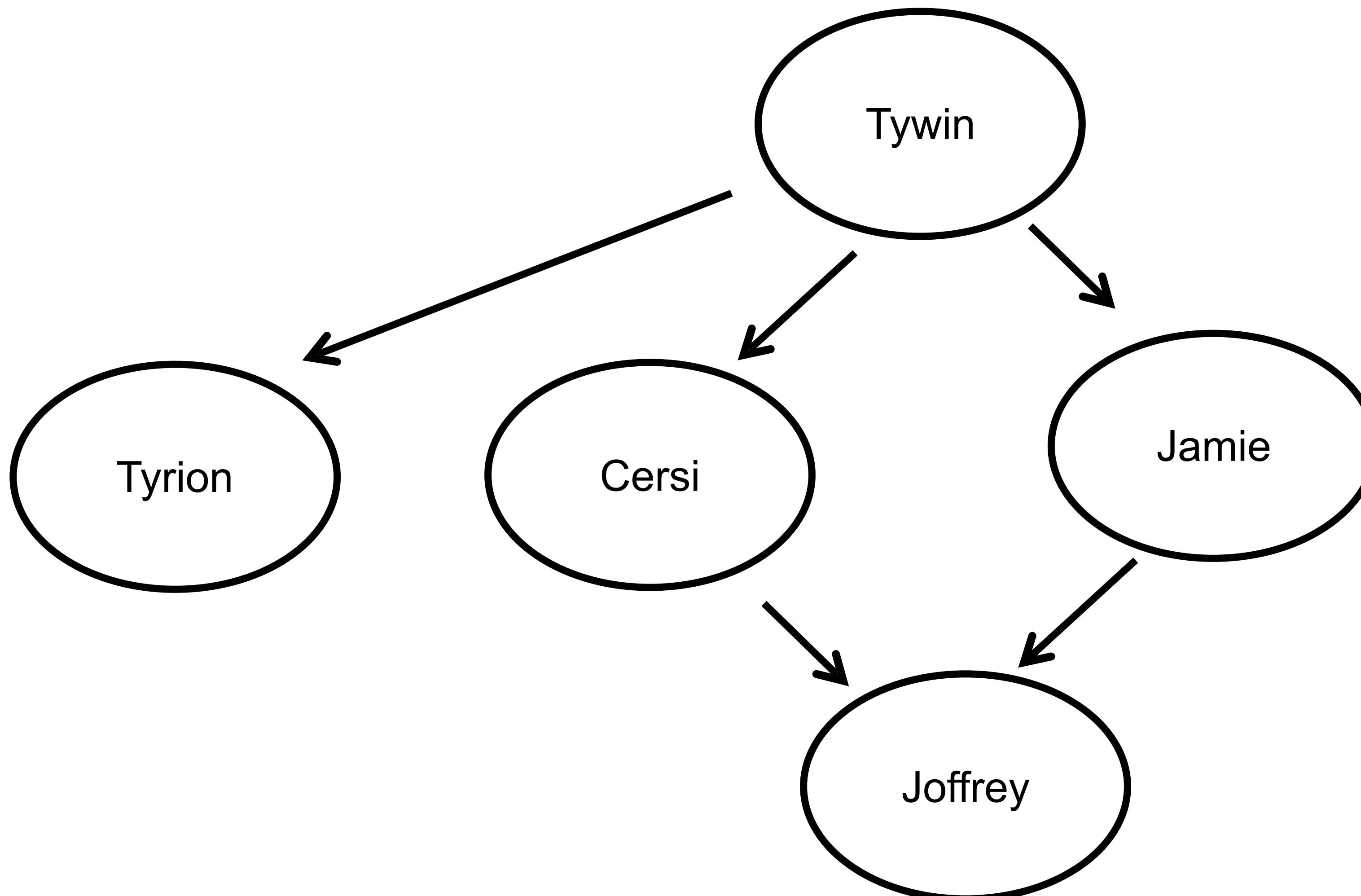
A photograph of a man and a woman looking at each other with a serious expression. The man has long hair and a beard, and the woman has blonde hair. They are standing in front of a wall with a painting of a tree.

We can have a family tree?

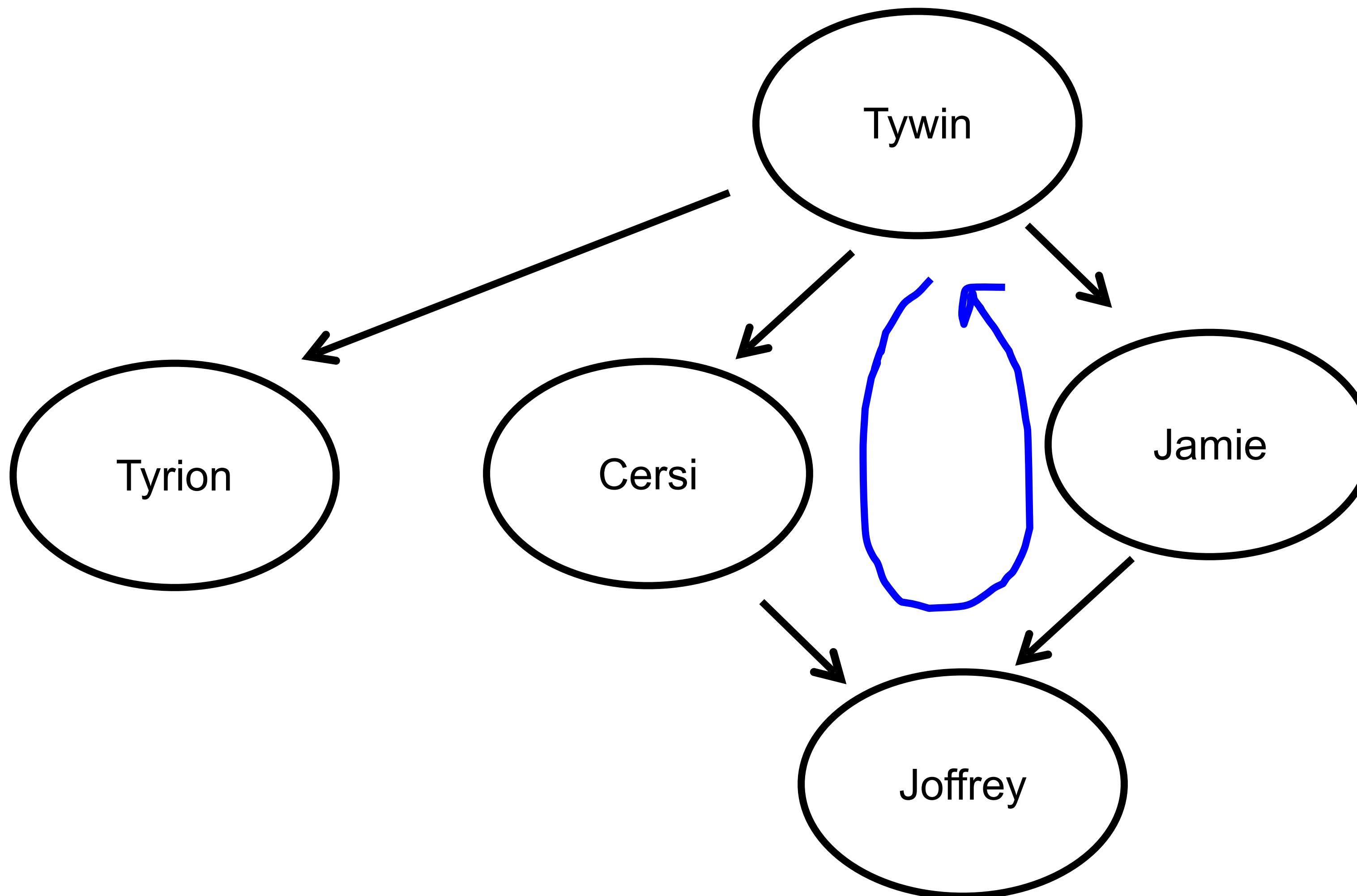
Family Tree



Not a Tree



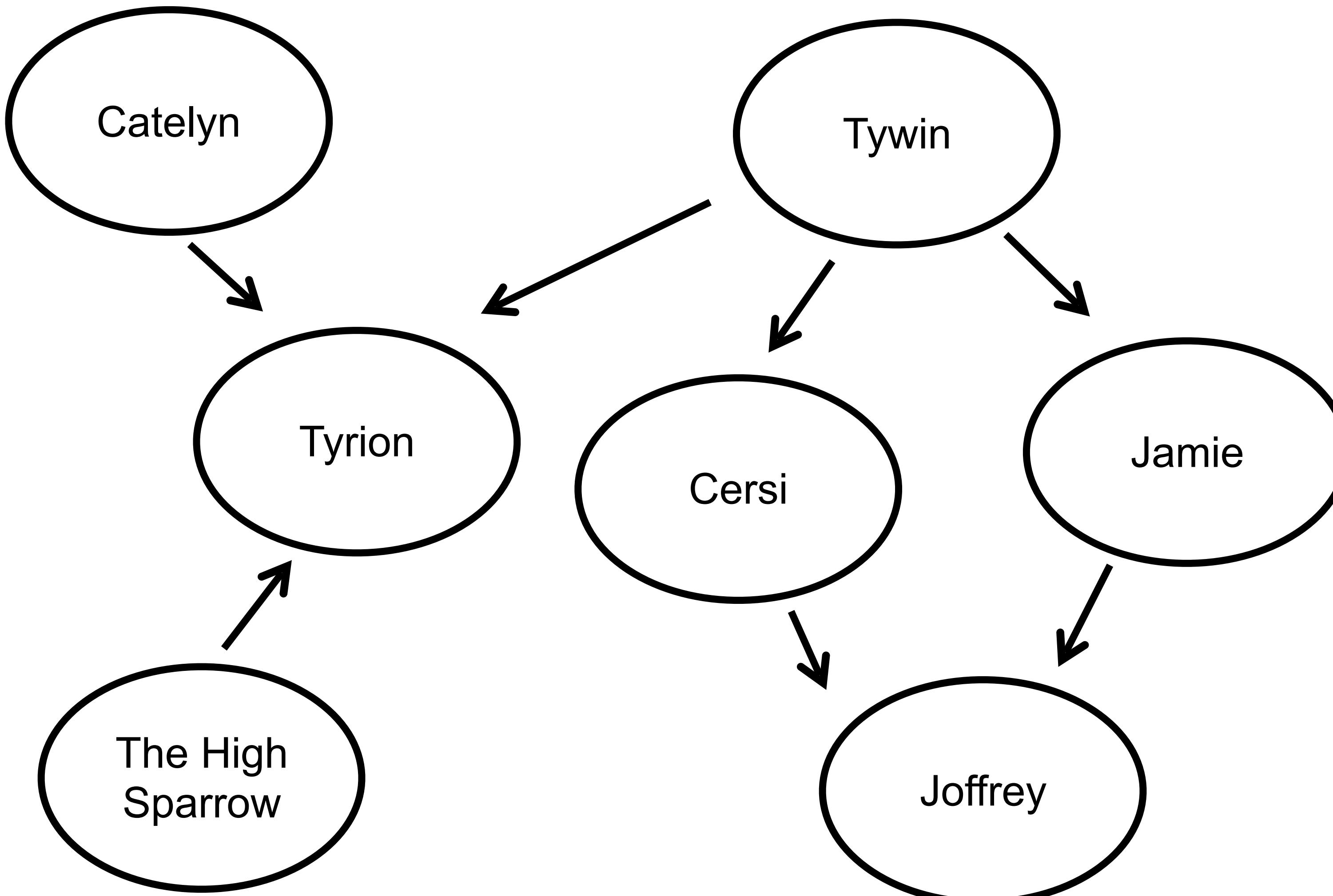
Not a Tree



A photograph of a man and a woman looking at each other. A white rectangular box with a black border is positioned in the lower right area of the image, containing the text.

We can have a family tree?
graph

Graphs Don't Have Roots



Simple Graph

```
struct Node{
    string value;
    Vector<Edge *> edges;
};

struct Edge{
    Node * start;
    Node * end;
};

struct Graph{
    Set<Node *> nodes;
    Set<Edge*> edges;
};
```

Simple Graph

```
struct Node{  
    string value;  
    vector<Edge *> edges;  
};
```

```
struct Edge{  
    Node * start;  
    Node * end;  
};
```

We allow for
more interesting
edges

```
struct Graph{  
    set<Node *> nodes;  
    set<Edge*> edges;  
};
```

Simple Graph

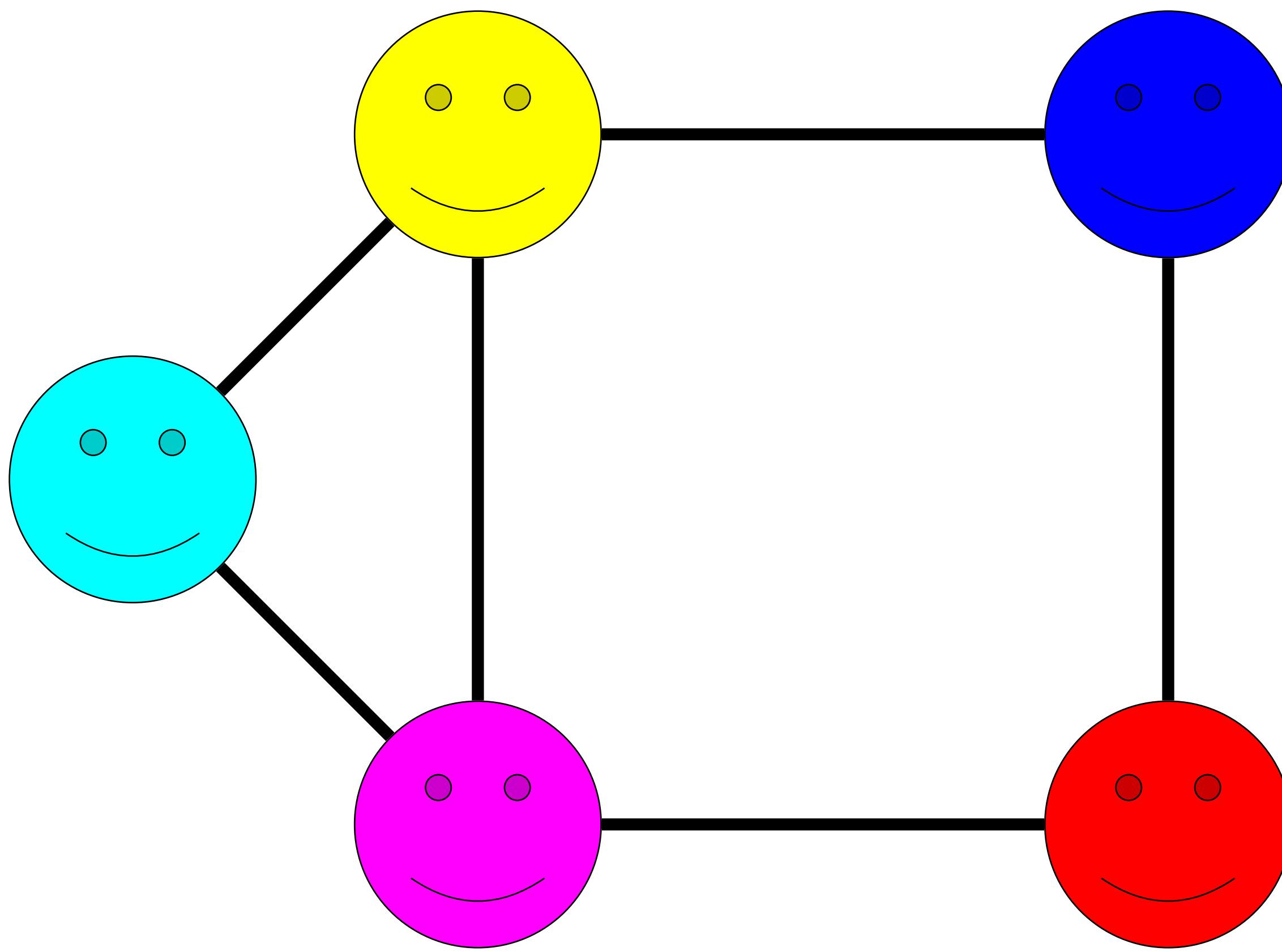
```
struct Node{  
    string value;  
    Vector<Edge *> edges;  
};
```

```
struct Edge {  
    Node * start;  
    Node * end;  
    double weight;  
};
```

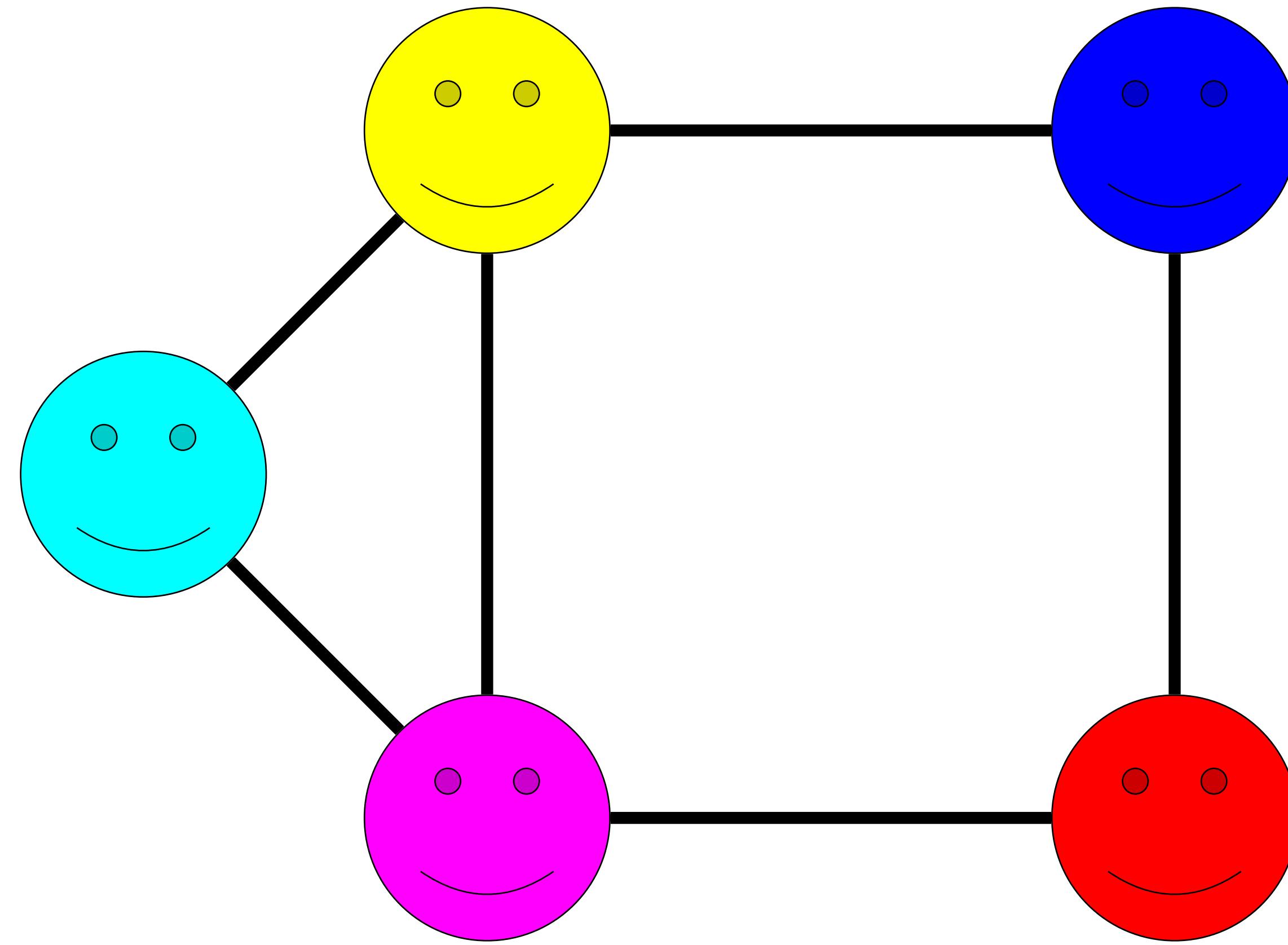
We allow for
more interesting
edges

```
struct Graph{  
    Set<Node *> nodes;  
    Set<Edge*> edges;
```

Simple Graph

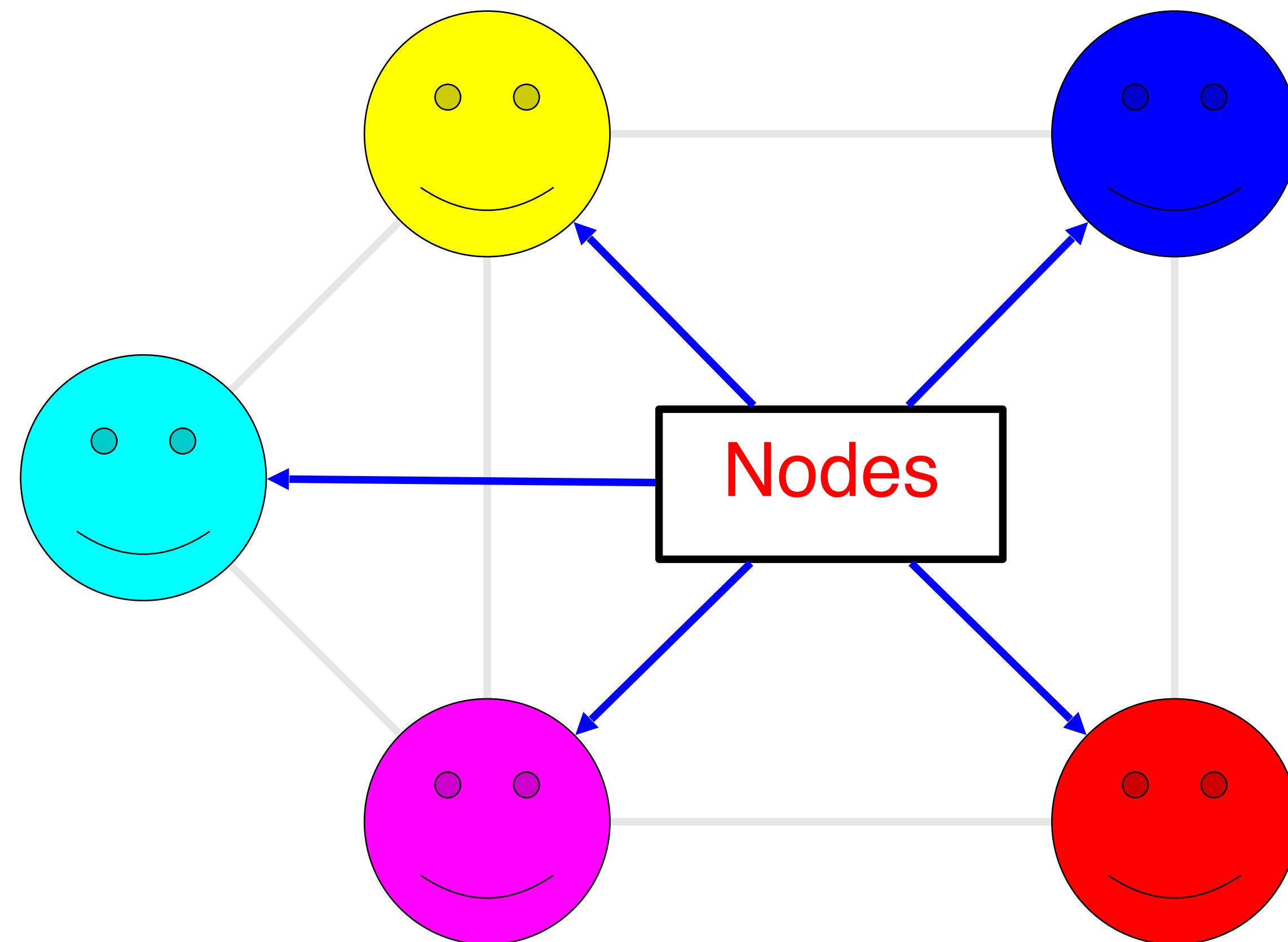


Simple Graph



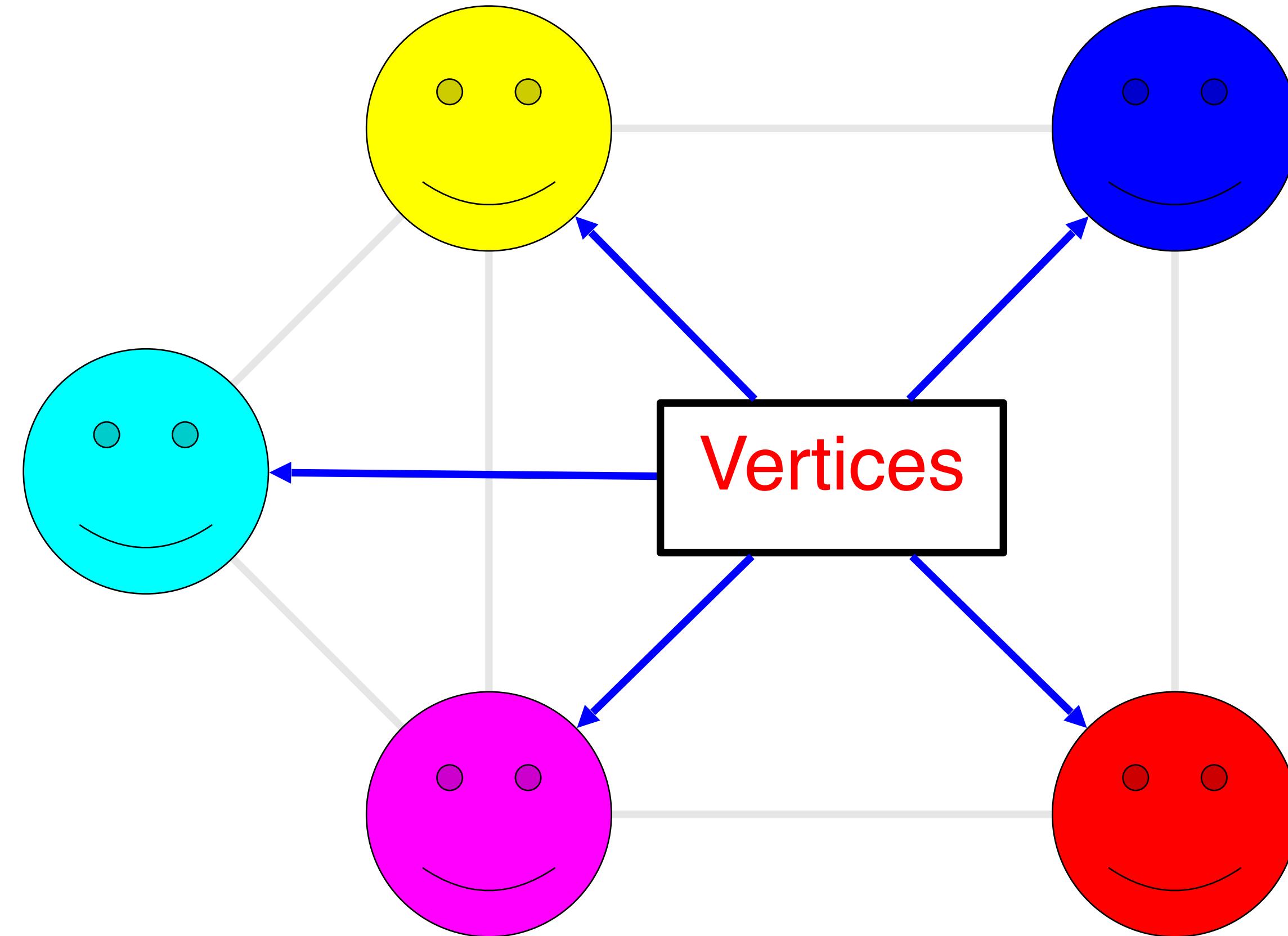
A graph consists of a set of **nodes** connected by
edges.

Graph Nodes



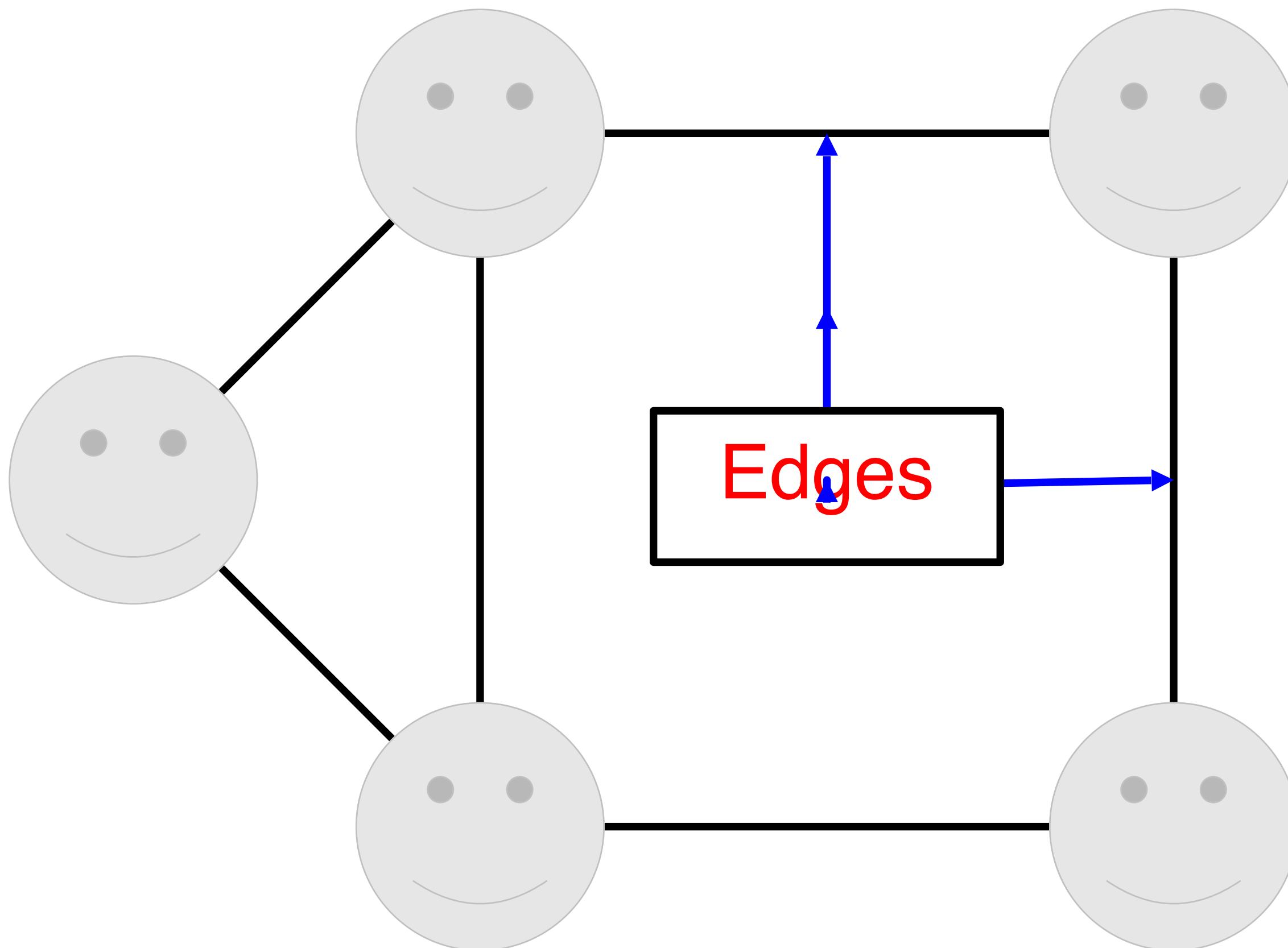
A graph consists of a set of **nodes** connected by **edges**.

Nodes are Also Called Vertices



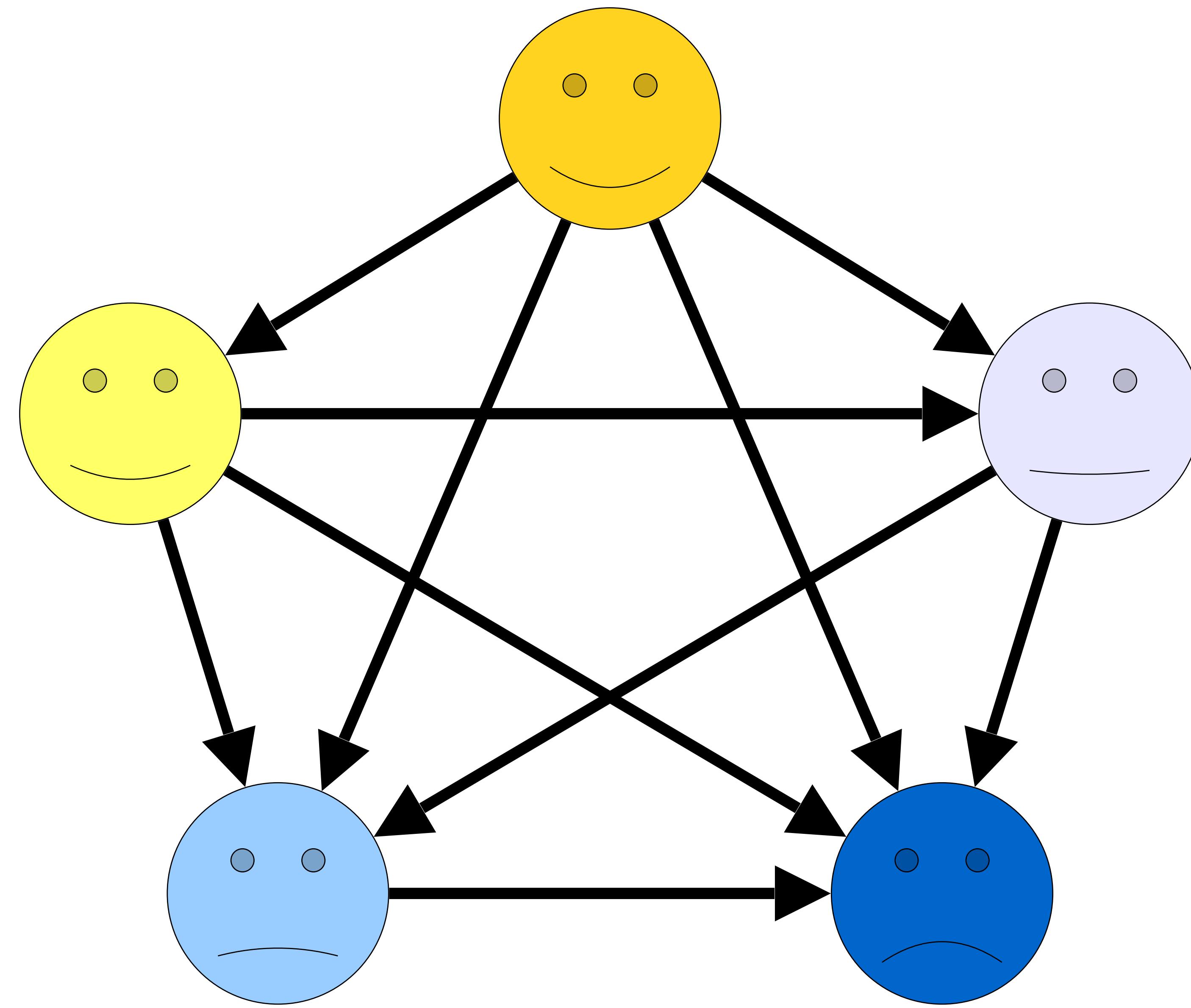
A graph consists of a set of **nodes** connected by
edges.

Graph Edges

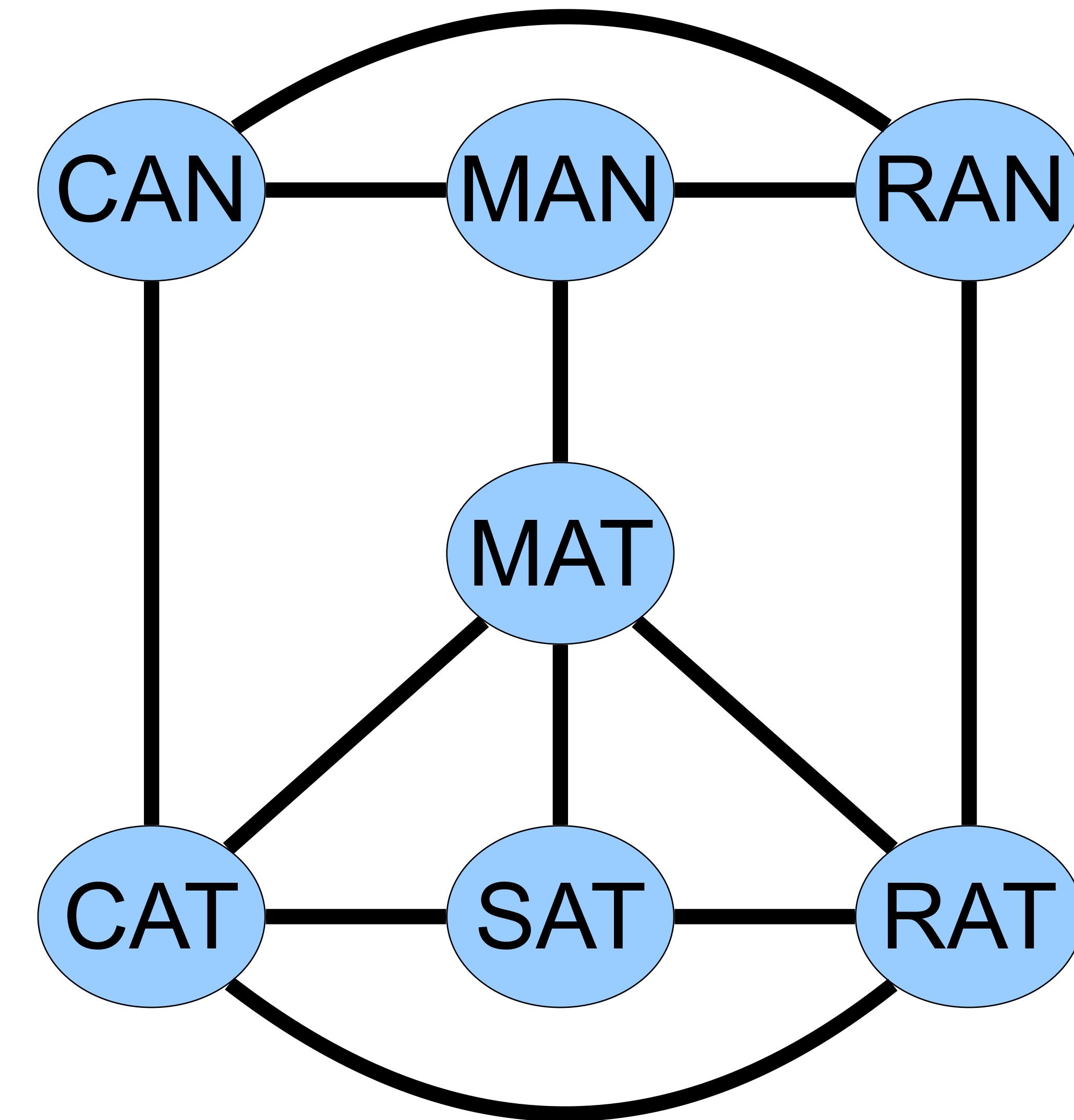


A graph consists of a set of **nodes** connected by **edges**.

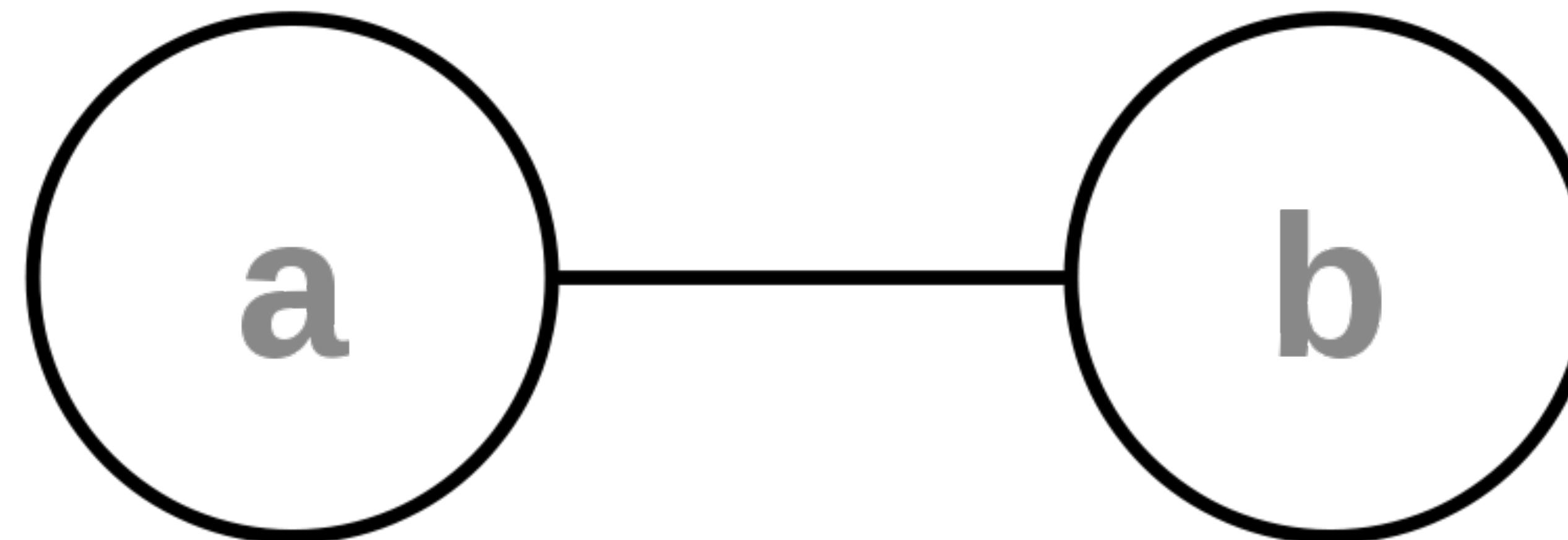
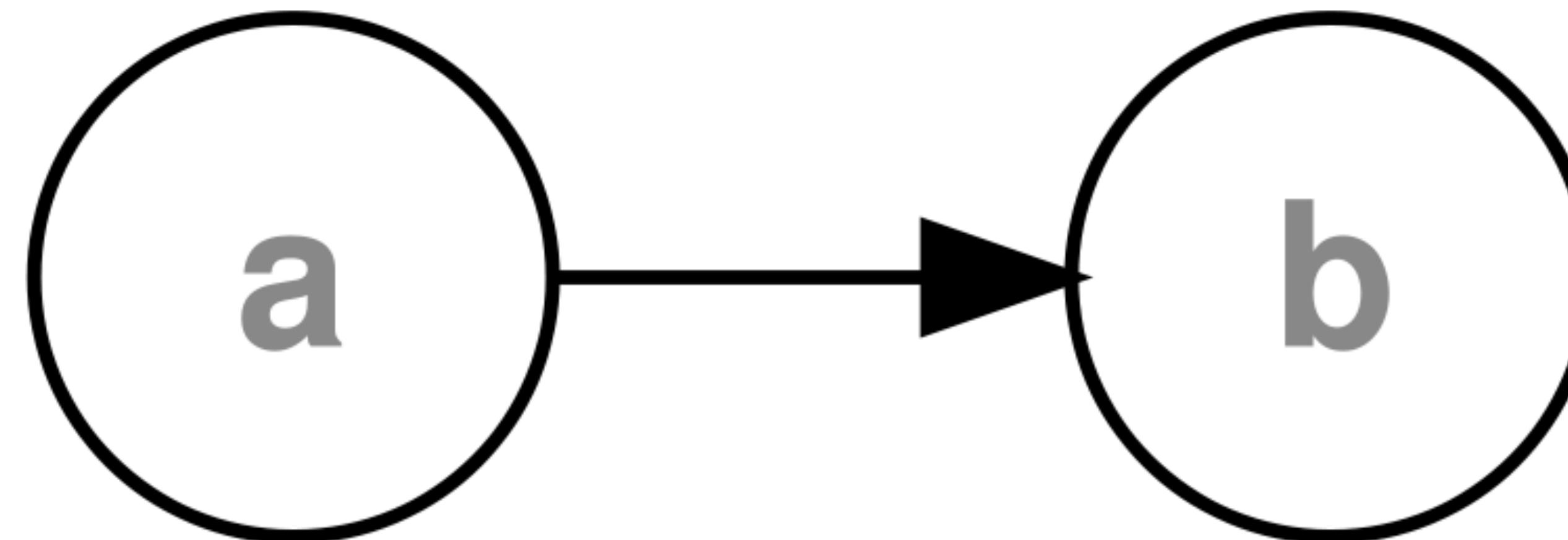
Directed Graph



Undirected Graph



Directed vs Undirected

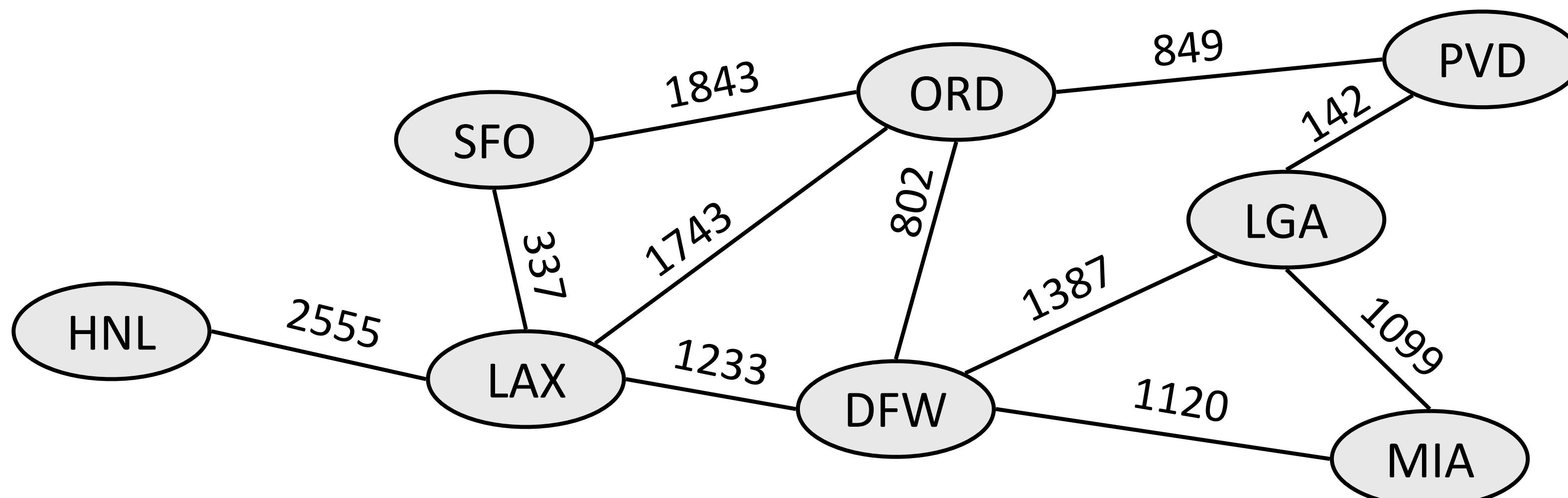


Weighted graphs

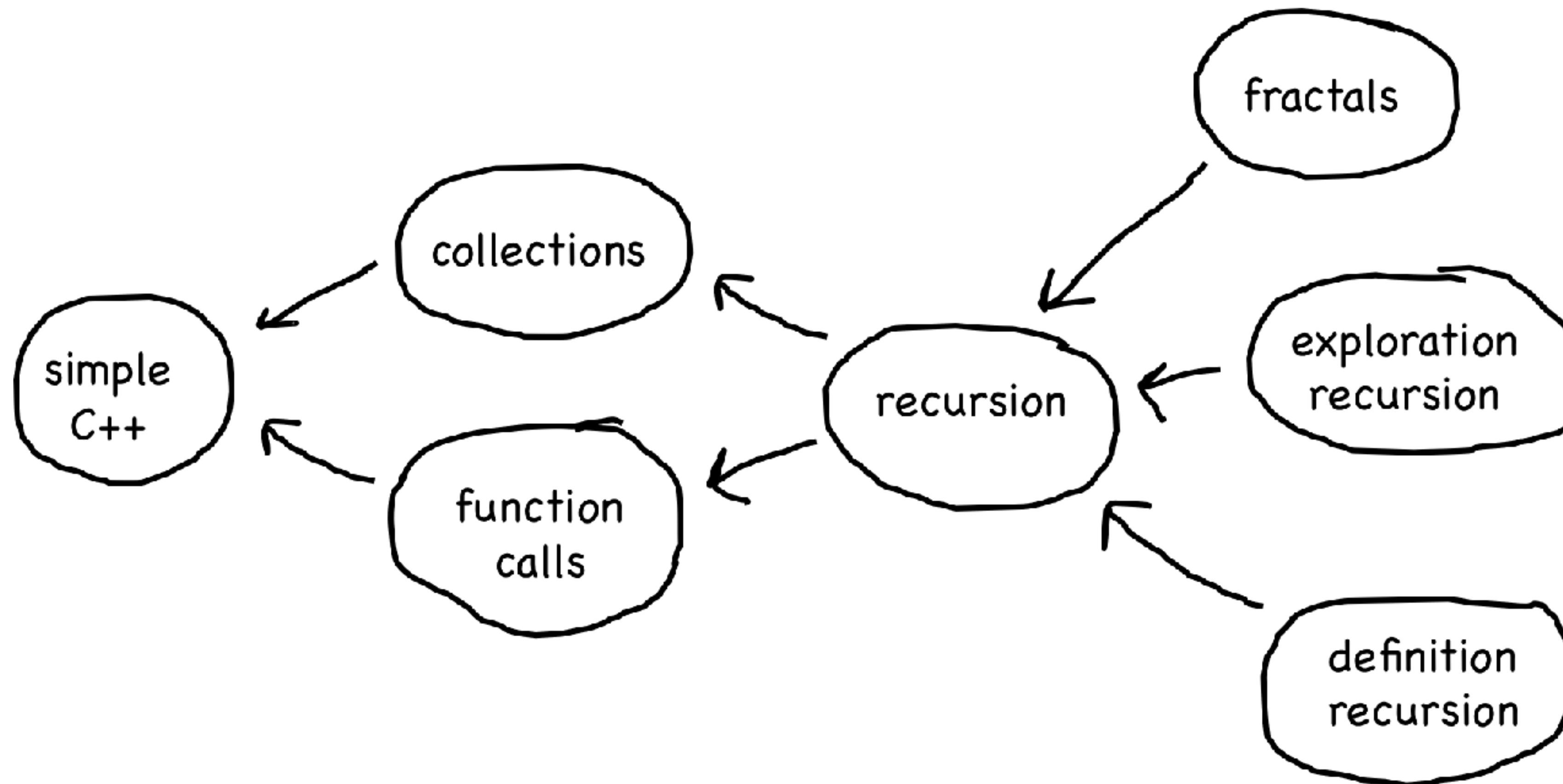


weight: Cost associated with a given edge.

example: graph of airline flights, weighted by miles between cities:



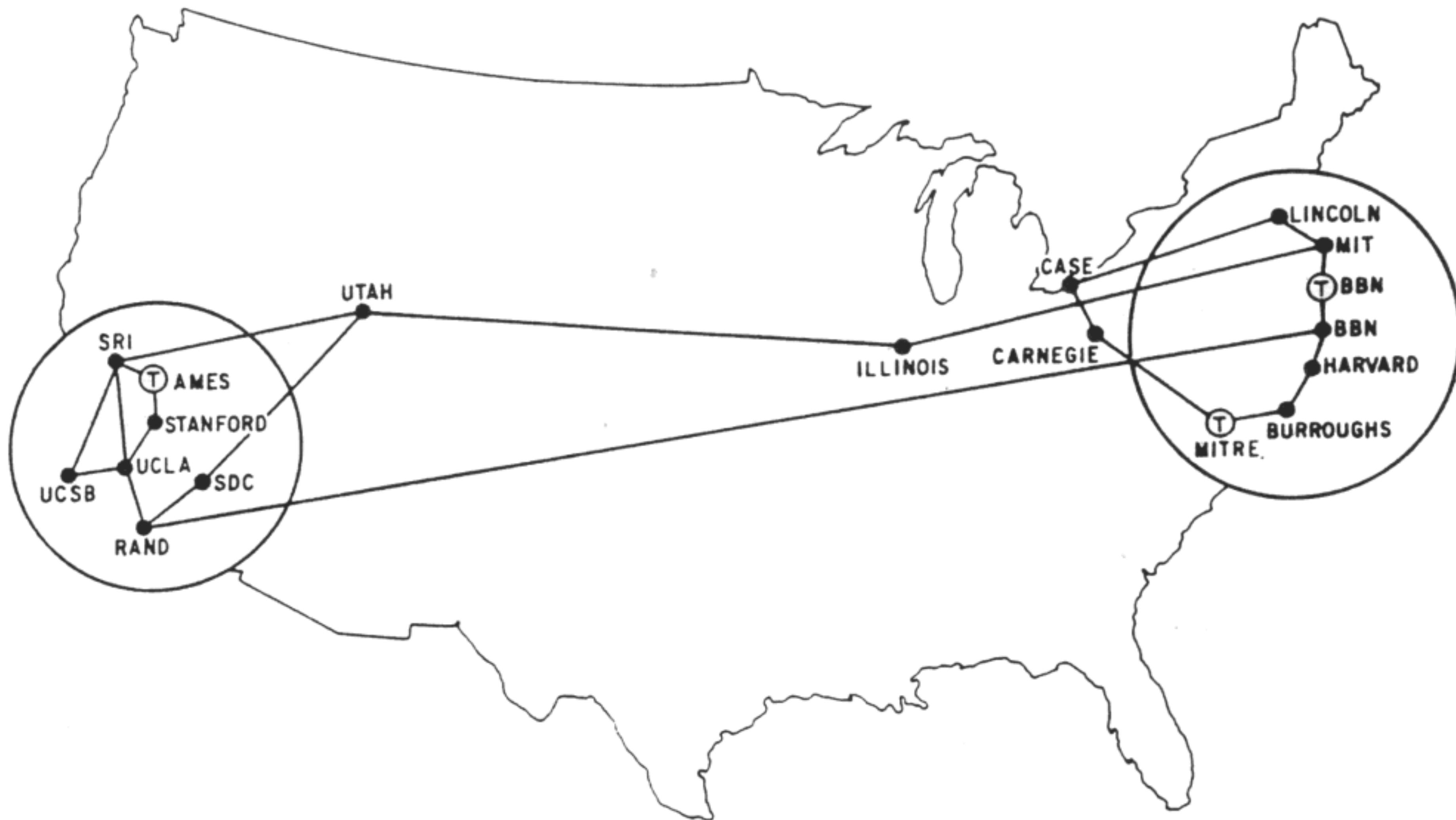
Prerequisite Graph



Social Network



The Internet

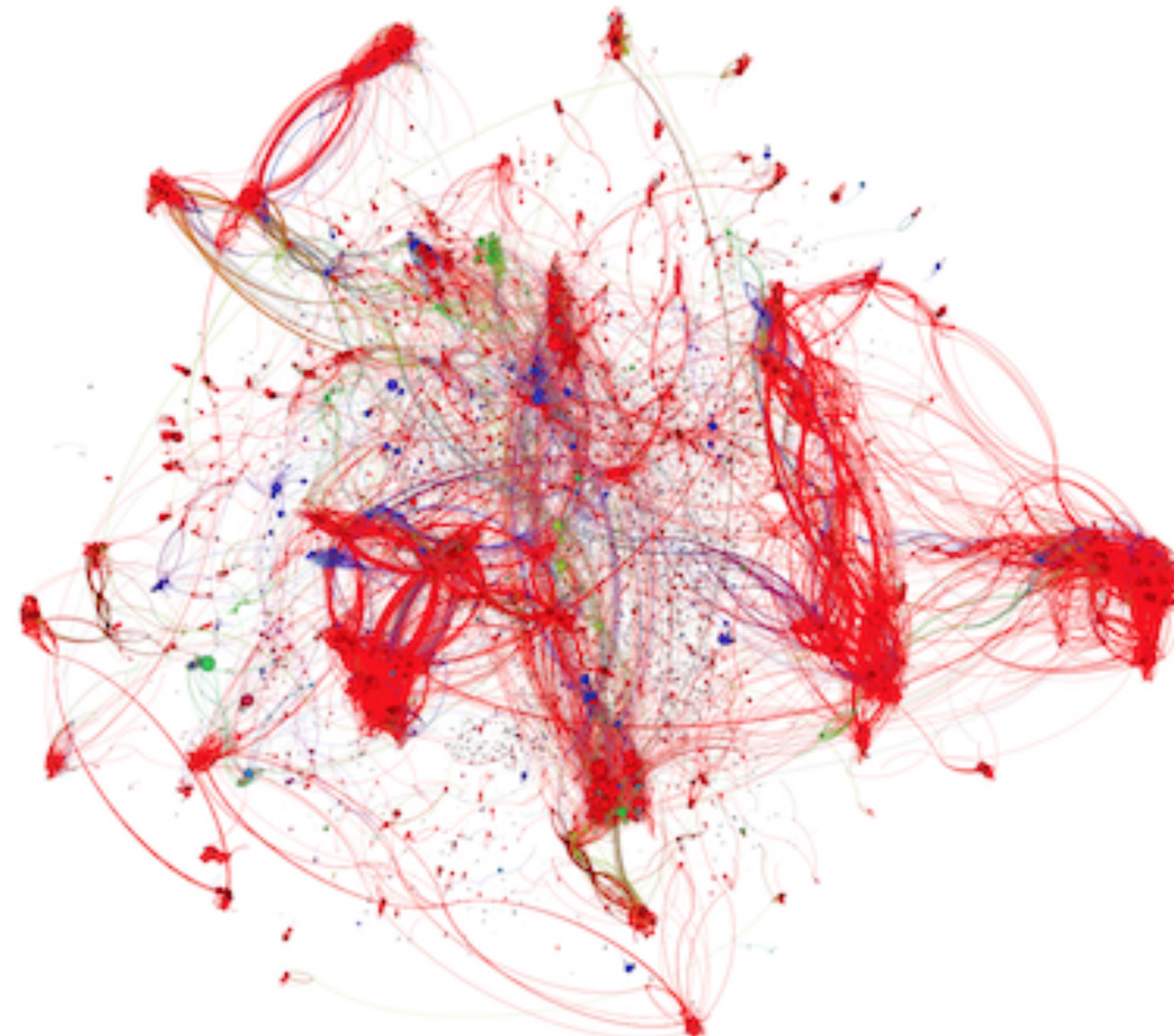


The Internet



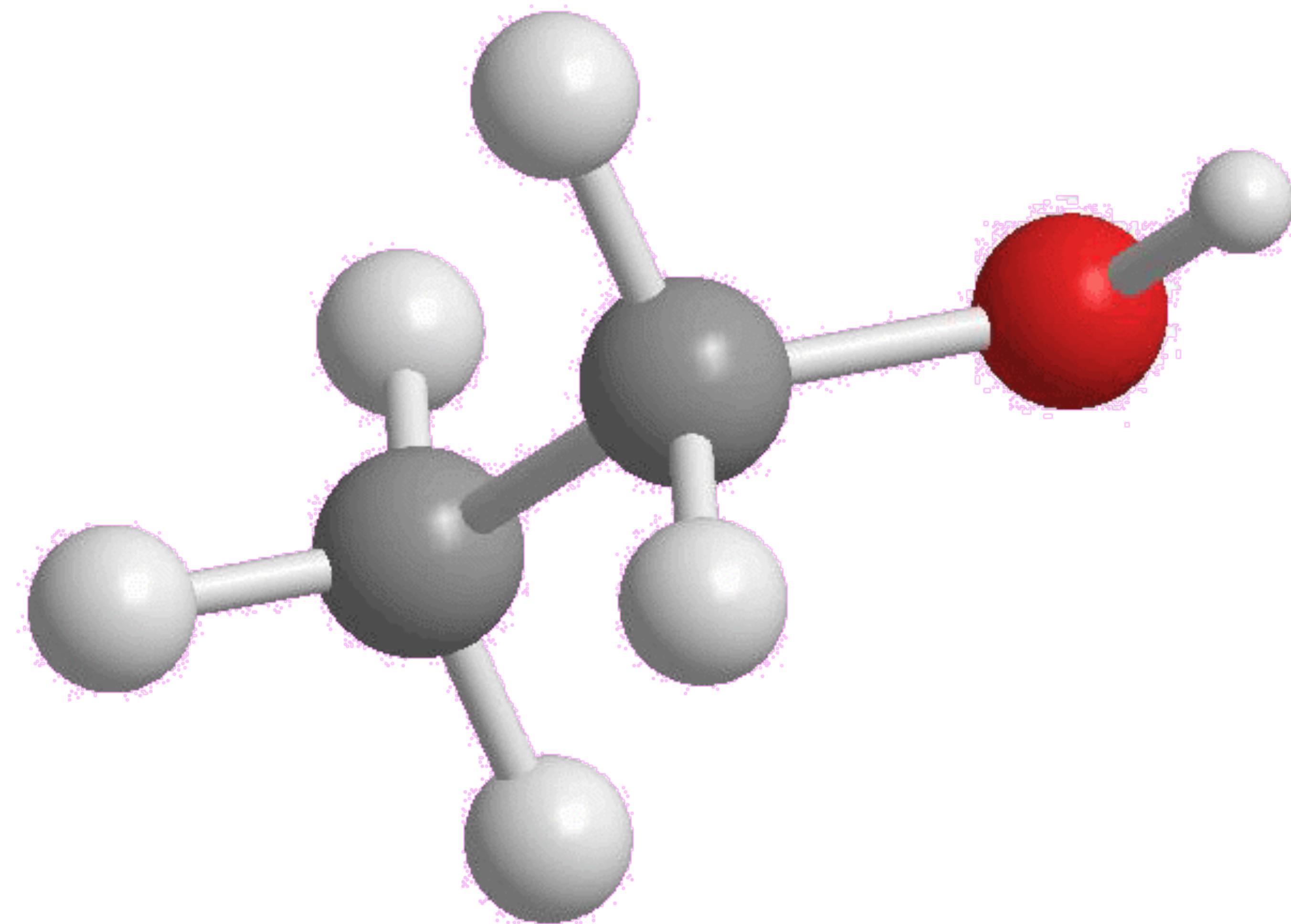
10 to 20 billion

CS Assignments

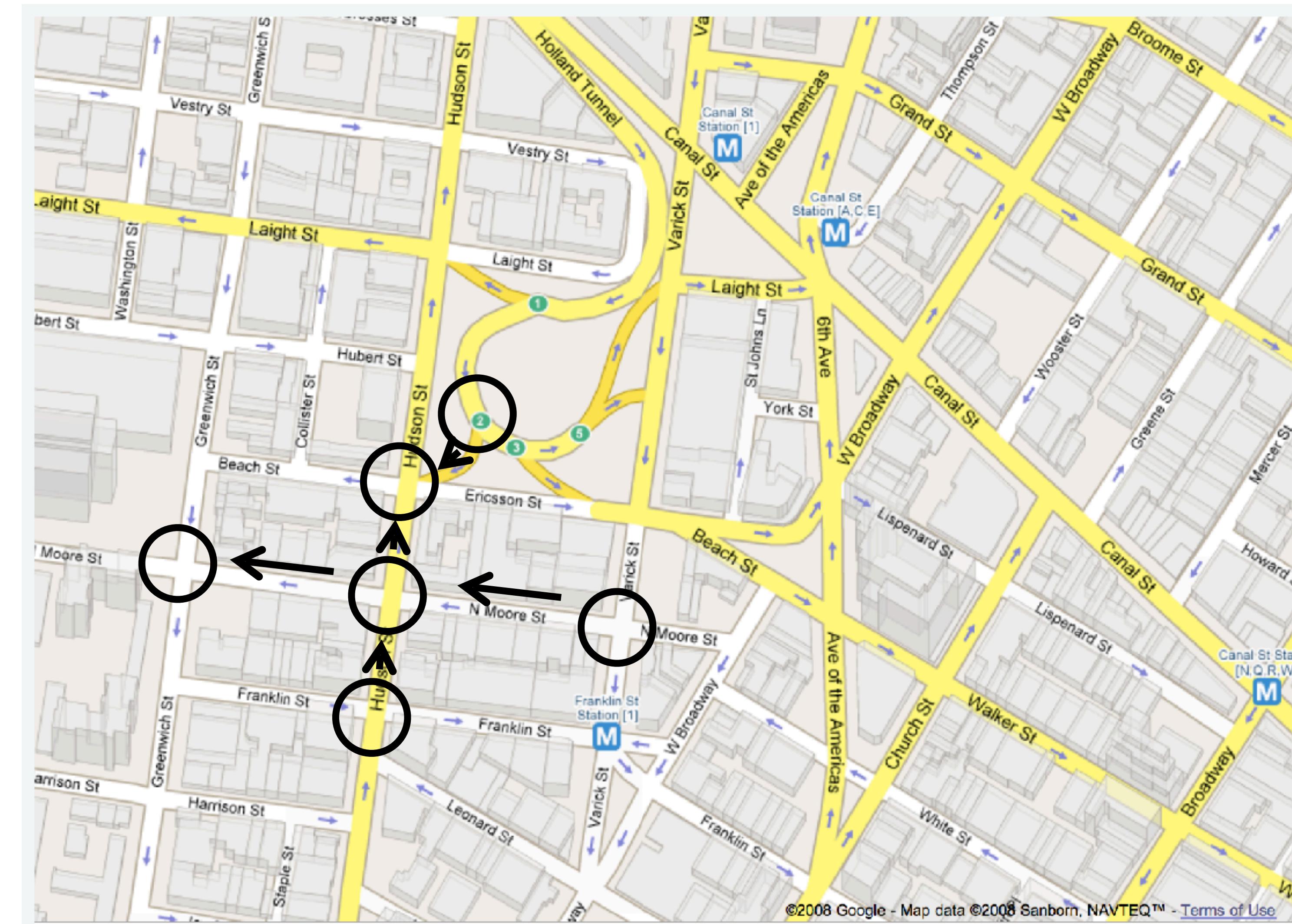


50,000 unique implementations of logistic regression in CS229

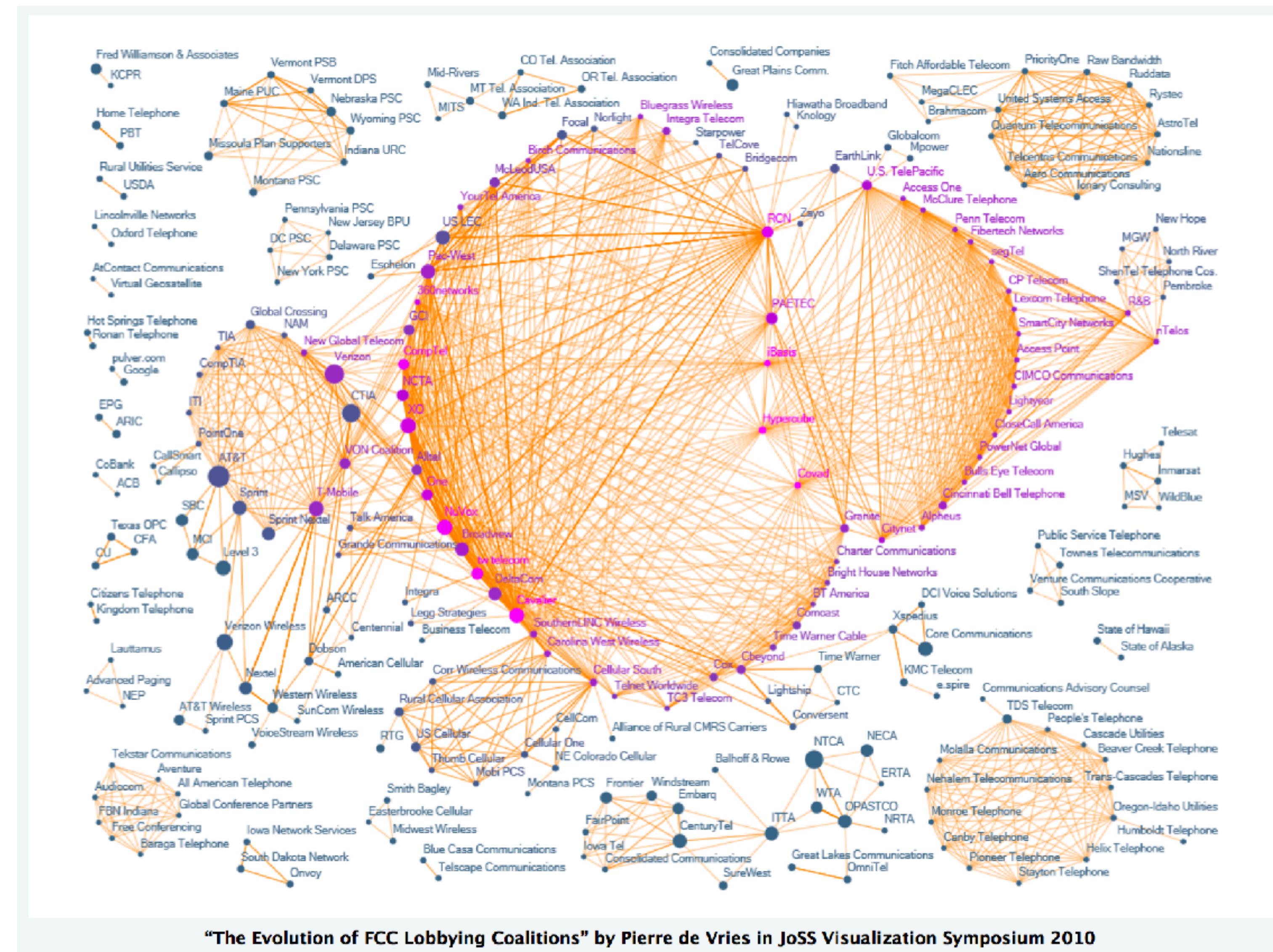
Chemical Bonds



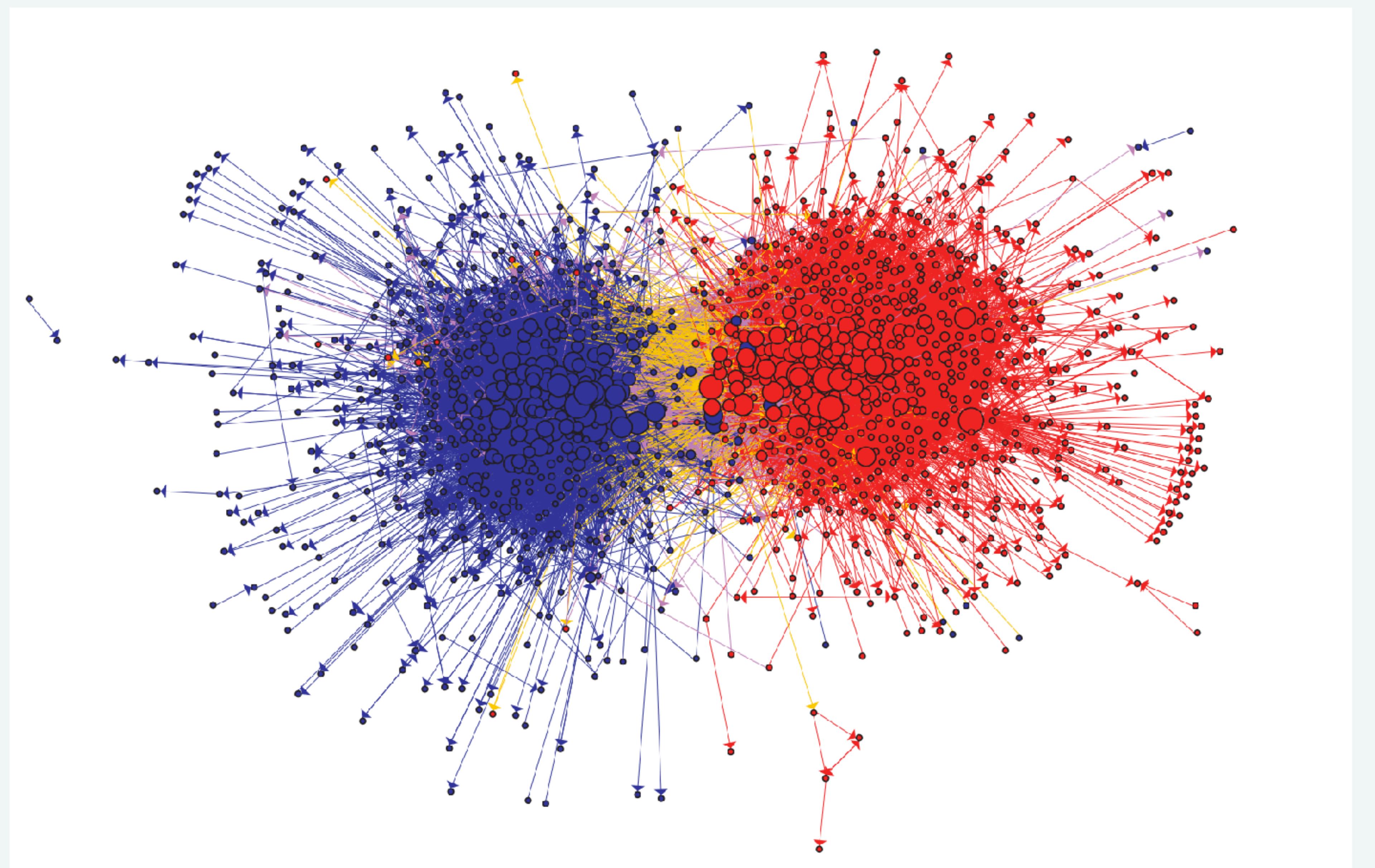
Road Map



Corruption



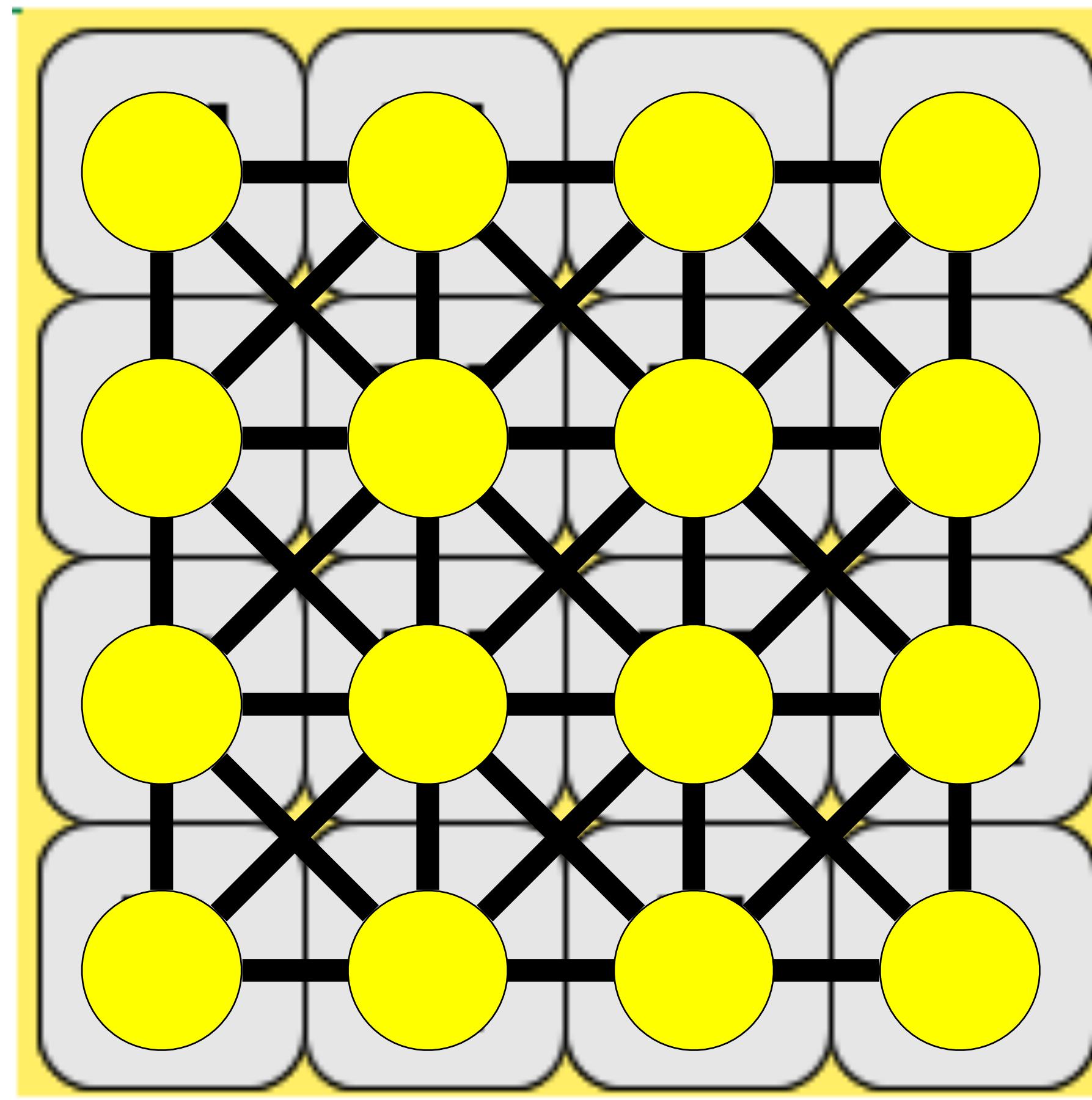
Partisanship



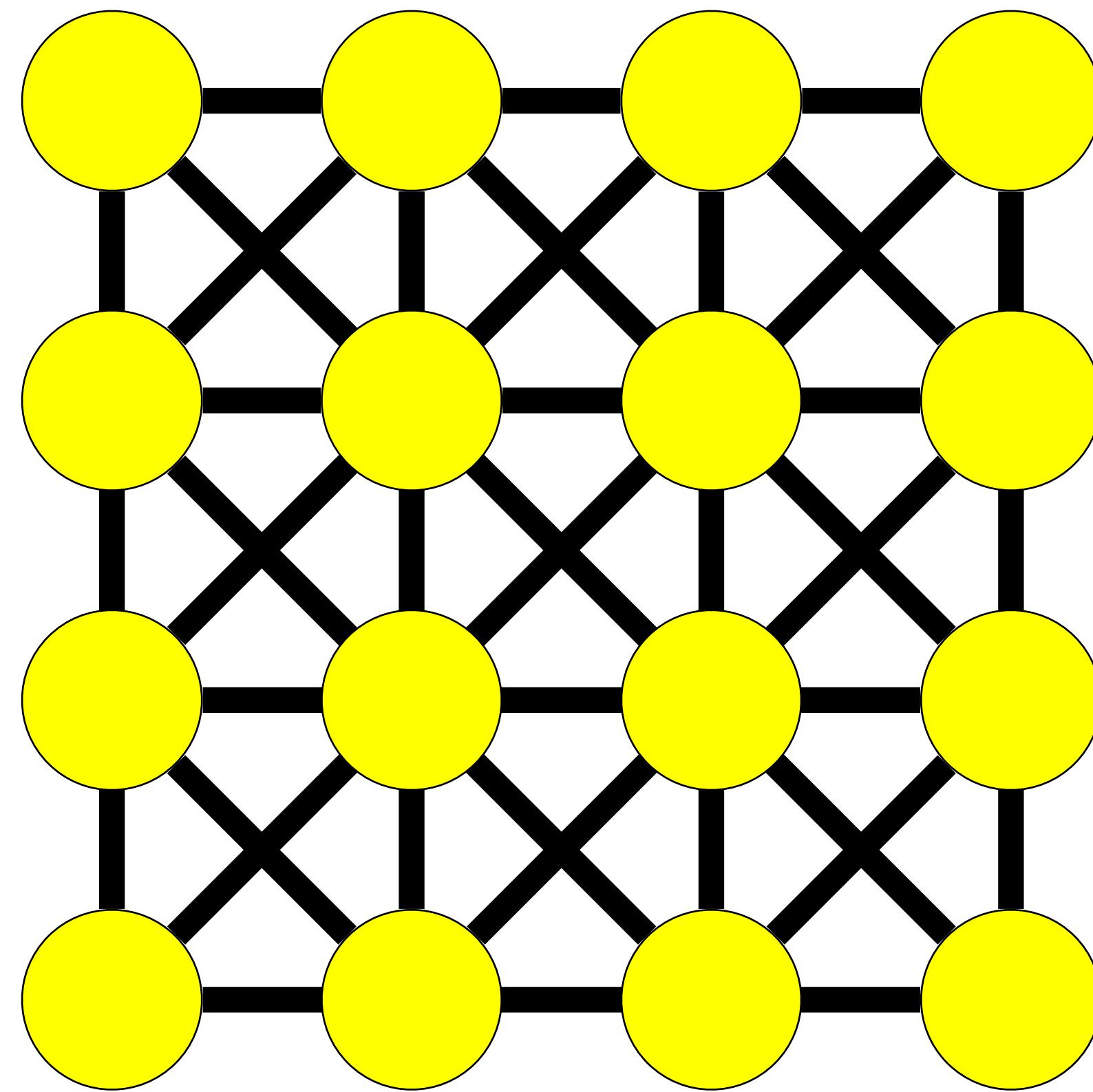
Boggle



Boggle



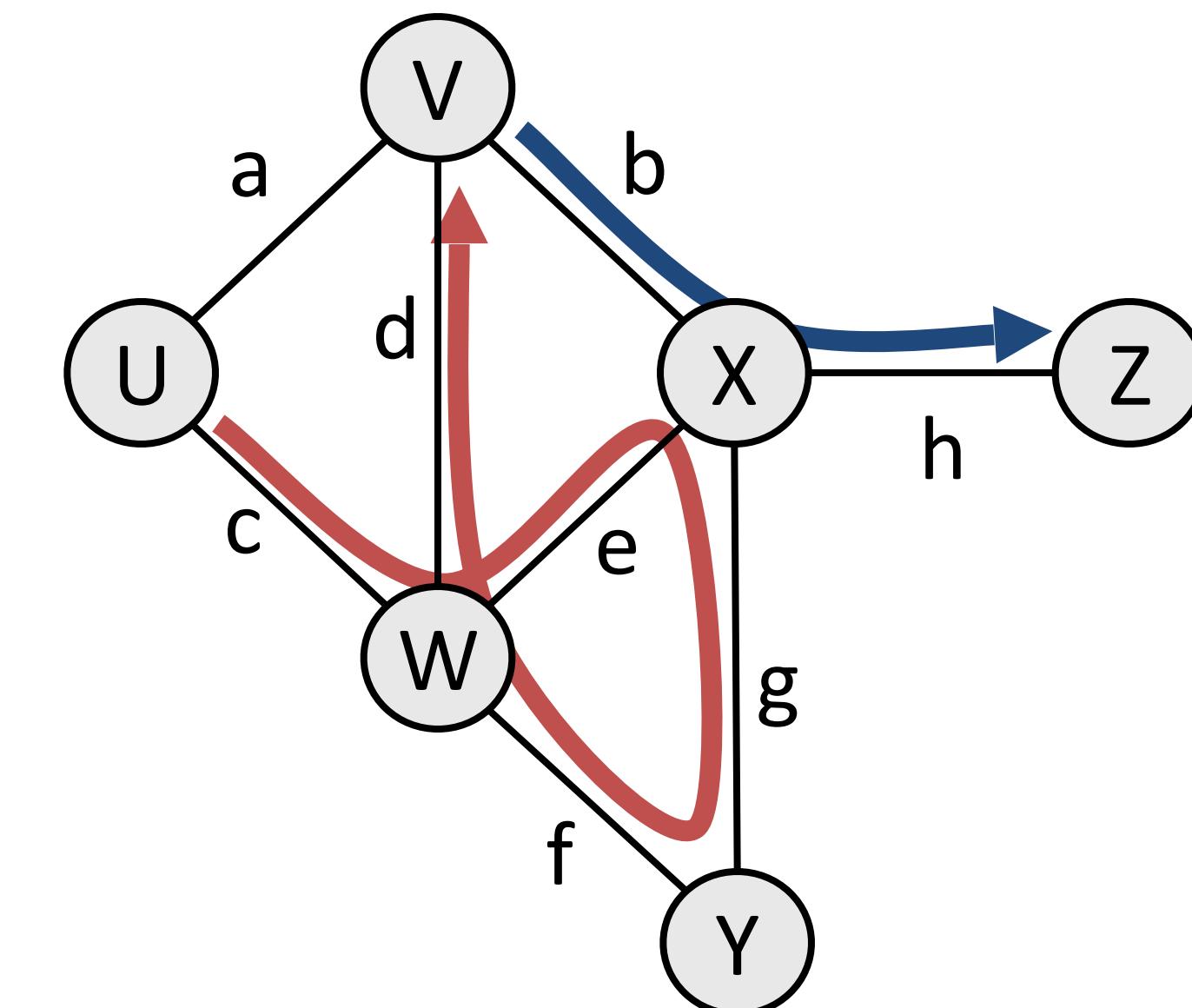
Boggle



Some terms:

Paths

- **path:** A path from vertex a to b is a sequence of edges that can be followed starting from a to reach b .
 - can be represented as vertices visited, or edges taken
 - example, one path from V to Z : $\{b, h\}$ or $\{V, X, Z\}$
 - What are two paths from U to Y ?
- **path length:** Number of vertices or edges contained in the path.
- **neighbor or adjacent:** Two vertices connected directly by an edge.
 - example: V and X



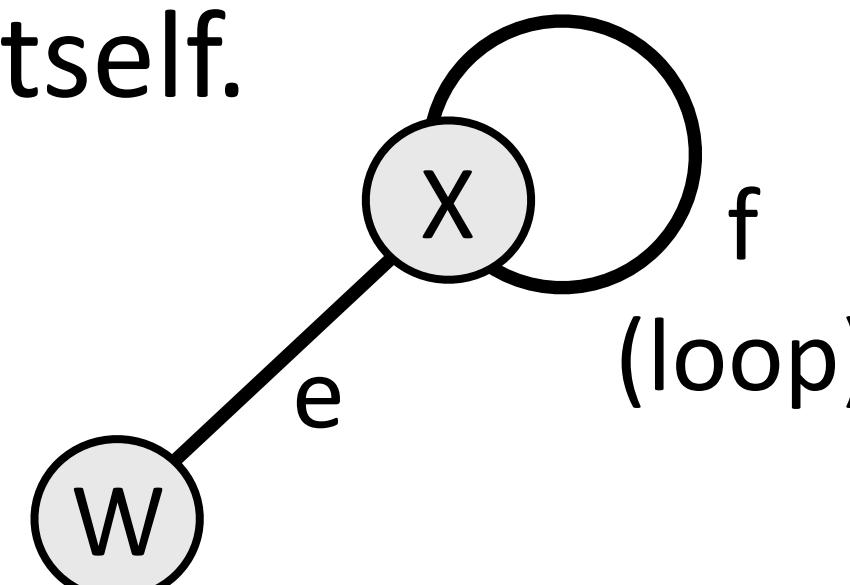
Loops and cycles

- **cycle:** A path that begins and ends at the same node.

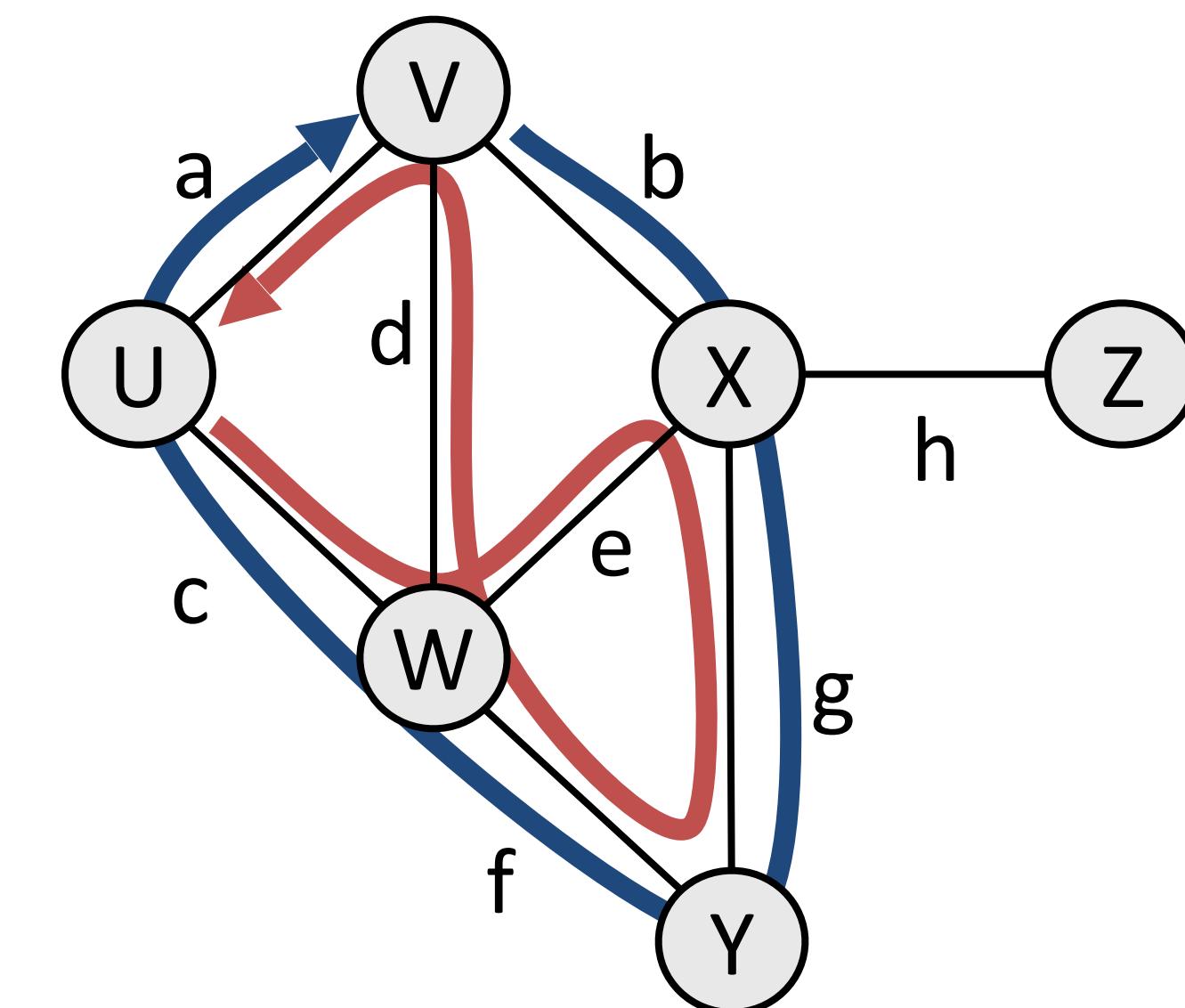
- example: {b, g, f, c, a} or {V, X, Y, W, U, V}.
 - example: {c, d, a} or {U, W, V, U}.

- **acyclic graph:** One that does not contain any cycles.

- **loop:** An edge directly from a node to itself.

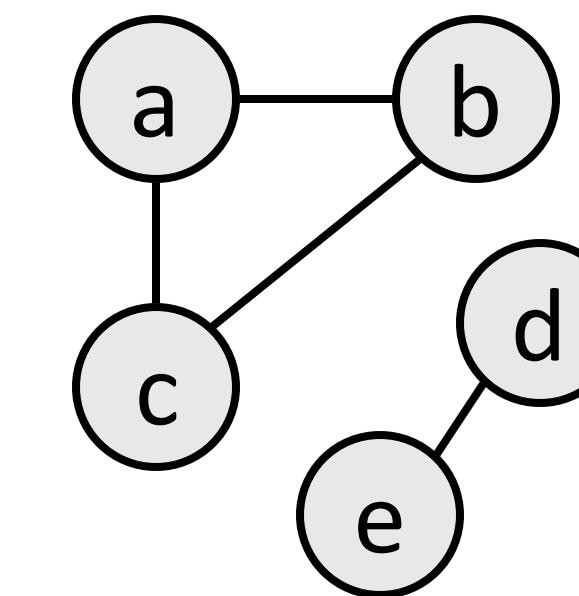
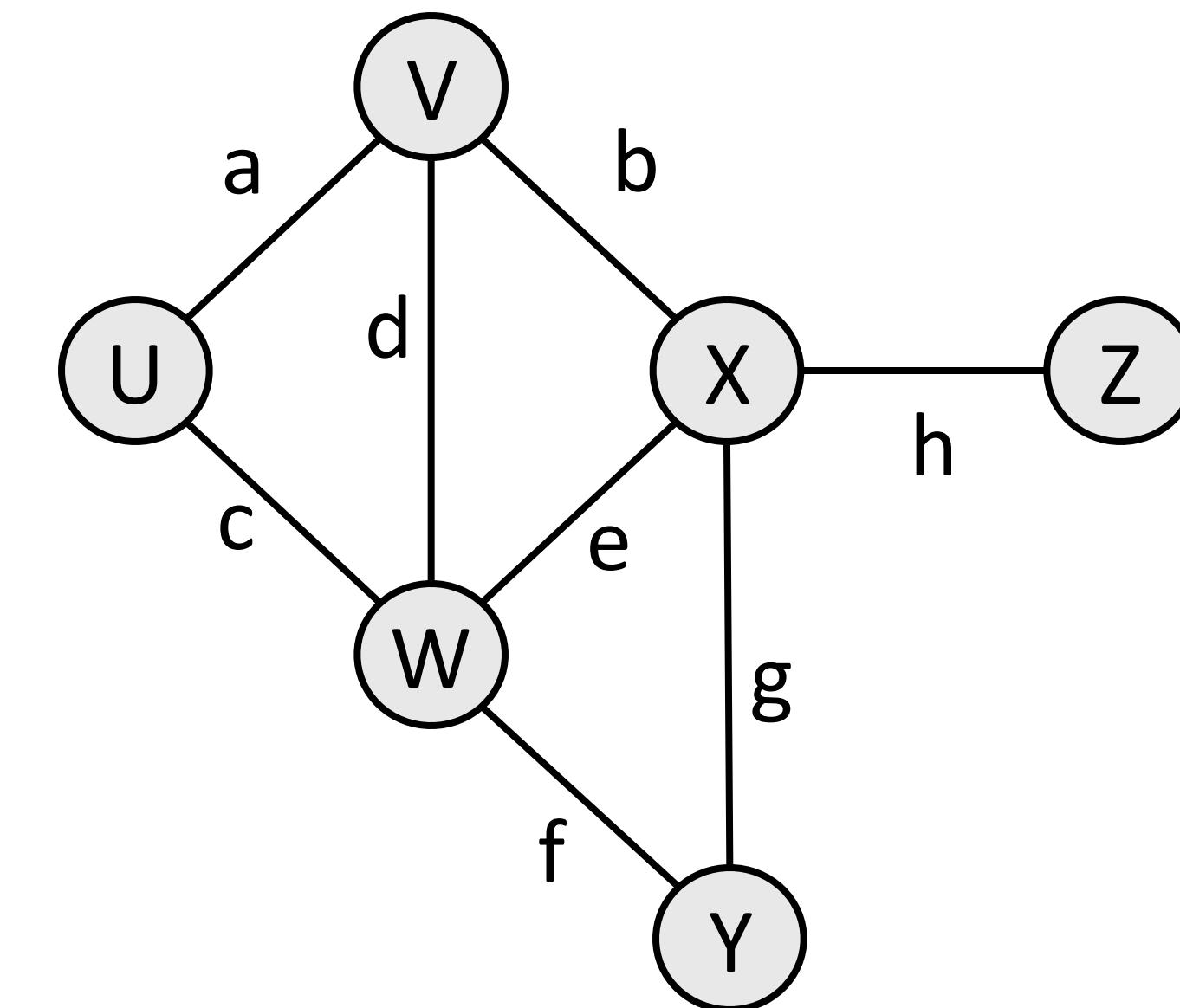
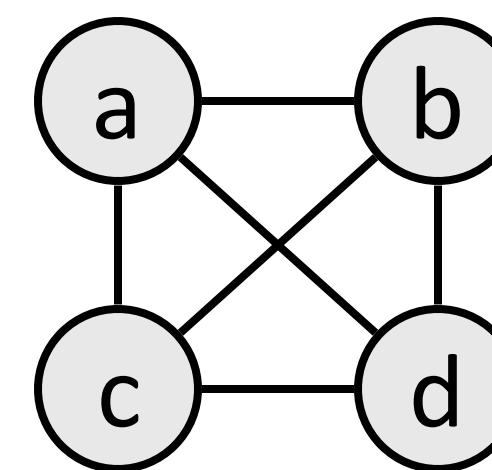


- Many graphs don't allow loops.



Reachability, connectedness

- **reachable**: Vertex a is *reachable* from b if a path exists from a to b .
- **connected**: A graph is *connected* if every vertex is reachable from every other.
- **complete**: If every vertex has a direct edge to every other.



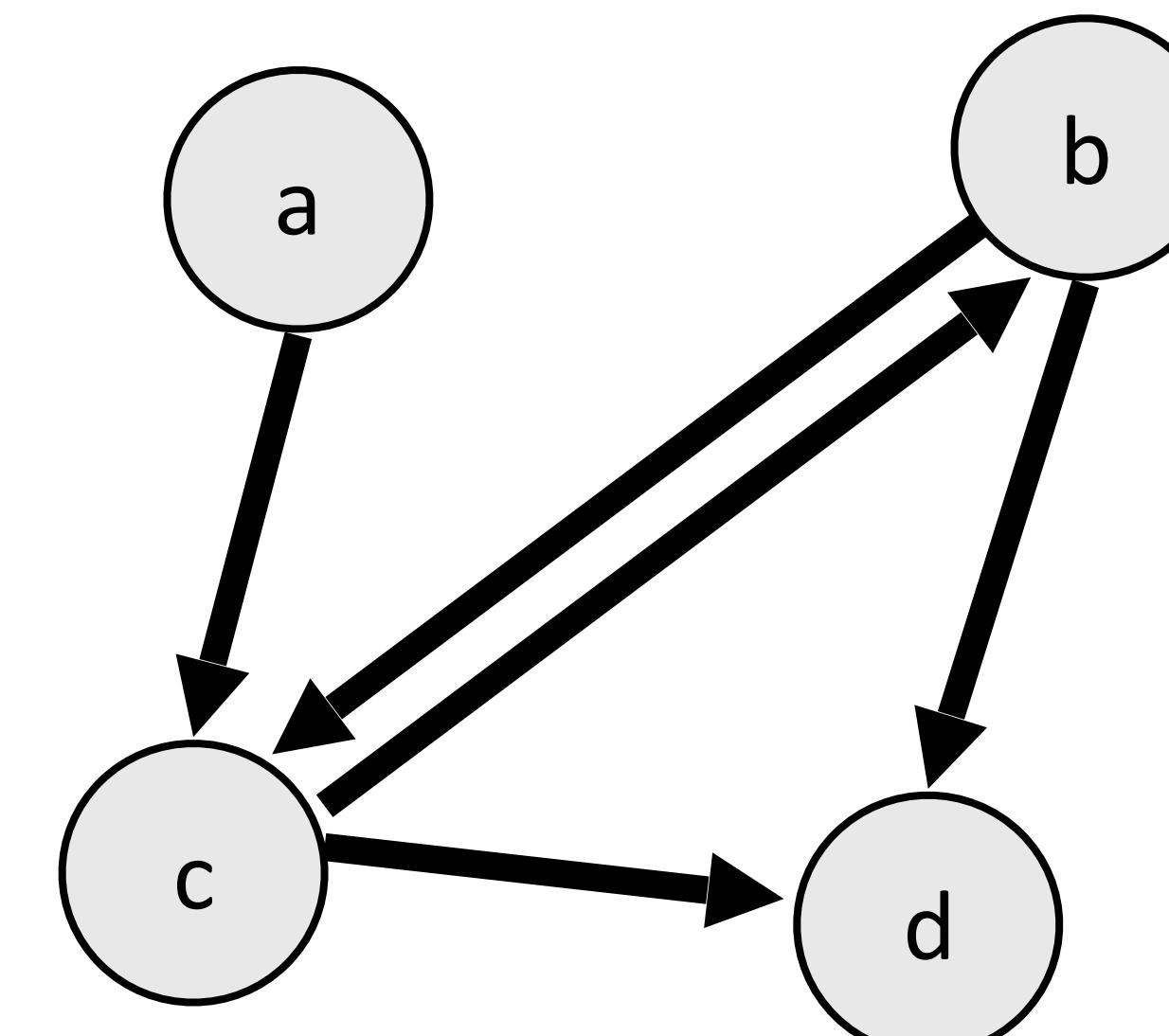
Stanford BasicGraph

The Stanford C++ library includes a BasicGraph class.

- Based on an older library class named Graph

You can construct a graph and add vertices/edges:

```
#include "basicgraph.h"  
...  
BasicGraph graph;  
graph.addVertex("a");  
graph.addVertex("b");  
graph.addVertex("c");  
graph.addVertex("d");  
graph.addEdge("a", "c");  
graph.addEdge("b", "c");  
graph.addEdge("c", "b");  
graph.addEdge("b", "d");  
graph.addEdge("c", "d");
```



BasicGraph members

```
#include "basicgraph.h"    // a directed, weighted graph
```

<code>g.addEdge(v1, v2);</code>	adds an edge between two vertexes
<code>g.addVertex(name);</code>	adds a vertex to the graph
<code>g.clear();</code>	removes all vertexes/edges from the graph
<code>g.getEdgeSet()</code> <code>g.getEdgeSet(v)</code>	returns all edges, or all edges that start at <code>v</code> , as a Set of pointers
<code>g.getNeighbors(v)</code>	returns a set of all vertices that <code>v</code> has an edge to
<code>g.getVertex(name)</code>	returns pointer to vertex with the given name
<code>g.getVertexSet()</code>	returns a set of all vertexes
<code>g.isNeighbor(v1, v2)</code>	returns true if there is an edge from vertex <code>v1</code> to <code>v2</code>
<code>g.isEmpty()</code>	returns true if queue contains no vertexes/edges
<code>g.removeEdge(v1, v2);</code>	removes an edge from the graph
<code>g.removeVertex(name);</code>	removes a vertex from the graph
<code>g.size()</code>	returns the number of vertexes in the graph
<code>g.toString()</code>	returns a string such as " <code>{a, b, c, a -> b}</code> "

BasicGraph members

```
#include "basicgraph.h"    // a directed, weighted graph
```

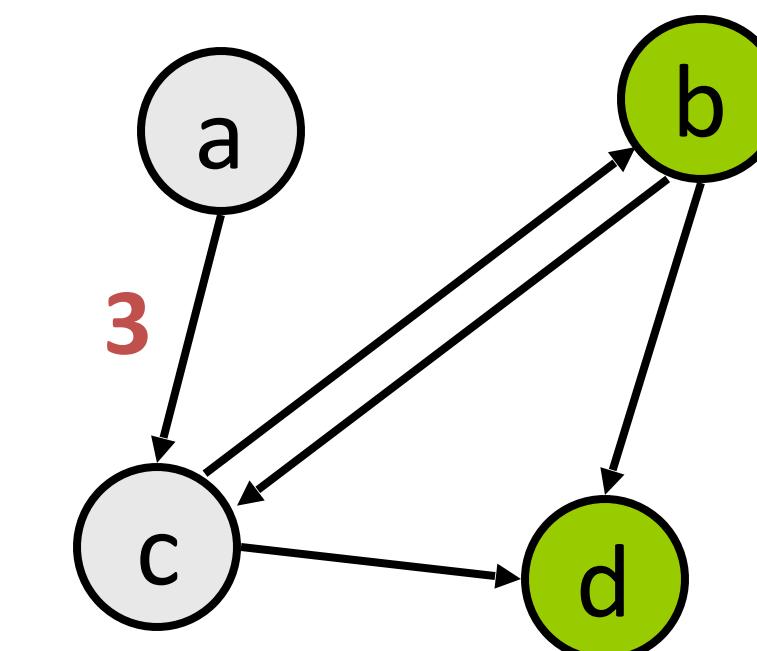
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<code>g.isNeighbor(v1, v2)</code>	returns true if there is an edge from vertex v1 to v2
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<code>g.toString()</code>	returns a string such as " <code>{a, b, c, a -> b}</code> "

Using BasicGraph

The graph stores a struct of information about each vertex/edge:

```
struct Vertex {  
    string name;  
    Set<Edge*> edges;  
    double cost;  
    // other stuff  
};
```

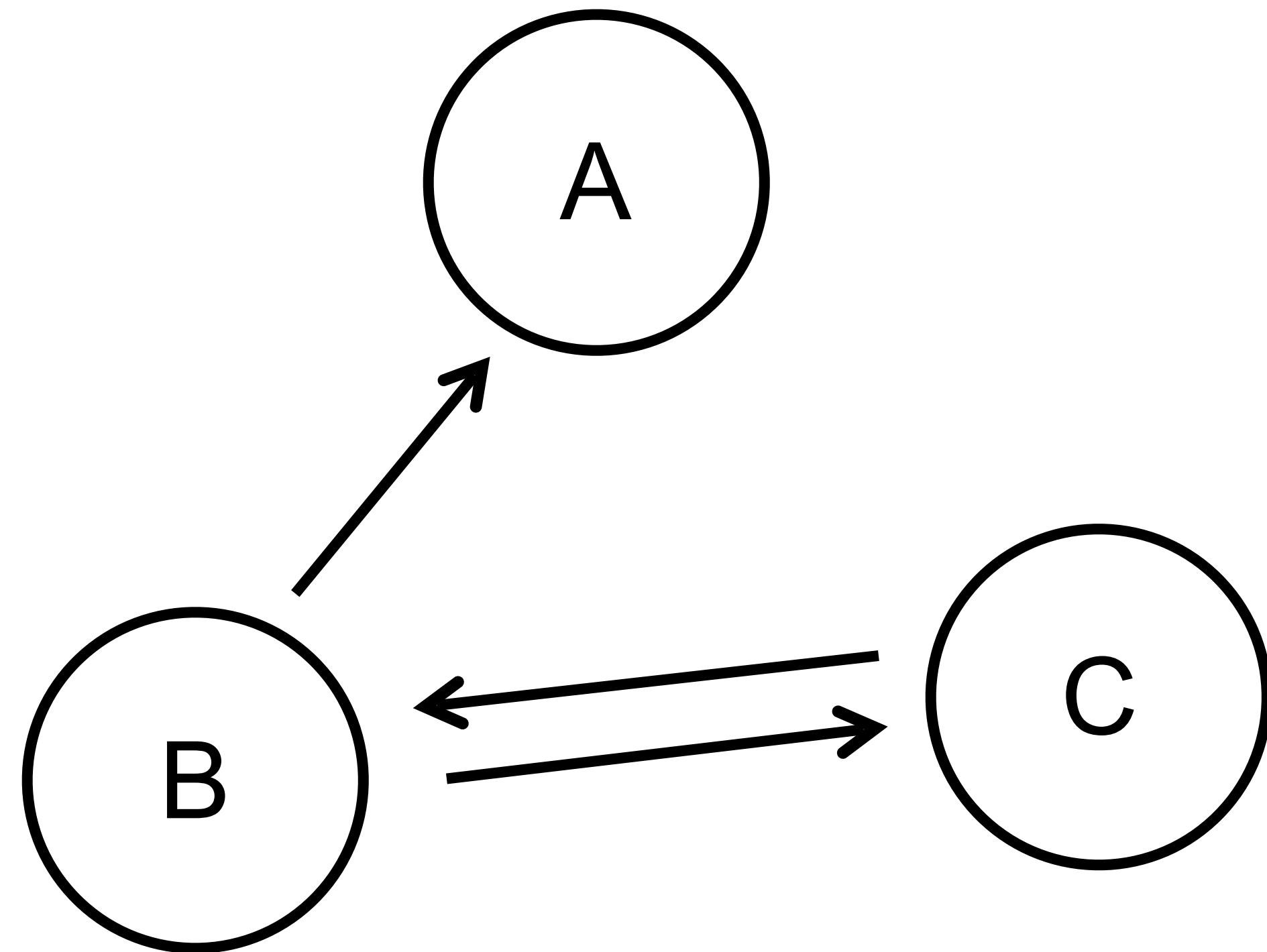
```
struct Edge {  
    Vertex* start;  
    Vertex* finish;  
    double weight;  
    // other stuff  
};
```



You can use these to help implement graph algorithms:

```
Vertex * vertC = graph.getVertex("c");  
Edge * edgeAC = graph.getEdge("a", "c");
```

Our First Graph

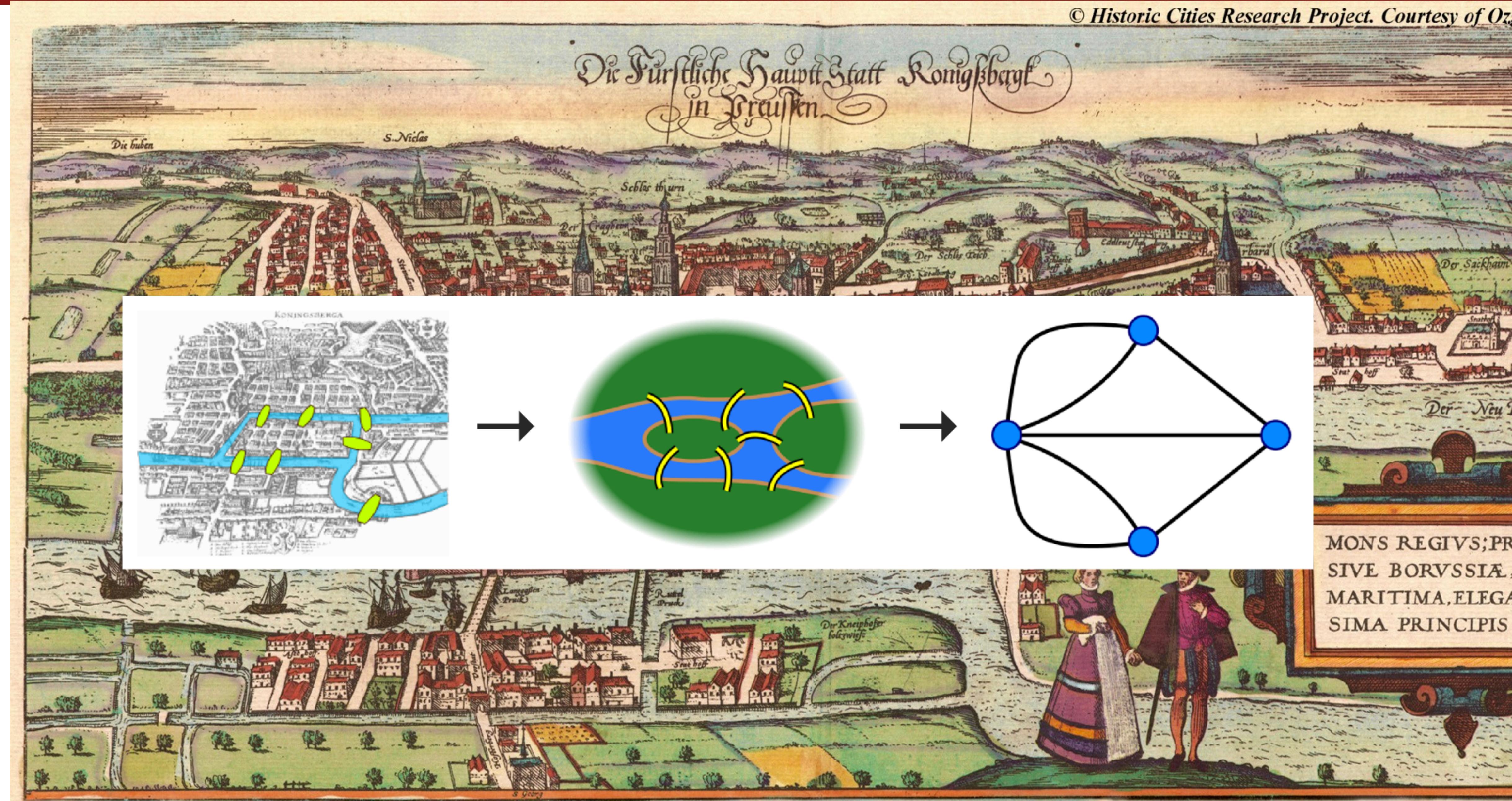


There are other representations...

... this is the one we are going to use.

Algorithms

© Historic Cities Research Project. Courtesy of Oz.

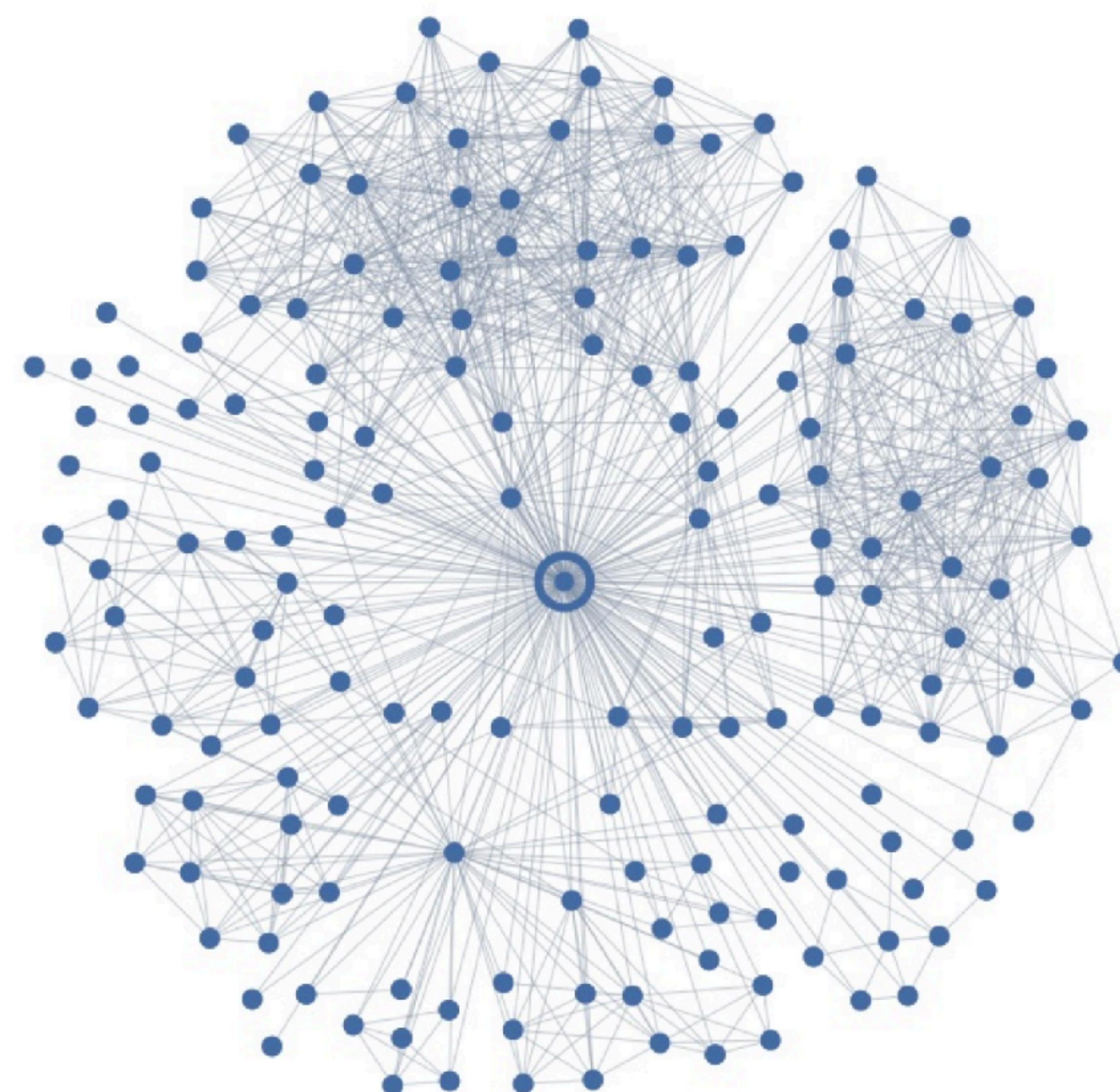


Who Do You Love

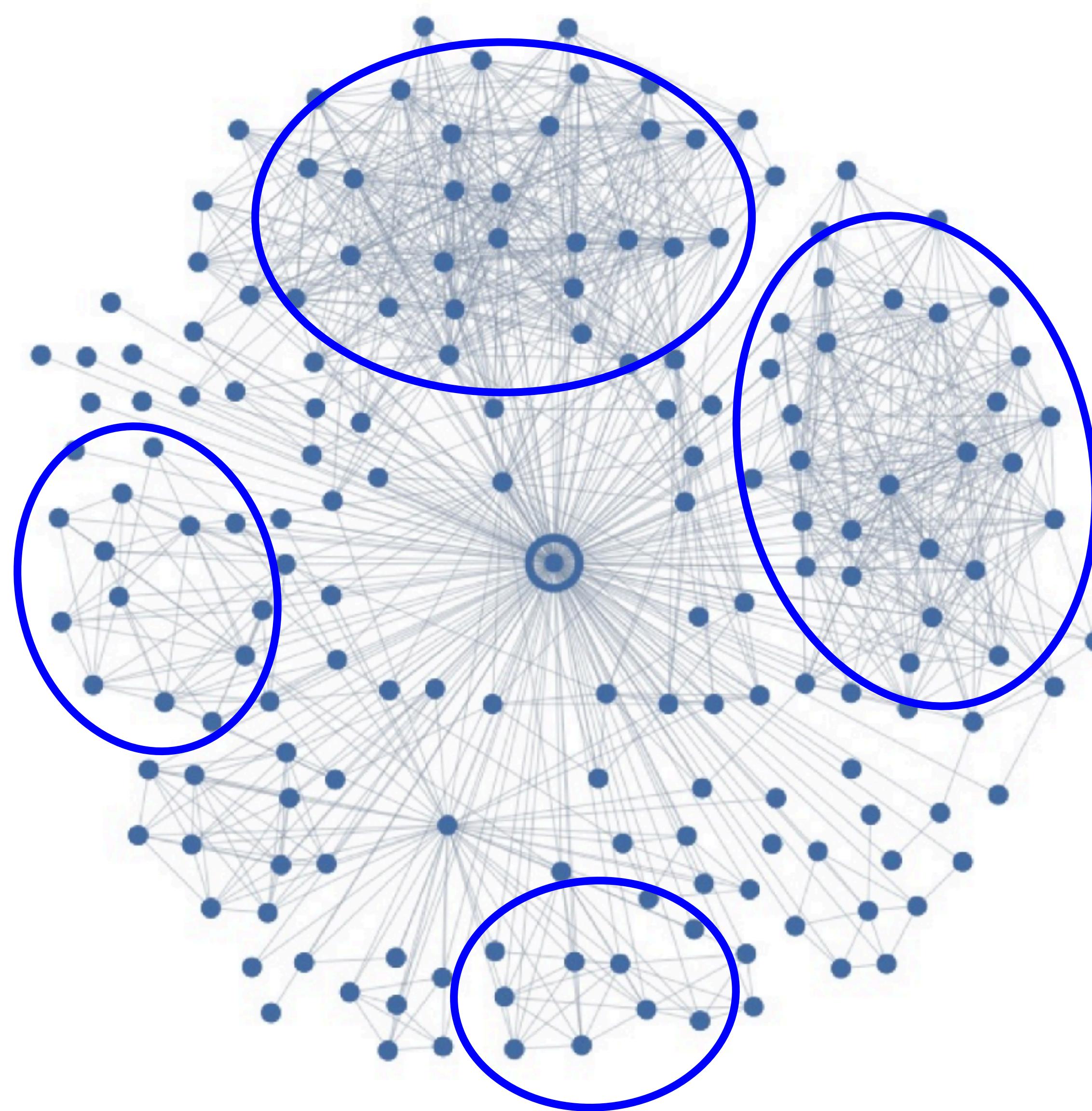
And how does Facebook know?



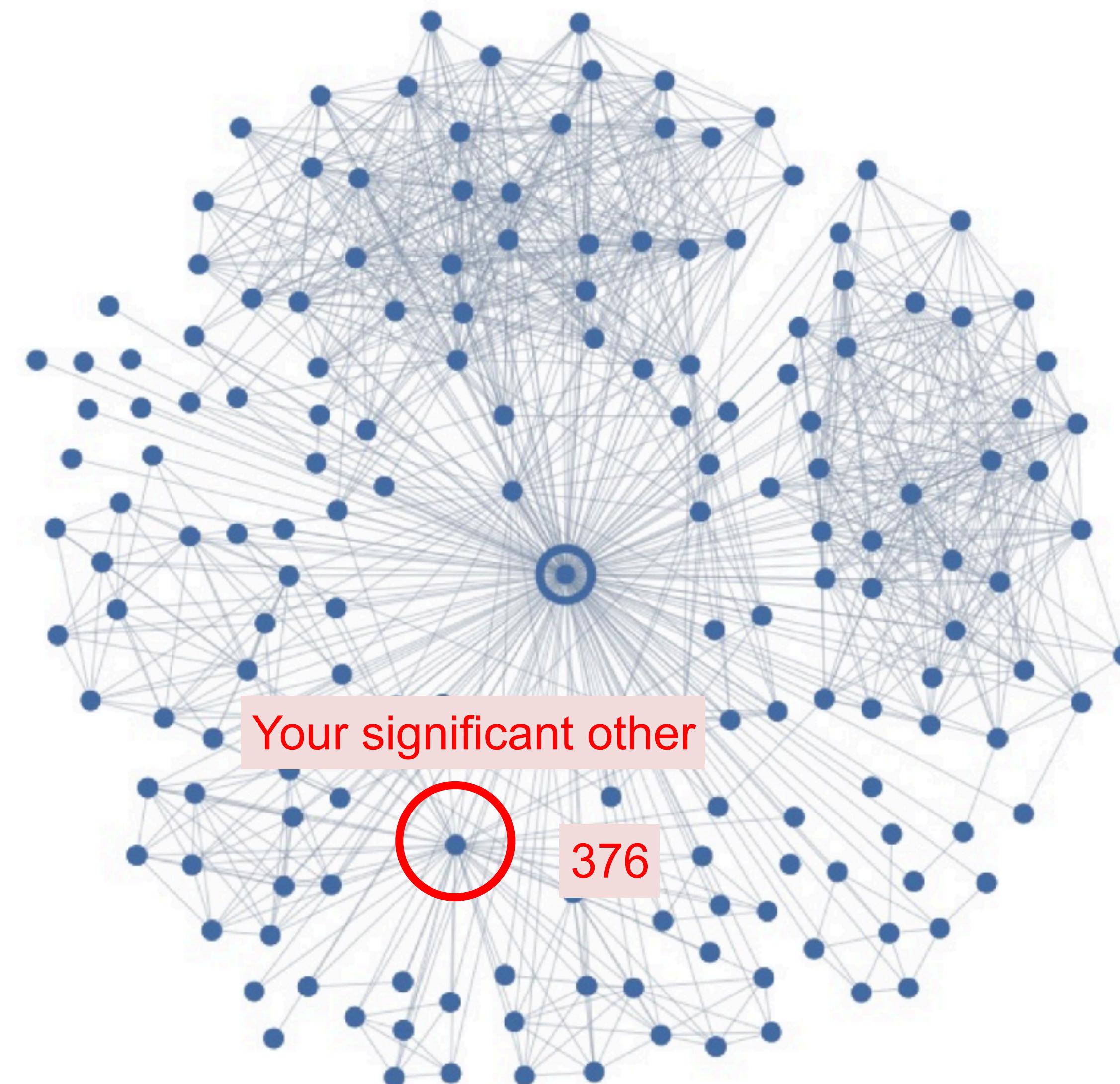
Ego Graph



Maybe I Love These People?



But I Actually Love This Person



Romance and Dispersion

Romantic Partnerships and the Dispersion of Social Ties: A Network Analysis of Relationship Status on Facebook

Lars Backstrom
Facebook Inc.

Jon Kleinberg
Cornell University

ABSTRACT

A crucial task in the analysis of on-line social-networking systems is to identify important people — those linked by strong social ties — within an individual's network neighborhood. Here we investigate this question for a particular category of strong ties, those involving spouses or romantic partners. We organize our analysis around a basic question: given all the connections among a person's friends, can you recognize his or her romantic partner from the network structure alone? Using data from a large sample of Facebook users, we find that this task can be accomplished with high accuracy, but doing so requires the development of a new measure of tie strength that we term 'dispersion' — the extent to which two people's mutual friends are not themselves well-connected. The results offer methods for identifying types of structurally significant people in on-line applications, and suggest a potential expansion of existing theories of tie strength.

Categories and Subject Descriptors: H.2.8 [Database Management]: Database applications—*Data mining*

Keywords: Social Networks; Romantic Relationships.

they see from friends [1], and organizing their neighborhood into conceptually coherent groups [23, 25].

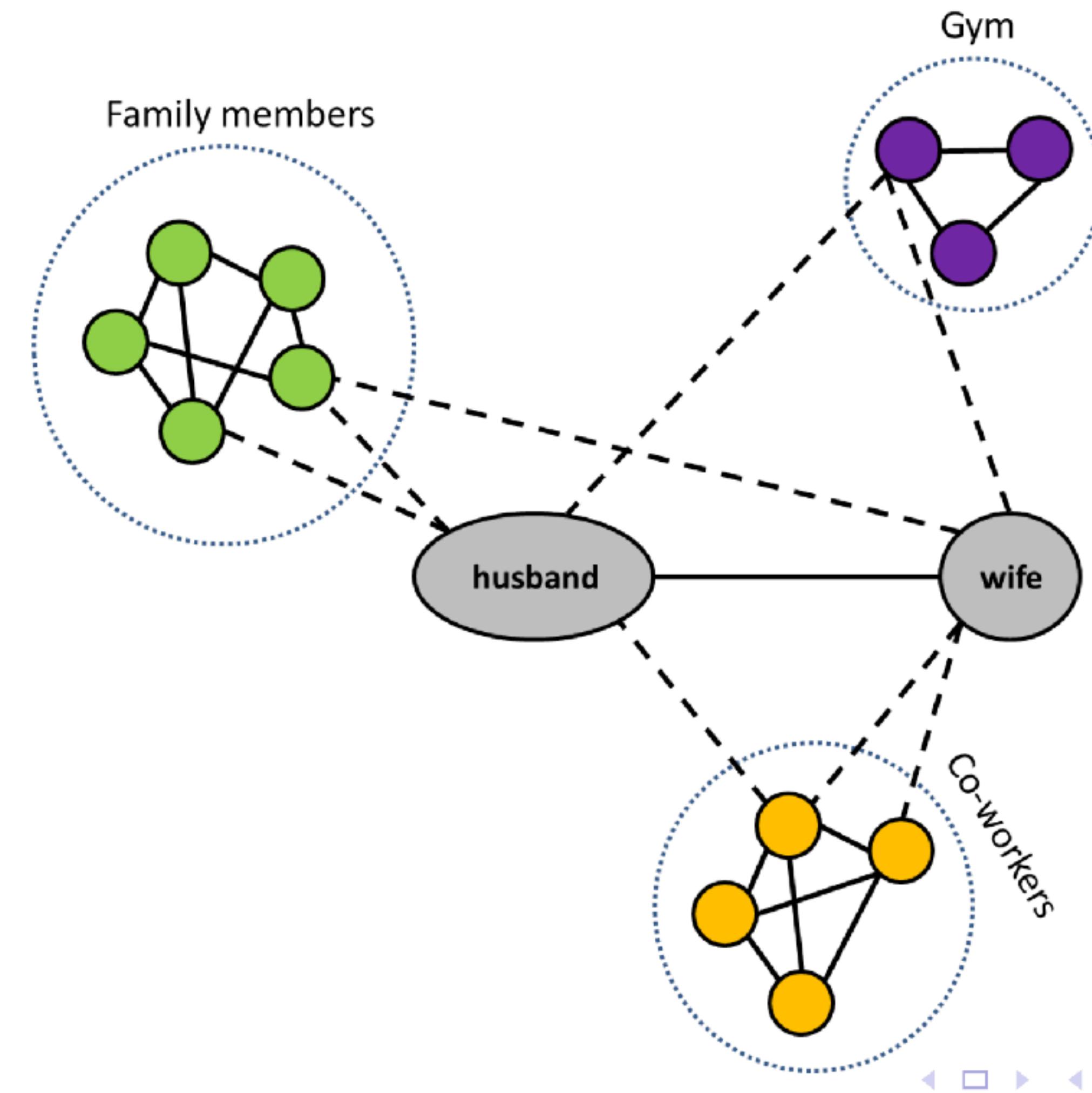
Tie Strength.

Tie strength forms an important dimension along which to characterize a person's links to their network neighbors. Tie strength informally refers to the 'closeness' of a friendship; it captures a spectrum that ranges from strong ties with close friends to weak ties with more distant acquaintances. An active line of research reaching back to foundational work in sociology has studied the relationship between the strengths of ties and their structural role in the underlying social network [15]. Strong ties are typically 'embedded' in the network, surrounded by a large number of mutual friends [6, 16], and often involving large amounts of shared time together [22] and extensive interaction [17]. Weak ties, in contrast, often involve few mutual friends and can serve as 'bridges' to diverse parts of the network, providing access to novel information [5, 15].

A fundamental question connected to our understanding of strong ties is to identify the most important person's social network ...

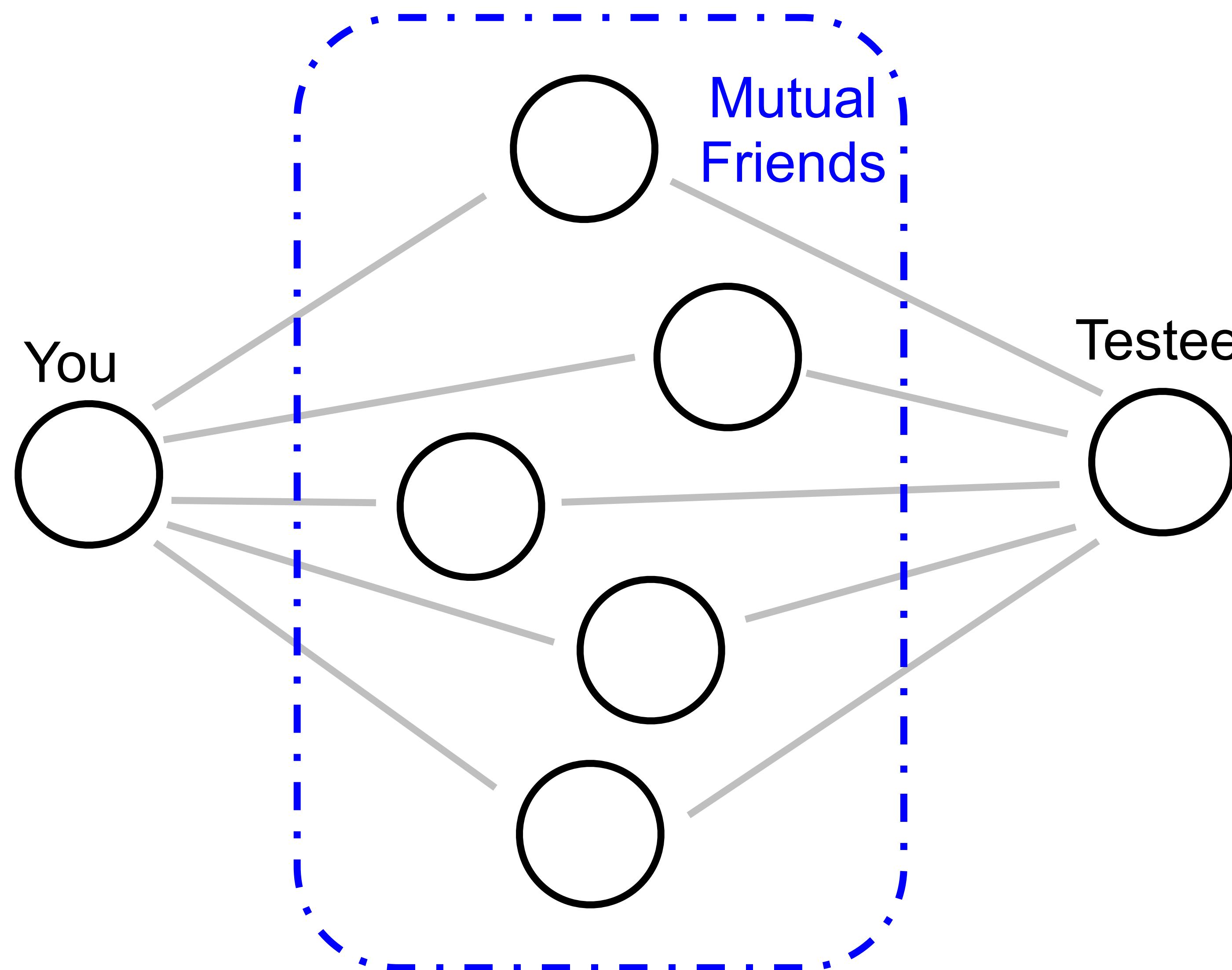
October 2013

Dispersion Insight



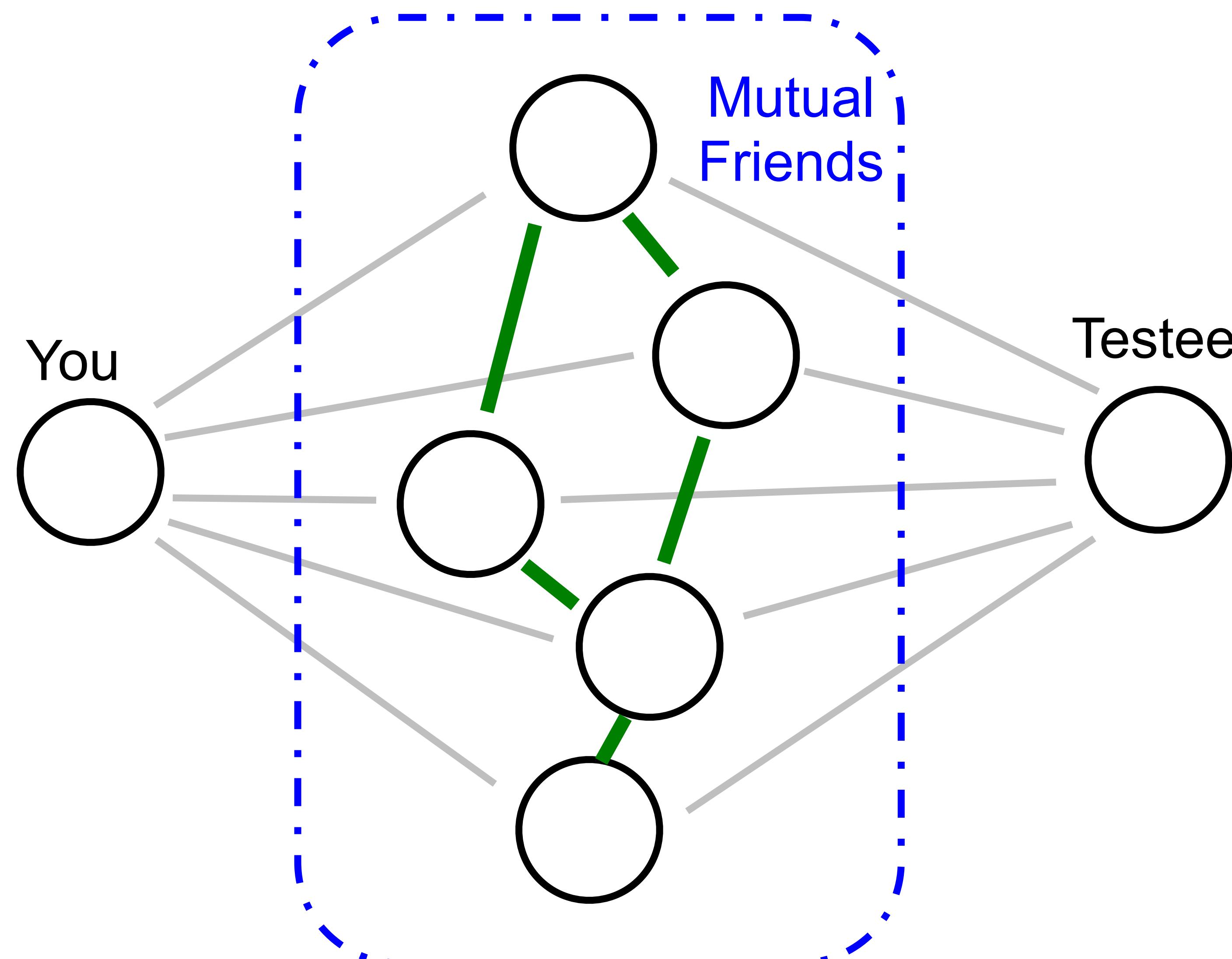
Dispersion: The extent to which two people's mutual friends are not directly connected

Dispersion



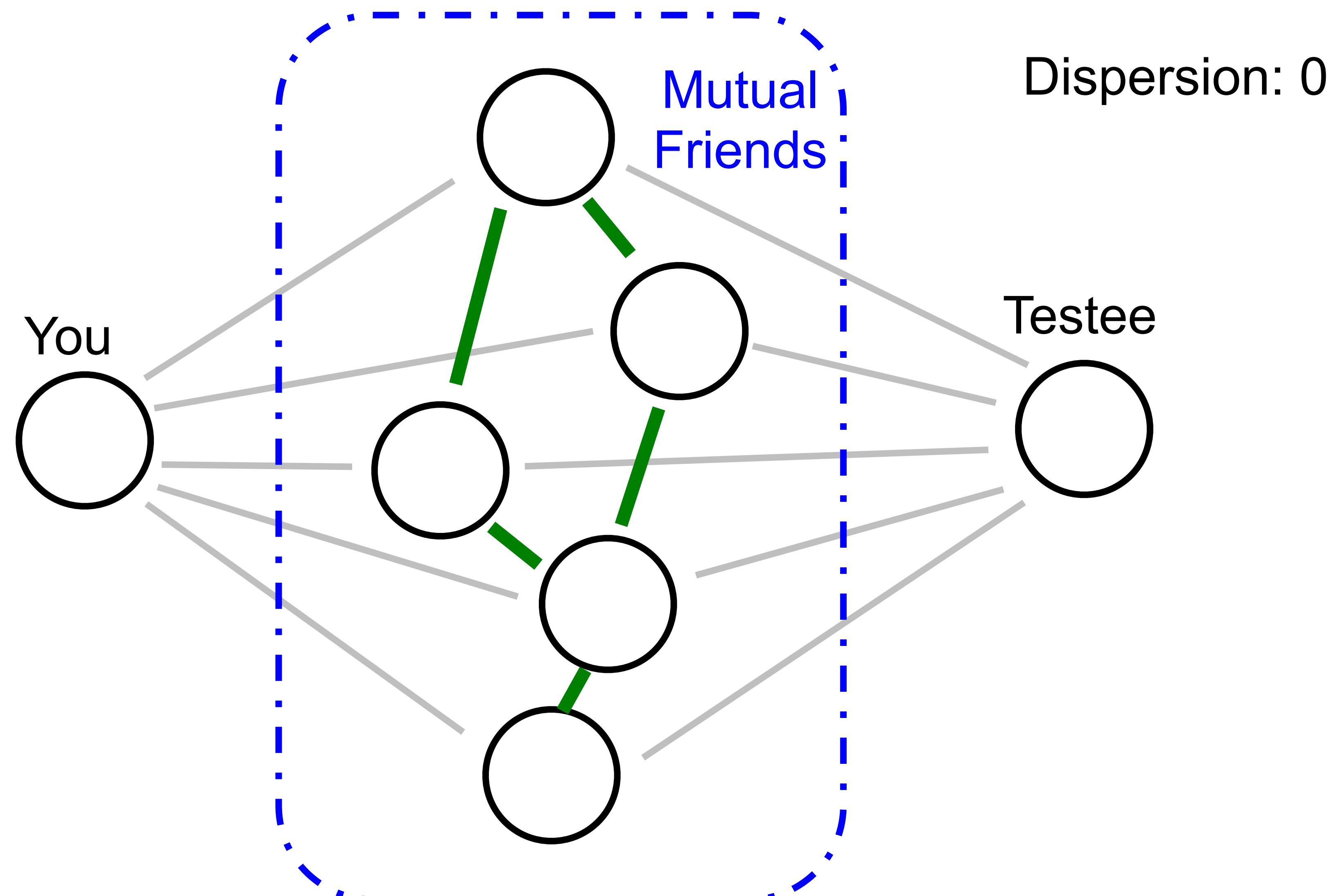
Dispersion: The extent to which two people's mutual friends are not directly connected

Dispersion



Dispersion: The extent to which two people's mutual friends are not directly connected

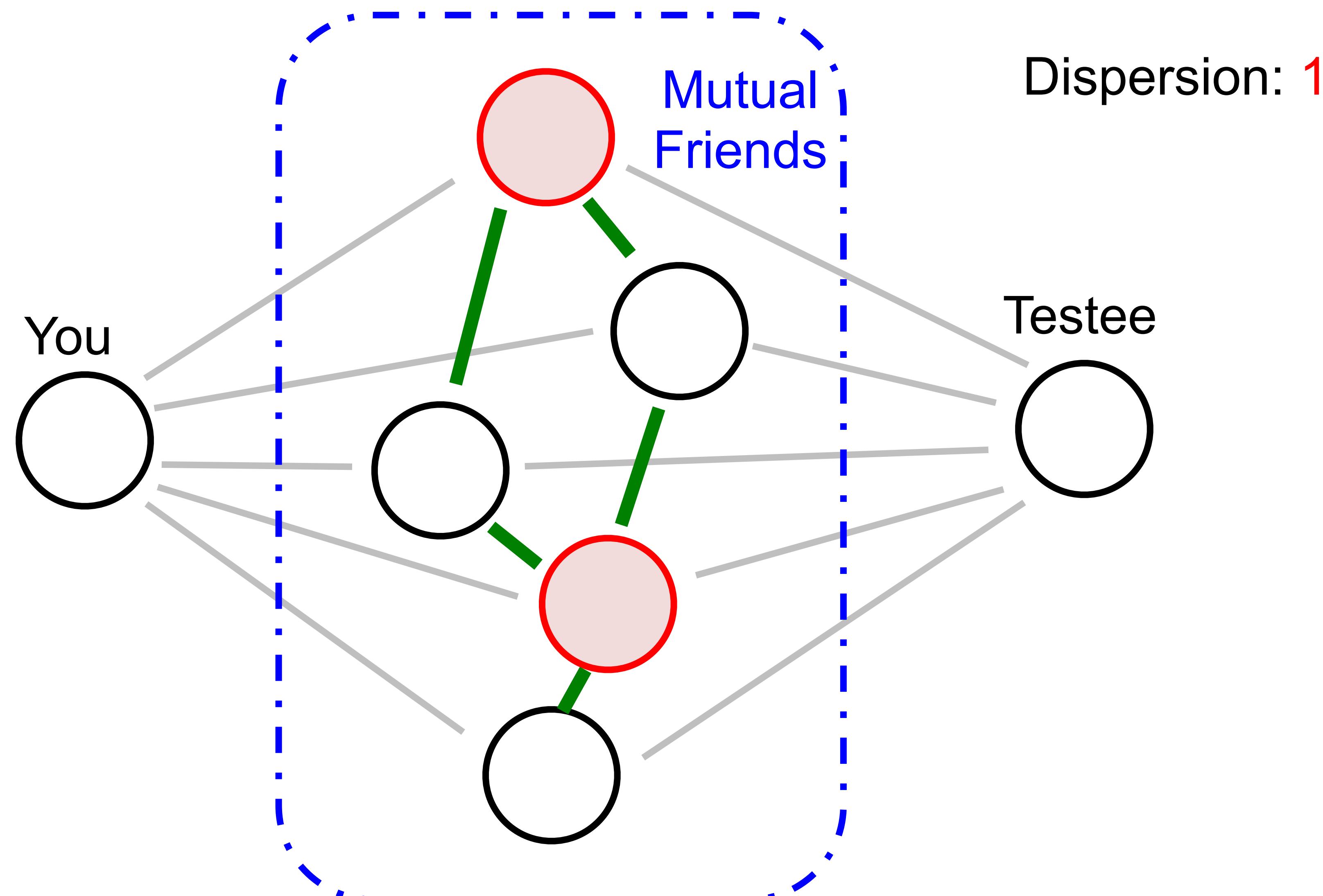
Dispersion



Dispersion: 0

Dispersion: The extent to which two people's mutual friends are not directly connected

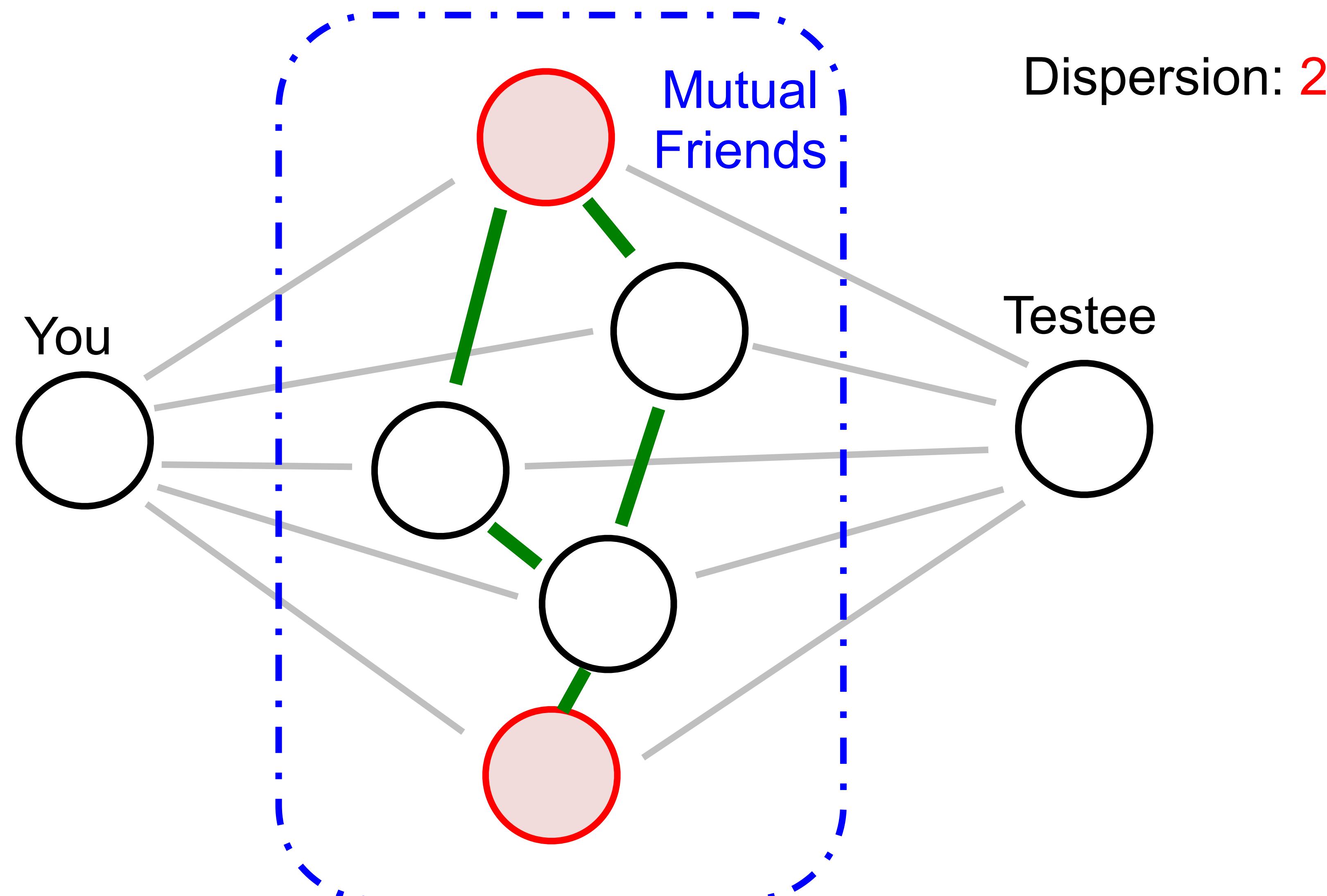
Dispersion



Dispersion: 1

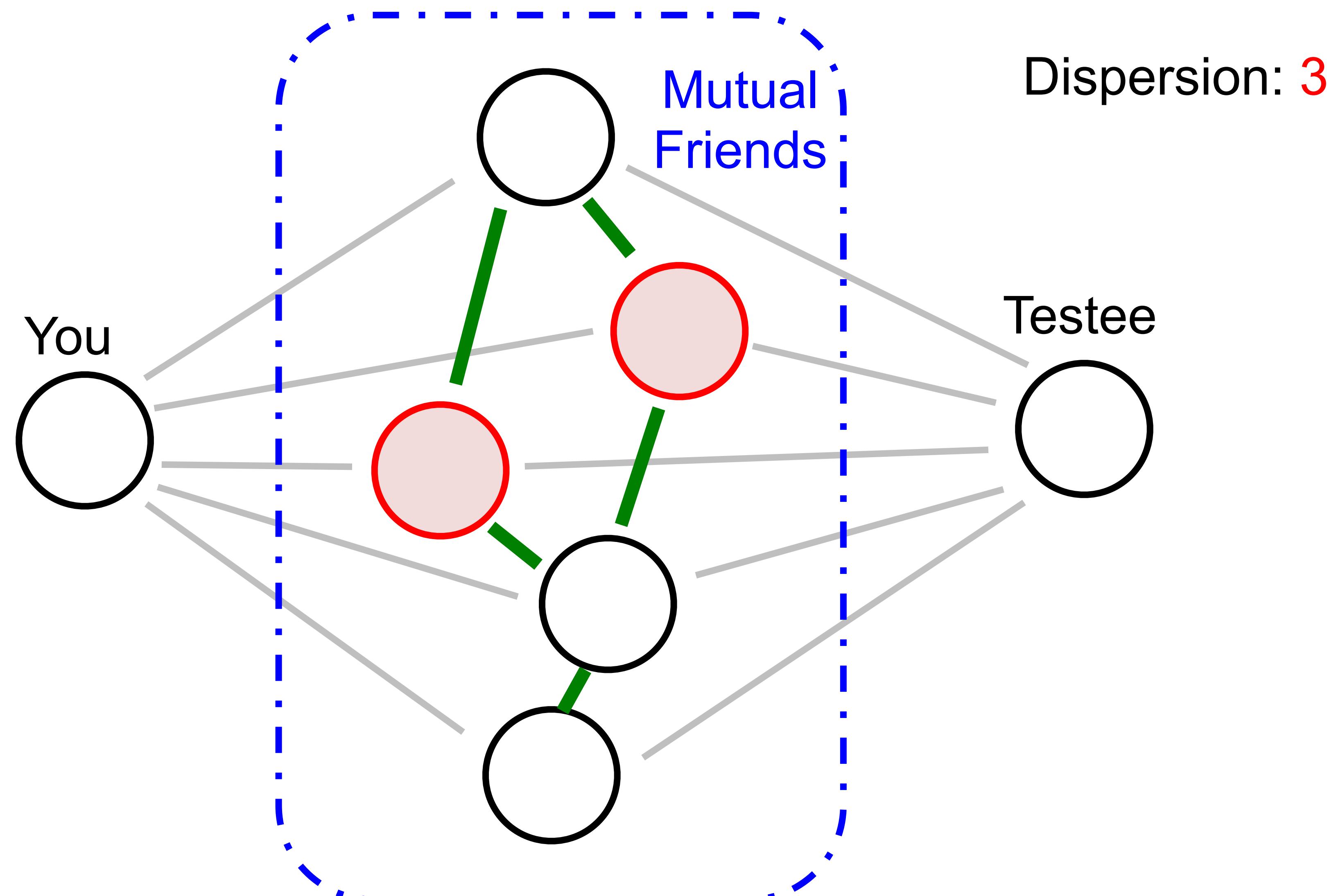
Dispersion: The extent to which two people's
mutual friends are not directly connected

Dispersion



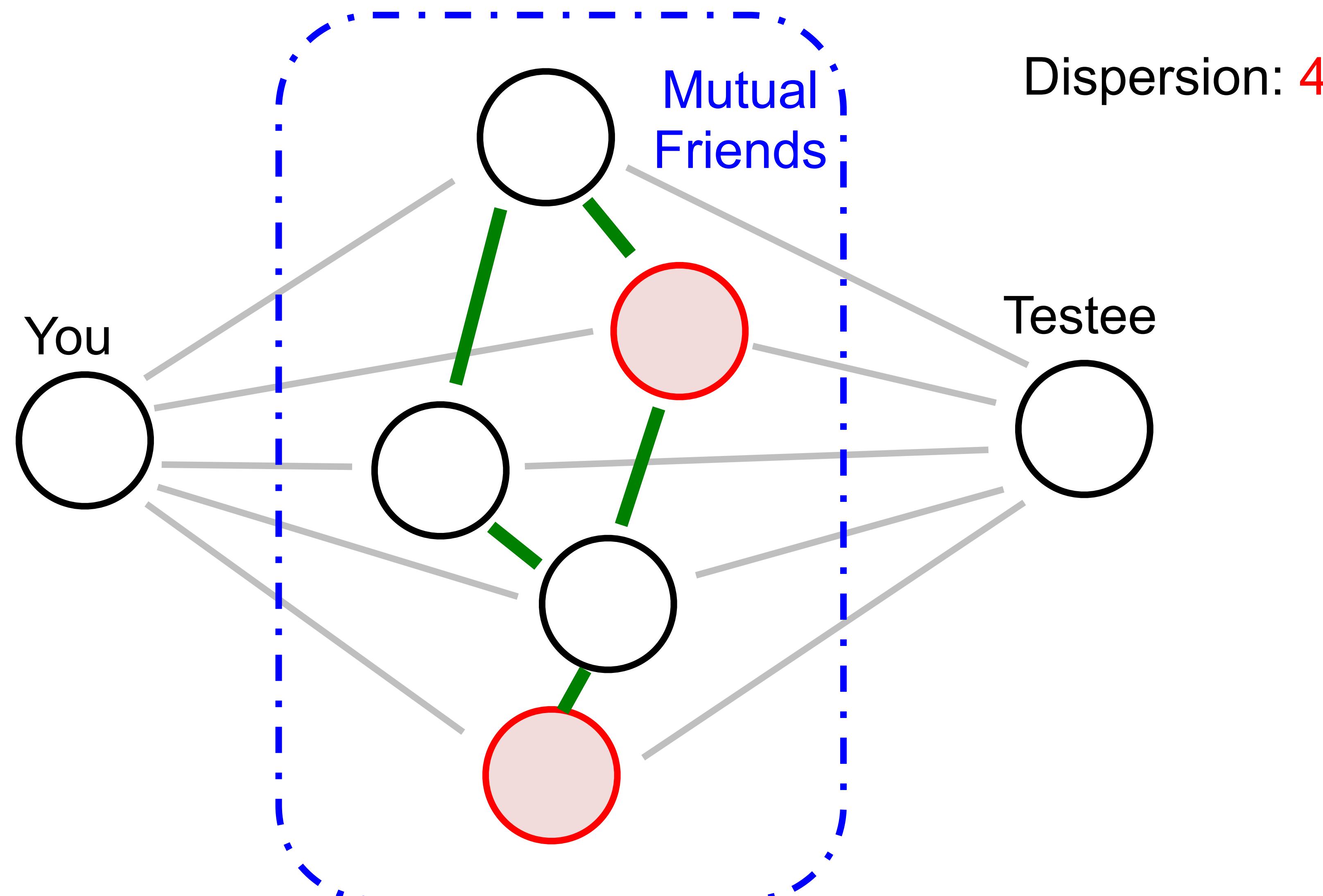
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Dispersion



Dispersion: The extent to which two people's mutual friends are not directly connected

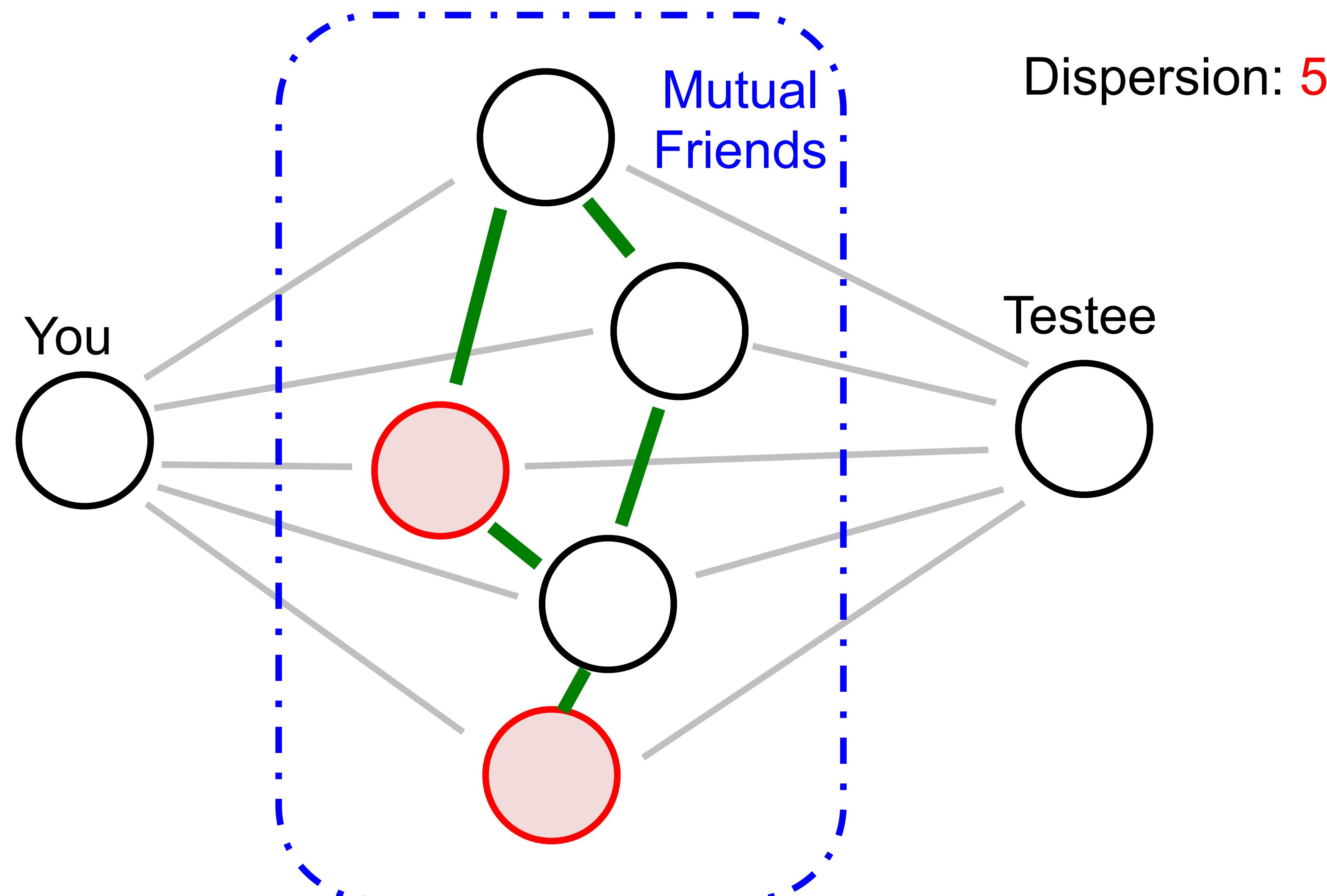
Dispersion



Dispersion: 4

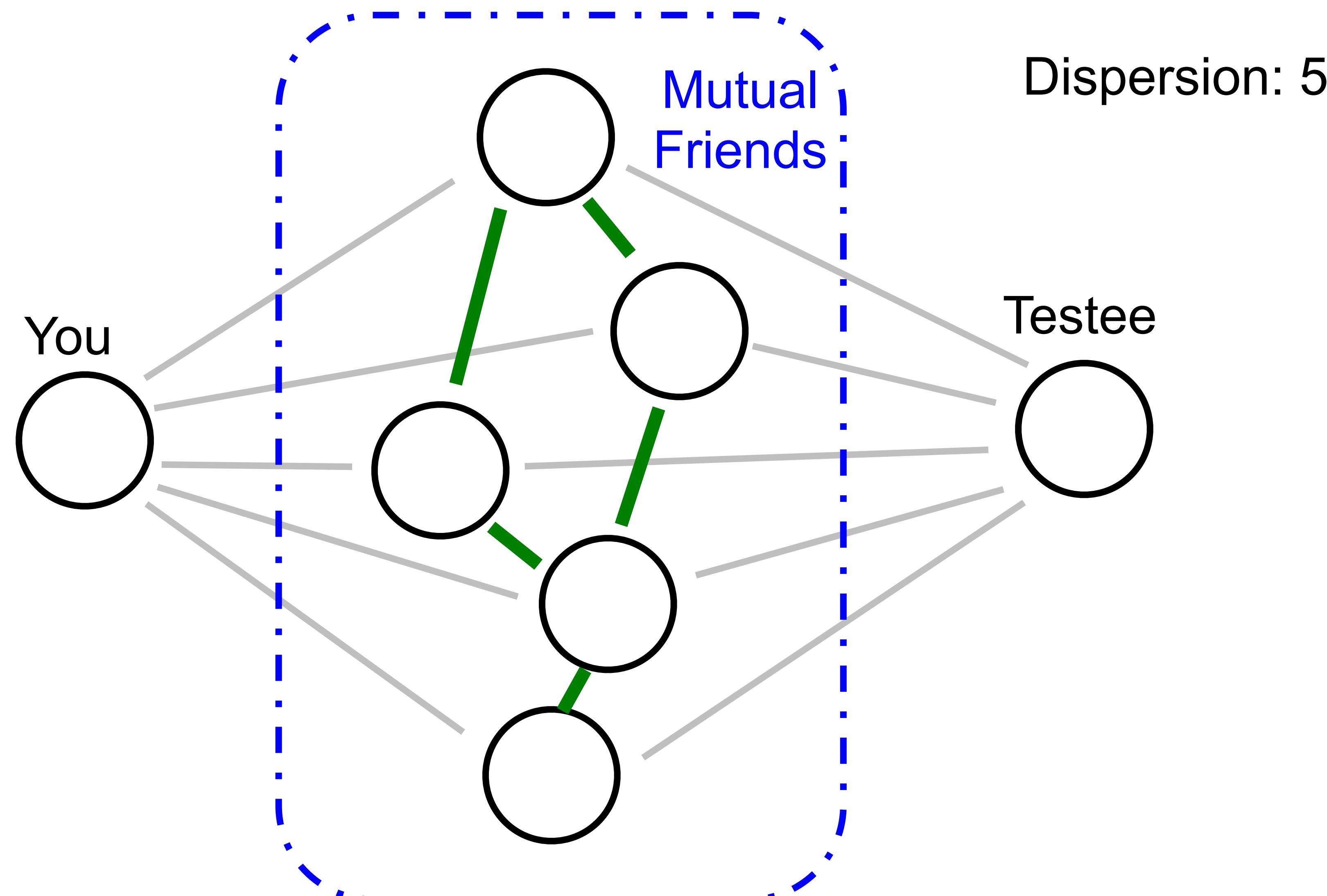
Dispersion: The extent to which two people's mutual friends are not directly connected

Dispersion



Dispersion: The extent to which two people's mutual friends are not directly connected

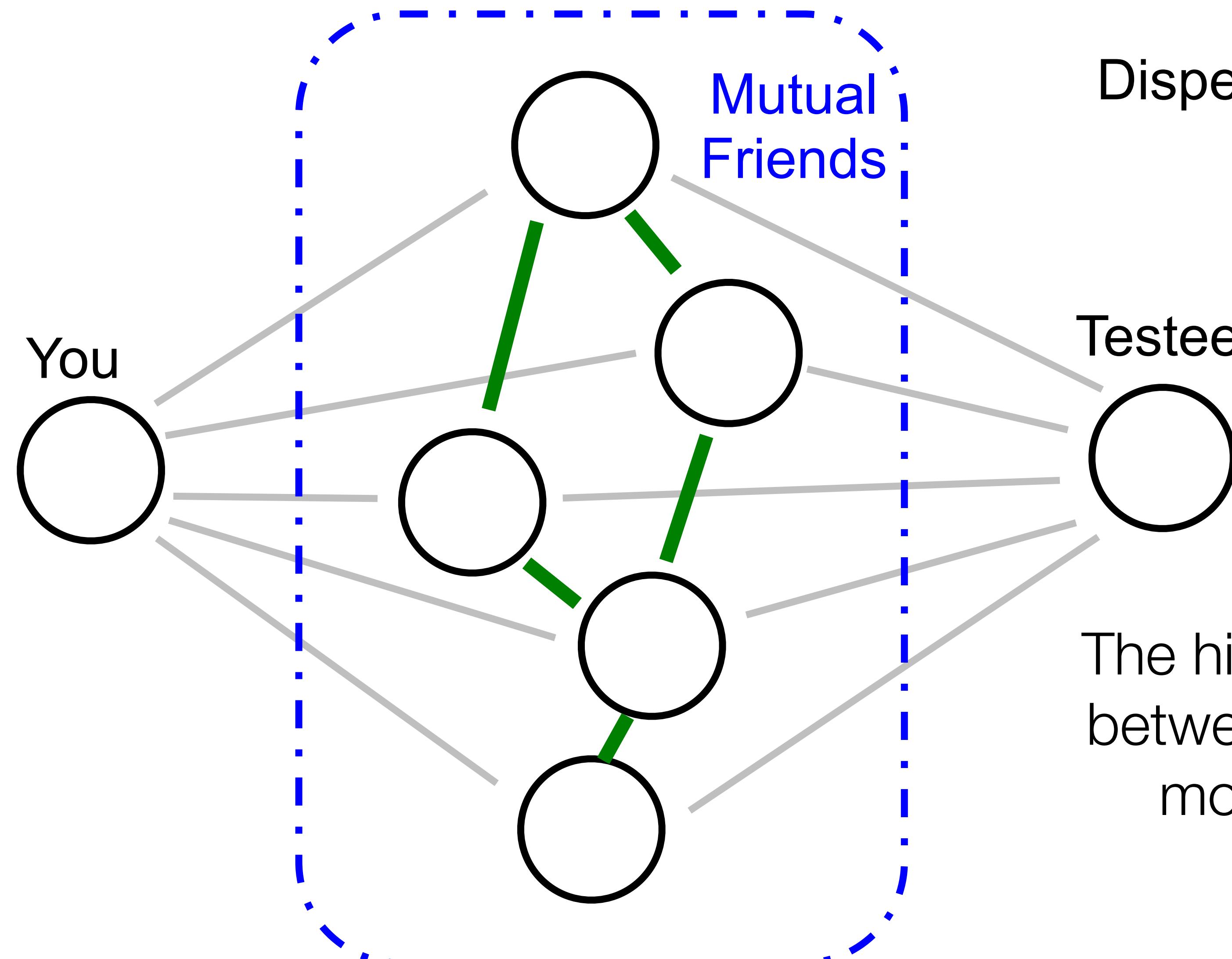
Dispersion



Dispersion: 5

Dispersion: The extent to which two people's
mutual friends are not directly connected

Dispersion



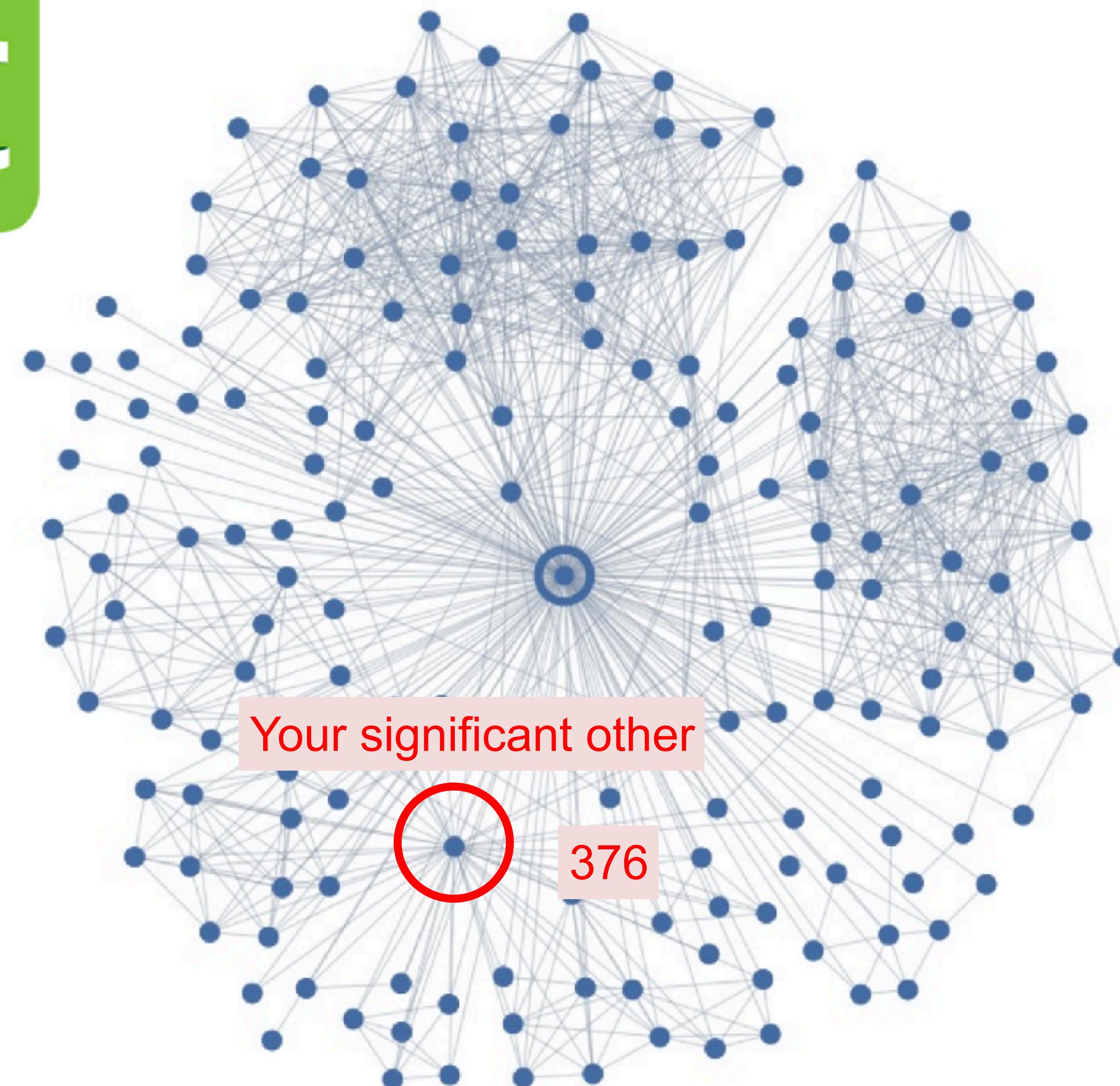
Dispersion: 5

Testee

The higher the dispersion
between two people, the
more likely they are
lovers!

Dispersion: The extent to which two people's
mutual friends are not directly connected

Who Do You Love?



References and Advanced Reading

References:

- Wikipedia on graphs: [https://en.wikipedia.org/wiki/Graph_\(discrete_mathematics\)](https://en.wikipedia.org/wiki/Graph_(discrete_mathematics))
- Wolfram Graph theory: <http://mathworld.wolfram.com/Graph.html>

Advanced Reading:

- Facebook graph API: <https://developers.facebook.com/docs/graph-api>
- Different graph lecture: <https://www.youtube.com/watch?v=yIWAB6CMYiY>

Extra Slides

Extra Slides