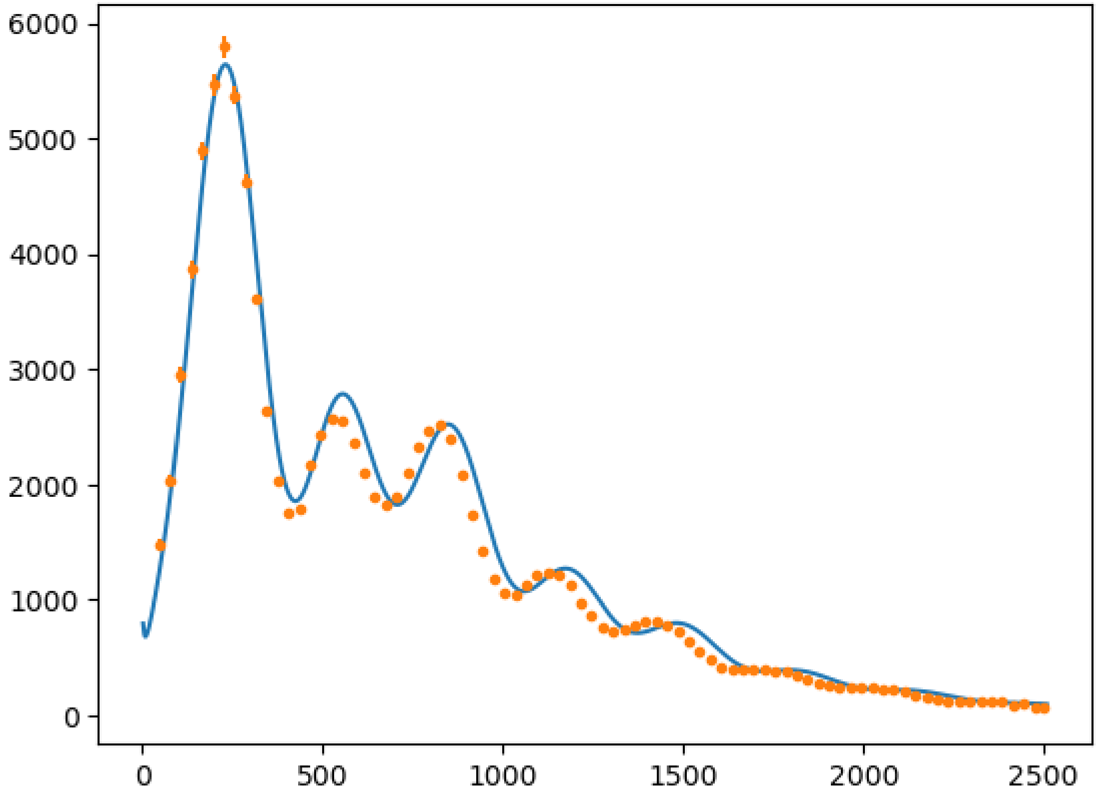
1)

In [88]:

```
# Sample script

import numpy as np
import camb
from matplotlib import pyplot as plt
import time


def get_spectrum(pars,lmax=3000,fix_tau=False):
    #print('pars are ',pars)
    H0=pars[0]
    ombh2=pars[1]
    omch2=pars[2]
    if fix_tau:
        tau=0.0540
        As=pars[3]
        ns=pars[4]
    else:
        tau=pars[3]
        As=pars[4]
        ns=pars[5]
    pars=camb.CAMBparams()
    pars.set_cosmology(H0=H0,ombh2=ombh2,omch2=omch2,mnu=0.06,omk=0,tau=tau)
    pars.InitPower.set_params(As=As,ns=ns,r=0)
    pars.set_for_lmax(lmax,lens_potential_accuracy=0)
    results=camb.get_results(pars)
    powers=results.get_cmb_power_spectra(pars,CMB_unit='muK')
    cmb=powers['total']
    tt=cmb[:,0]    #you could return the full power spectrum here if you wantec
    return tt[2:]


plt.ion()

pars=np.asarray([60,0.02,0.1,0.05,2.00e-9,1.0])
planck=np.loadtxt('COM_PowerSpect_CMB-TT-full_R3.01.txt',skiprows=1)
ell=planck[:,0]
spec=planck[:,1]
errs=0.5*(planck[:,2]+planck[:,3])
model=get_spectrum(pars)
model=model[:len(spec)]
resid=spec-model
chisq=np.sum( (resid/errs)**2)
print("chisq is ",chisq," for ",len(resid)-len(pars)," degrees of freedom.")
#read in a binned version of the Planck PS for plotting purposes
planck_binned=np.loadtxt('COM_PowerSpect_CMB-TT-binned_R3.01.txt',skiprows=1)
errs_binned=0.5*(planck_binned[:,2]+planck_binned[:,3]);
plt.clf()
plt.plot(ell,model)
plt.errorbar(planck_binned[:,0],planck_binned[:,1],errs_binned,fmt='.')
plt.show()
```

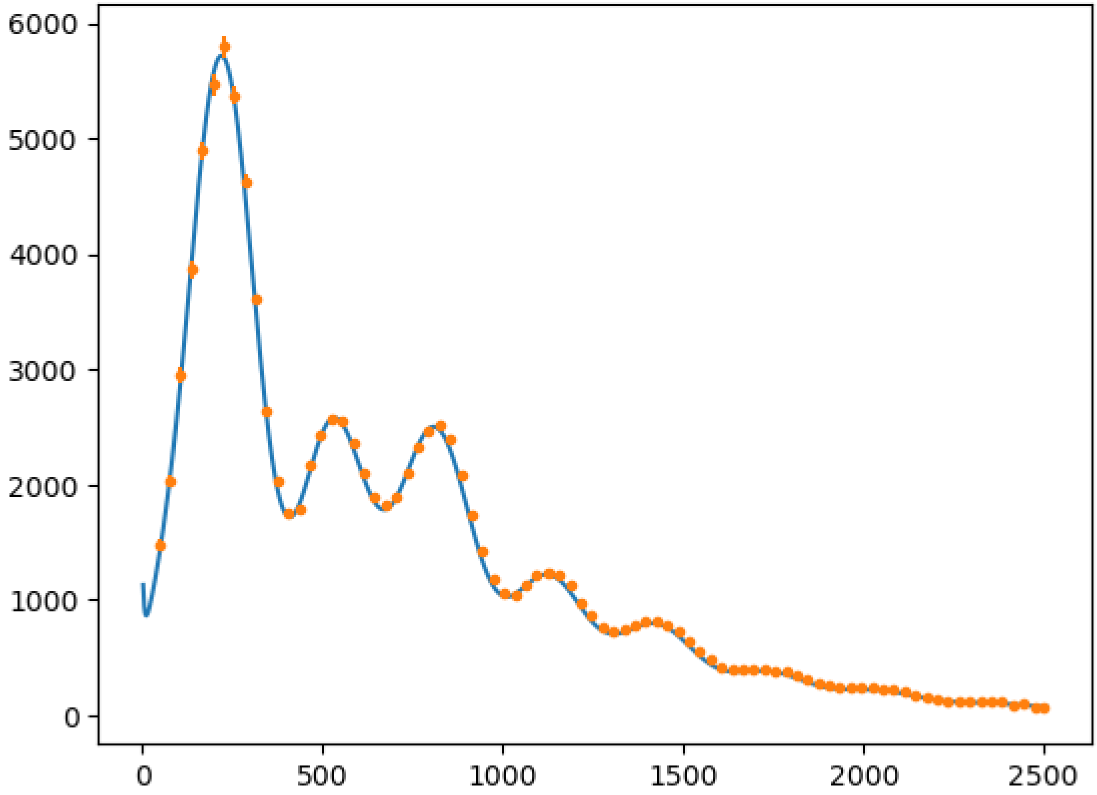
```
chisq is  15267.937435709791  for  2501  degrees of freedom.
```



In [89]:

```
pars=np.asarray([69,0.022,0.12,0.06,2.1e-9,0.95])
model=get_spectrum(pars)
model=model[:len(spec)]
resid=spec-model
chisq=np.sum( (resid/errs)**2)
print("chisq is ",chisq," for ",len(resid)-len(pars)," degrees of freedom.")
#read in a binned version of the Planck PS for plotting purposes
errs_binned=0.5*(planck_binned[:,2]+planck_binned[:,3]);
plt.clf()
plt.plot(ell,model)
plt.errorbar(planck_binned[:,0],planck_binned[:,1],errs_binned,fmt='.')
plt.show()
```

```
chisq is  3272.203673904469  for  2501  degrees of freedom.
```



In [9]:

```
print(3272.2/len(planck))
```

```
1.3052253689668927
```

For the initial parameters got a chi-squared of 15267.9 which is not a good fit as it is much higher than the degrees of freedom. Using the values given in the problem we obtain a chi-squared value of 3272.2 which is better only 1.3 times the degrees of freedom.

2)

In [90]:

```
def calc_num_der(fun,p,lendata,fixed=False):
    grad=np.zeros([lendata,len(p)], dtype=np.float64)
    y=fun(p,fix_tau=fixed)[:len(spec)]
    for i in range(len(p)):
        dx = 0.01*p[i]
        dx_array = np.zeros(np.shape(p), dtype=np.float64)
        dx_array[i] = dx
        grad[:,i]=(fun(p+dx_array,fix_tau=fixed)[:len(spec)] - fun(p-dx_array,f
    return y,grad

for j in range(6):
    pred1,grad1=calc_num_der(get_spectrum,pars,len(spec))
    chi2=sum(np.array(((pred1-spec)/errs))**2)
    r1=spec-pred1
    r1=np.matrix(r1).T
    grad1=np.matrix(grad1)
    lhs1=grad1.T@grad1
    rhs1=grad1.T@r1
    dp1=np.linalg.pinv(lhs1)@(rhs1)
    for jj in range(len(pars)):
        pars[jj]=pars[jj]+dp1[jj]
    print(pars,chi2)
```

```
[6.90000000e+01 2.20000000e-02 1.20000000e-01 6.00000000e-02
 2.11779228e-09 9.50000000e-01] 3272.203673904469
[6.90000000e+01 2.20000000e-02 1.20000000e-01 6.00000000e-02
 2.11779232e-09 9.50000000e-01] 3059.3763413101424
[6.90000000e+01 2.20000000e-02 1.20000000e-01 6.00000000e-02
 2.11779232e-09 9.50000000e-01] 3059.3760330128175
[6.90000000e+01 2.20000000e-02 1.20000000e-01 6.00000000e-02
 2.11779232e-09 9.50000000e-01] 3059.3760330159143
[6.90000000e+01 2.20000000e-02 1.20000000e-01 6.00000000e-02
 2.11779232e-09 9.50000000e-01] 3059.3760330159057
[6.90000000e+01 2.20000000e-02 1.20000000e-01 6.00000000e-02
 2.11779232e-09 9.50000000e-01] 3059.3760330159057
```

In [17]:

```
print(pars)
N = np.mean((spec-pred1)**2)
print(N)
par_errs=np.sqrt(N*np.diag(np.linalg.inv(lhs1)))
print(par_errs)

f = open("planck_fit_params.txt", "x")
f.write('Best-fit parameters: {}\n'.format(np.array2string(pars)))
f.write('Errors in parameters: {}'.format(np.array2string(par_errs)))
f.close()
```

```
[6.90000000e+01 2.20000000e-02 1.20000000e-01 6.00000000e-02
 2.11779232e-09 9.50000000e-01]
32885.267283882284
[1.73048608e+00 3.58805686e-04 3.63701363e-03 4.57945090e-02
 1.83574203e-10 1.10856860e-02]
```

3)

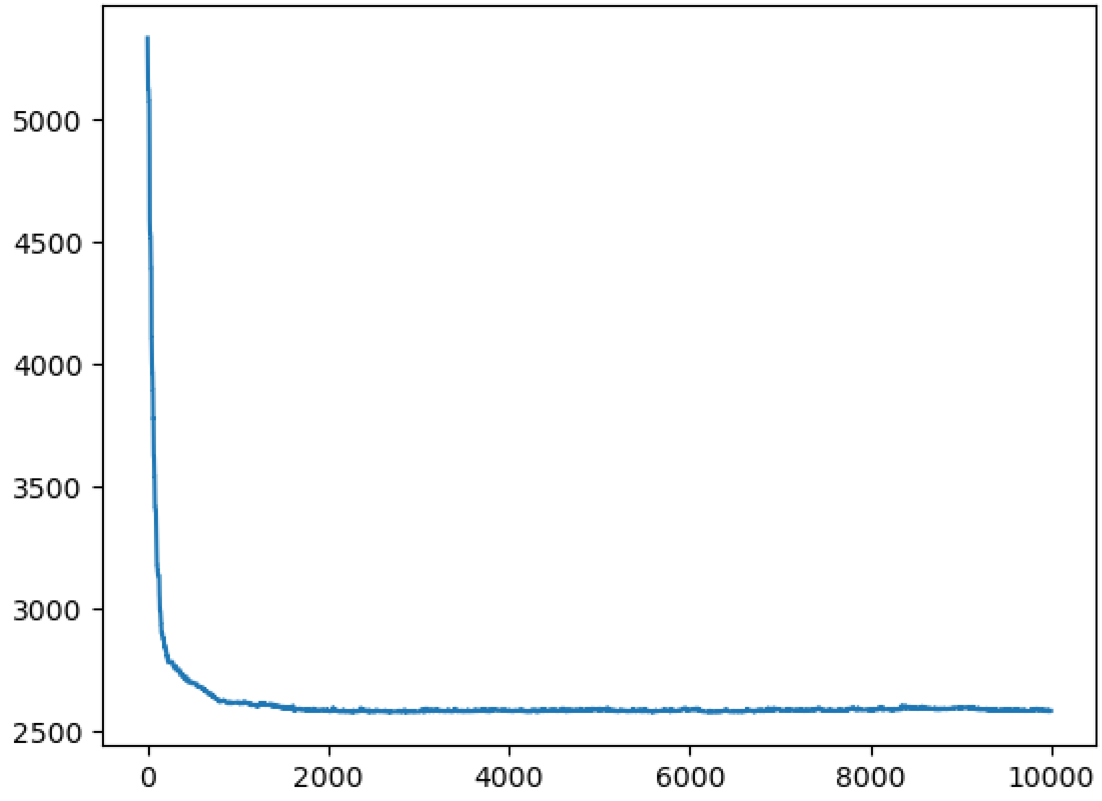
In [36]:

```
from datetime import datetime
import sys
import warnings
warnings.filterwarnings('ignore')
warnings.simplefilter('ignore')
from IPython.utils import io

def get_chi2(pars,y,errs):
    with io.capture_output() as captured:
        pred=get_spectrum(pars)[:len(y)]
    return sum(np.array(((pred-y)/errs))**2)

mcpars=pars
steps=10000
chains=np.zeros([steps,len(mcpars)+1])
accepted=0
mcpars=mcpars+np.random.randn(len(mcpars))*par_errs
chi2=get_chi2(mcpars,spec,errs)
chains[0,:]=np.append(chi2,mcpars)
print(datetime.now().strftime("\n%H:%M:%S.%f"))
try:
    for i in range(1,steps):
        pars_new=mcpars+0.05*np.random.randn(len(mcpars))*par_errs
        chi2_new=get_chi2(pars_new,spec,errs)
        prob=np.exp(-0.5*(chi2_new-chi2))
        accept=prob>np.random.rand(1)
        if accept:
            accepted+=1
            mcpars=pars_new
            chi2=chi2_new
        chains[i,:]=np.append(chi2,mcpars)
        if i%50==0:
            print('Progress: {}% | Accepted: {}% | Chi2: {}'.format(np.round((
                   np.round((accepted/i)*100,1),np.round(chi2,1)), end="\r", fl
        i+=1
except KeyboardInterrupt:
    pass
np.savetxt("planck_chain.txt",chains,delimiter=',')
print(datetime.now().strftime("\n%H:%M:%S.%f"))
```

```
02:08:10.081581
Progress: 99.5% | Accepted: 23.0% | Chi2: 2584.0
03:43:18.957924
```

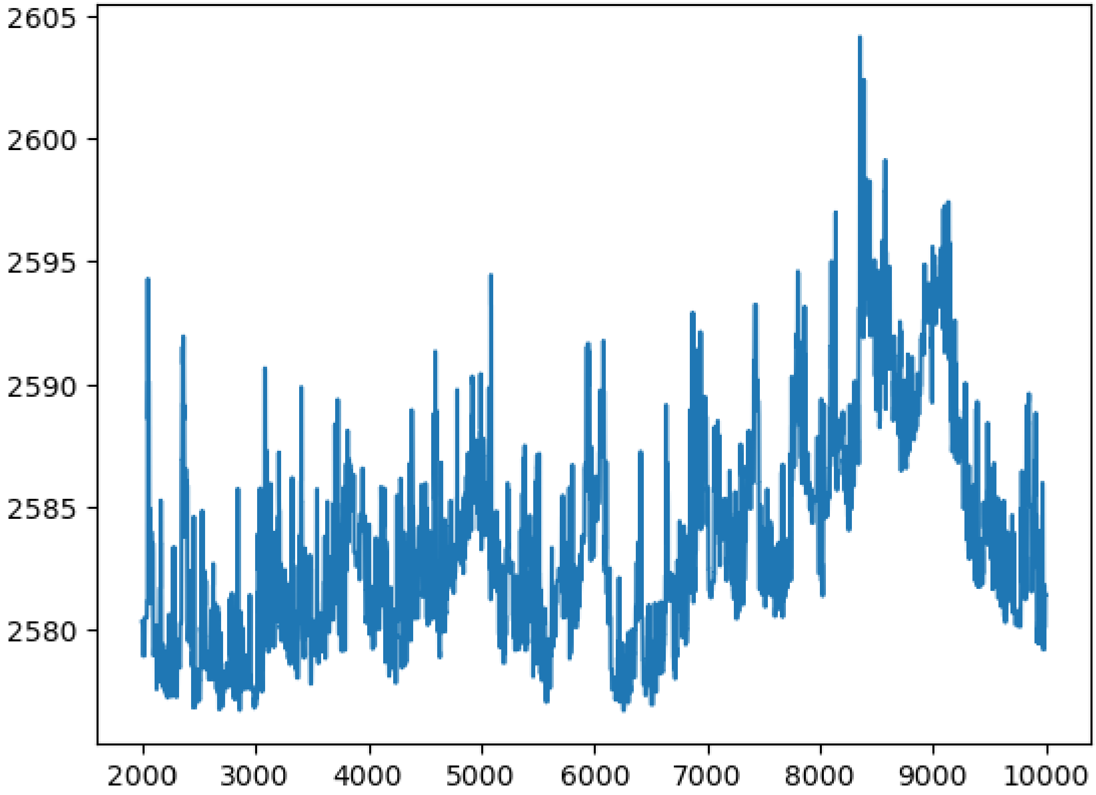
In [38]:
```
plt.plot(range(steps),chains[:,0])
```

Out[38]:
```
[<matplotlib.lines.Line2D at 0x156c93400>]
```



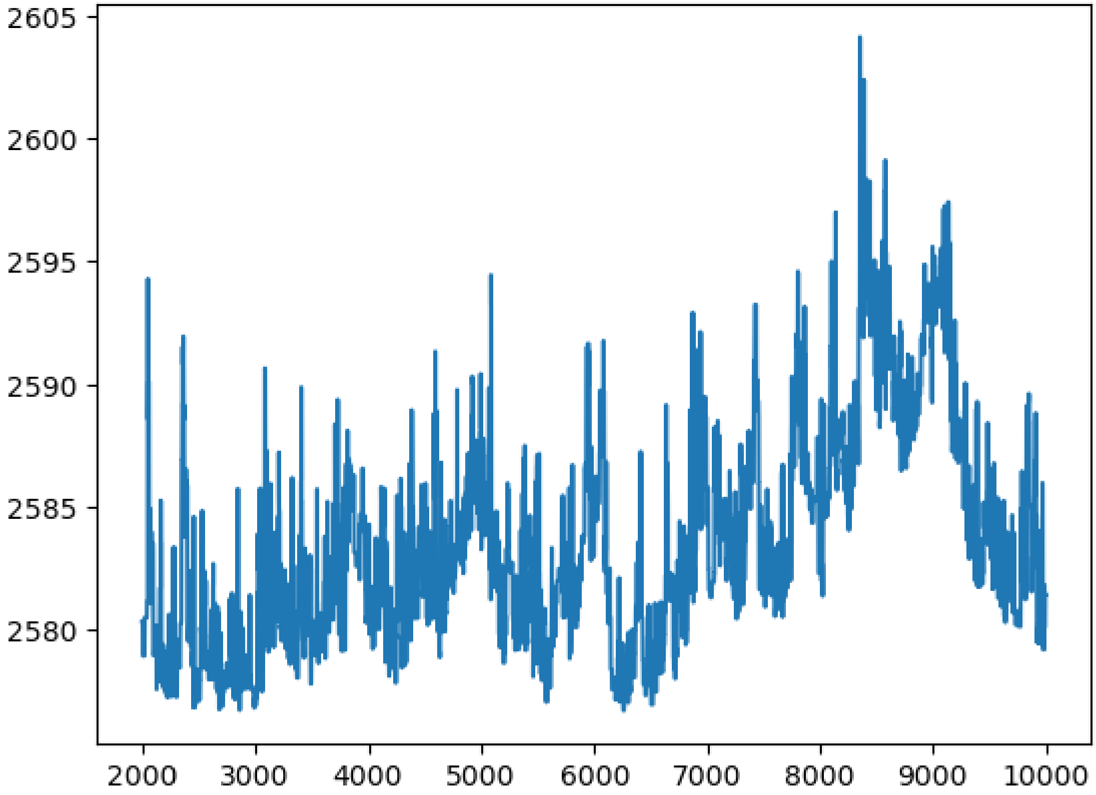
In [39]:
```
plt.plot(range(steps)[2000:],chains[2000:,0])
```

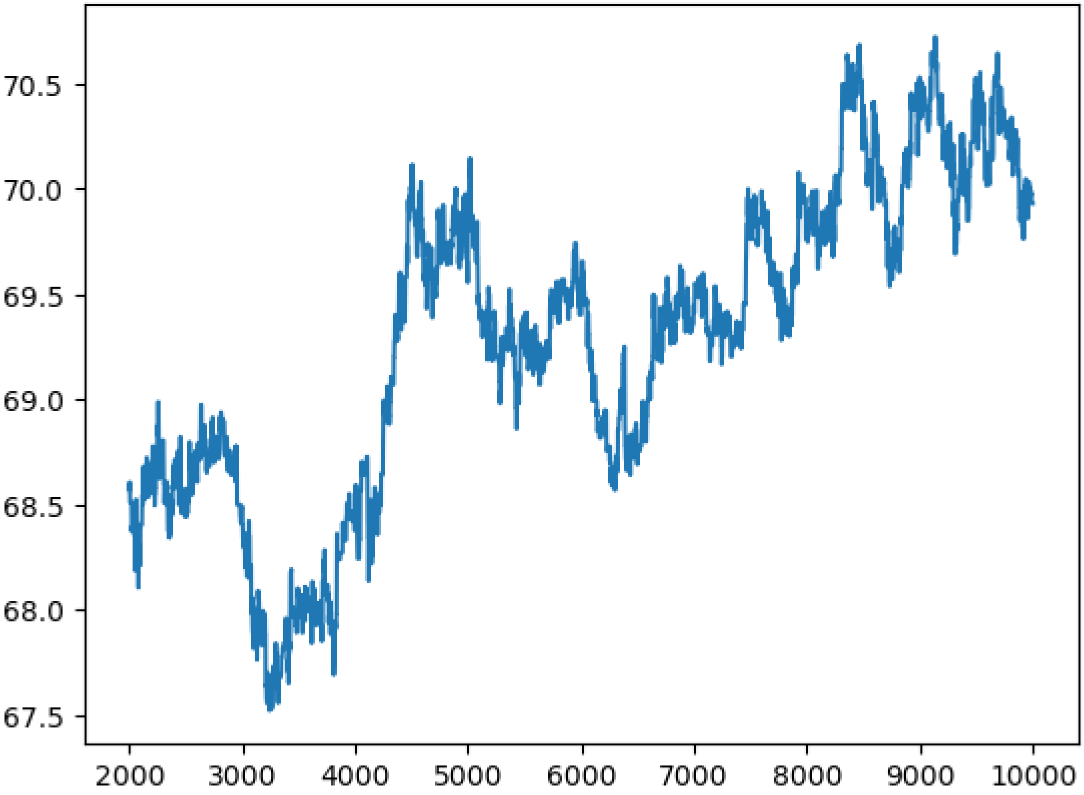
Out[39]:
```
[<matplotlib.lines.Line2D at 0x156c5d340>]
```

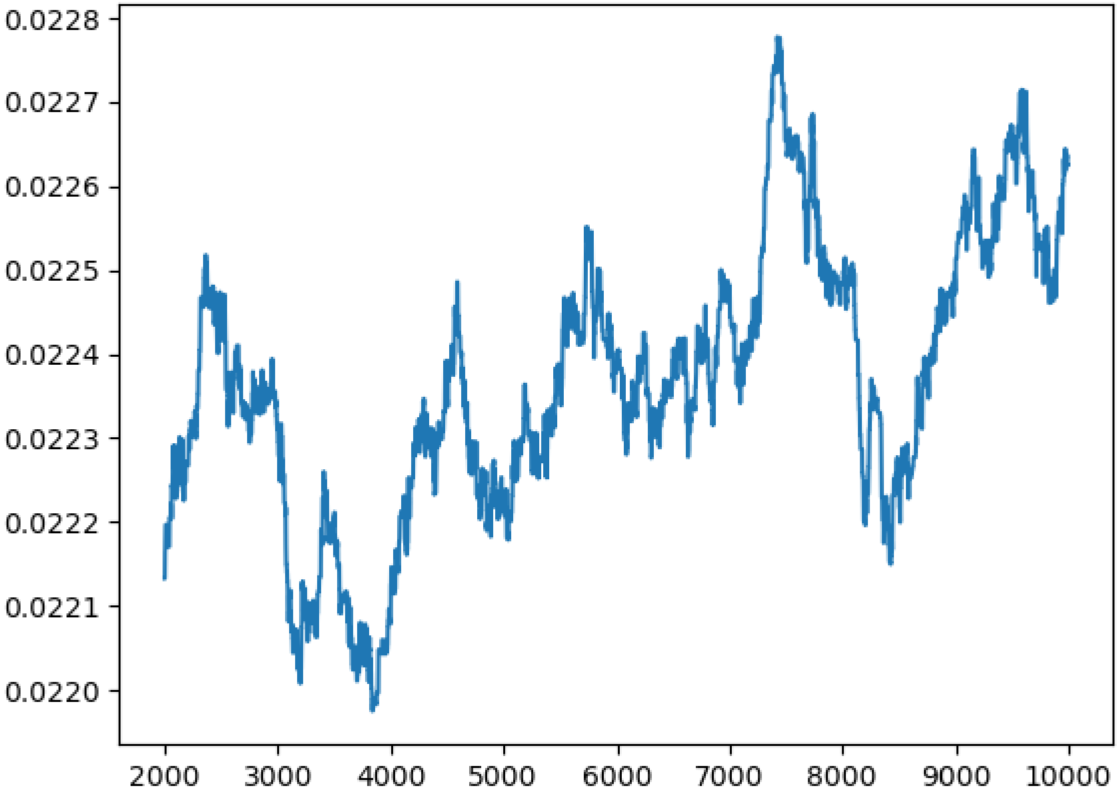


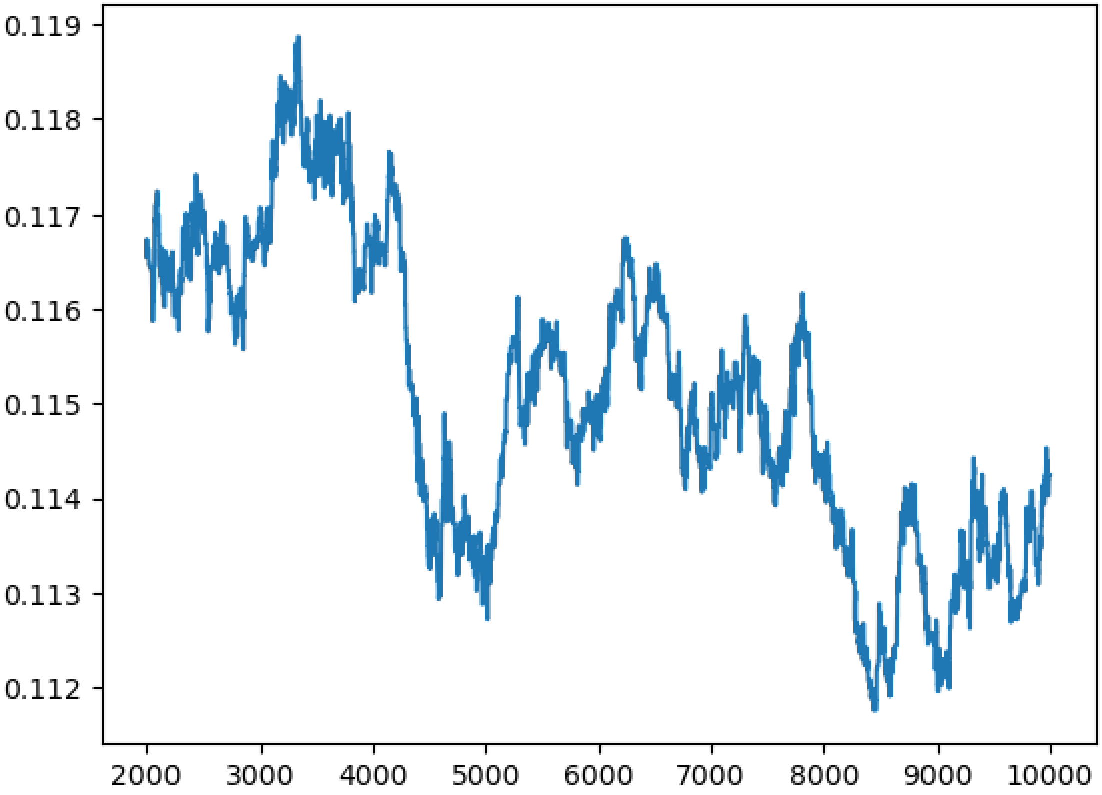
In [49]:

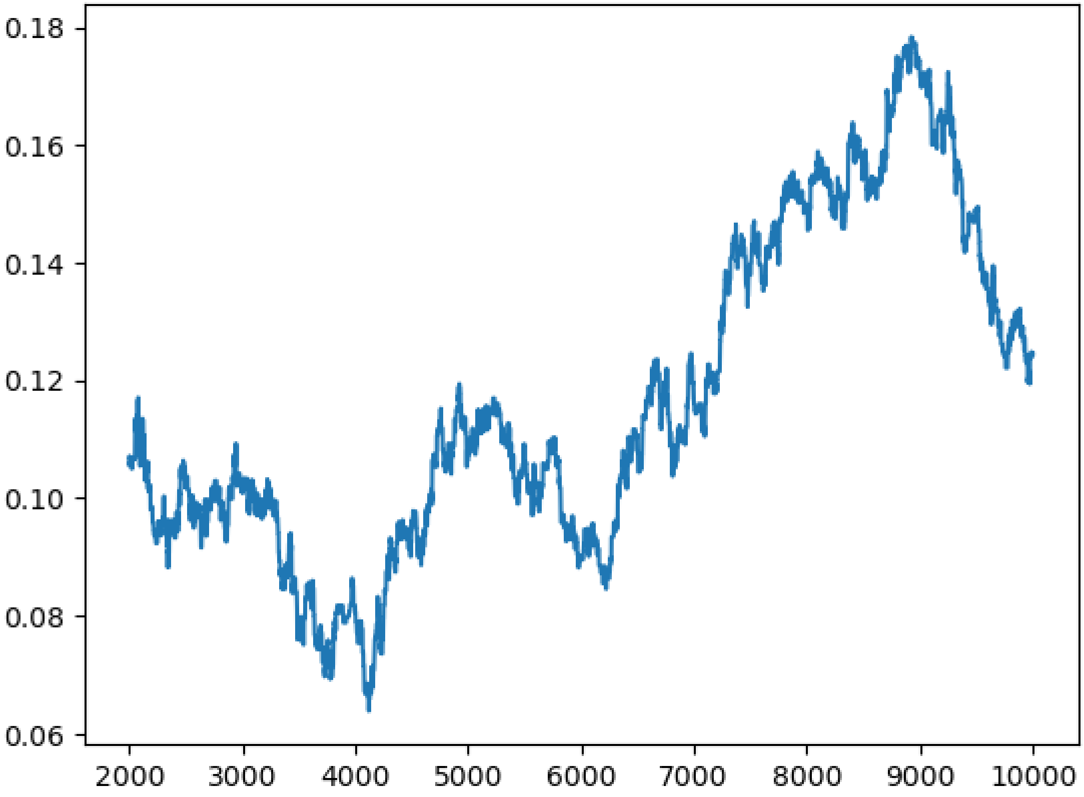
```
for i in range(6):
    plt.plot(range(steps)[2000:],chains[2000:,i])
    plt.show()
```

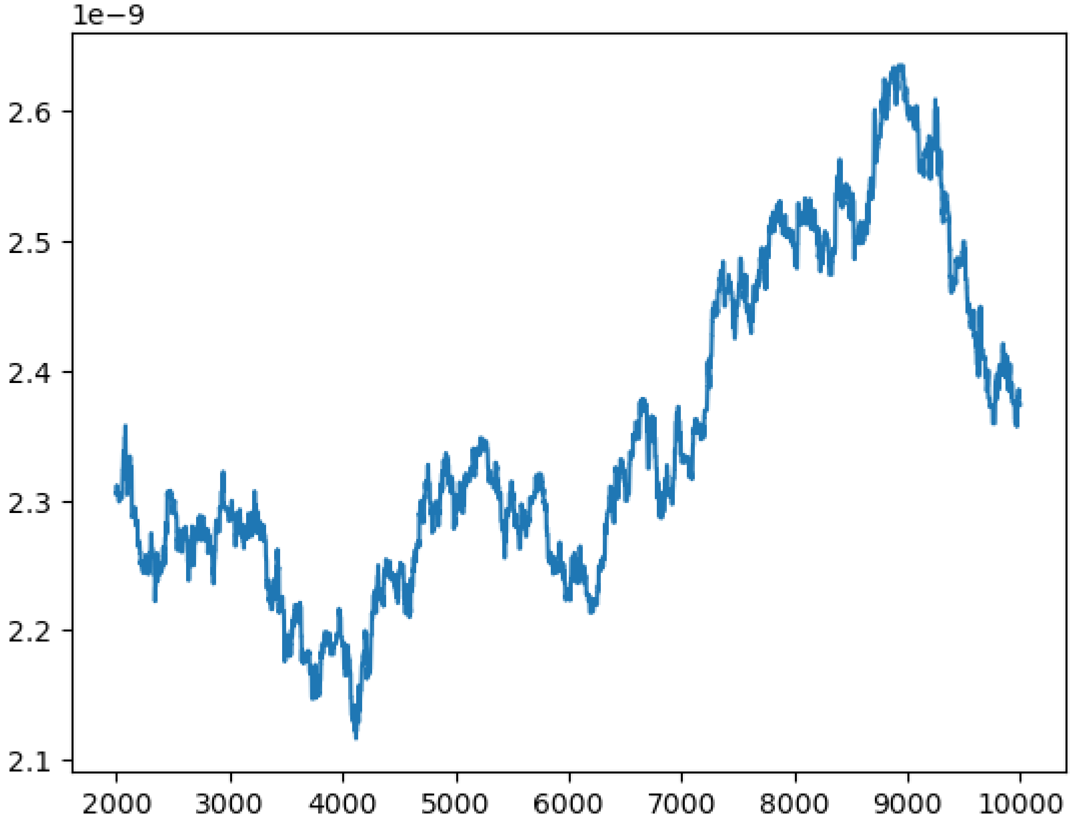



70.5
70.0
69.5
69.0
68.5
68.0
67.5
2000
3000
4000
5000
6000
7000
8000
9000
10000


0.0228
0.0227
0.0226
0.0225
0.0224
0.0223
0.0222
0.0221
0.0220
2000
3000
4000
5000
6000
7000
8000
9000
10000


0.119
0.118
0.117
0.116
0.115
0.114
0.113
0.112
2000
3000
4000
5000
6000
7000
8000
9000
10000


0.18
0.16
0.14
0.12
0.10
0.08
0.06
2000
3000
4000
5000
6000
7000
8000
9000
10000



In [58]:

```
final_pars = []
for i in range(1,np.shape(chains)[1]):
    val=np.mean(chains[:,i])
    scat=np.std(chains[:,i])
    final_pars.append([val,scat])
    print([val,scat])
```

```
[68.9698835114894, 1.0292952982205967]
[0.022132277253063482, 0.000575507025484718]
[0.11504521137584829, 0.0016655387029129887]
[0.11365470455314812, 0.026171587103233886]
[2.332186713082096e-09, 1.217928601906132e-10]
[0.9805359165875569, 0.006424800827851004]
```

In [68]:

```
h = final_pars[0][0]/100
h_err = final_pars[0][1]/100
h2_err = 2*(h_err/h)
temp1 = (final_pars[1][1]/final_pars[1][0]+h2_err)*(final_pars[1][0]/h**2)
temp2 = (final_pars[2][1]/final_pars[2][0]+h2_err)*(final_pars[2][0]/h**2)
print(1-final_pars[1][0]/h**2-final_pars[2][0]/h**2,temp1+temp2)
```

```
0.7116208464106972 0.013318649038606564
```

Does not seem like the chains are converged as they still have pattern to them and do not appear like noise. Calculated value for dark energy of 0.71 ± 0.01.

4)

In [92]:

```
pars=np.asarray([69,0.022,0.12,0.06,2.1e-9,0.95])
fix_pars=np.asarray([pars[0],pars[1],pars[2],pars[4],pars[5]])
```

```
for j in range(6):
    pred1,grad1=calc_num_der(get_spectrum,fix_pars,len(spec), True)
    chi2=sum(np.array(((pred1-spec)/errs))**2)
    r1=spec-pred1
    r1=np.matrix(r1).T
    grad1=np.matrix(grad1)
    lhs1=grad1.T@grad1
    rhs1=grad1.T@r1
    dp1=np.linalg.pinv(lhs1)@(rhs1)
    for jj in range(len(fix_pars)):
        fix_pars[jj]=fix_pars[jj]+dp1[jj]
    print(fix_pars,chi2)
```

```
[6.90000000e+01 2.20000000e-02 1.20000000e-01 2.09265201e-09
 9.50000000e-01] 2992.352698026743
[6.9000000e+01 2.2000000e-02 1.2000000e-01 2.0926521e-09 9.5000000e-01] 3059.3
32996493575
[6.9000000e+01 2.2000000e-02 1.2000000e-01 2.0926521e-09 9.5000000e-01] 3059.3
320185444713
[6.9000000e+01 2.2000000e-02 1.2000000e-01 2.0926521e-09 9.5000000e-01] 3059.3
320185527587
[6.9000000e+01 2.2000000e-02 1.2000000e-01 2.0926521e-09 9.5000000e-01] 3059.3
320185527587
[6.9000000e+01 2.2000000e-02 1.2000000e-01 2.0926521e-09 9.5000000e-01] 3059.3
320185527587
```

In [93]:

```
N = np.mean((spec-pred1)**2)
print(N)
par_errs=np.sqrt(N*np.diag(np.linalg.inv(lhs1)))
print(par_errs)
```

```
32859.862070636555
[1.31637550e+00 2.86353704e-04 2.87430374e-03 1.67976238e-11
 7.54696105e-03]
```

In [99]:

```
warnings.filterwarnings('ignore')
warnings.simplefilter('ignore')

def get_chi2(pars,y,errs):
    with io.capture_output() as captured:
        pred=get_spectrum(pars,fix_tau=True)[:len(y)]
    return sum(np.array(((pred-y)/errs))**2)

# mcpars=fix_pars
mcpars=save_chains[-1,:]
steps=10000
chains=np.zeros([steps,len(mcpars)+1])
accepted=0
mcpars=mcpars+np.random.randn(len(mcpars))*par_errs
chi2=get_chi2(mcpars,spec,errs)
chains[0,:]=np.append(chi2,mcpars)
print(datetime.now().strftime("\n%H:%M:%S.%f"))
try:
    for i in range(1,steps):
        pars_new=mcpars+0.10*np.random.randn(len(mcpars))*par_errs
        chi2_new=get_chi2(pars_new,spec,errs)
        prob=np.exp(-0.5*(chi2_new-chi2))
        accept=prob>np.random.rand(1)
        if accept:
            accepted+=1
            mcpars=pars_new
            chi2=chi2_new
        chains[i,:]=np.append(chi2,mcpars)
        if i%50==0:
            print('Progress: {}% | Accepted: {}% | Chi2: {}'.format(np.round((
                   np.round((accepted/i)*100,1),np.round(chi2,1)), end="\r", fl
        i+=1
except KeyboardInterrupt:
    pass
np.savetxt("planck_chain_tauprior1.txt",chains,delimiter=',')
print(datetime.now().strftime("\n%H:%M:%S.%f"))
```

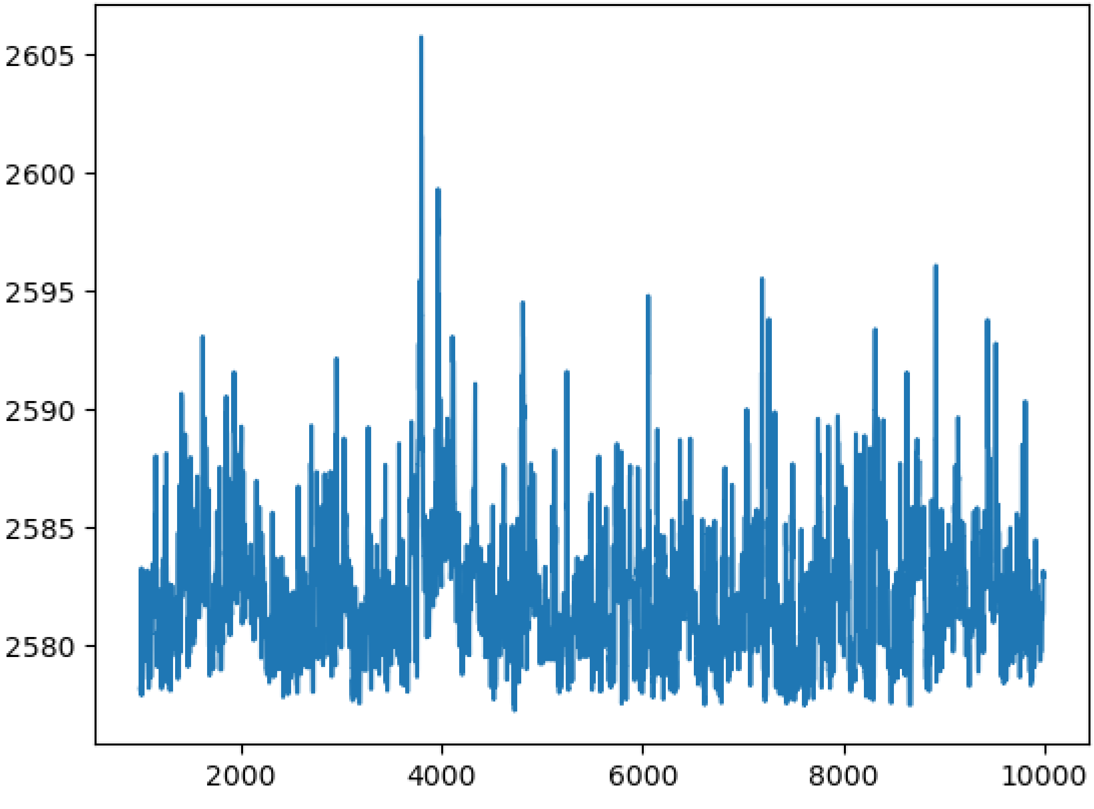
```
05:16:25.946081
Progress: 99.5% | Accepted: 26.6% | Chi2: 2581.1
06:58:47.220892
```

In [102…

```
plt.plot(range(steps)[1000:],chains[1000:,0])
```

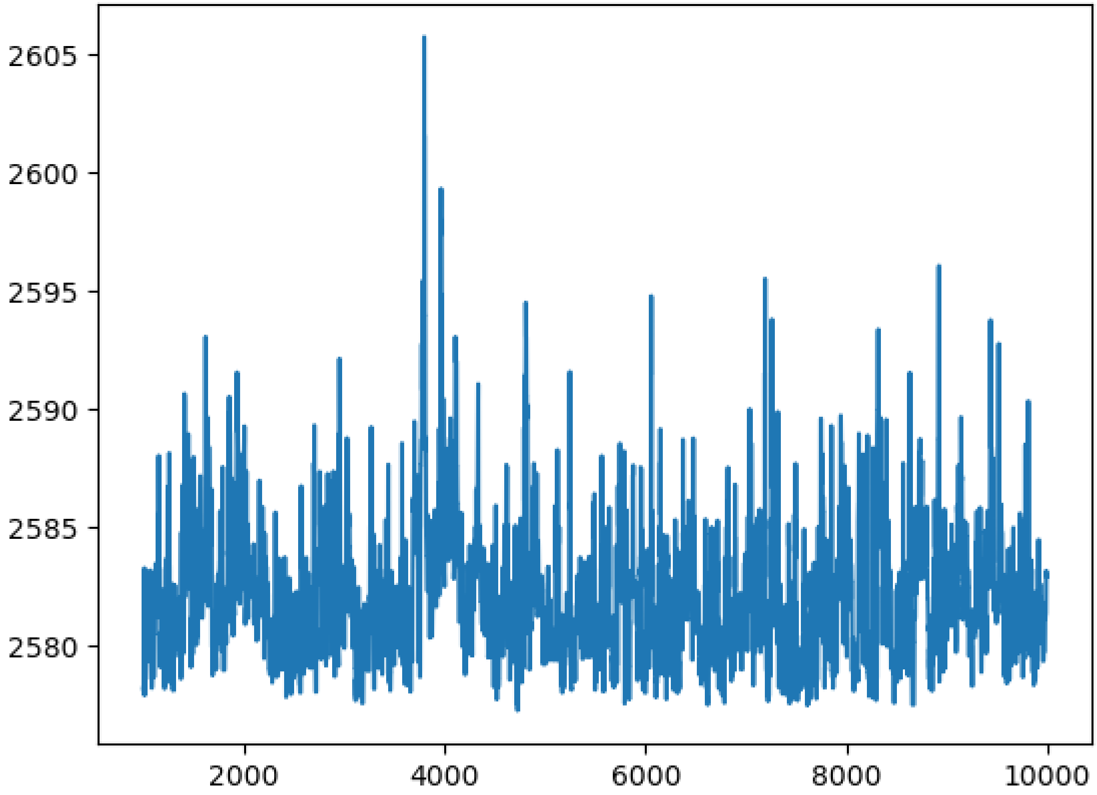
Out[102]:

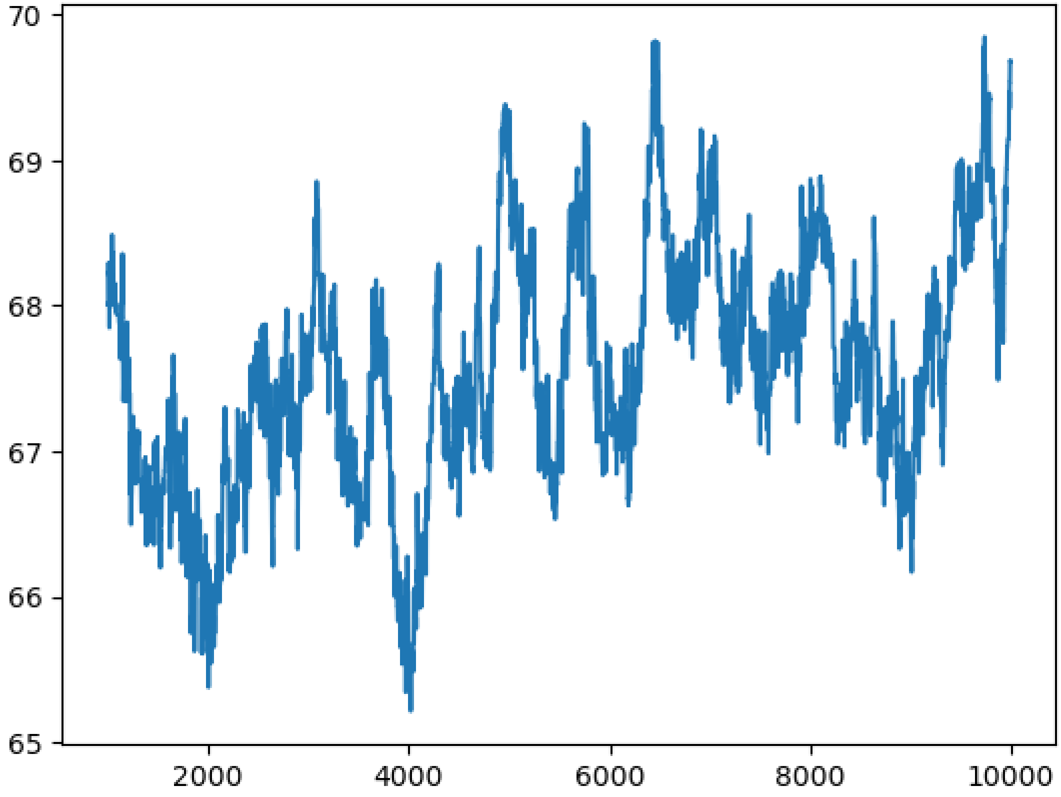
```
[<matplotlib.lines.Line2D at 0x15795a790>]
```

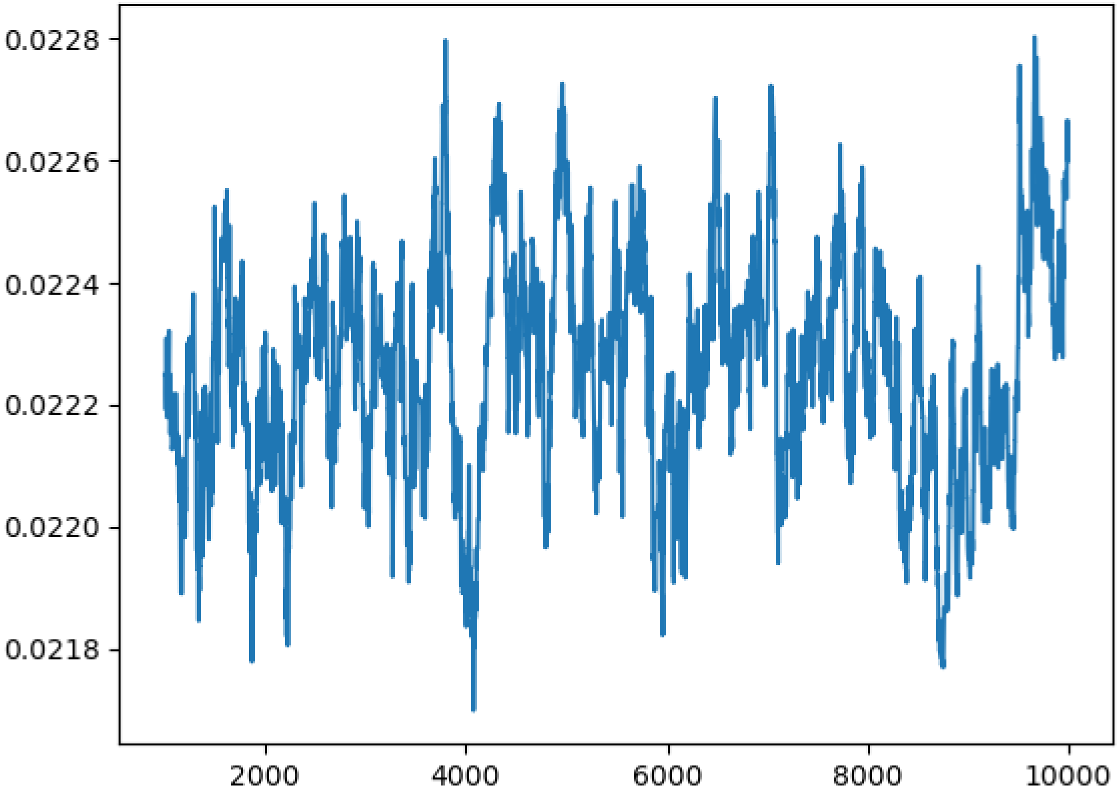


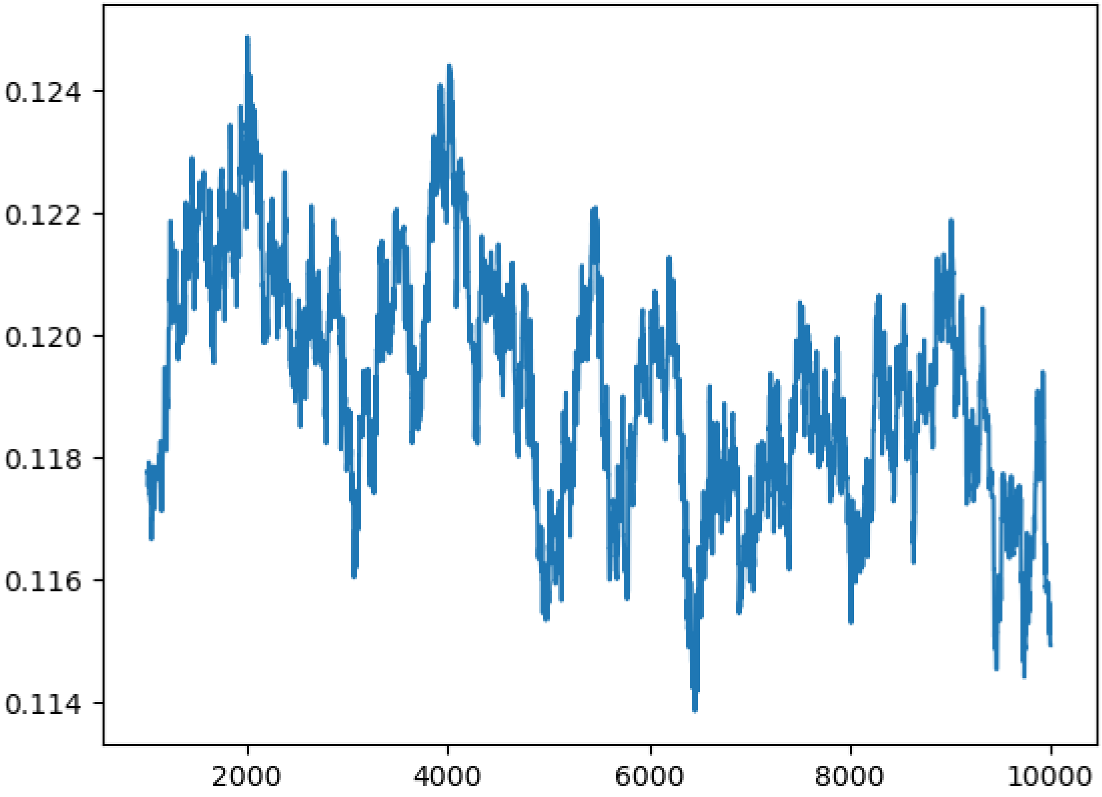
In [104…

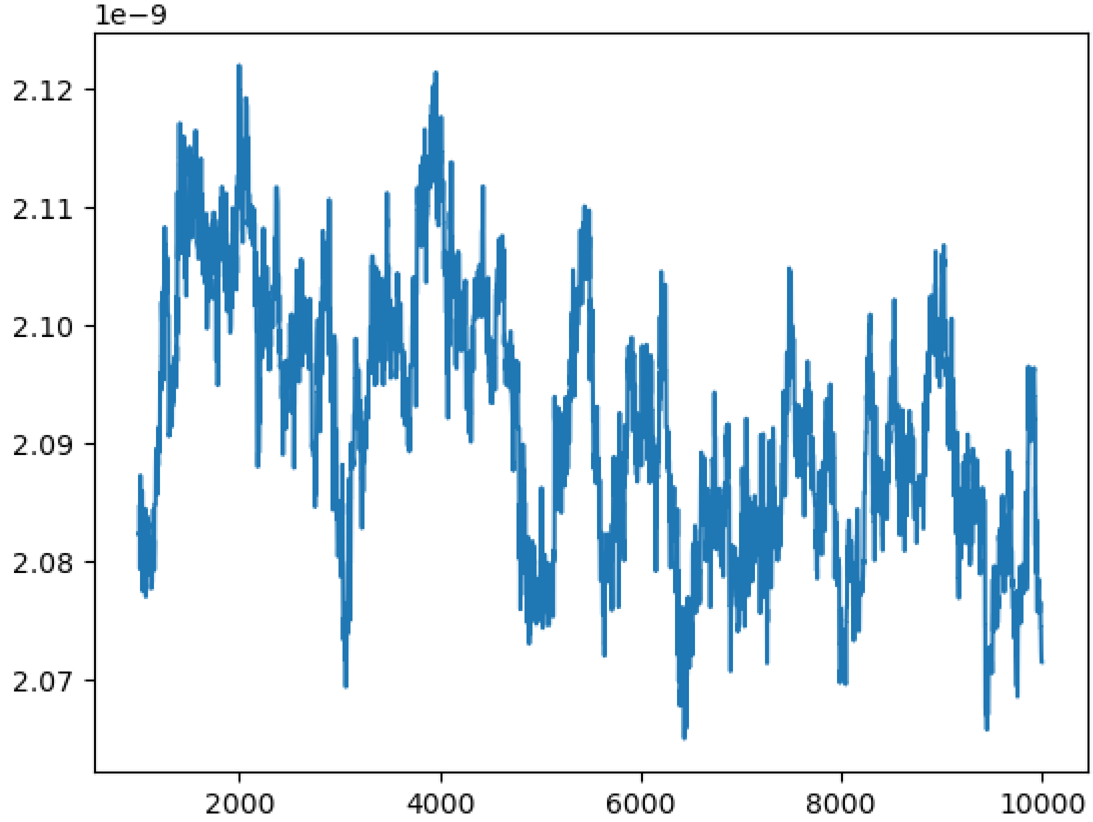
```
for i in range(6):
    plt.plot(range(steps)[1000:],chains[1000:,i])
    plt.show()
```

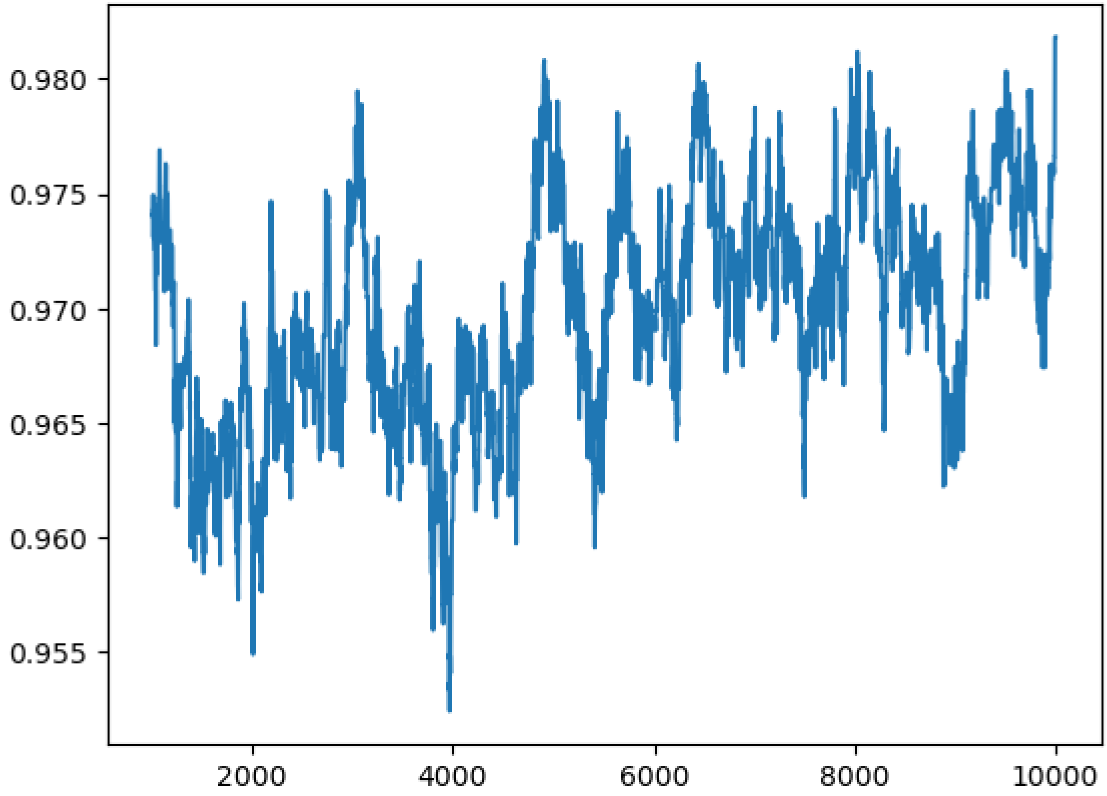

2605
2600
2595
2590
2585
2580
2000
4000
6000
8000
10000


70
69
68
67
66
65
2000
4000
6000
8000
10000


0.0228
0.0226
0.0224
0.0222
0.0220
0.0218
2000
4000
6000
8000
10000


0.124
0.122
0.120
0.118
0.116
0.114
2000
4000
6000
8000
10000





In [106…

```
save_chains=chains
```

Even with fixed tau does not seem to be converging.