

# Proof of Convergence

September 17, 2021

Current diffusion NMF update rule for X (with mask  $M$ ):

$$X \leftarrow X \cdot \frac{(M \cdot D)K^TV^T}{(M \cdot XVK)K^TV^T} \quad (1)$$

Given that V stays fixed while X is being updated can combine  $K^T$  and  $V^T$  into one  $\hat{V}$ :

$$\hat{V} = VK \quad (2)$$

Then the update rule becomes:

$$X \leftarrow X \cdot \frac{(M \cdot D)\hat{V}^T}{(M \cdot X\hat{V})\hat{V}^T} \quad (3)$$

Which is identical to the form showed in <https://arxiv.org/pdf/1612.06037.pdf>

(With  $W = X$ ,  $H = \hat{V}$ ,  $X = D$ ,  $V = M$ ):

$$W \leftarrow W \cdot \frac{(V \cdot X)H^T}{(V \cdot WH)H^T} \quad (4)$$

and can proved to converge on a local minimum using their methods plus the ones from original Lee and Seung paper.

Current diffusion NMF update rule for V (with mask M):

$$V \leftarrow V \cdot \frac{X^T(M \cdot D)K^T}{X^T(M \cdot XVK)K^T} \quad (5)$$

Not sure how to get rid of the  $K^T$  in order to simplify? Might have to be proved another way