Diffusion NMF Algorithms

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1 Standard Multiplicative Update

Algorithm 1 Input: D, K, number of components

$X \leftarrow$ randomized n x components matrix
$V \leftarrow$ randomized components x m matrix

repeat

$X \leftarrow X \cdot \frac{DK^TV^T}{XVKK^TV^T}$

$V \leftarrow V \cdot \frac{X^TDK^T}{X^TXVKK^T}$

until Convergence

return X, V

2 Modified Hoyer Projection Algorithm

Algorithm 2 Input: D, K, number of components, sparseness

$X \leftarrow$ randomized n x components matrix
$V \leftarrow$ randomized components x m matrix

Project each row of V to be non negative, have L1 norm based on desired sparseness, and unit L2 norm

repeat

$X \leftarrow X \cdot \frac{DK^TV^T}{XVKK^TV^T}$

$V \leftarrow V + \eta_V \cdot (X^TDK^T - X^TXVKK^T)$
$V \leftarrow$ Project each row of V' to be non negative, have L1 norm based no desired sparseness, and L2 norm of 1

until convergence

return X, V

3 Multiplicative Update with Projection

Algorithm 3 Input: D, K, number of components, sparseness

$X \leftarrow$ randomized n x components matrix
$V \leftarrow$ randomized components x m matrix

repeat

$X \leftarrow X \cdot \frac{DK^TV^T}{XVKK^TV^T}$

$V \leftarrow V \cdot \frac{X^TDK^T}{X^TXVKK^T}$

$V \leftarrow$ Project V to be non negative, have L1 norm based on desired sparseness, and L2 norm of 1

until Convergence

return X, V

4 Modified Hoyer Sparse Coding Algorithm

Algorithm 4 Input: D, K, number of components, λ (sparseness parameter)

$X \leftarrow$ randomized n x components matrix
$V \leftarrow$ randomized components x m matrix
repeat
$X \leftarrow X + \eta_X \cdot (DK^TV^T - XVKK^TV^T)$

Any negative values in X are set to 0

$V \leftarrow V \cdot \frac{X^TDK^T}{X^TXVKK^T+\lambda}$

Rescale each row of V to have unit L2 norm

until convergence

5 Two phase algorithm

Algorithm 5 Input: D, K, number of components, sparseness

Run Algorithm 1 (Standard Multiplicative Update) starting with random intial X, V

Run Algorithm 2 (Hoyer Projection) with the output from algorithm 1 as input (starting point)

return X,V from algorithm 2 output

6 Alternating Least Squares

$X \leftarrow$ randomized n x components matrix
$V \leftarrow$ randomized components x m matrix

repeat

•Solve for X:

$$\binom{K^T H^T}{\sqrt{\eta} I_k} X^T = \binom{D^T}{0_{kxm}}$$

Where I_k is a k x k identity matrix and 0_{kxm} is a zero matrix of size k x m

•Solve for V:

$$\binom{X}{\sqrt{\beta} e_{1xk}} V = \binom{D}{0_{1xn}} K^{-1}$$

Where e_{1xk} is a row vector with all components equal to 1 and 0_{1xn} is a zero vector

Set all negative values in V equal to 0

until convergence