

# Concepts of Programming Languages

Course: CSE2120 Edition: 2019-2020

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## 1.8. Week 8: Lazy Evaluation



In this week's assignments, you will be implementing an interpreter with *lazy evaluation*, based on [chapter 17.1](#) of the book.

The interpreter is for a language with built-in support for lists and recursion (i.e., recursion is no longer treated as syntactic sugar). The language does not have built-in support for mutation.

## 1 Features to Implement

This week's interpreter is a variation of the environment-based interpreter from [week 4](#). You can probably reuse and adapt some of the code from that interpreter, but be aware of the differences.

### 1.1 Lazy Evaluation

Implement lazy evaluation using thunks as described in the book.

You should use the classes summarized [below](#).

Your interpreter should use *memoization*: once a thunked computation has been interpreted, the interpreter memoizes the value so that each thunk is never interpreted more than once.

Choose sensible *strictness points*. For example, evaluation of expressions in the function position of a function application expression should be strict. You should implement a *strict* function that takes a value as input and, if the input value is a thunk, evaluates the expression to a value and returns the result. Evaluation of expressions in the argument positions of a function application expression should be lazy.

For example, an expression `(f e1 e2 e3)` should:

- evaluate the expression `f` to a function closure `v`;
- construct a thunk value for each `e1`, `e2`, and `e3`, and
- evaluate the body of the function closure `v` using the environment of the closure, updated with [bindings](#) for each argument thunk.

Your language should also be extended with a *force* construct that adds first-class support for strictness to your otherwise lazy language. An expression `(force e)` should be treated as a strictness point that eagerly and *recursively* coerces all thunks into values.

### 1.2 Recursion as a Construct

Implement first-class support for recursion in your language.

The semantics of *letrec* is similar to the *letrec* construct from [week 5](#). That is, it should use the "create, update, use" approach from [the book](#).

You should implement *letrec* by exploiting Scala's support for mutation; that is, you can mutate the *Bind* objects in environments directly in Scala, instead of returning a new store representing the mutated memory state.

You should use *uninitializedv* for environment locations that have been created but not yet updated.

*letrec* should interpret binding expressions lazily (call-by-need).

### 1.3 Lists and Laziness

Like in previous weeks, your language should support linked lists, using *nil* for the empty list, and *cons* for constructing a single link of a list.

#### 1.3.1 Lazy Interpretation of *cons*

Interpreting a `(cons e1 e2)` expression should thunk each of its two argument expressions, where `e1` is an expression that computes the head of the list, and `e2` is an expression that computes the tail of a list.

#### 1.3.2 Strictness

The *strict* function should evaluate thunks to values. You should use this function to execute previously delayed computations when needed.

#### 1.3.3 Force

Lists are composed of (possibly thunked) computations. Ensure that your implementation of *force* recursively applies strictness to list elements.

*force* should be implemented such that the result of the following program is the value `ConsV(NumV(1), NilV())`:

```
(force ((lambda (x y) (cons x y)) 1 nil))
```

## 2 Examples

### 2.1 Stream of Ones

The following expression constructs a lazily computed infinite list of `1`s and returns the first 10 of these:

```
(letrec ((ones (cons 1 ones))
  (take
    (lambda (n xs)
      (if (num= n 0)
          nil
          (cons (head xs) (take (- n 1) (tail xs)))))))
  (force (take 10 ones)))
```

The *take* function constructs a finite list with the first `n` element of a list `xs` (or fails if `xs` is shorter than `n`).

### 2.2 Natural Numbers

The following expression constructs an infinite series containing all of the natural numbers, and takes the first 10:

```
(letrec ((mk-nats (lambda (n) (cons n (mk-nats (+ n 1)))))
  (nats (mk-nats 0))
  (take
    (lambda (n xs)
      (if (num= n 0)
          nil
          (cons (head xs) (take (- n 1) (tail xs)))))))
  (force (take 10 nats)))
```

An alternative approach is to use a *zip-with* function and the *ones* stream from earlier:

```
(letrec ((ones (cons 1 ones))

  (nats (cons 0 (zip-with plus ones nats)))

  (plus (lambda (n m) (+ n m)))

  (zip-with
    (lambda (f xs ys)
      (if (is-nil xs)
          nil
          (cons
            (f (head xs) (head ys))
            (zip-with f (tail xs) (tail ys))))))

  (take
    (lambda (n xs)
      (if (num= n 0)
          nil
          (cons (head xs) (take (- n 1) (tail xs)))))))
  (force (take 10 nats)))
```

### 2.3 Factorial Numbers

We can reuse the *zip-with* function to construct the infinite series of factorial numbers:

```
(letrec ((ones (cons 1 ones))

  (nats (cons 0 (zip-with plus ones nats)))

  (facs (cons 1 (zip-with times (tail nats) facs)))

  (plus (lambda (n m) (+ n m)))
  (times (lambda (n m) (* n m)))

  (zip-with
    (lambda (f xs ys)
      (if (and (is-nil xs) (is-nil ys))
          nil
          (cons
            (f (head xs) (head ys))
            (zip-with f (tail xs) (tail ys))))))

  (take
    (lambda (n xs)
      (if (num= n 0)
          nil
          (cons (head xs) (take (- n 1) (tail xs)))))))
  (force (take 10 facs)))
```

## 3 Grammar

```
module lazy-functions

imports Common

context-free syntax

Expr.NumExt      = INT          // integer literals
Expr.TrueExt     = [true]
Expr.FalseExt    = [false]
Expr.IdExt       = ID

Expr.UnOpExt     = [[UnOp] [Expr]]
Expr.BinOpExt    = [[BinOp] [Expr] [Expr]]

UnOp.MIN         = [-]
UnOp.NOT         = [not]
UnOp.FORCE       = [force]
UnOp.HEAD       = [head]
UnOp.TAIL        = [tail]
UnOp.ISNIL      = [is-nil]
UnOp.ISLIST     = [is-list]

BinOp.PLUS       = [+]
BinOp.MULT       = [*]
BinOp.MINUS      = [-]
BinOp.AND        = [and]
BinOp.OR         = [or]
BinOp.NUMEQ      = [=num=]
BinOp.WMLT       = [=num<]
BinOp.WMGT       = [=num>]
BinOp.CONS       = [cons]

Expr.IfExt       = [(if [Expr] [Expr] [Expr])]

Expr.FdExt       = [(lambda ([ID*]) [Expr])]
Expr.AppExt      = [[[Expr] [Expr*]]]
Expr.LetExt      = [[let ([LetBind*]) [Expr]]]
Expr.LetRecExt   = [[letrec ([LetBind*]) [Expr]]]

Expr.NilExt      = [nil]
Expr.ListExt     = [[list [Expr*]]]

LetBind.LetBind = [[ID] [Expr]]
```

## 4 Classes

### 4.1 Abstract Syntax

```
sealed abstract class ExprExt
case class TrueExt() extends ExprExt
case class FalseExt() extends ExprExt
case class NumExt(num: Int) extends ExprExt
case class BinOpExt(s: String, l: ExprExt, r: ExprExt) extends ExprExt
case class UnOpExt(s: String, e: ExprExt) extends ExprExt
case class IfExt(c: ExprExt, t: ExprExt, e: ExprExt) extends ExprExt
case class ListExt(l: List[ExprExt]) extends ExprExt
case class NilExt() extends ExprExt
case class AppExt(f: ExprExt, args: List[ExprExt]) extends ExprExt
case class IdExt(c: String) extends ExprExt
case class FdExt(params: List[String], body: ExprExt) extends ExprExt
case class LetExt(binds: List[LetBindExt], body: ExprExt) extends ExprExt
case class LetRecExt(binds: List[LetBindExt], body: ExprExt) extends ExprExt

case class LetBindExt(name: String, value: ExprExt)

object ExprExt {
  val binOps = Set("+", "*", "-", "and", "or", "num=", "num<", "num>", "cons")
  val unOps = Set("-", "not", "force", "head", "tail", "is-nil", "is-list")
  val reservedWords = binOps ++ unOps ++ Set("if", "lambda", "let", "true", "false", "nil", "list", "letrec")
}
```

### 4.2 Desugared Syntax

```
sealed abstract class ExprC
case class TrueC() extends ExprC
case class FalseC() extends ExprC
case class NumC(num: Int) extends ExprC
case class PlusC(l: ExprC, r: ExprC) extends ExprC
case class MultC(l: ExprC, r: ExprC) extends ExprC
case class IfC(c: ExprC, t: ExprC, e: ExprC) extends ExprC
case class EqNumC(l: ExprC, r: ExprC) extends ExprC
case class LtC(l: ExprC, r: ExprC) extends ExprC
case class AppC(f: ExprC, args: List[ExprC]) extends ExprC
case class IdC(c: String) extends ExprC
case class FdC(params: List[String], body: ExprC) extends ExprC
case class LetRecC(binds: List[LetBindC], body: ExprC) extends ExprC
case class ForceC(e: ExprC) extends ExprC
case class NilC() extends ExprC
case class ConsC(hd: ExprC, tl: ExprC) extends ExprC
case class HeadC(e: ExprC) extends ExprC
case class TailC(e: ExprC) extends ExprC
case class IsNilC(e: ExprC) extends ExprC
case class IsListC(e: ExprC) extends ExprC

case class LetBindC(name: String, value: ExprC)
```

### 4.3 Values

```
sealed abstract class Value
case class NumV(v: Int) extends Value
case class BoolV(v: Boolean) extends Value

import java.util.UUID.randomUUID // for generating a random ID and hash code

case class ClosV(f: FdC, env: List[Bind]) extends Value {
  override def toString: String = s"ClosV($f, <env>)"
  override def hashCode(): Int = randomUUID().hashCode()
}

case class ThunkV(var value: Either[(ExprC, List[Bind]), Value]) extends Value {
  override def toString: String = value match {
    case Left((e, _)) => s"ThunkV($e, <env>)"
    case Right(v) => s"ThunkV($v)"
  }
  override def hashCode(): Int = randomUUID().hashCode()
}

case class ConsV(head: Value, tail: Value) extends Value
case class NilV() extends Value
case class UninitializedV() extends Value
```

The *Either[(ExprC, List[Bind]), Value]* type is used to represent a thunk that either records an expression and the environment under which that expression is closed or a value.

The *var* annotation on the *value* argument of a *ThunkV* denotes a mutable field.

Since environments may be cyclic, the *ClosV* and *ThunkV* case classes are defined to override the *toString* and *hashCode* methods, which might otherwise attempt a divergent traversal of a cyclic environment.

### 4.4 Other

```
case class Bind(name: String, var value: Value)
type Environment = List[Bind]
```

Note that *bindings* are now mutable to allow the updates of *LetRec* values.

## 5 Exceptions

Specific exceptions should be created that *inherit* from the given abstract exceptions.

Creating specific exceptions for each case makes debugging a lot easier and are more informative.

Throw only exceptions derived from *ParseException* in the parser, *DesugarException* in the desugarer, and so on.

```
abstract class ParseException extends RuntimeException
abstract class DesugarException extends RuntimeException
abstract class InterException extends RuntimeException
```