

Error analysis for a general optimization fit:

Minimize: $F(\vec{x}, \vec{u})$

data (independent) \swarrow

\nwarrow parameters

Then for fitted \vec{u} ,

$$\nabla_{\vec{u}} F(\vec{x}, \vec{u}) = \vec{0}$$

Thus, we have a solution surface: the set of (\vec{x}, \vec{u}) pairs exhibiting the above property.

Within the surface, we know that

$$\frac{\partial F}{\partial u_i} = 0,$$

Thus, $d\left(\frac{\partial F}{\partial u_i}\right) = 0$

$$\sum_j \frac{\partial^2 F}{\partial u_i \partial u_j} du_j + \sum_k \frac{\partial^2 F}{\partial u_i \partial x_k} dx_k = 0,$$

or,

$$\sum_j \frac{\partial^2 F}{\partial u_i \partial u_j} du_j = - \sum_k \frac{\partial^2 F}{\partial u_i \partial x_k} dx_k$$

Let $M_{ij} = \frac{\partial^2 F}{\partial u_i \partial u_j}$, $V_{ik} = \frac{\partial^2 F}{\partial u_i \partial x_k}$.

Then

$$du_j = \cancel{(M^{-1}V)} - \sum_k (M^{-1}V)_{jk} dx_k;$$

thus, locally,

$$\frac{\partial u_j}{\partial x_k} = -(M^{-1}V)_{jk}.$$

Now you have partial derivatives, even though you don't have explicit expressions for your parameters!

General error propagation:

For x_1, \dots, x_m with standard errors s_1, \dots, s_m ,

if $\textcircled{A} y = f(x_1, \dots, x_m)$ then

$$s_y^2 = \sum_{i=1}^m \left(\frac{df}{dx_i} \right)^2 s_i^2$$

Put it together....

Fit for muon data. 0

A ~~least-squares~~ fit usually minimizes

$$\sum_i (y_i - \hat{y}_i)^2$$

where y_i is the i^{th} data point and \hat{y}_i is the predicted value of y_i .

However, because you are ^{essentially} fitting a ~~discrete~~ probability distribution, I recommend minimizing ~~the~~ χ^2 ~~statistic~~

Statistic:

$$\chi^2 = \sum_i \frac{(y_i - \hat{y}_i)^2}{\hat{y}_i}$$