CS 4610/5335 – Lecture 3
Representations and Transformations

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1/26/22

Material adapted from:
1. Robert Platt, CS 4610/5335
2. Peter Corke, Robotics, Vision and Control
3. Oussama Khatib, Stanford CS 223A

Announcements

Office hours:

Tue	1 – 3 PM	Isaac	Virtual (on Khoury OH)
Wed	7 – 9 PM	Shuo	Virtual
Thu	10 AM – Noon	John	Virtual
Fri	3:30 – 5:30 PM	Lawson	Hybrid (also in 513 ISEC)

- lec02 slides updated

- Ex0 + onboarding questionnaire due Fri
- Ex1 will be posted on Fri/Sat/Sun

- First project “session” will be during lecture on 2/2 (Wed)

Recap: Transformations

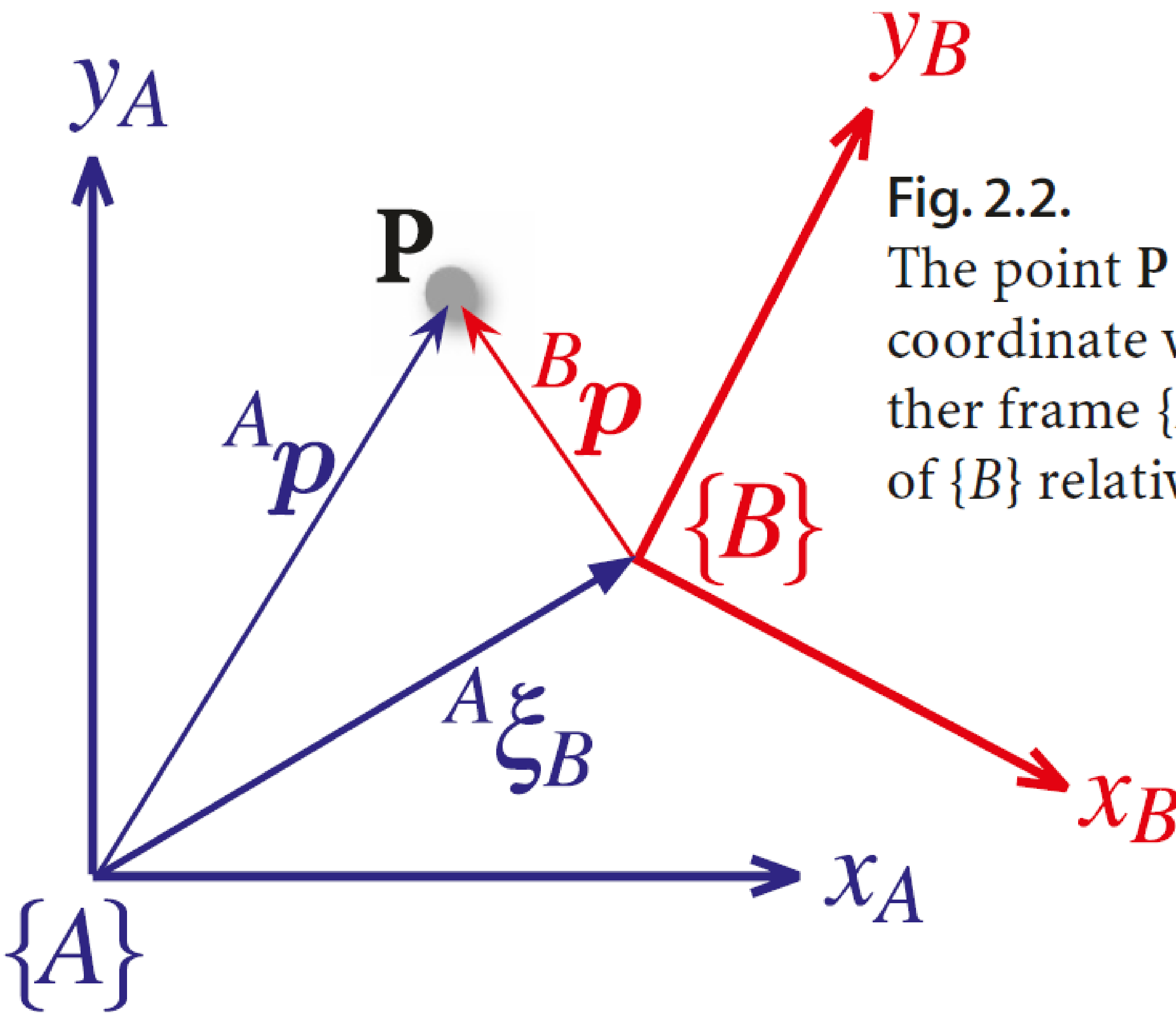


Fig. 2.2.
The point **P** can be described by coordinate vectors relative to either frame $\{A\}$ or $\{B\}$. The pose of $\{B\}$ relative to $\{A\}$ is $^A\xi_B$

Transformation notation

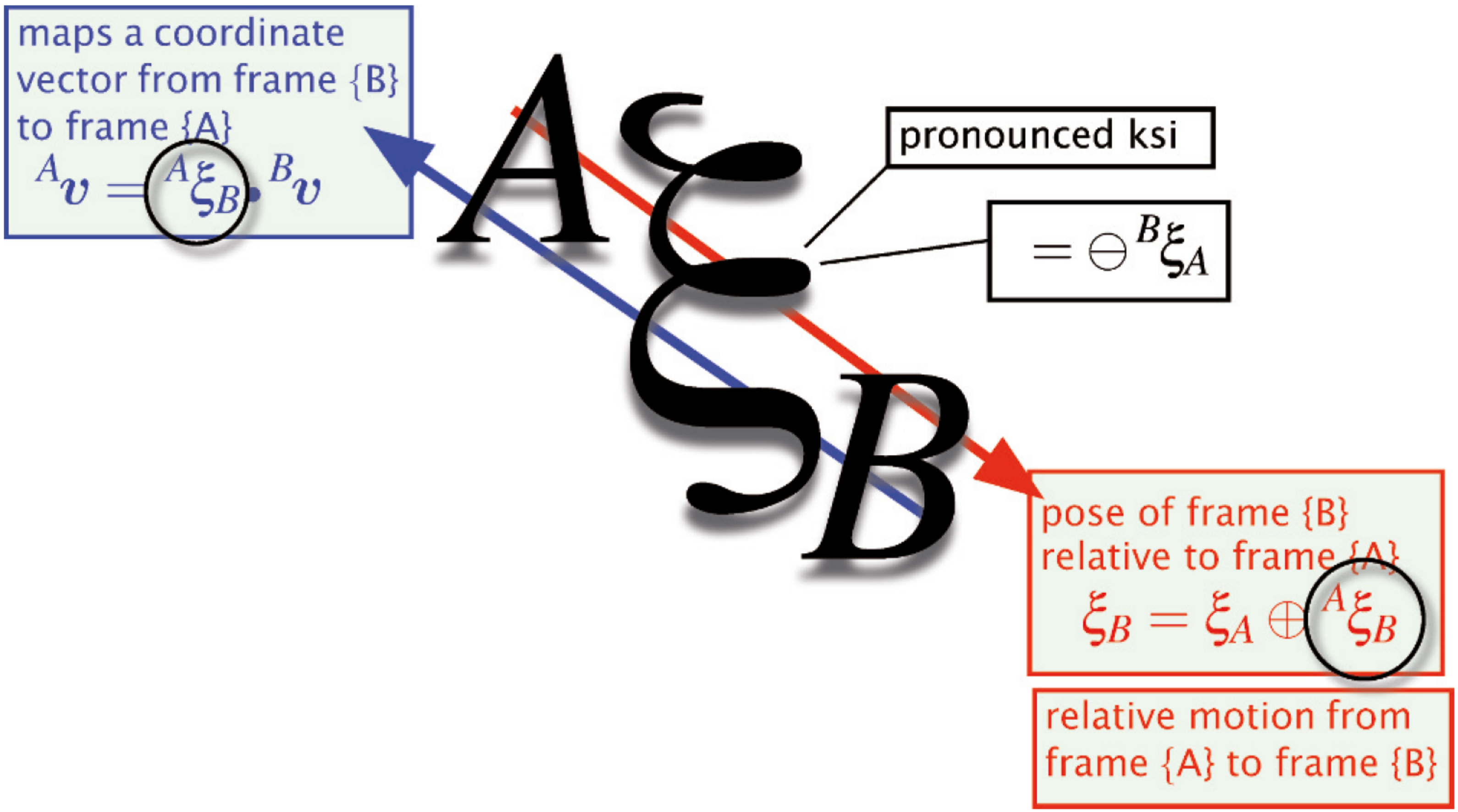


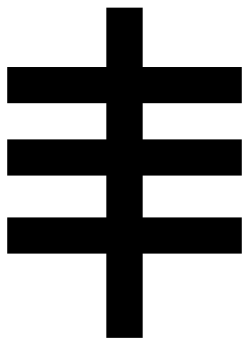
Fig. 2.19. Everything you need to know about pose

It's all Greek to me

Upper Case Ξ ξ lower case

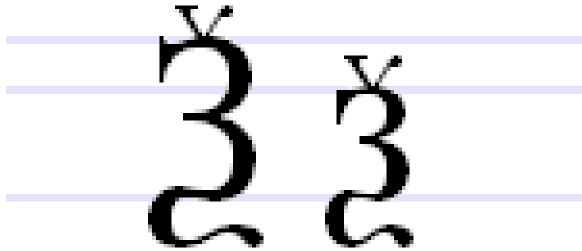
Xi

It's all Greek to me



Samekh (Phoenician)

Xi



Ksi (early Cyrillic)

It's all Greek to me

Νν	Ξξ	Οο
Nu (13)	Xi (14)	Omicron (15)

It's all Greek to me

Νν Ξξ Οο

Nu (13) Xi (14) Omicron (15)

Not to be confused with:

Chi (22) Psi (23) Zeta (6)

Χχ Ψψ Ζζ

It's all Greek to me

Νν Ξξ Οο

Nu (13 / 50)

Xi (14 / 60)

Omicron (15 / 70)

Not to be confused with:

Chi (22 / 600)

Psi (23 / 700)

Zeta (6 / 7)

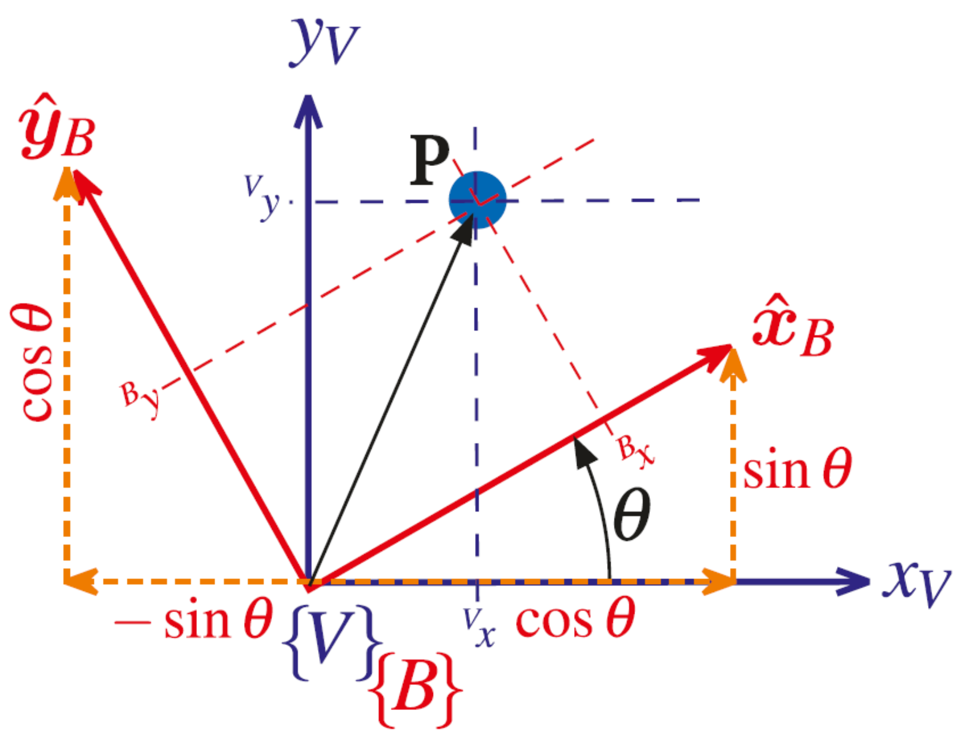
Χχ Ψψ Ζζ

Recap: 2-D rotation matrix

Relationship between bases:

$$\hat{\boldsymbol{x}}_B = \cos\theta \hat{\boldsymbol{x}}_V + \sin\theta \hat{\boldsymbol{y}}_V$$

$$\hat{\boldsymbol{y}}_B = -\sin\theta \hat{\boldsymbol{x}}_V + \cos\theta \hat{\boldsymbol{y}}_V$$



Coordinate transformation:

$$\begin{pmatrix} {}^V x \\ {}^V y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} {}^B x \\ {}^B y \end{pmatrix}$$

To obtain rotation matrix from V to B:
- Express each B axis unit vector in V coordinates
- Insert each as a <u>column</u> vector

$${}^V R_B = \begin{pmatrix} {}^V \hat{x}_B & {}^V \hat{y}_B \end{pmatrix}$$

$$\begin{pmatrix} {}^V x \\ {}^V y \end{pmatrix} = {}^V \boldsymbol{R}_B \begin{pmatrix} {}^B x \\ {}^B y \end{pmatrix}$$

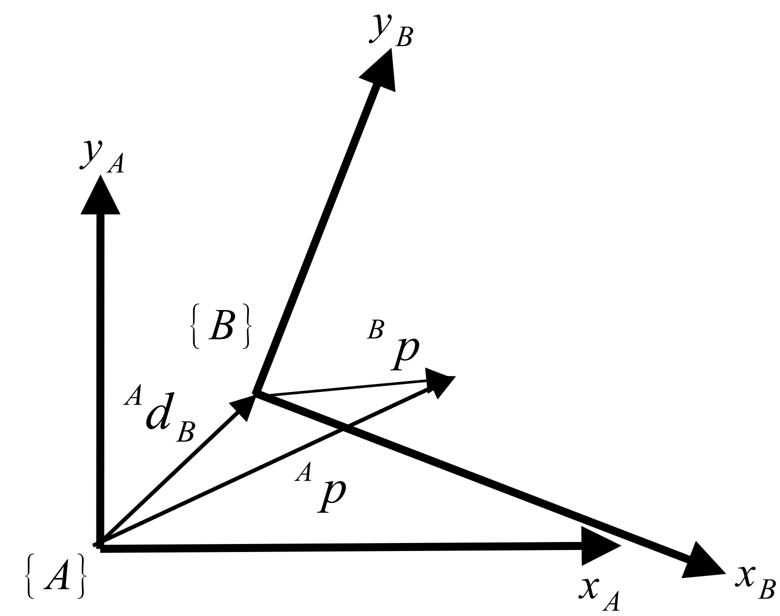
Recap: 2-D homogeneous transformation

$$^{A}p = {}^{A}R_{B}\,{}^{B}p + {}^{A}d_{B}$$

$$= \begin{pmatrix} {}^{A}R_{B} & {}^{A}d_{B} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} {}^{B}p \\ 1 \end{pmatrix}$$

always zeros

always one



$$\begin{pmatrix} {}^{A}x \\ {}^{A}y \\ 1 \end{pmatrix} = \begin{pmatrix} {}^{A}\boldsymbol{R}_{B} & \boldsymbol{t} \\ \boldsymbol{0}_{1\times 2} & 1 \end{pmatrix} \begin{pmatrix} {}^{B}x \\ {}^{B}y \\ 1 \end{pmatrix}$$

$$^{A}\tilde{\boldsymbol{p}} = \begin{pmatrix} {}^{A}\boldsymbol{R}_{B} & \boldsymbol{t} \\ \boldsymbol{0}_{1\times 2} & 1 \end{pmatrix} {}^{B}\tilde{\boldsymbol{p}}$$

$$= {}^{A}\boldsymbol{T}_{B}\,{}^{B}\tilde{\boldsymbol{p}}$$

Homogeneous transformation: Inverse

Can also derive it from the forward Homogeneous transform:

$$^{B}p = {}^{B}R_{A}\,{}^{A}p + {}^{B}d_{A}$$

$$^{A}p = {}^{B}R_{A}^{T}\left({}^{B}p - {}^{B}d_{A}\right) = \left({}^{B}T_{A}\right)^{-1}\begin{pmatrix} {}^{B}p \\ 1 \end{pmatrix}$$

where $\left({}^{B}T_{A}\right)^{-1} = \begin{pmatrix} {}^{B}R_{A}^{T} & -{}^{B}R_{A}^{T}\,{}^{B}d_{A} \\ 0 & 1 \end{pmatrix}$

Summary: 2-D rotation

$$^{X}\boldsymbol{R}_{Y}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

is a 2-dimensional rotation matrix with some special properties:

- it is *orthonormal* (also called *orthogonal*) since each of its columns is a unit vector and the columns are orthogonal.
- the columns are the unit vectors that define the axes of the rotated frame Y with respect to X and are by definition both unit-length and orthogonal.
- it belongs to the special orthogonal group of dimension 2 or $\boldsymbol{R} \in \mathbf{SO}(2) \subset \mathbb{R}^{2\times 2}$. This means that the product of any two matrices belongs to the group, as does its inverse.
- its *determinant* is $+1$, which means that the length of a vector is unchanged after transformation, that is, $\|^{Y}\boldsymbol{p}\| = \|^{X}\boldsymbol{p}\|, \forall\theta$.
- the inverse is the same as the transpose, that is, $\boldsymbol{R}^{-1} = \boldsymbol{R}^{T}$.

Summary: 2-D transformation

$$T = \begin{pmatrix} \cos\theta & -\sin\theta & x \\ \sin\theta & \cos\theta & y \\ 0 & 0 & 1 \end{pmatrix}$$

A concrete representation of relative pose ξ is $\xi \sim T \in \mathrm{SE}(2)$ and $T_1 \oplus T_2 \mapsto T_1 T_2$ which is standard matrix multiplication

$$T_1 T_2 = \begin{pmatrix} R_1 & t_1 \\ \mathbf{0}_{1\times2} & 1 \end{pmatrix} \begin{pmatrix} R_2 & t_2 \\ \mathbf{0}_{1\times2} & 1 \end{pmatrix} = \begin{pmatrix} R_1 R_2 & t_1 + R_1 t_2 \\ \mathbf{0}_{1\times2} & 1 \end{pmatrix}$$

One of the algebraic rules from page 21 is $\xi \oplus 0 = \xi$. For matrices we know that $TI = T$, where I is the identify matrix, so for pose $0 \mapsto I$ the identity matrix. Another rule was that $\xi \ominus \xi = 0$. We know for matrices that $TT^{-1} = I$ which implies that $\ominus T \mapsto T^{-1}$

$$T^{-1} = \begin{pmatrix} R & t \\ \mathbf{0}_{1\times2} & 1 \end{pmatrix}^{-1} = \begin{pmatrix} R^T & -R^T t \\ \mathbf{0}_{1\times2} & 1 \end{pmatrix}$$

For a point described by $\tilde{p} \in \mathbb{P}^2$ then $T \cdot \tilde{p} \mapsto T\tilde{p}$ which is a standard matrix-vector product.

Outline

✔ Vector / matrix refresher

✔ SO(2): 2-D rotation / orientation

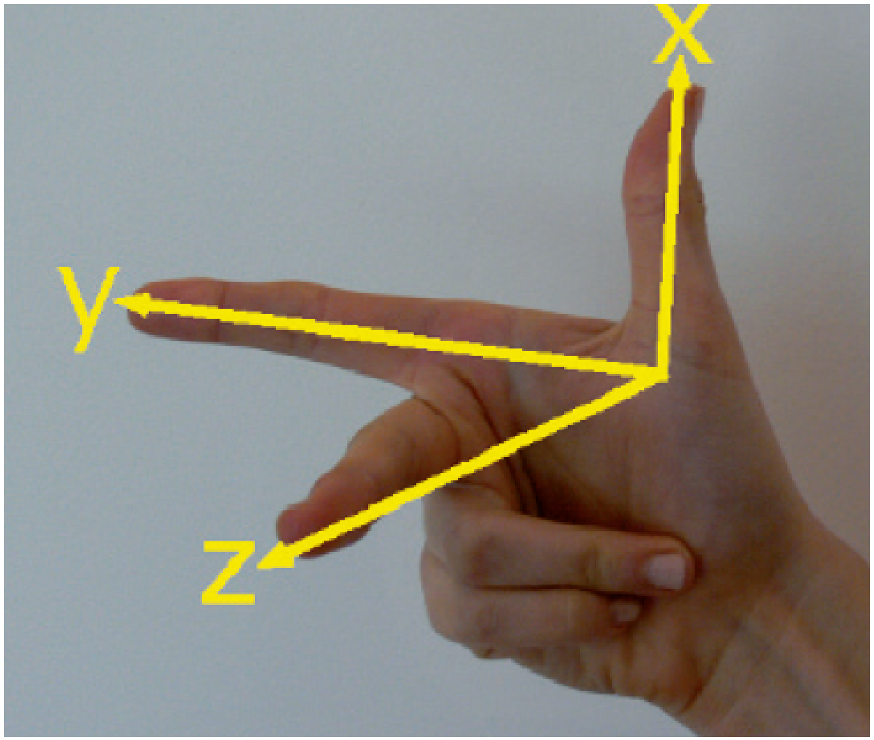
✔ SE(2): 2-D transformations

SO(3): 3-D rotation / orientation

SE(3): 3-D transformations

More representations for 3-D rotations (time permitting)
- Euler angles
- Axis-angle representations
- Quaternions

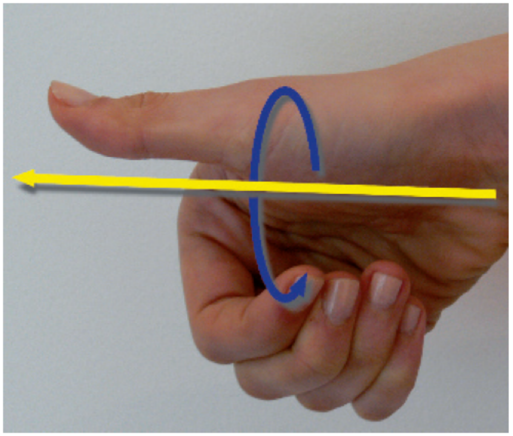
3-D rotation



Right-hand rule. A right-handed coordinate frame is defined by the first three fingers of your right hand which indicate the relative directions of the *x*-, *y*- and *z*-axes respectively.

$$\hat{\boldsymbol{z}} = \hat{\boldsymbol{x}} \times \hat{\boldsymbol{y}}, \;\; \hat{\boldsymbol{x}} = \hat{\boldsymbol{y}} \times \hat{\boldsymbol{z}}; \;\; \hat{\boldsymbol{y}} = \hat{\boldsymbol{z}} \times \hat{\boldsymbol{x}}$$

3-D rotation



Rotation about a vector. Wrap your right hand around the vector with your thumb (your x-finger) in the direction of the arrow. The curl of your fingers indicates the direction of increasing angle.

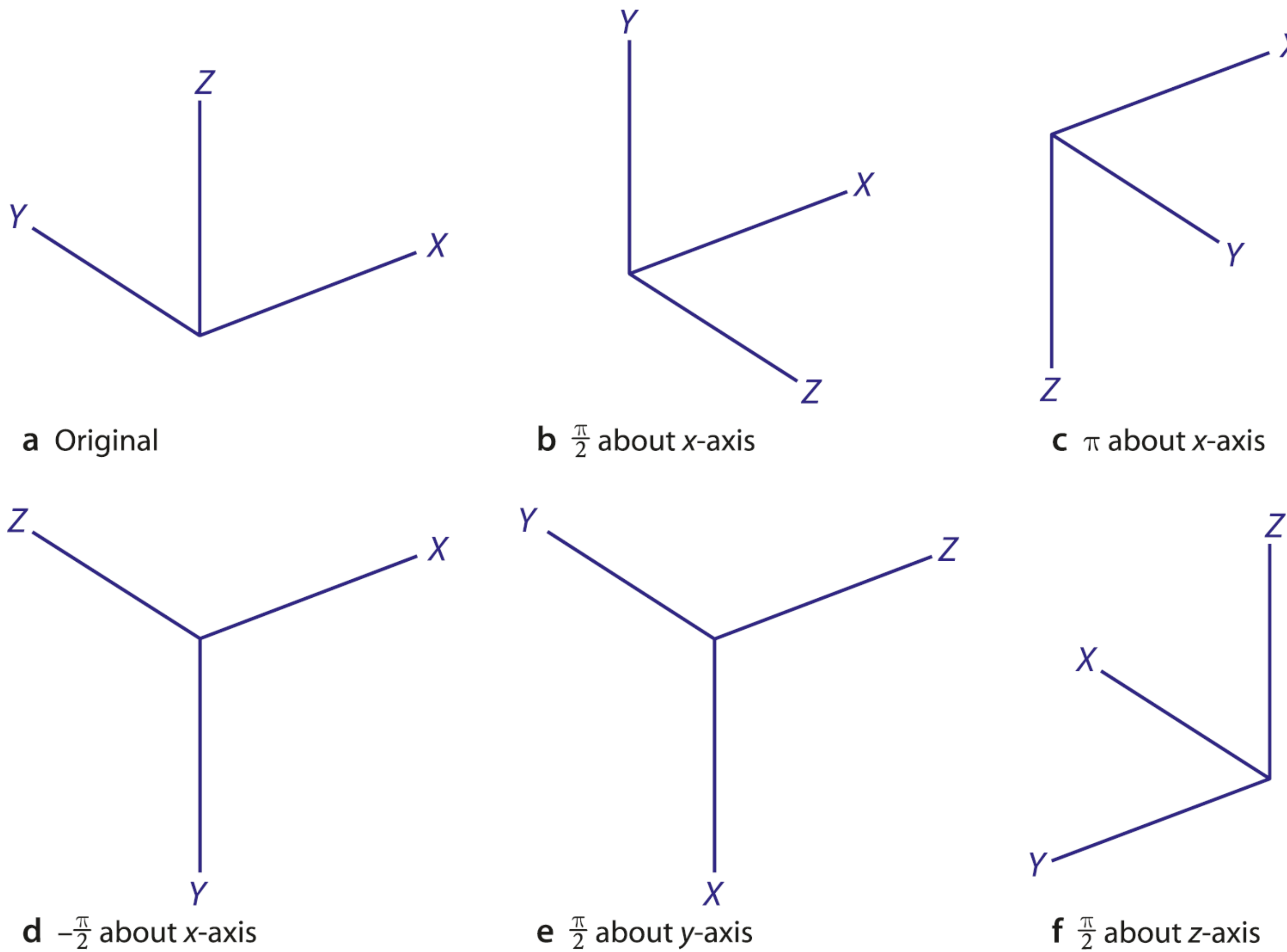


Fig. 2.11.
Rotation of a 3D coordinate frame. **a** The original coordinate frame, **b–f** frame **a** after various rotations as indicated

3-D rotation

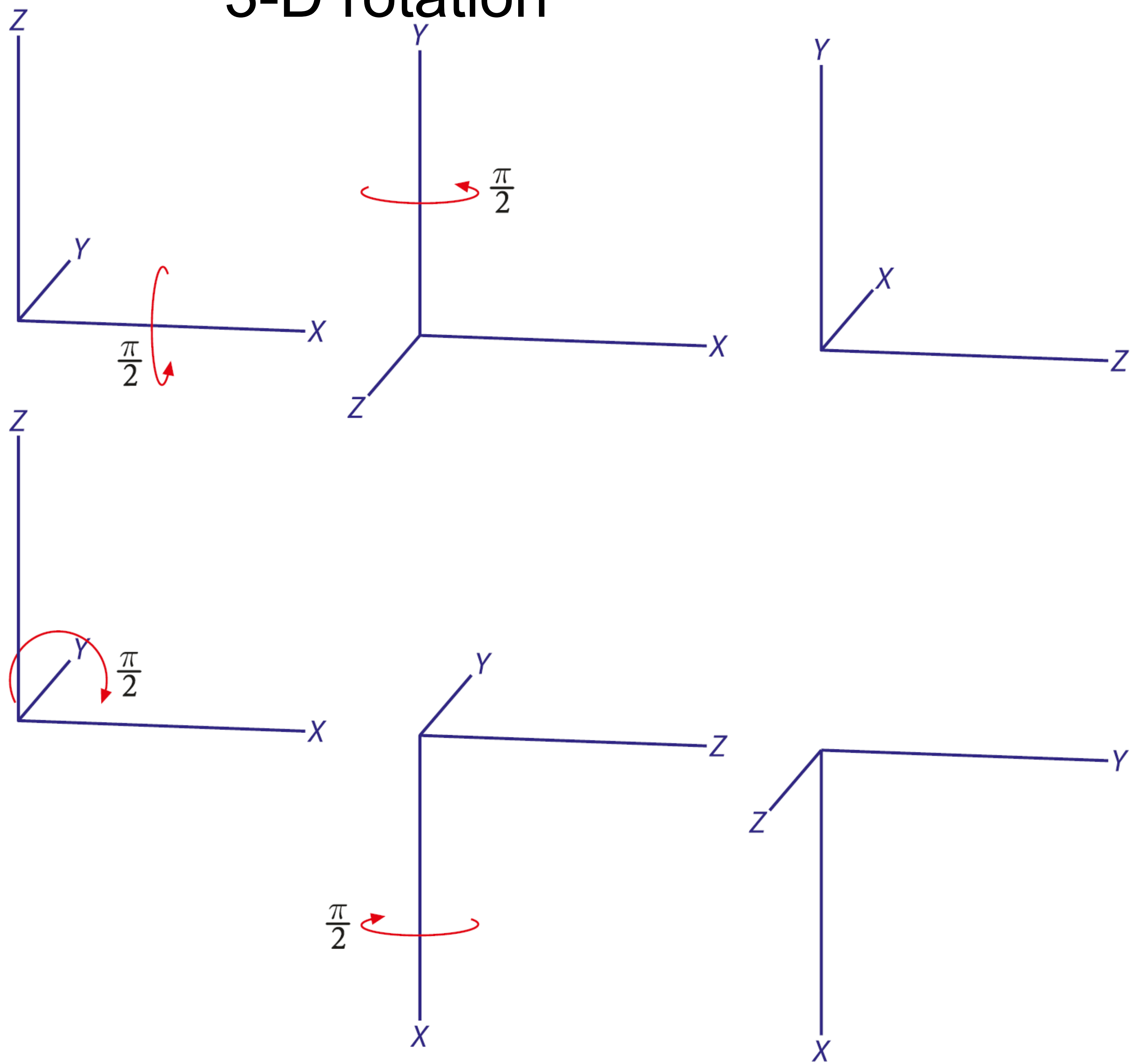


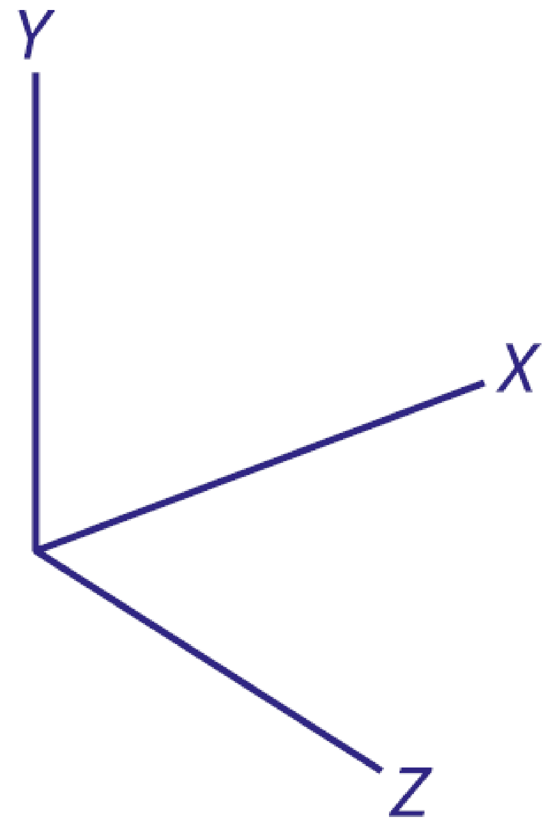
Fig. 2.12. Example showing the noncommutativity of rotation. In the top row the coordinate frame is rotated by $\frac{\pi}{2}$ about the x-axis and then $\frac{\pi}{2}$ about the y-axis. In the bottom row the order of rotations has been reversed. The results are clearly different

Rotation about XYZ axes

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$R_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



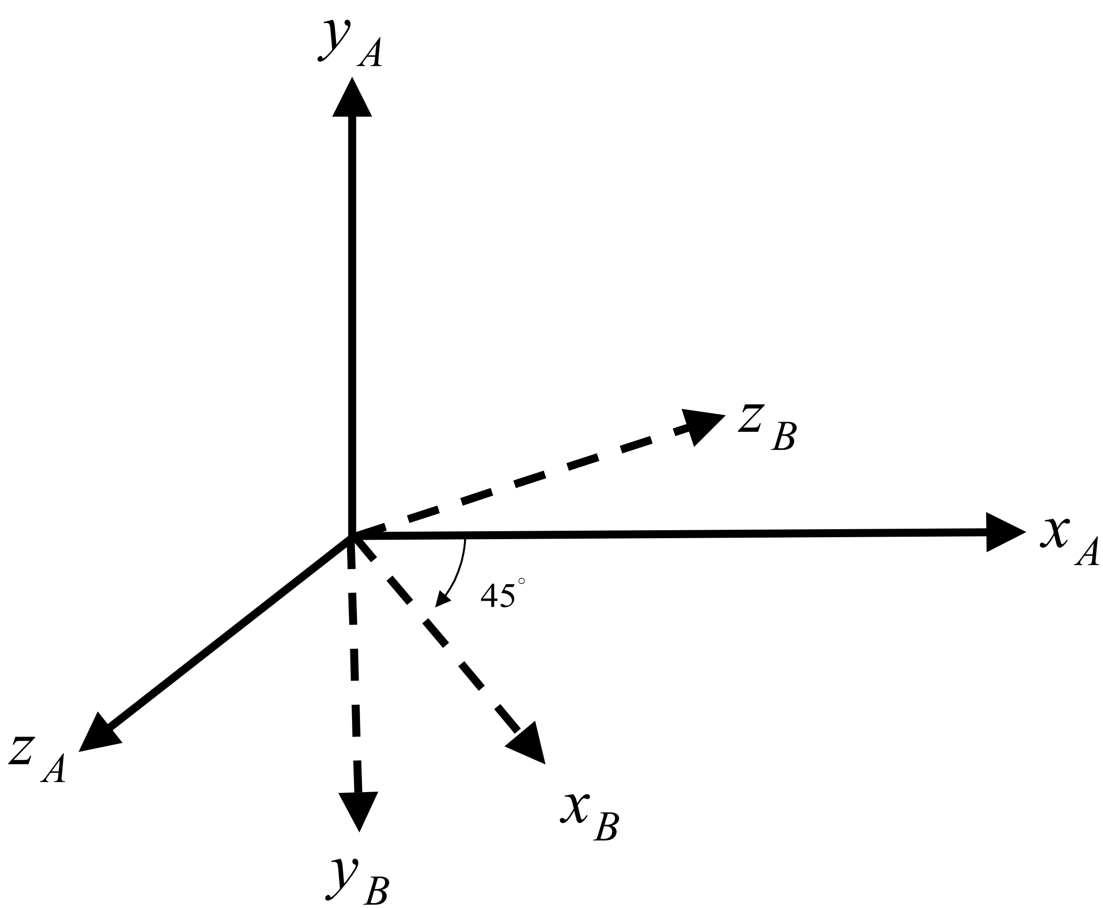
Summary: 3-D rotation

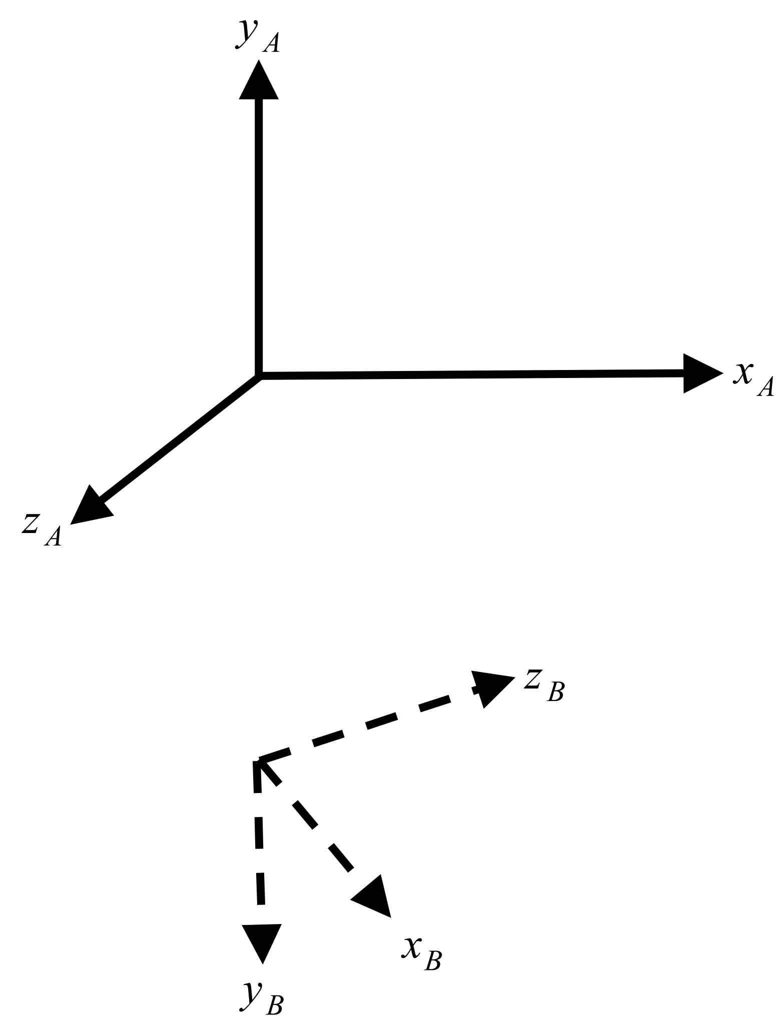
$$\begin{pmatrix} {}^{A}x \\ {}^{A}y \\ {}^{A}z \end{pmatrix} = {}^{A}\boldsymbol{R}_{B} \begin{pmatrix} {}^{B}x \\ {}^{B}y \\ {}^{B}z \end{pmatrix}$$

A 3-dimensional rotation matrix ${}^{X}\boldsymbol{R}_{Y}$ has some special properties:

- it is *orthonormal* (also called *orthogonal*) since each of its columns is a unit vector and the columns are orthogonal.
- the columns are the unit vectors that define the axes of the rotated frame Y with respect to X and are by definition both unit-length and orthogonal.
- it belongs to the special orthogonal group of dimension 3 or $\boldsymbol{R} \in \mathbf{SO}(3) \subset \mathbb{R}^{3\times 3}$. This means that the product of any two matrices within the group also belongs to the group, as does its inverse.
- its determinant is $+1$, which means that the length of a vector is unchanged after transformation, that is, $\|{}^{Y}\boldsymbol{p}\| = \|{}^{X}\boldsymbol{p}\|, \forall\theta$.
- the inverse is the same as the transpose, that is, $\boldsymbol{R}^{-1} = \boldsymbol{R}^{T}$.

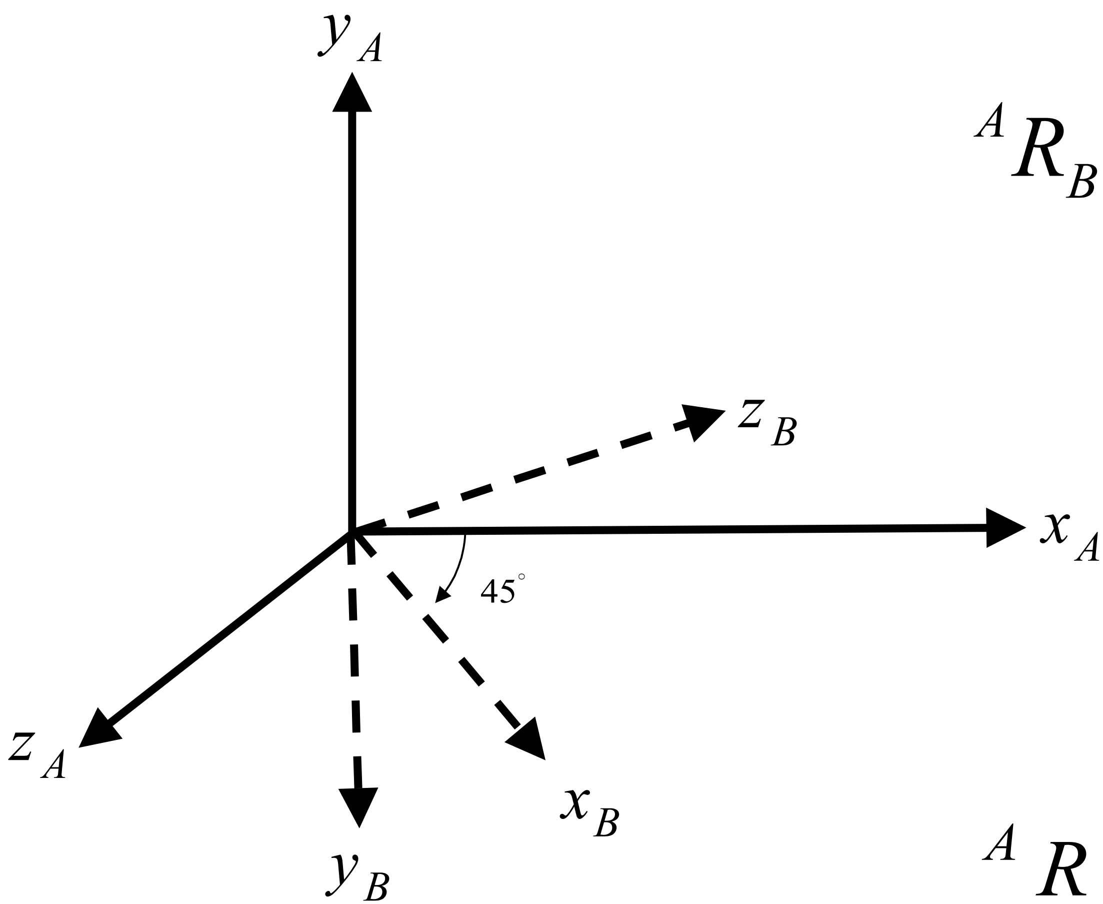
Example





Find ${}^{A}R_{B}$

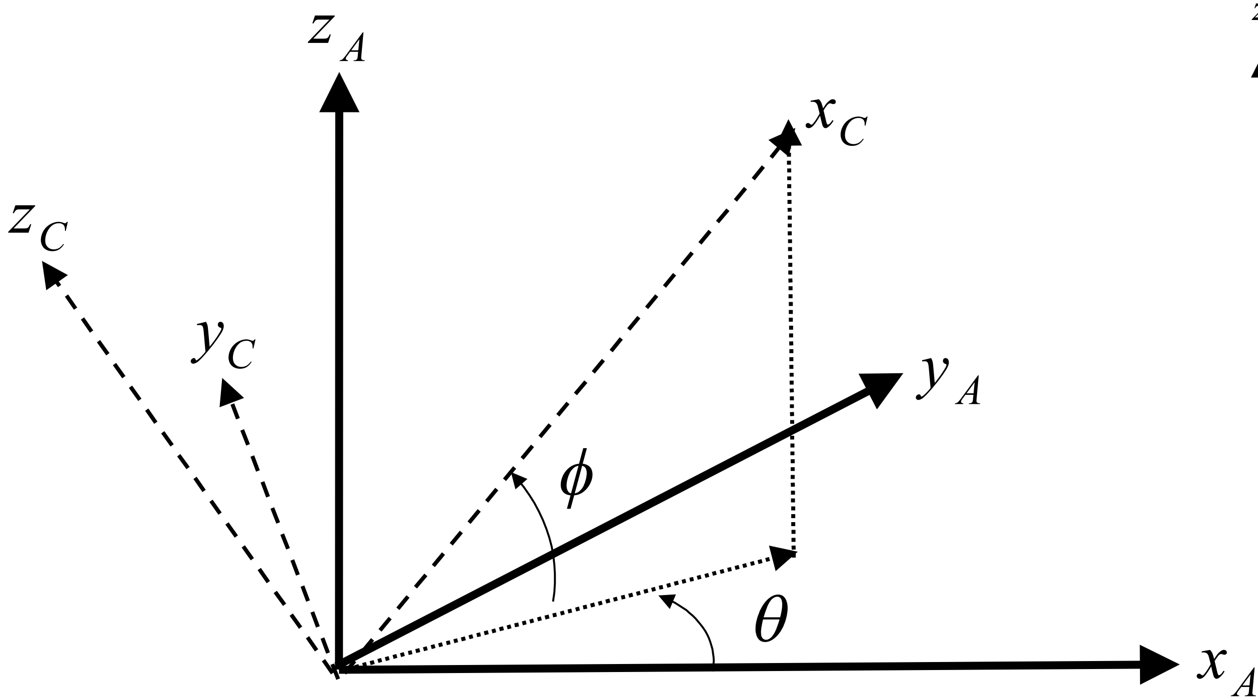
Example

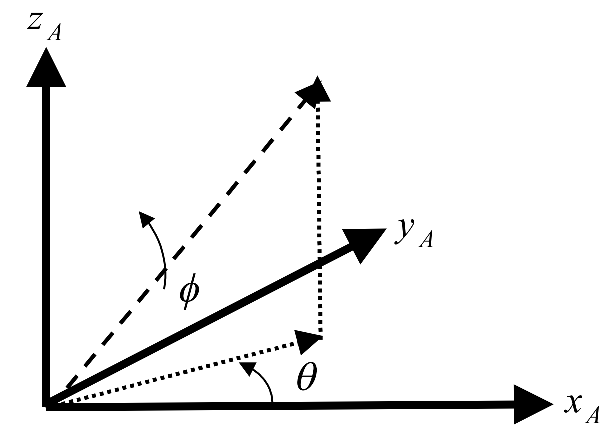


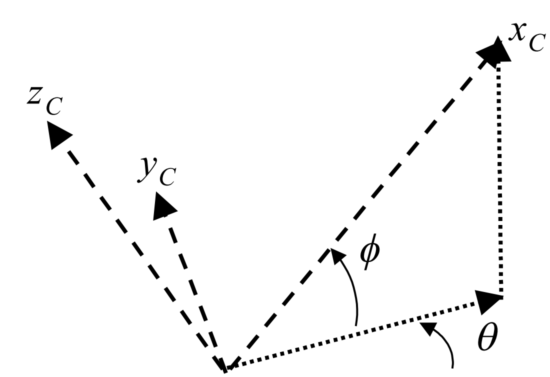
$$ {}^{A}R_{B} = \begin{pmatrix} {}^{A}\hat{x}_{B} & {}^{A}\hat{y}_{B} & {}^{A}\hat{z}_{B} \end{pmatrix} $$

$$ {}^{A}R_{B} = \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & -1 & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{pmatrix} $$

Example

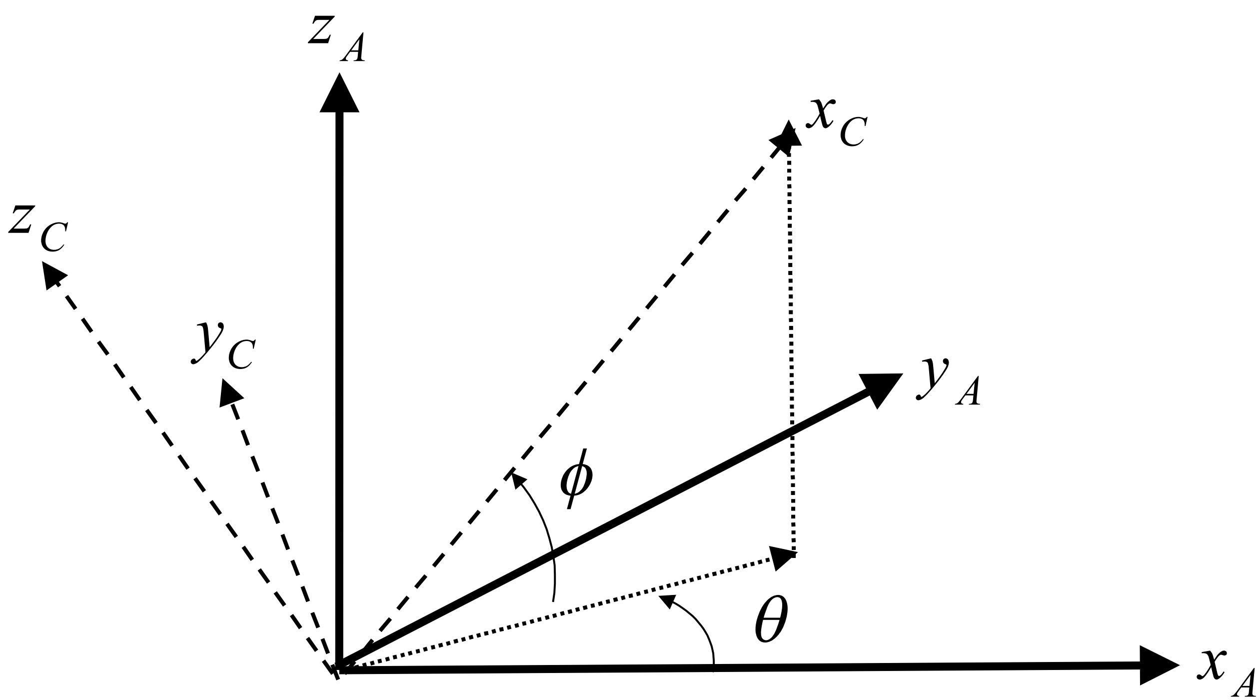






Find $^{A}R_{C}$

Example



$$
{}^{A}R_{C} = {}^{A}R_{B}\,{}^{B}R_{C} = \begin{pmatrix} c_\theta & -s_\theta & 0 \\ s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_\varphi & 0 & -s_\varphi \\ 0 & 1 & 0 \\ s_\varphi & 0 & c_\varphi \end{pmatrix} = \begin{pmatrix} c_\theta c_\varphi & -s_\theta & -c_\theta s_\varphi \\ s_\theta c_\varphi & c_\theta & -s_\theta s_\varphi \\ s_\varphi & 0 & c_\varphi \end{pmatrix}
$$

Outline

✔ Vector / matrix refresher

✔ SO(2): 2-D rotation / orientation

✔ SE(2): 2-D transformations

✔ SO(3): 3-D rotation / orientation

SE(3): 3-D transformations

More representations for 3-D rotations (time permitting)
- Euler angles
- Axis-angle representations
- Quaternions

3-D transformation

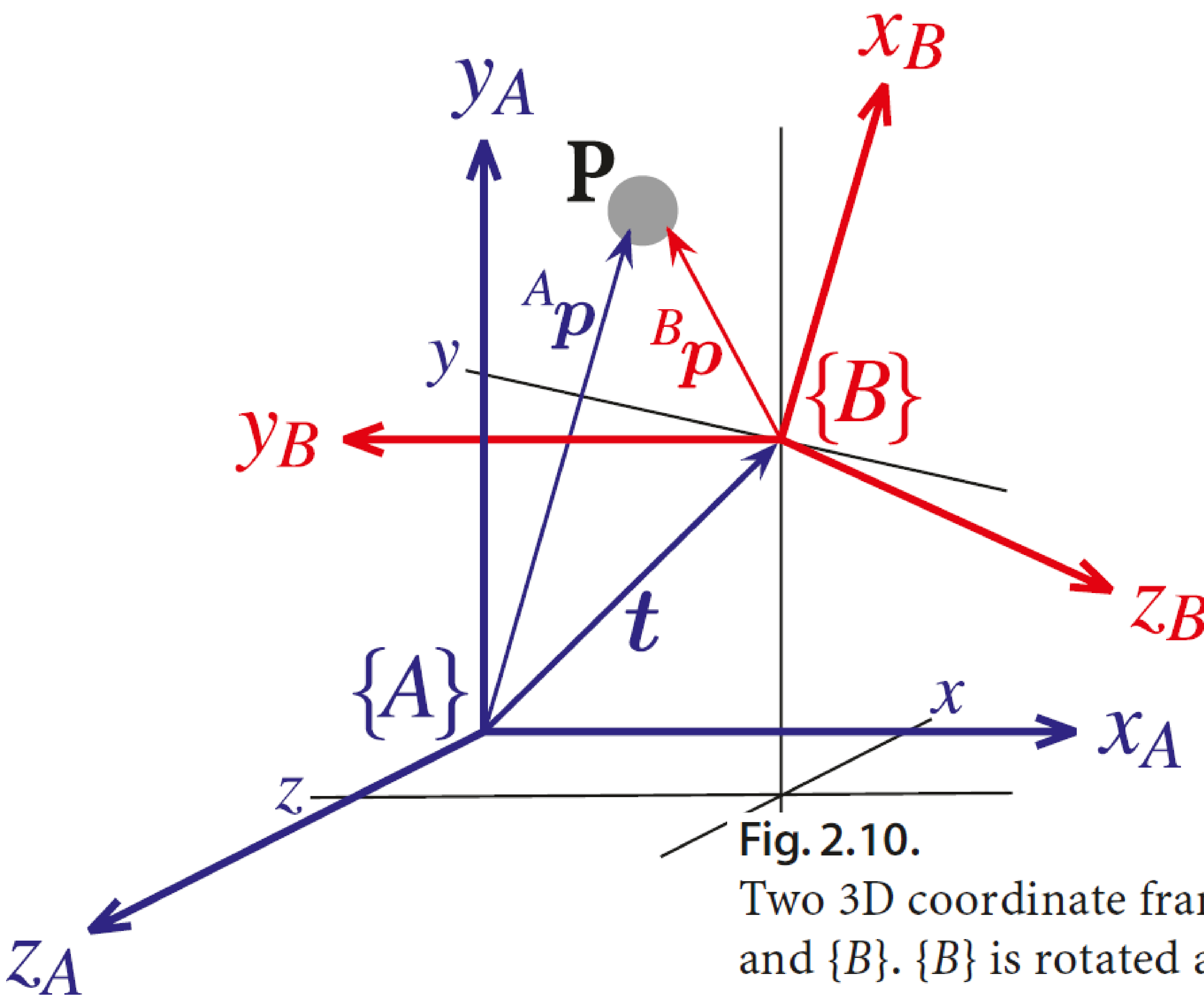


Fig. 2.10.
Two 3D coordinate frames $\{A\}$ and $\{B\}$. $\{B\}$ is rotated and translated with respect to $\{A\}$

3-D transformation

$$\begin{pmatrix} {}^{A}x \\ {}^{A}y \\ {}^{A}z \\ 1 \end{pmatrix} = \begin{pmatrix} {}^{A}\boldsymbol{R}_B & \boldsymbol{t} \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix} \begin{pmatrix} {}^{B}x \\ {}^{B}y \\ {}^{B}z \\ 1 \end{pmatrix}$$

$$\begin{aligned} {}^{A}\tilde{\boldsymbol{p}} &= \begin{pmatrix} {}^{A}\boldsymbol{R}_B & \boldsymbol{t} \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix} {}^{B}\tilde{\boldsymbol{p}} \\ &= {}^{A}\boldsymbol{T}_B \, {}^{B}\tilde{\boldsymbol{p}} \end{aligned}$$

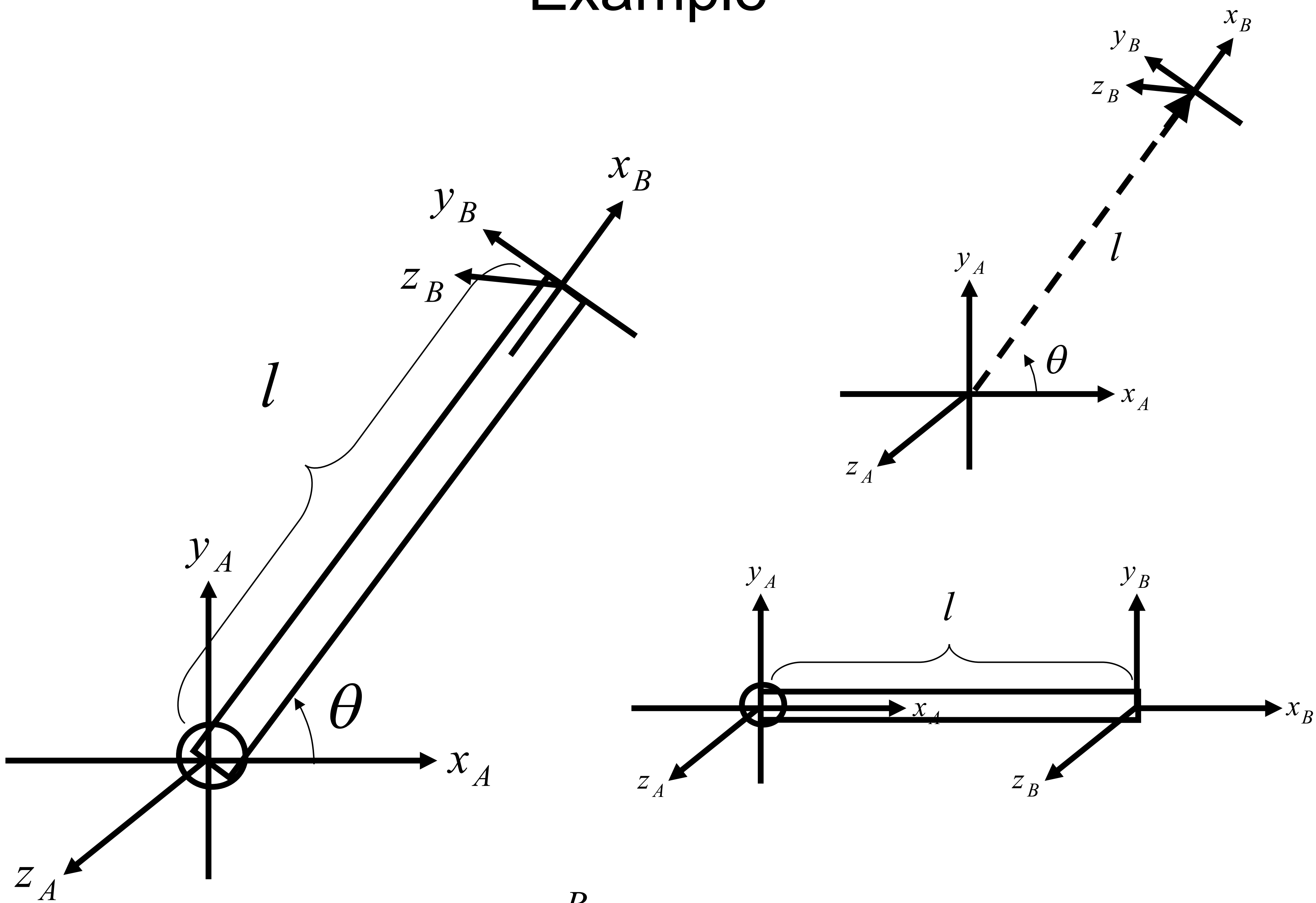
A concrete representation of relative pose is $\xi \sim \boldsymbol{T} \in \mathbf{SE}(3)$ and $\boldsymbol{T}_1 \oplus \boldsymbol{T}_2 \mapsto \boldsymbol{T}_1\boldsymbol{T}_2$ which is standard matrix multiplication.

$$\boldsymbol{T}_1\boldsymbol{T}_2 = \begin{pmatrix} \boldsymbol{R}_1 & \boldsymbol{t}_1 \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix} \begin{pmatrix} \boldsymbol{R}_2 & \boldsymbol{t}_2 \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix} = \begin{pmatrix} \boldsymbol{R}_1\boldsymbol{R}_2 & \boldsymbol{t}_1 + \boldsymbol{R}_1\boldsymbol{t}_2 \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix} \tag{2.24}$$

One of the rules of pose algebra from page 21 is $\xi \oplus 0 = \xi$. For matrices we know that $\boldsymbol{T}\boldsymbol{I} = \boldsymbol{T}$, where $\boldsymbol{I}$ is the identify matrix, so for pose $0 \mapsto \boldsymbol{I}$ the identity matrix. Another rule of pose algebra was that $\xi \ominus \xi = 0$. We know for matrices that $\boldsymbol{T}\boldsymbol{T}^{-1} = \boldsymbol{I}$ which implies that $\ominus \boldsymbol{T} \mapsto \boldsymbol{T}^{-1}$

$$\boldsymbol{T}^{-1} = \begin{pmatrix} \boldsymbol{R} & \boldsymbol{t} \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix}^{-1} = \begin{pmatrix} \boldsymbol{R}^T & -\boldsymbol{R}^T\boldsymbol{t} \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix} \tag{2.25}$$

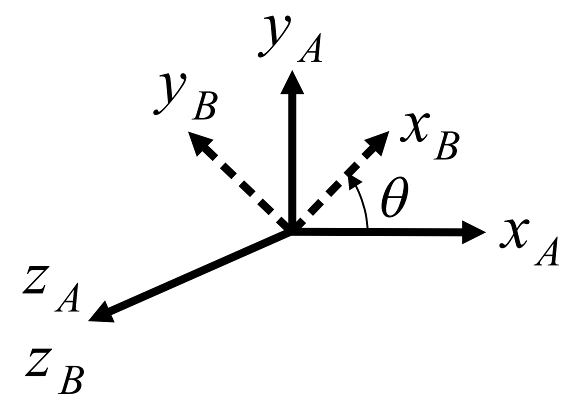
Example



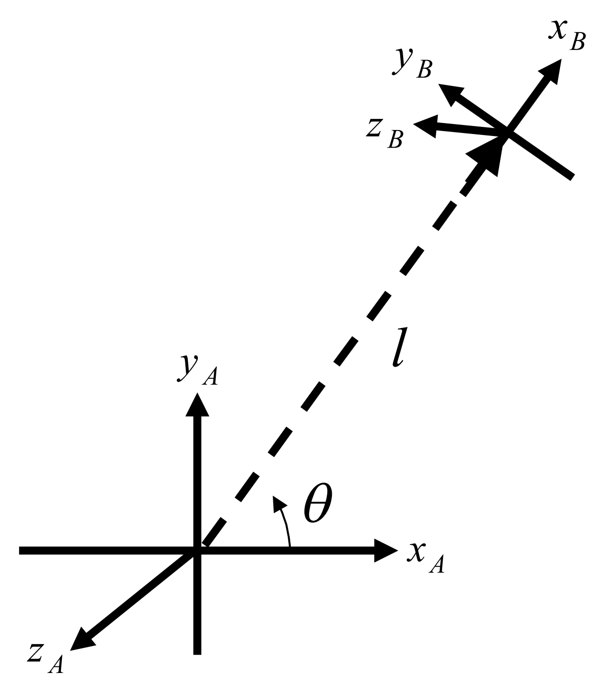
Find $^{B}T_{A}$

Example

Find ${}^{B}T_{A}$



$$
{}^{A}R_{B} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

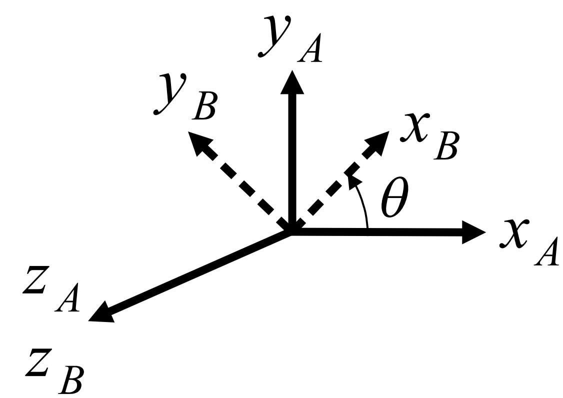


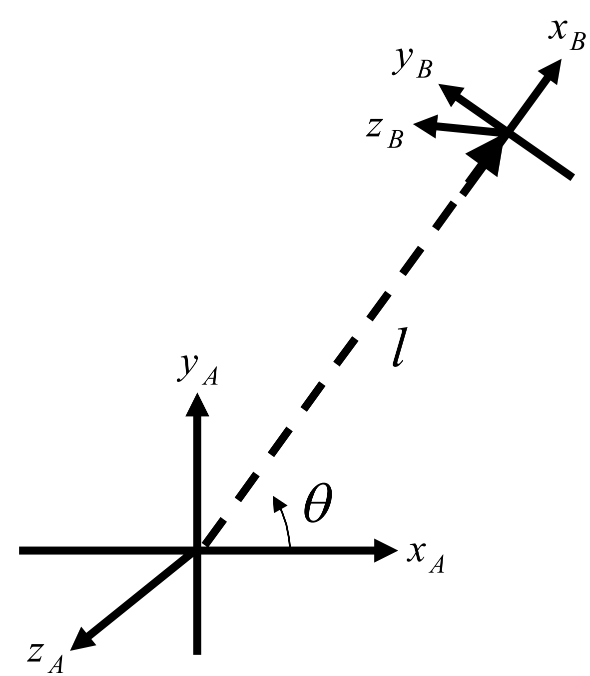
Example

Find ${}^{B}T_{A}$

$$ {}^{A}R_{B} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

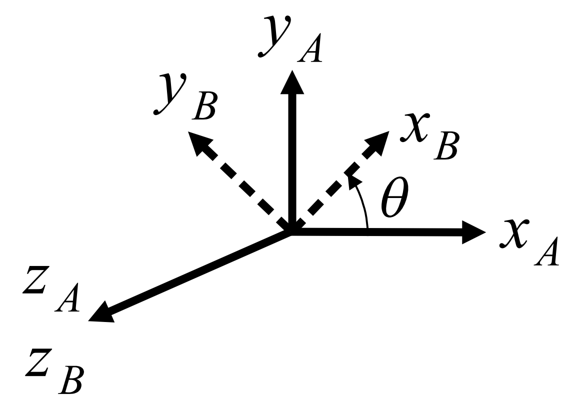
$$ {}^{B}d_{A} = \begin{pmatrix} -l \\ 0 \\ 0 \end{pmatrix} $$





Example

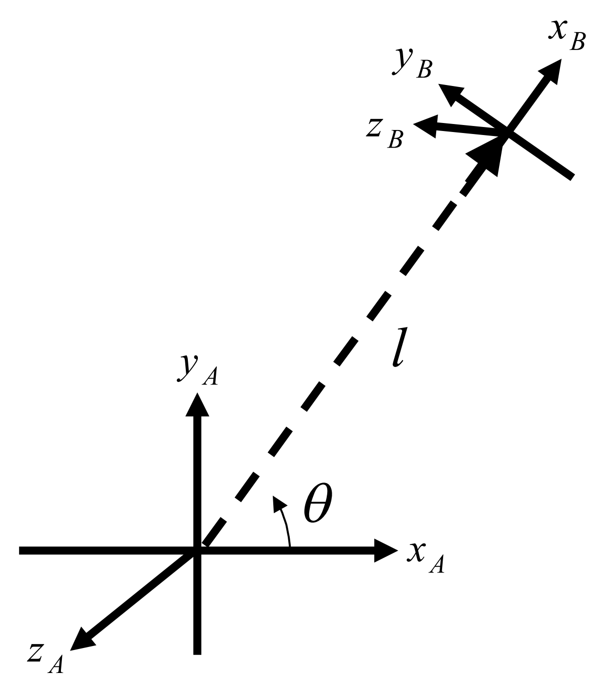
Find ${}^{B}T_{A}$



$$ {}^{A}R_{B} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

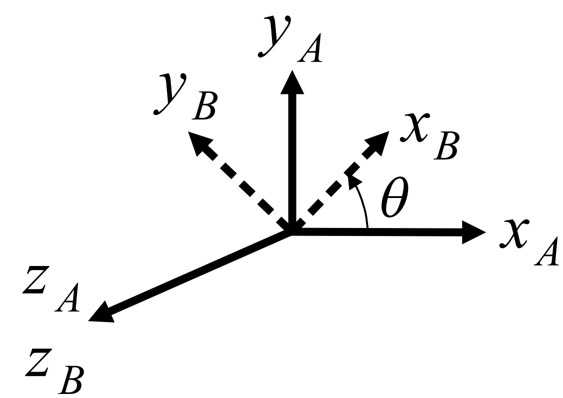
$$ {}^{B}d_{A} = \begin{pmatrix} -l \\ 0 \\ 0 \end{pmatrix} $$

$$ {}^{B}T_{A} = \begin{pmatrix} {}^{B}R_{A} & {}^{B}d_{A} \\ 0 & 1 \end{pmatrix} $$



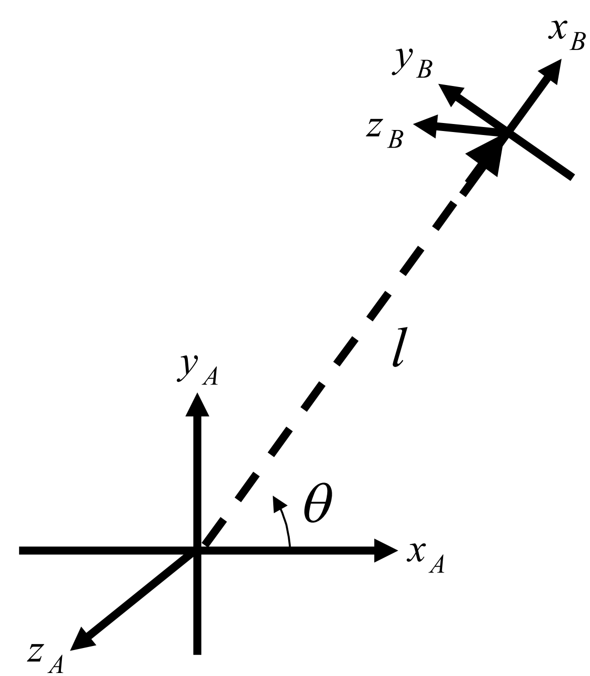
Example

Find ${}^{B}T_{A}$



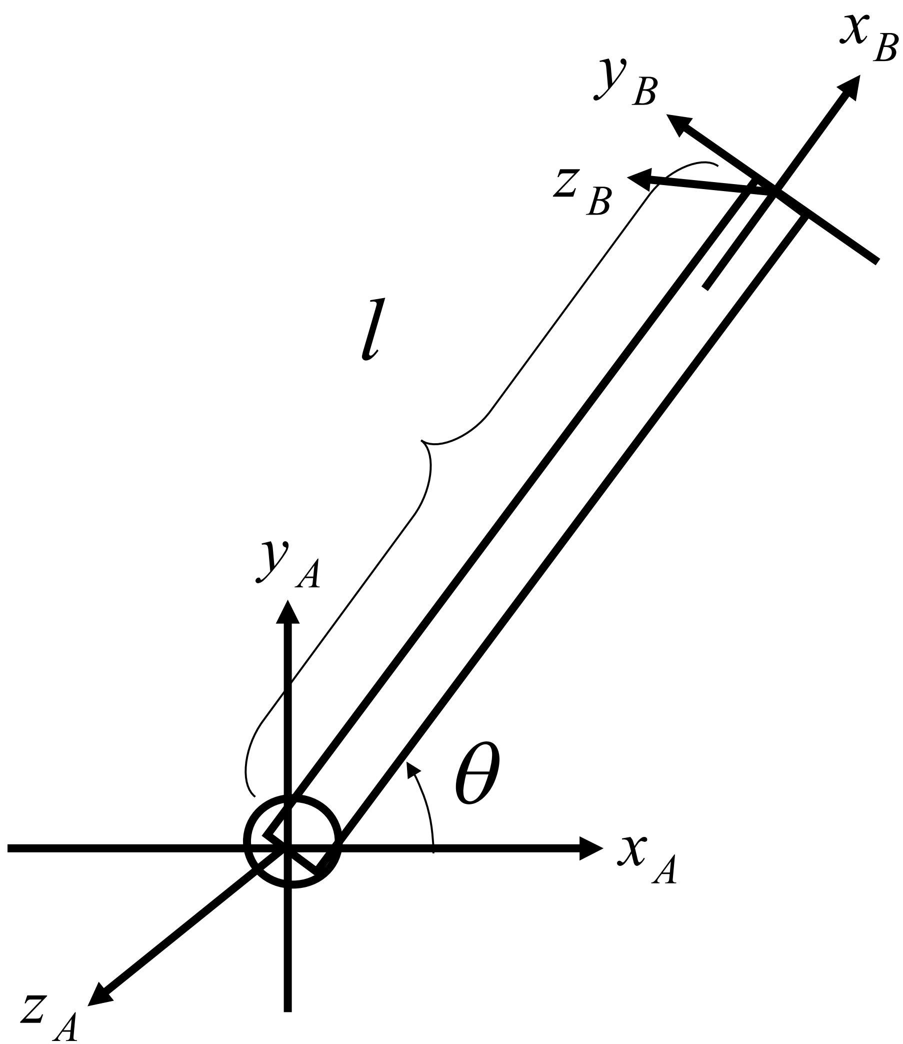
$$ {}^{A}R_{B} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{B}d_{A} = \begin{pmatrix} -l \\ 0 \\ 0 \end{pmatrix} \qquad {}^{B}T_{A} = \begin{pmatrix} {}^{B}R_{A} & {}^{B}d_{A} \\ 0 & 1 \end{pmatrix} $$



$$ {}^{B}T_{A} = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 & -l \\ -\sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Example



$$^{B}T_{A} = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 & -l \\ -\sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

How about $^{A}T_{B}$?

Example

$$ {}^{B}T_{A}{}^{-1} = \begin{pmatrix} {}^{B}R_{A}{}^{T} & -{}^{B}R_{A}{}^{T}\,{}^{B}d_{A} \\ 0 & 1 \end{pmatrix} $$

Example

$$ {}^{B}T_{A}^{\;-1} = \begin{pmatrix} {}^{B}R_{A}^{\;T} & -{}^{B}R_{A}^{\;T}\,{}^{B}d_{A} \\ 0 & 1 \end{pmatrix} $$

$$ {}^{A}R_{B} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

Example

$$ {}^{B}T_{A}{}^{-1} = \begin{pmatrix} {}^{B}R_{A}{}^{T} & -{}^{B}R_{A}{}^{T}\,{}^{B}d_{A} \\ 0 & 1 \end{pmatrix} \qquad {}^{A}R_{B} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{B}d_{A} = \begin{pmatrix} -l \\ 0 \\ 0 \end{pmatrix} \qquad -{}^{A}R_{B}\,{}^{B}d_{A} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -l \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} l\cos(\theta) \\ l\sin(\theta) \\ 0 \end{pmatrix} $$

Example

$$ {}^{B}T_{A}^{-1} = \begin{pmatrix} {}^{B}R_{A}^{T} & -{}^{B}R_{A}^{T}\,{}^{B}d_{A} \\ 0 & 1 \end{pmatrix} $$

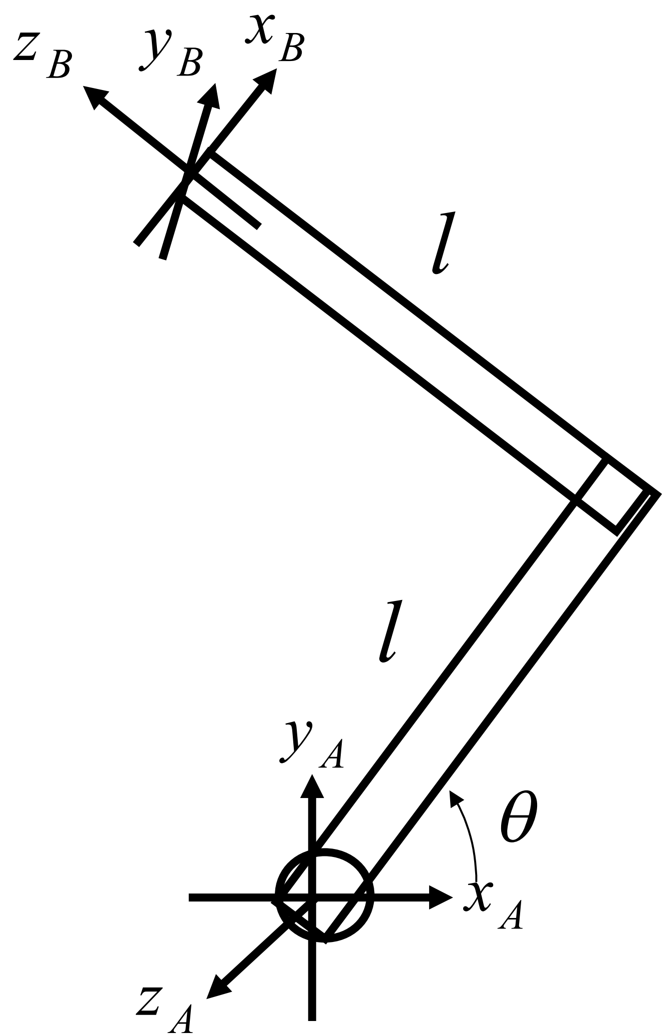
$$ {}^{A}R_{B} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{B}d_{A} = \begin{pmatrix} -l \\ 0 \\ 0 \end{pmatrix} $$

$$ -{}^{A}R_{B}\,{}^{B}d_{A} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -l \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} l\cos(\theta) \\ l\sin(\theta) \\ 0 \end{pmatrix} $$

$$ {}^{B}T_{A}^{-1} = {}^{A}T_{B} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & l\cos(\theta) \\ \sin(\theta) & \cos(\theta) & 0 & l\sin(\theta) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Example



This arm rotates about the z_A axis

Find ${}^{A}T_{B}$ and ${}^{B}T_{A}$

Summary: 3-D transformation

$$\begin{pmatrix} {}^{A}x \\ {}^{A}y \\ {}^{A}z \\ 1 \end{pmatrix} = \begin{pmatrix} {}^{A}\boldsymbol{R}_B & \boldsymbol{t} \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix} \begin{pmatrix} {}^{B}x \\ {}^{B}y \\ {}^{B}z \\ 1 \end{pmatrix}$$

$$\begin{aligned} {}^{A}\tilde{\boldsymbol{p}} &= \begin{pmatrix} {}^{A}\boldsymbol{R}_B & \boldsymbol{t} \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix} {}^{B}\tilde{\boldsymbol{p}} \\ &= {}^{A}\boldsymbol{T}_B \, {}^{B}\tilde{\boldsymbol{p}} \end{aligned}$$

A concrete representation of relative pose is $\xi \sim \boldsymbol{T} \in \mathbf{SE}(3)$ and $\boldsymbol{T}_1 \oplus \boldsymbol{T}_2 \mapsto \boldsymbol{T}_1 \boldsymbol{T}_2$ which is standard matrix multiplication.

$$\boldsymbol{T}_1 \boldsymbol{T}_2 = \begin{pmatrix} \boldsymbol{R}_1 & \boldsymbol{t}_1 \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix} \begin{pmatrix} \boldsymbol{R}_2 & \boldsymbol{t}_2 \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix} = \begin{pmatrix} \boldsymbol{R}_1 \boldsymbol{R}_2 & \boldsymbol{t}_1 + \boldsymbol{R}_1 \boldsymbol{t}_2 \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix} \tag{2.24}$$

One of the rules of pose algebra from page 21 is $\xi \oplus 0 = \xi$. For matrices we know that $\boldsymbol{T}\boldsymbol{I} = \boldsymbol{T}$, where $\boldsymbol{I}$ is the identify matrix, so for pose $0 \mapsto \boldsymbol{I}$ the identity matrix. Another rule of pose algebra was that $\xi \ominus \xi = 0$. We know for matrices that $\boldsymbol{T}\boldsymbol{T}^{-1} = \boldsymbol{I}$ which implies that $\ominus \boldsymbol{T} \mapsto \boldsymbol{T}^{-1}$

$$\boldsymbol{T}^{-1} = \begin{pmatrix} \boldsymbol{R} & \boldsymbol{t} \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix}^{-1} = \begin{pmatrix} \boldsymbol{R}^T & -\boldsymbol{R}^T \boldsymbol{t} \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix} \tag{2.25}$$

Feedback

Piazza thread: 1/26 Lec 03 Feedback

Please post your answers to the following anonymously.

1. What did you like so far?

2. What was unclear?

3. Have you read the Shakey paper (for Ex0) yet?
   If so, did you enjoy it?

4. Any additional feedback / comments?