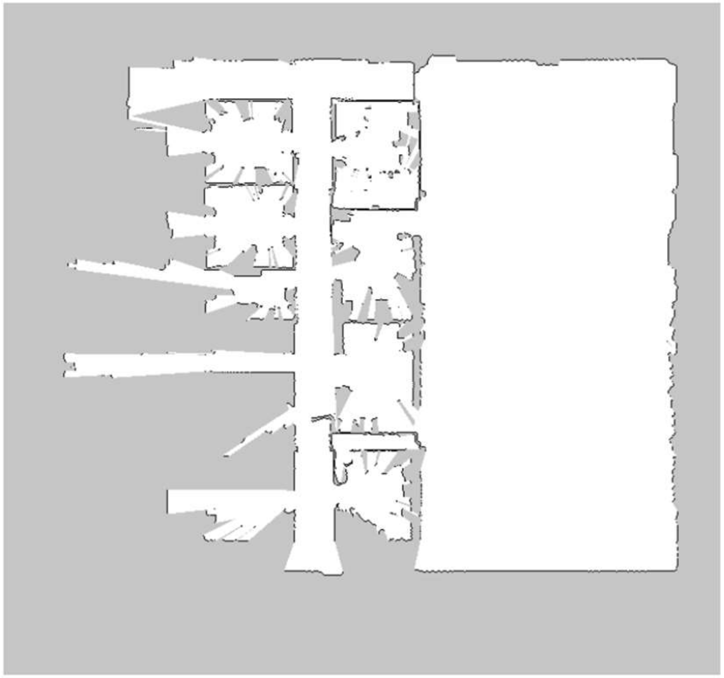
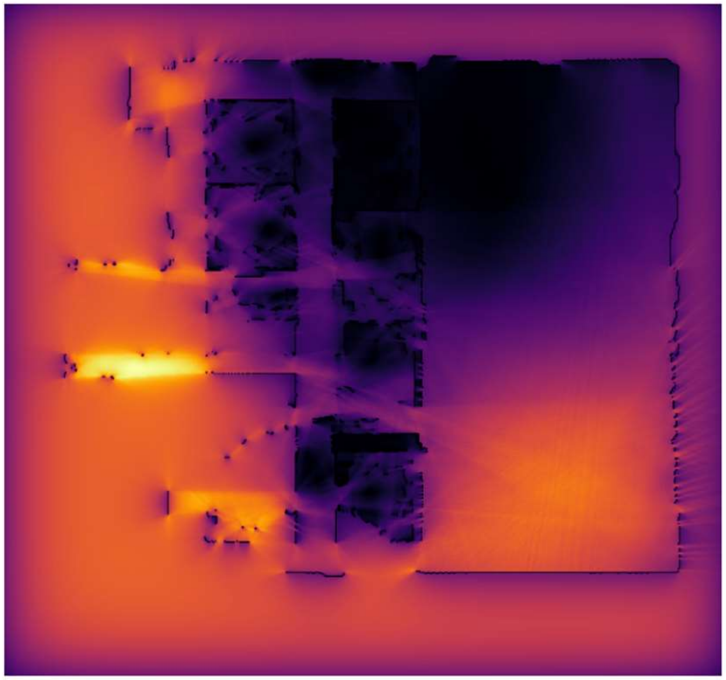
EECE 5550: Mobile Robotics





Lecture 21: Robotic Exploration

Plan of the day

- The exploration problem
- Frontier-based exploration
- A brief introduction to information theory
- Information-based exploration

References

- B. Yamauchi: “A Frontier-Based Approach for Autonomous Exploration”

- F. Bourgault, A.A. Makarenko, S.B. Williams: “Information Based Adaptive Exploration”

- H. Carrillo, P. Dames, V. Kumar, J.A. Castellanos: “Autonomous Robotic Exploration Using Occupancy Grid Maps and Graph SLAM Based on Shannon and Renyi Entropy”

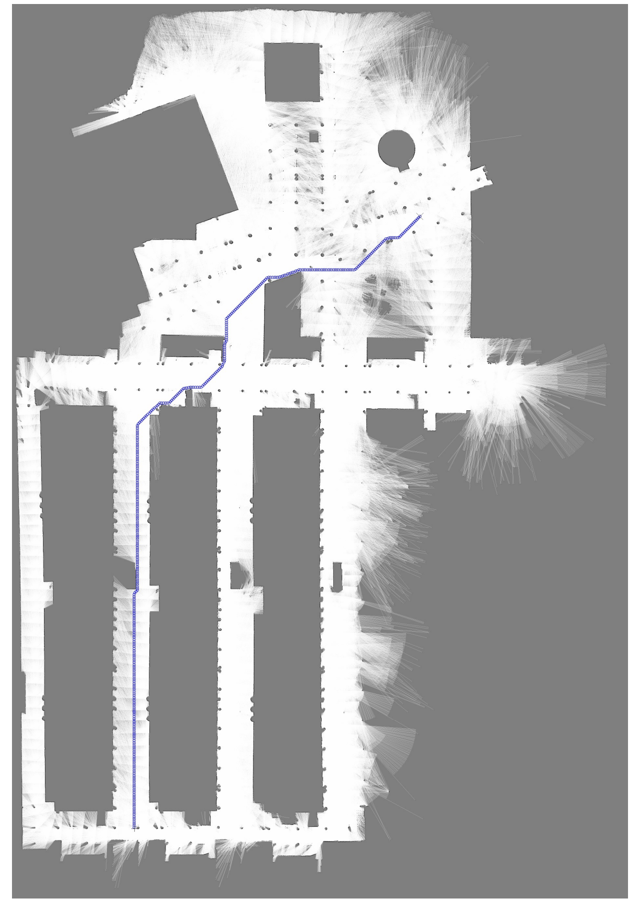
- C.E. Shannon: “A Mathematical Theory of Communication”

The Exploration Problem

Goal: Build a complete map of the environment as quickly as possible.

Recall: SLAM and mapping algorithms enable us to build a map from sensor measurements, but they don't directly tell us *what measurements to collect*.

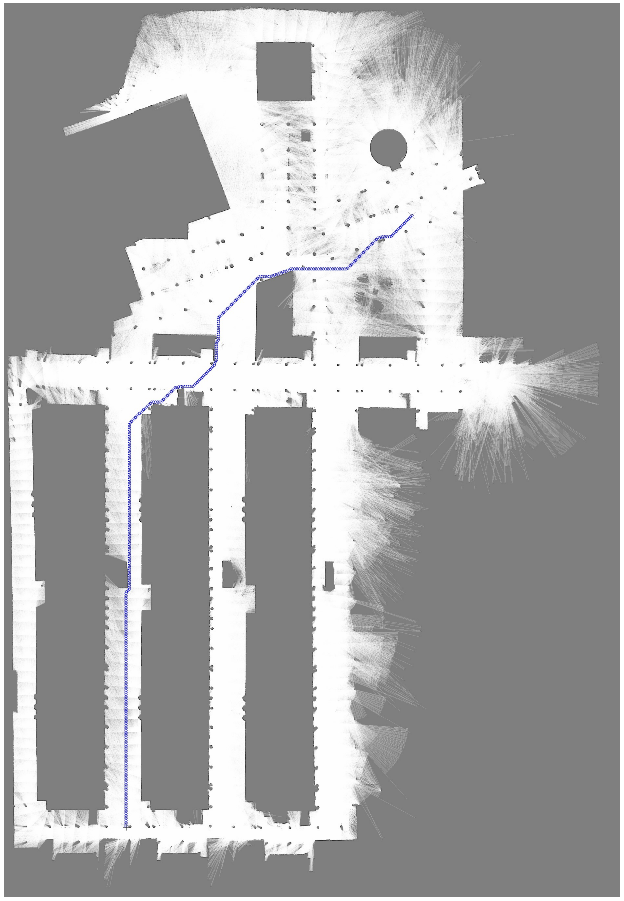
Fundamental question: How should we decide *what measurements to collect,* given that we only have partial knowledge of the environment as we are building the map?



Frontier-based exploration

Main idea [Yamauchi]**:** To gain the most new information about the world, move to the boundary between the open space and unexplored territory.

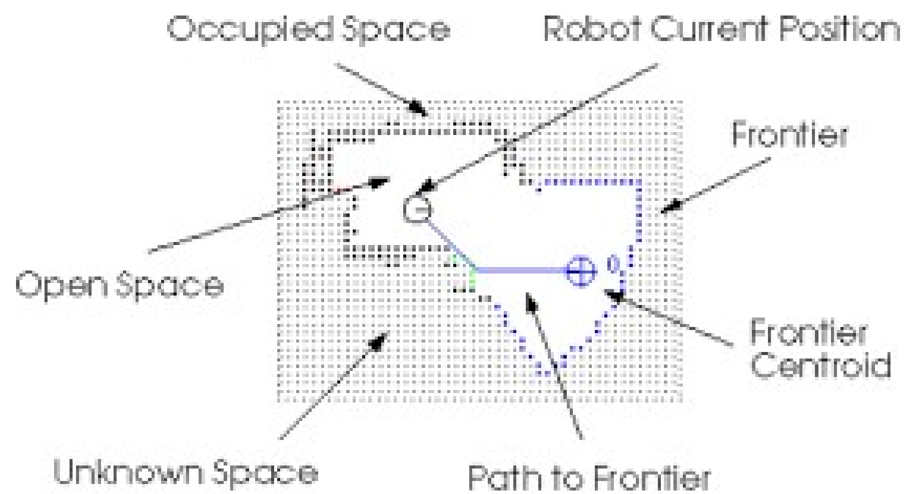
Key question: How can we formalize this notion of a "boundary" between free and unexplored space?



Frontier extraction in occupancy grids

Occupancy grids are particularly nice for frontier-based exploration, because they admit a simple approach to classifying space as *free, occupied, and unknown*:

- **Free:** p(occ) < prior
- **Occupied:** p(occ) > prior
- **Unknown:** p(occ) = prior



We then define the set of *boundary cells* to be the set of all grid cells that are:

- Unoccupied
- Adjacent to an unknown cell

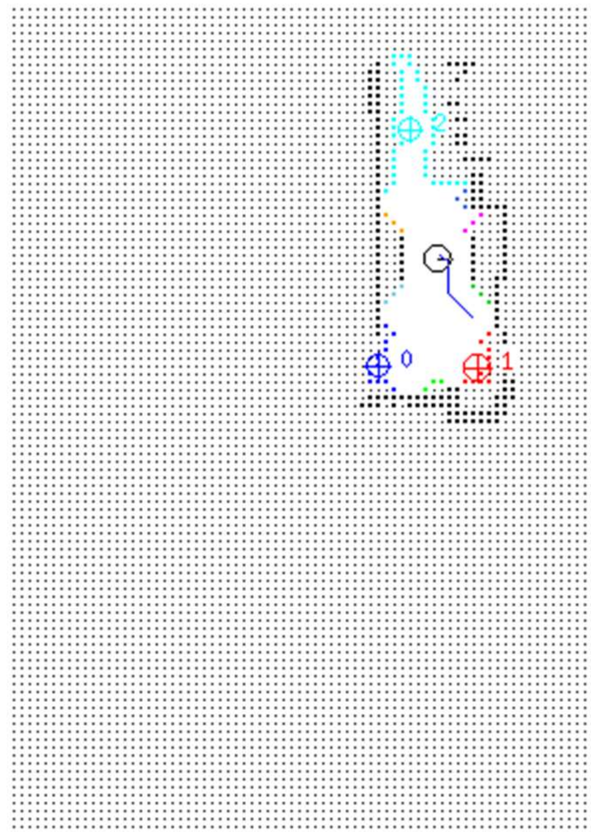
Finally, we define a *frontier* to be a maximal connected set of boundary cells

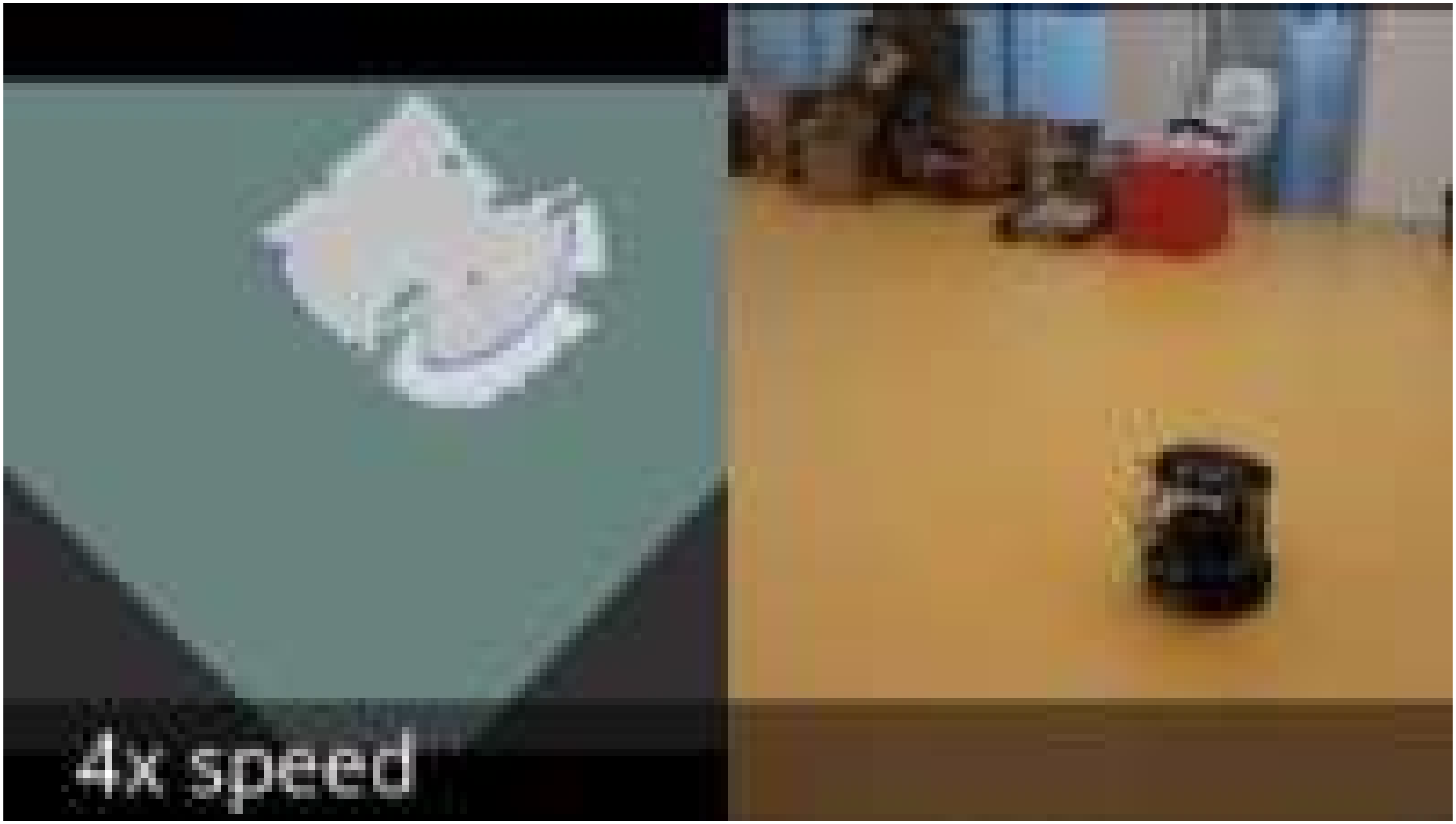
A (Very) Simple Frontier-Based Exploration Algorithm

while frontiers exist, repeat:

1. Plan a path to the nearest one
2. Follow the plan to collect measurements
3. Update map & frontiers

end while





https://youtu.be/op0L0LyGNwY

Frontier-Based Exploration in Occupancy Grids

Pro: Super simple ☺!

But: This approach doesn't directly address map *quality*: that is, how (*un*)*certain* we are about the final map

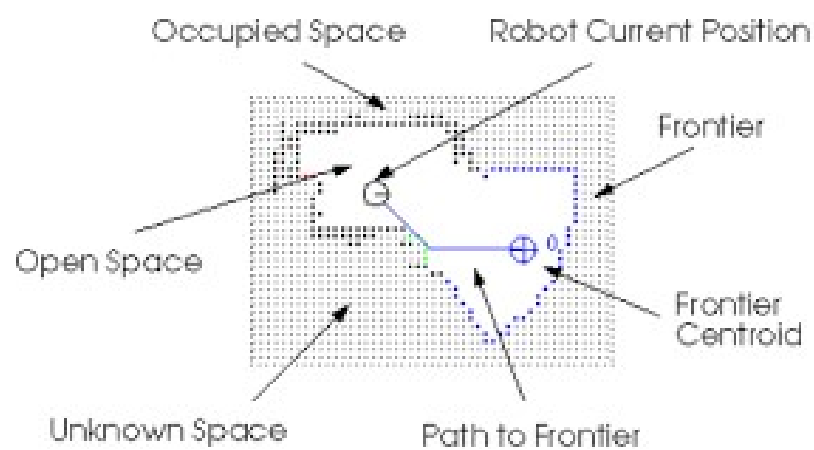
We might like to control this directly …

Frontier-Based Exploration

while frontiers exist, repeat:

1. Plan a path to the nearest one
2. Follow the plan to collect measurements
3. Update map & frontiers

end while



Measuring map quality

Goal: We would like to develop a measure of "quality" for occupancy grid maps.

Key question: What should this actually *be*?

One possible approach: Recall that (probabilistic) occupancy grids encode a *belief* $p(m|z_{1:m})$ over over the map m given measurements $z_{1:m}$

- **Initially:** We (typically) have a uniform prior $p(m_i) = .5$ for all cells
  ⇒Intuitively: we are completely ignorant about the state of the world

- **After mapping:** Ideally, we want either $p(m_i) \approx 1$ $p(m_i) \approx 0$ for all reachable m_i
  ⇒Intuitively: We are highly certain about the status of all cells

⇒This suggests that we think about "quality" in terms of *posterior uncertainty*

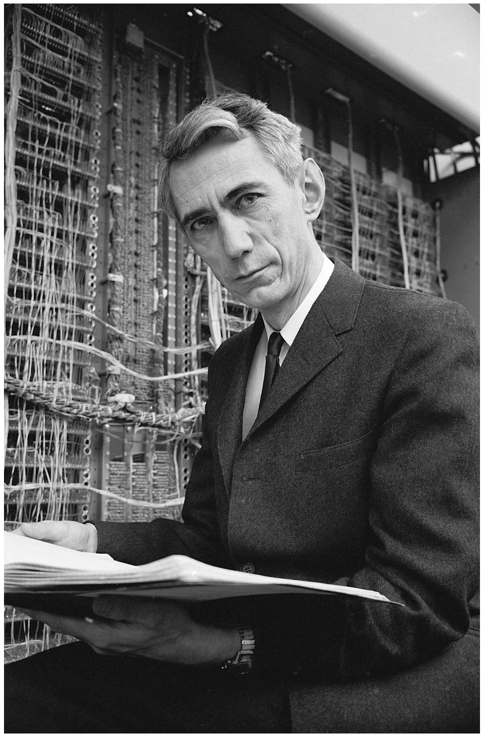
But: How to do *this*?

Quantifying Uncertainty: Entropy

In information theory, the canonical measure of "uncertainty" associated with a random variable X is *entropy*.

If X is a discrete random variable taking values in the set $\{x_1, \dots, x_N\}$, its entropy is given by:

$$H(X) = -\sum_{i=1}^{N} P(x_i) \log P(x_i)$$



Claude E. Shannon

How does entropy measure “uncertainty”?

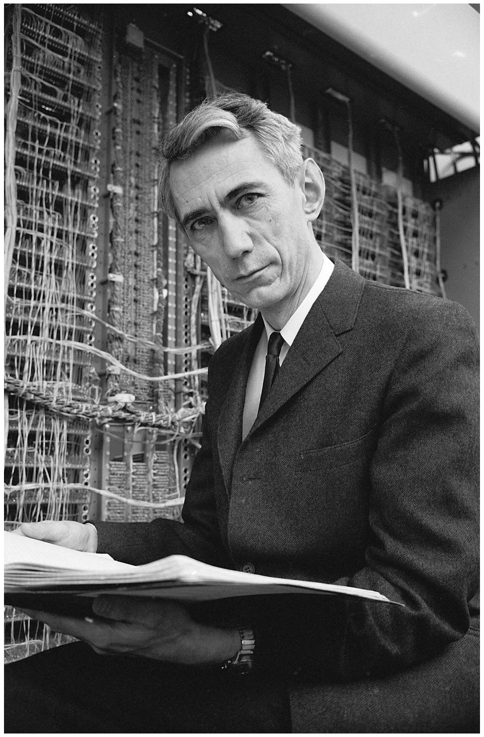
Shannon’s key insight: The amount of “information” obtained by observing the outcome of a random process is directly related to how *unlikely* (or “surprising”) that outcome is.

Intuition: If I know that $P(X = x_i) = 1$, then drawing a random realization of X equal to x_i is not very interesting …

(Conversely, finding out that you just won the lottery is *very* interesting!)

Key question: Can we make this intuition *quantitative*?

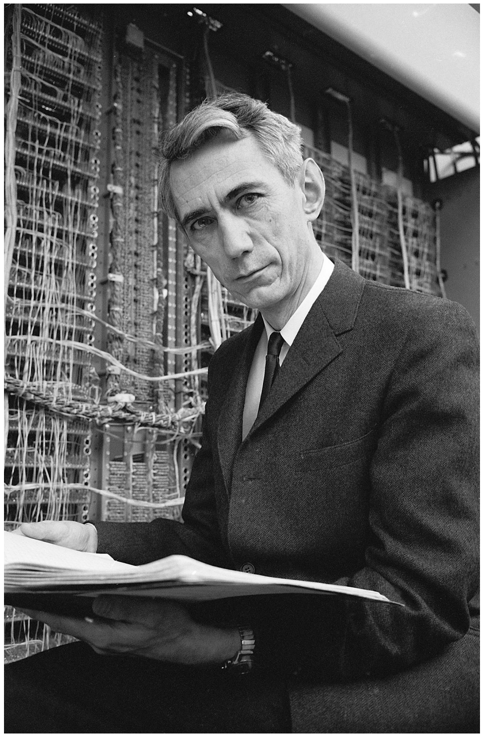
(That is: can we come up with a good way of *measuring* the “information content” $I(x)$ of a random event x?)



Claude E. Shannon

How does entropy measure “uncertainty”?

We would like to develop a measure $I(A)$ that assigns to a random event *A* a measure of how “informative” observing that event is.



Claude E. Shannon

One approach: Let’s specify what properties might we like *I* to posses

Information axioms: Let *A* be a random event.

1. Nonnegativity: $I(A) \geq 0$
2. Monotonicity: $I(A)$ is *monotonically decreasing* in *p*(*A*).
   (Observing a *more likely* event is *less informative / surprising*).
3. If *p*(*A*) = 1 then $I(A) = 0$
   (Observing an almost-sure outcome provides *zero* information)
4. Additivity: If *B* is another event independent of *A*, then:

$$I(A \cap B) = I(A) + I(B)$$

Key point: Shannon showed that any function *I* satisfying 1-4 must be of the form:

$$\boxed{I(A) = k \log p(A)}$$

for k > 0.

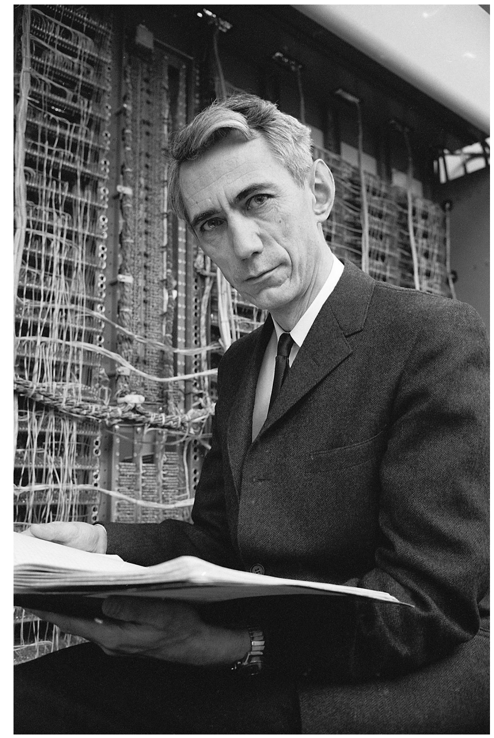
How does entropy measure “uncertainty”?

Now let’s recall the definition of entropy:

$$H(X) = -\sum_{i=1}^{N} P(x_i) \log P(x_i)$$

$$= \underbrace{\sum_{i=1}^{N} P(x_i)}_{\text{Expectation over } \textit{all} \text{ possible realizations of } X} \cdot \underbrace{(-\log P(x_i))}_{\substack{\text{Information / surprise of} \\ \text{observing realization } X = x_i}}$$

Therefore: The entropy $H(X)$ of a random variable *X* is measuring the *expected information / surprise* of observing a realization of *X*.



Claude E. Shannon

Conditional entropy and mutual information

We can also use the notion of entropy to measure how "informative" one random variable is with respect to another.

Suppose that *X* and *Y* are jointly distributed, with $P(X, Y) = P(Y|X)P(X)$, and that I measure $Y = y$. How could I quantify how much observing *Y* has reduced my uncertainty about *X*?

Before measurement: My belief over *X* was simply the *prior* $p(X)$.
After measurement: My belief over *X* is the *posterior* $p(X|Y = y)$.

Therefore: The reduction in my uncertainty over *X* due to measuring *Y* = *y* is:

$$H(X) - H(X|Y = y)$$

It follows that the *expected reduction* in *X*'s uncertainty after observing *Y* is:

$$H(X) - E_Y[H(X|Y = y)]$$

Conditional entropy and mutual information

Recall: The *expected reduction* in *X*'s uncertainty after observing *Y* is:

$$H(X) - E_Y[H(X|Y = y)]$$

The second term above is called the *conditional entropy*, denoted $H(X|Y)$:

$$H(X|Y) \triangleq E_Y[H(X|Y = y)]$$

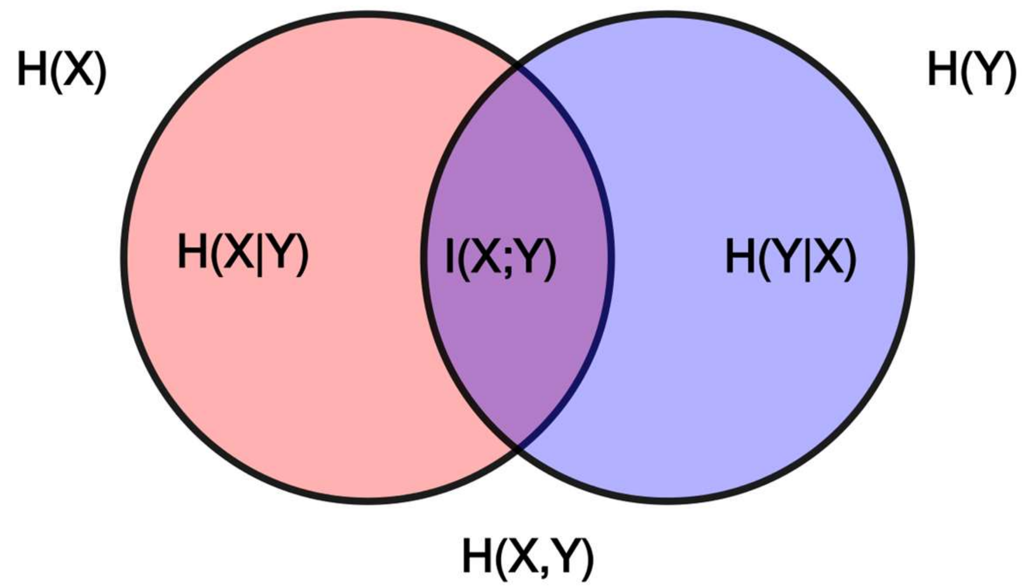
Similarly, the expected reduction in *X*'s uncertainty after observing *Y* is called the *mutual information* between *X* and *Y*, denoted $I(X;Y)$:

$$\boxed{I(X;Y) \triangleq H(X) - H(X|Y)}$$

⇒ The mutual information $I(X;Y)$ quantifies how much observing *Y* tells me about *X*.

Conditional entropy and mutual information

$$I(X;Y) \triangleq H(X) - H(X|Y)$$



Entropy in occupancy grid maps

Recall: We wanted to measure the "quality" of an occupancy grid map using a notion of map *uncertainty*.

The preceding discussion suggests that we use *entropy* as a performance measure.

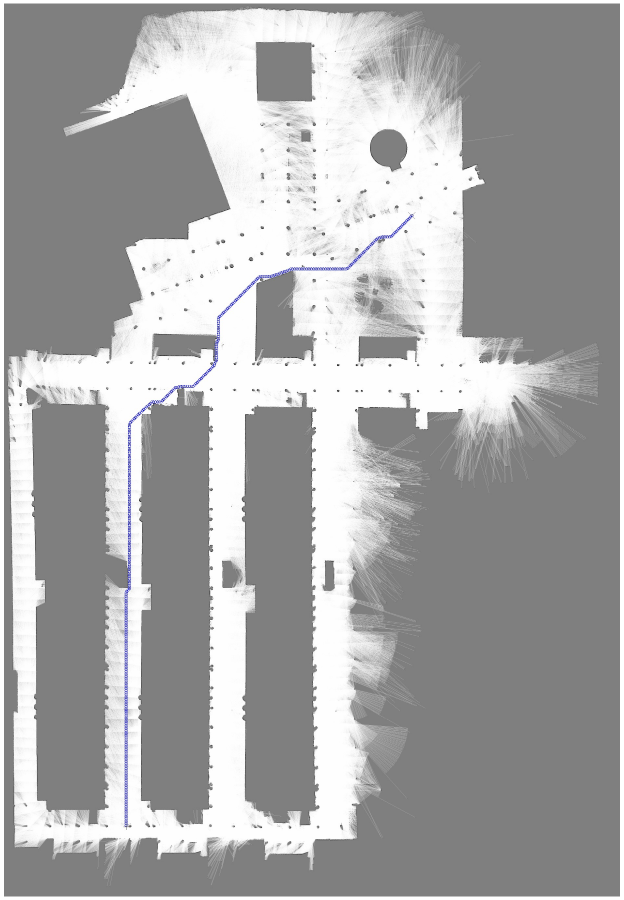
NB: The independence assumption

$$p(m) = \prod_i p(m_i)$$

implies:

$$H(m) = \sum_i H(m_i)$$

Intuitively: the uncertainty of the *entire map* is the *sum* of the uncertainties of each of its cells



Information-based exploration

Recall: The goal of exploration is to quickly build a high-quality map.

Given a *current* belief $p(M)$ for the map, how do we determine where to scan next?

Simple greedy strategy: Select the scan that *maximizes the decrease in map entropy*:

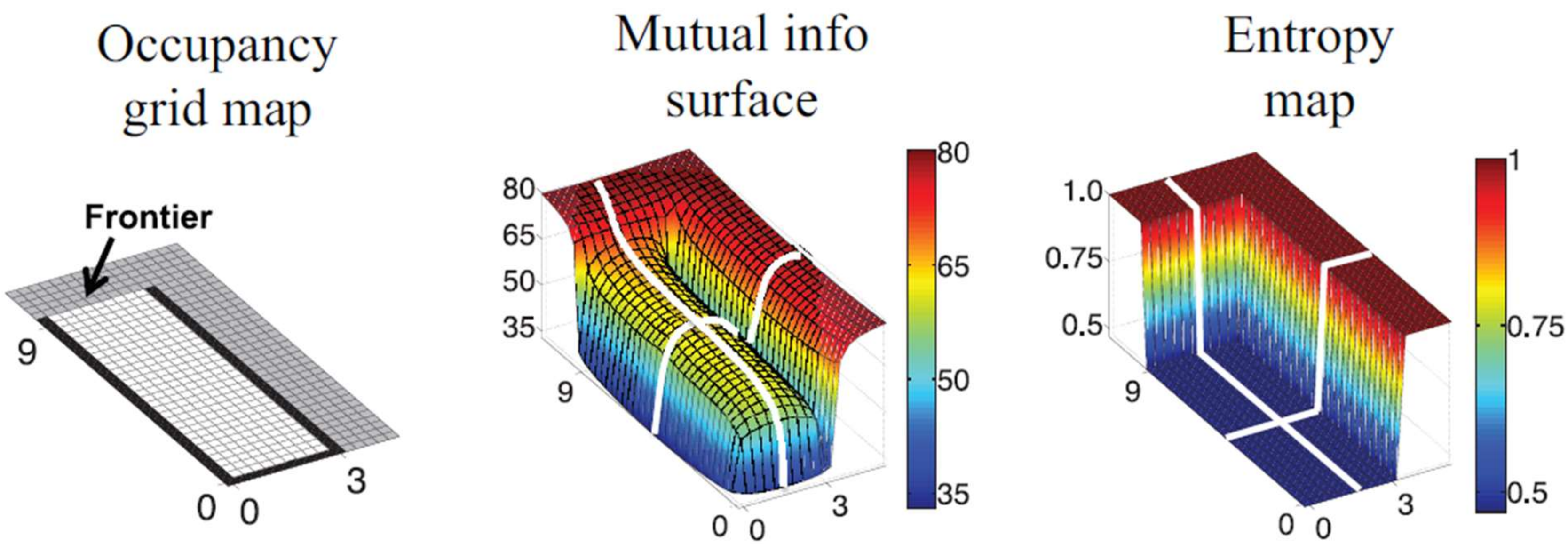
$$x_k^* = argmax_{x \in SE(d)} \boxed{H(M) - H(M|Z(x))} \longleftarrow I(M; Z(x))$$

where here "$Z(x)$" means "a scan taken at pose *x*".

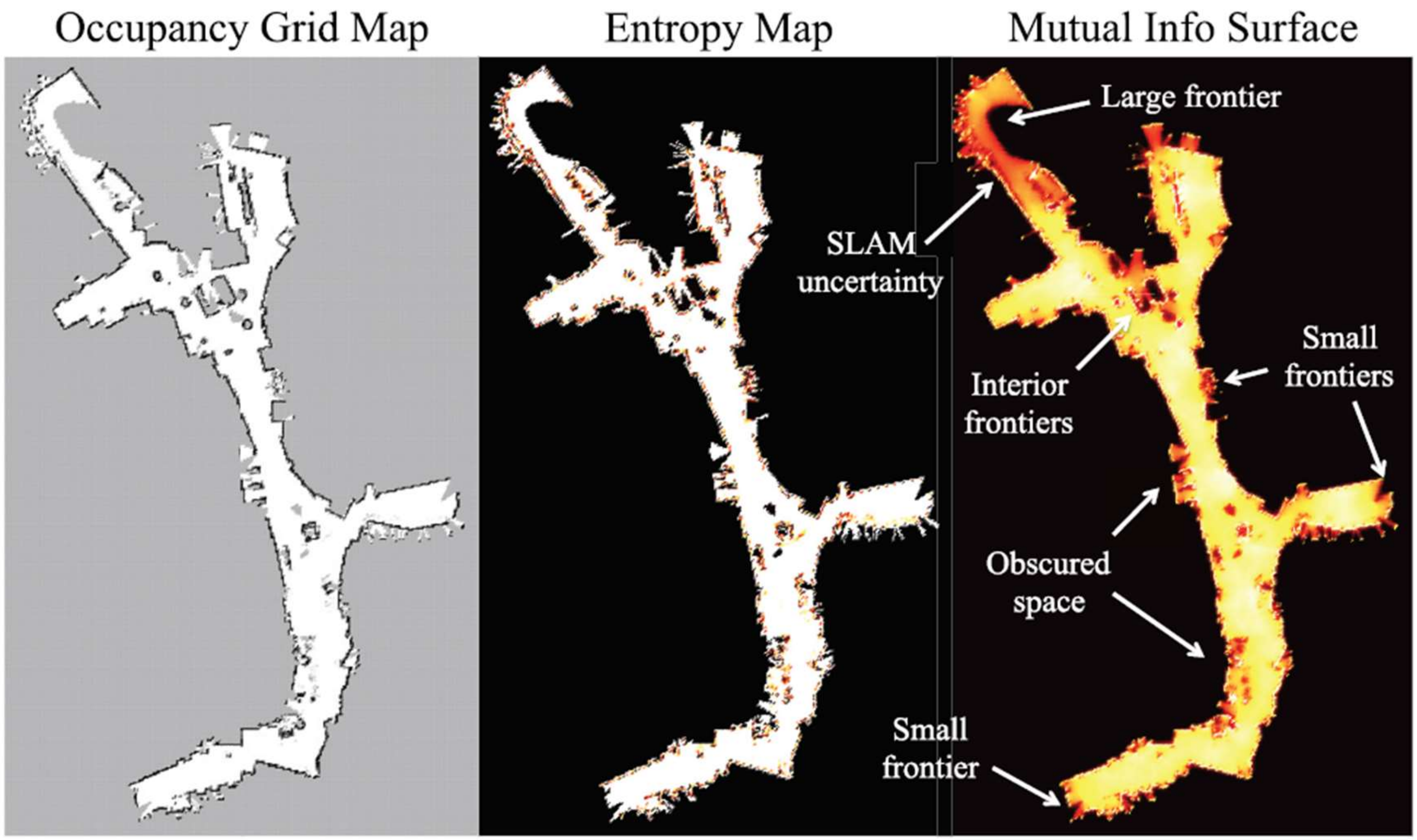
Equivalently: Choose the scan that *maximizes mutual information*:

$$x_k^* = argmax_{x \in SE(d)} I(M; Z(x))$$

Information-based exploration

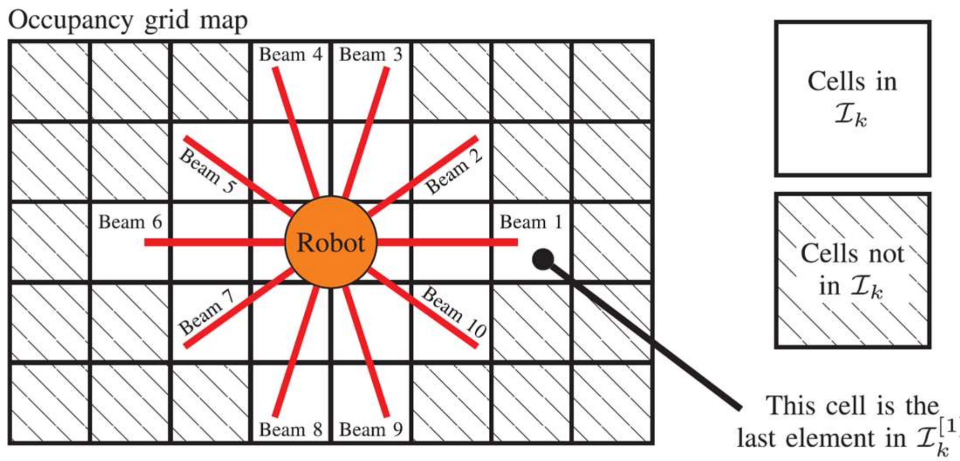


Information-based exploration



Sensor footprints

Recall: We only update the occupancy probabilities of cells that lie in the sensor footprint



Therefore: Decrease in map entropy is equal to decrease in entropy of cells in the sensor footprint:

$$H(M) - H(M|Z(x_k)) = H\left(M_{I_k}\right) - H\left(M_{I_k}\middle|Z(x_k)\right) = I\left(M_{I_k}; Z(x_k)\right)$$

Payoff: To calculate the value of a scan, we only need to consider the (relatively few) cells in the scan's sensor footprint

A simple information-based exploration algorithm

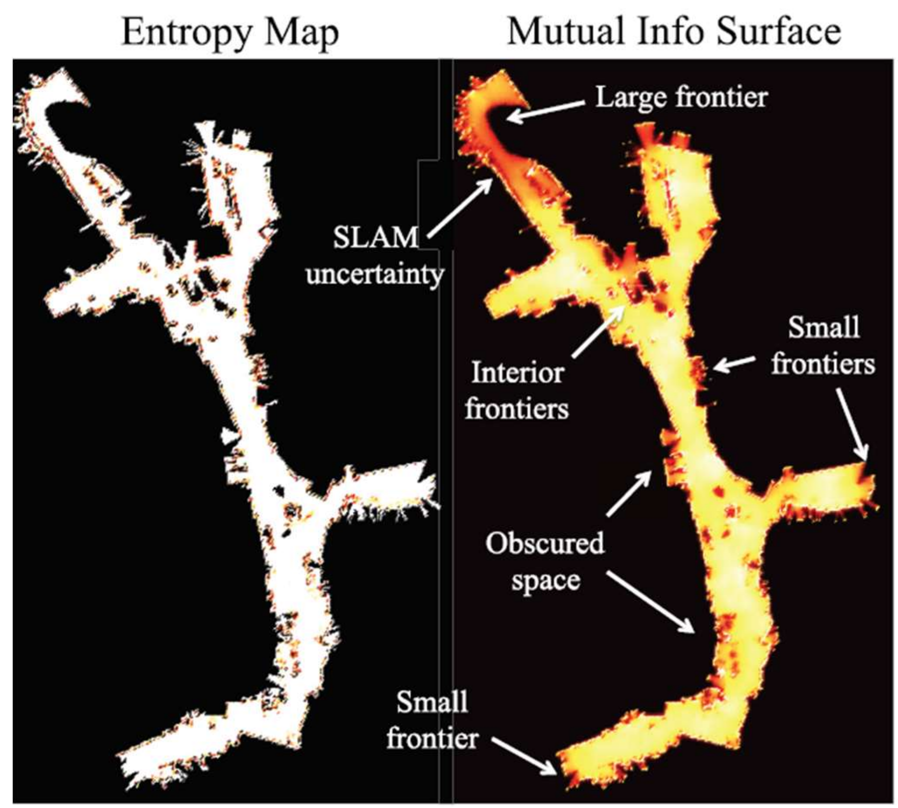
repeat

1. Calculate the next most-informative pose x_k^* at which to scan:

$$x_k^* = argmax_{x \in SE(d)} \; I\left(M_{I_k}; Z(x)\right)$$

2. Collect measurements
3. Update map

until (termination condition)



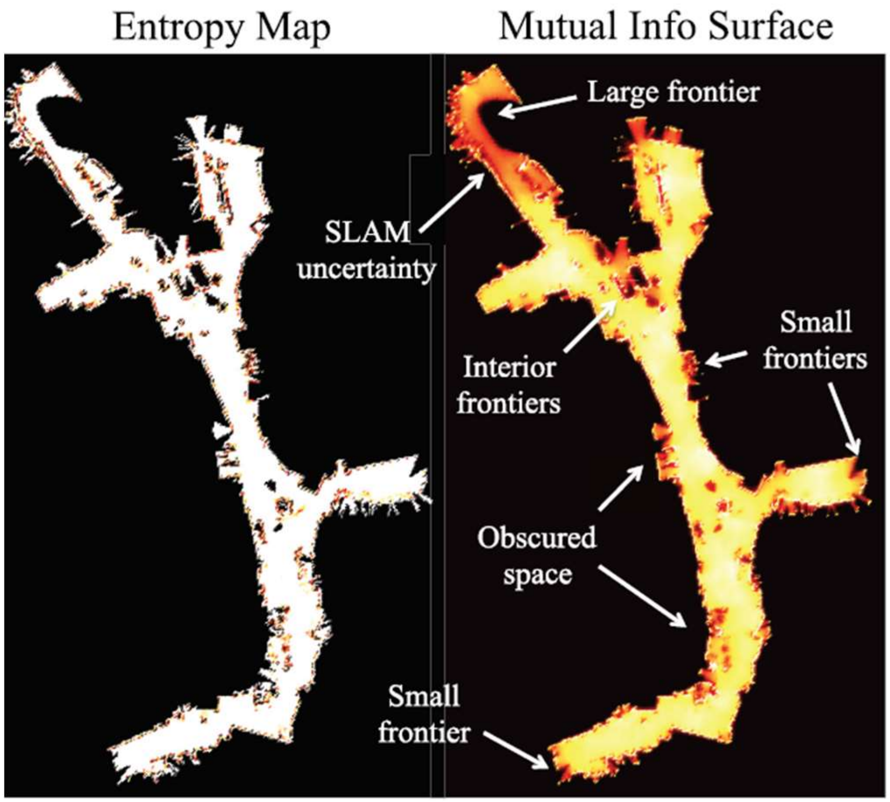
Practicalities

1. Computing the *exact* most-informative pose requires searching over *every possible* pose:

   $$x_k^* = argmax_{x \in SE(d)} \ I\left(M_{I_k}; Z(x)\right)$$

   This might get expensive …

2. Rather than planning for *single* measurements, we might like to design (approximately) optimal *trajectories*



MPC for information-based exploration

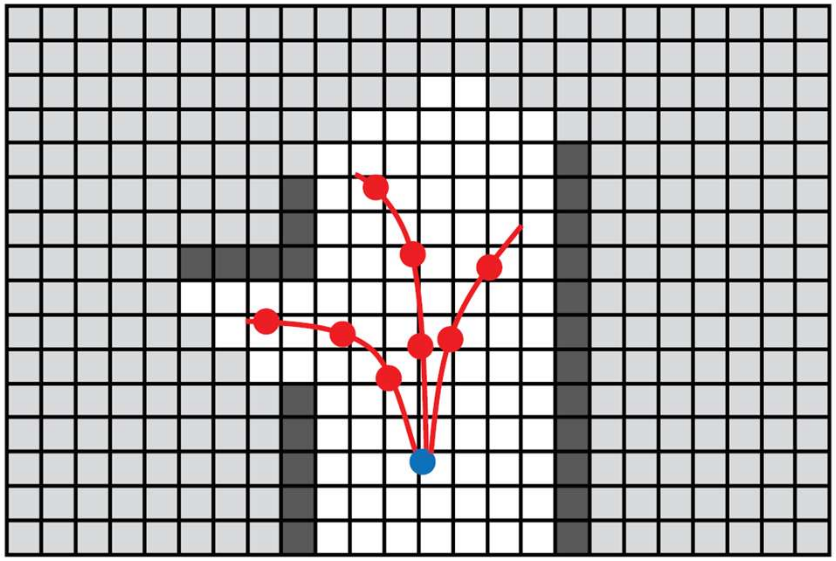
repeat

1. Calculate a (k-step, feasible) *trajectory* $x_{1:k}$ that (approximately) maximizes MI:

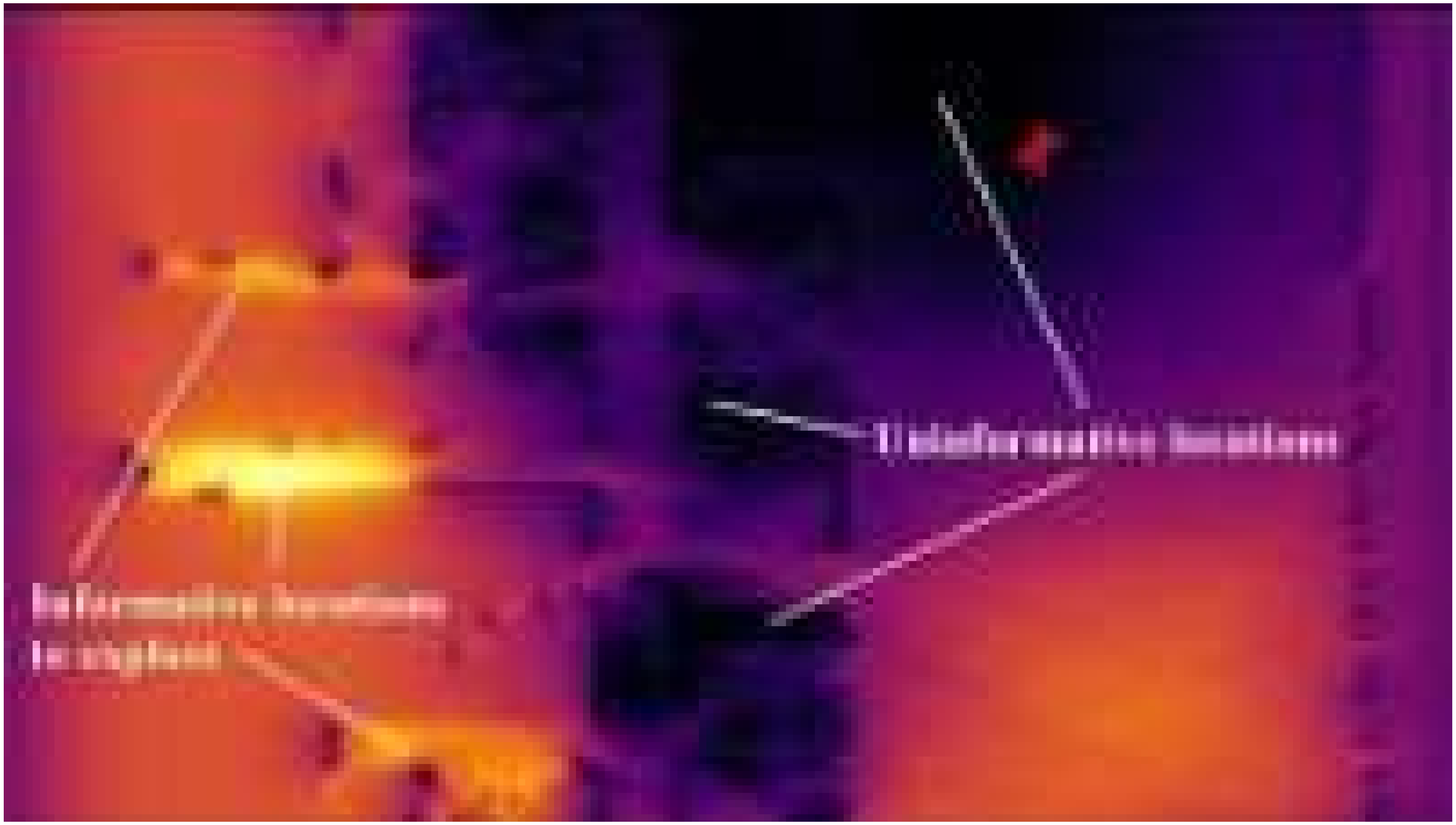
$$x^*_{1:k} = argmax_{x_i \in SE(d)} \; I(M; Z(x_{1:k}))$$

2. Execute first stage of plan
3. Update map

until (termination condition)



Information-based exploration



https://youtu.be/j_O1vOCrUME

Information-based exploration on MIT RACECAR

https://youtu.be/6Ia0conjKMQ