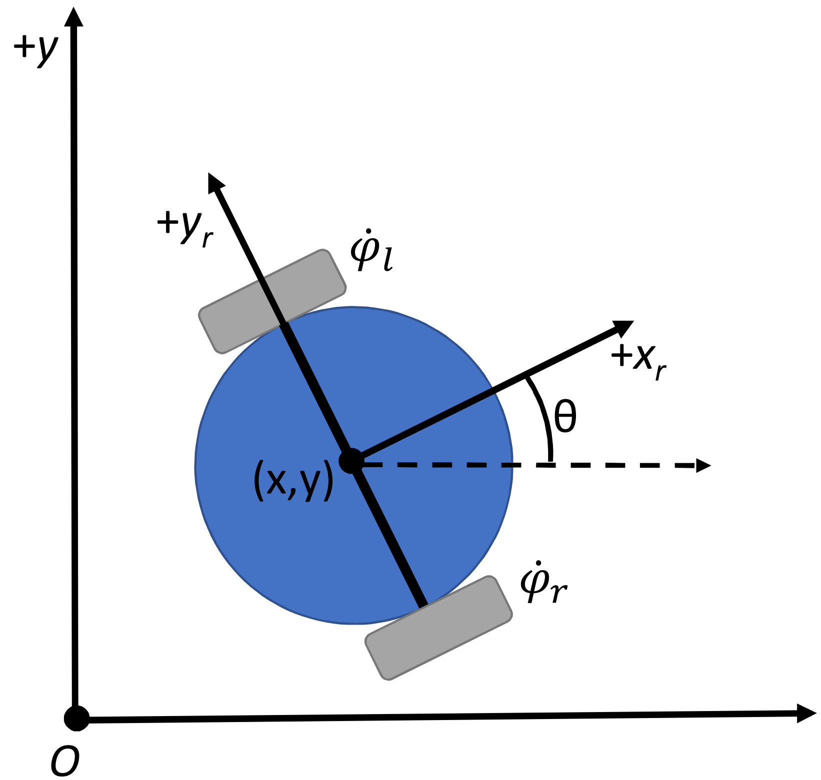
EECE 5550: Mobile Robotics



Lecture 7: Basic robot kinematics and sensing

The Story So Far

Mathematical foundations → weeks 1–2

Computational tools → week 3

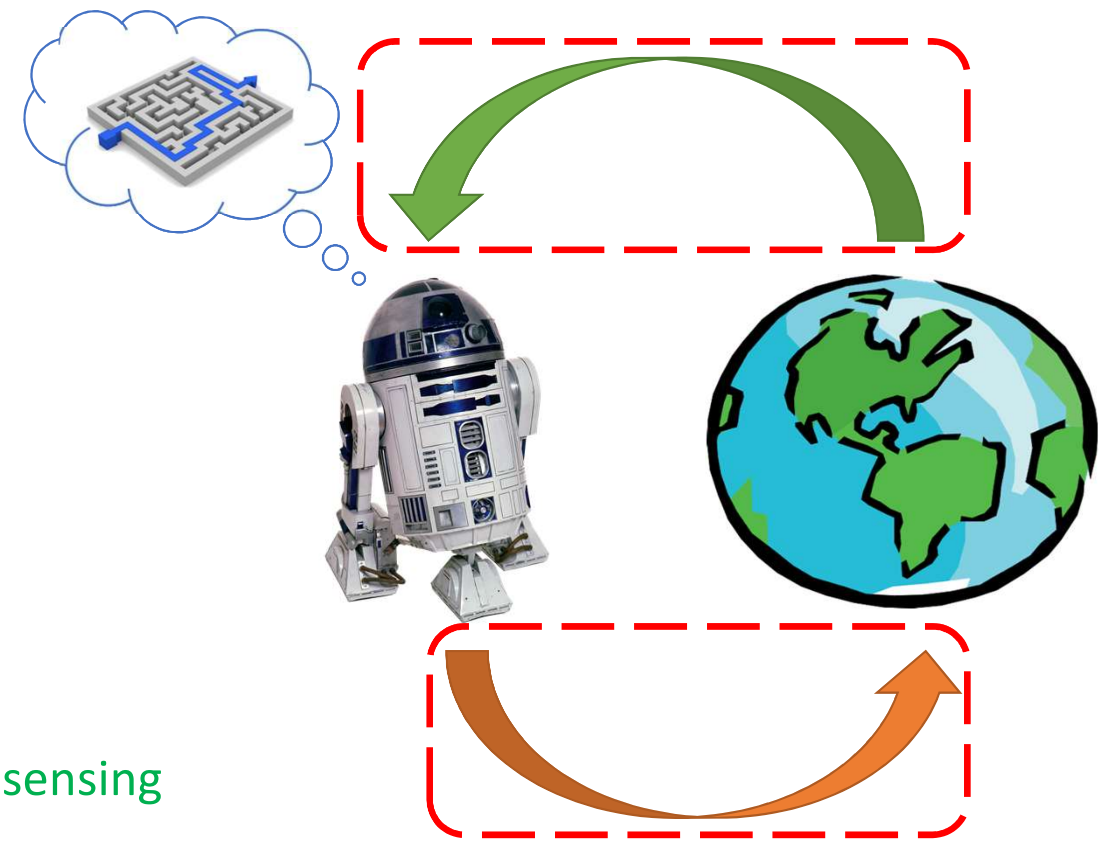
Actual robots (yay ☺!) → weeks 4–13

Week	Topics (tentative)
1	Coordinate transformations & geometry
2	Lie groups & probability theory
3	Computational tools: Linux, Git, Ros
4	Sensing, kinematics & feature extraction
5	Probabilistic robotics & Bayesian filtering
6	Robotic mapping & localization
7	Optimization & SLAM
8	Planning algorithms and graph search
9	Motion planning & obstacle avoidance
10	Feedback, optimal, and model-predictive control
11	Planning under uncertainty
12	Robotic exploration
13	Final presentations

Recap: The Central Dogma of Robotics

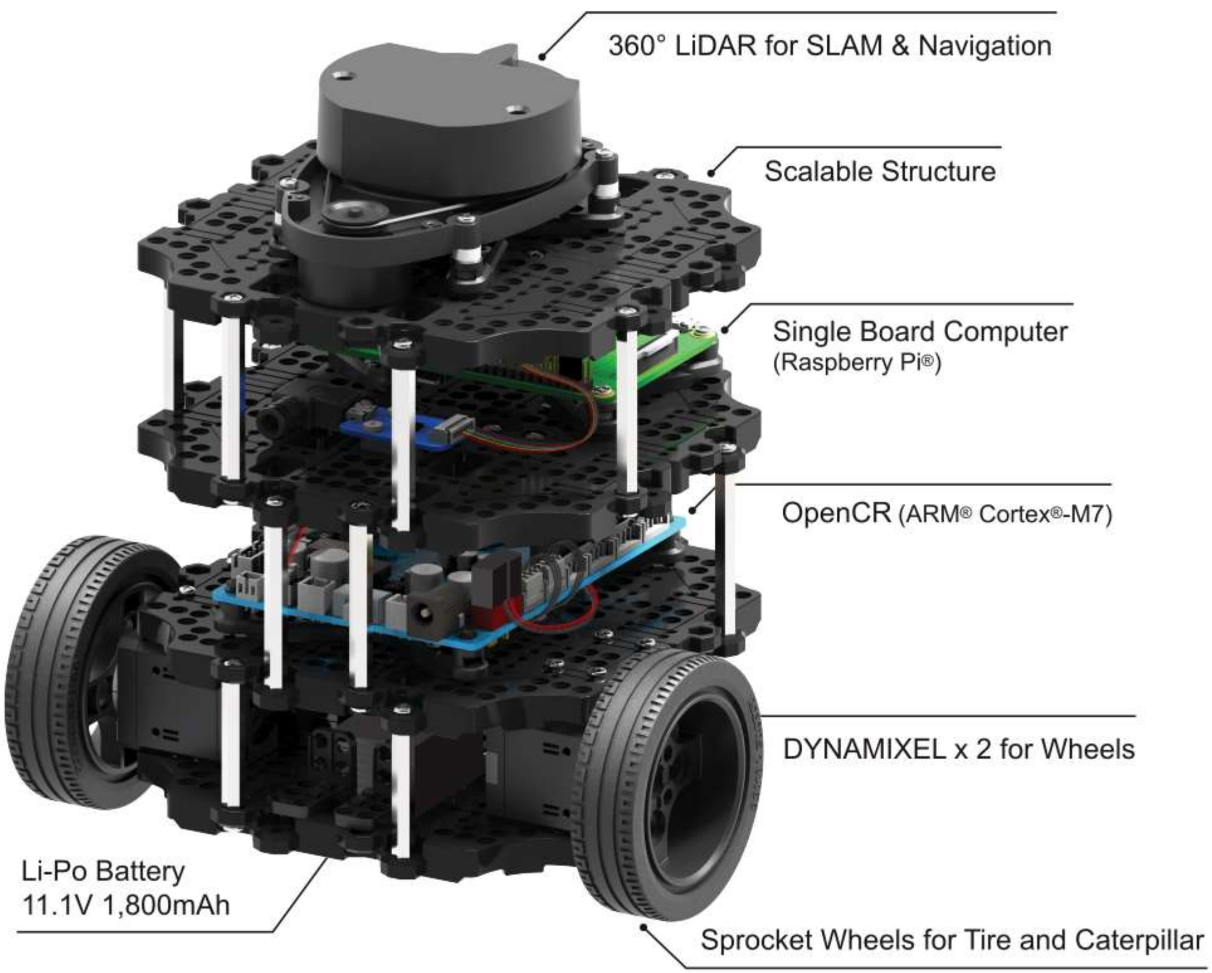
Sense → Think → Act

- Sense: Process sensor data to construct a model of the world
- Think: Construct a plan to move from the current state to the goal state
- Act: Control actuators to execute plan



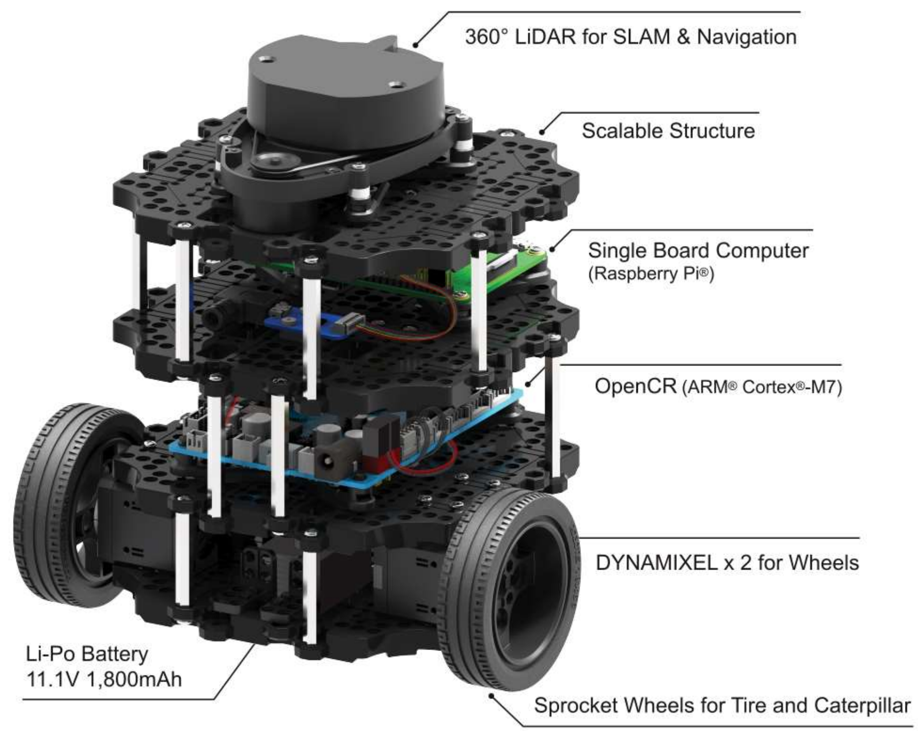
Today: Basic models of robot kinematics & sensing

Our platform: Turtlebot3 Burger

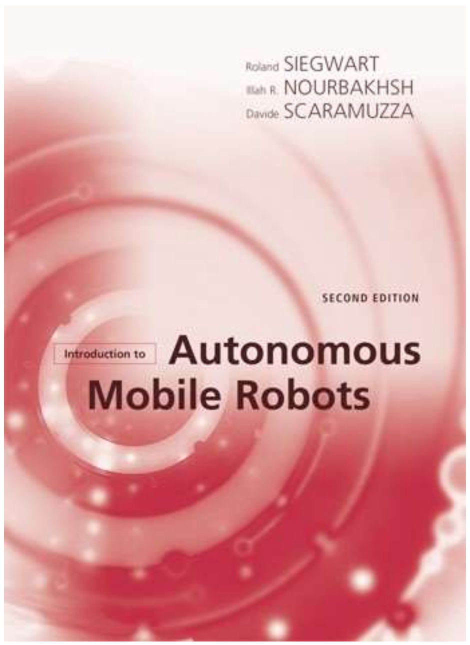


Plan of the day

- Basic kinematics of wheeled vehicles
- Mobile robot sensors
  - Wheel encoders
  - Laser scanners
  - Cameras



References



Autonomous Mobile Robots

Handbook of Robotics

- Secs. 3.2 and 4.1 of *Autonomous Mobile Robots*
- Chps. 17 & 22 of *Handbook of Robotics*

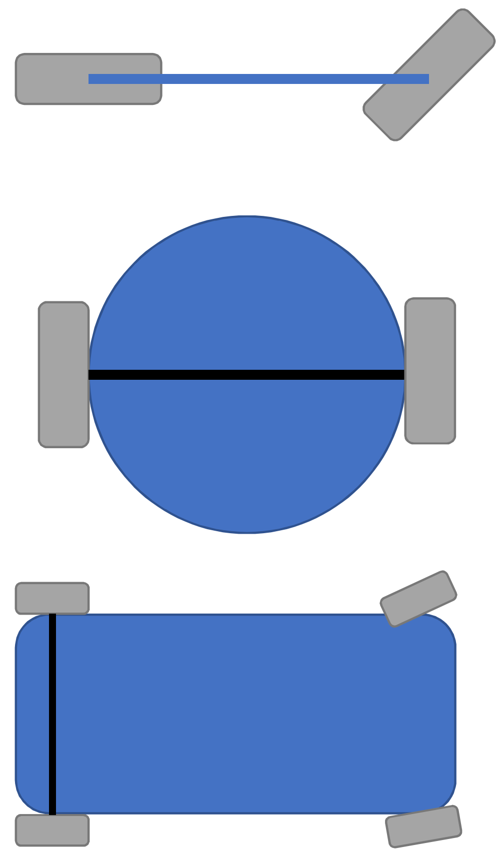
Kinematics of wheeled vehicles

LOTS of different possible designs for wheeled robots

Examples:

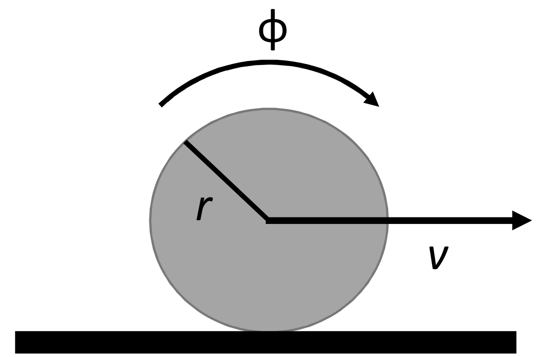
- Bicycle model
- Differential drive (2 or 3 wheels)
- Ackerman steering (car)

Main question: How does wheel geometry relate to robot motion (i.e. kinematics)?



Basic modeling assumptions for wheels

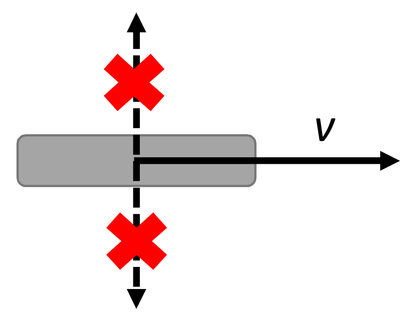
Main assumption: We assume that all wheels rotate without slipping



- *Forward* velocity v is determined by wheel radius r and angular velocity $\dot{\varphi}$:

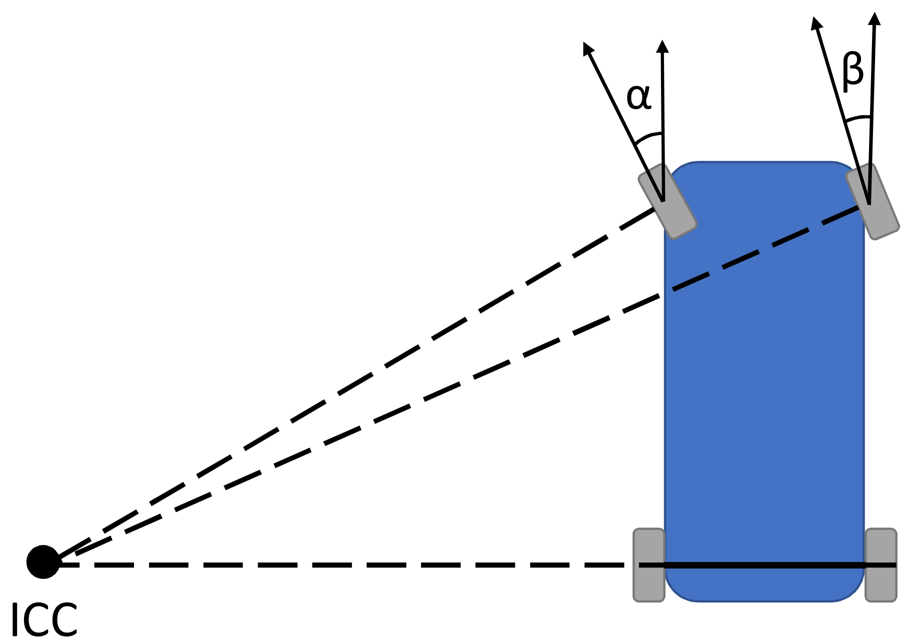
$$v = r\dot{\varphi}$$

- *Lateral* motion is prohibited: no side-slip!



Key observation: These conditions actually correspond to *constraints* on possible robot motion!

Wheel constraints on robot motion

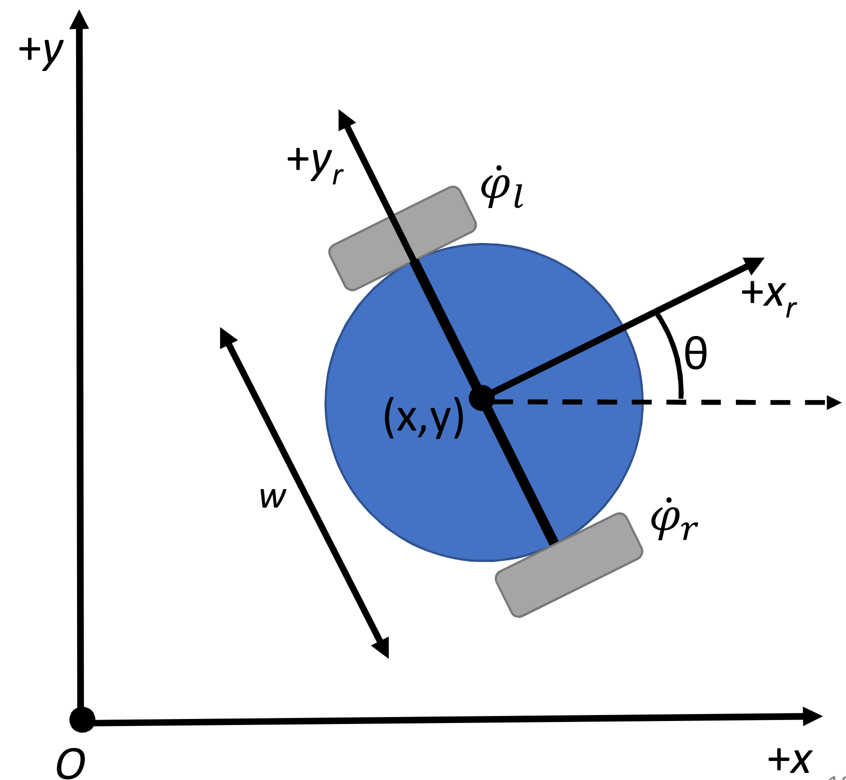


In order for each wheel to roll without slipping, they must all have a common *instantaneous center of curvature*

Kinematics of differential drive robots

Suppose we have a differential drive robot with:

- Wheels of radius *r*
- Wheel track of width *w*
- Pose (x,y,θ) with respect to the world frame *O* as shown in the diagram



Key question: What is the relation between the angular velocities $\dot{\varphi}_l$ and $\dot{\varphi}_r$ of the left and right wheels, and the velocity $v = (\dot{x}, \dot{y}, \dot{\theta})$ of the robot in the world frame *O*?

(Assume that positive values of $\dot{\varphi}_l$ and $\dot{\varphi}_r$ correspond to forward motion of the robot)

Kinematics of differential drive robots

Let's consider the robot's body-centric coordinate system.

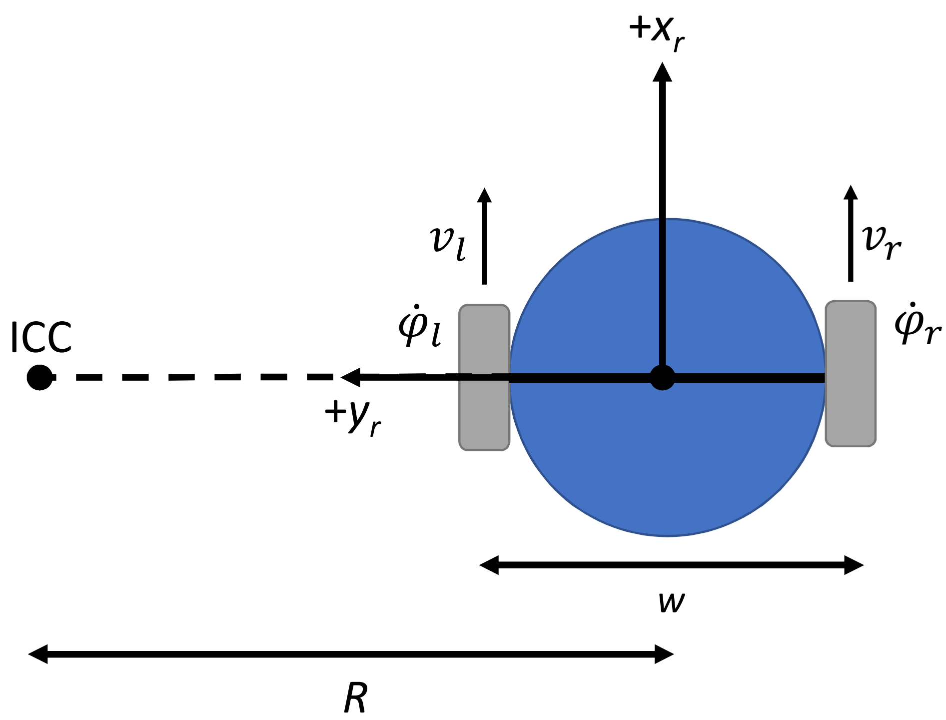
Recall that:

- Both wheels must have a common ICC
- There is no lateral motion ($\dot{y}_r = 0$)

⇒The left and right wheels are following *circular arcs* of radii $R - \frac{w}{2}$ and $R + \frac{w}{2}$

⇒ The forward velocities of the left and right wheels satisfy:

$$v_l = \dot{\theta}_r\left(R - \frac{w}{2}\right)$$
$$v_r = \dot{\theta}_r\left(R + \frac{w}{2}\right)$$



Kinematics of differential drive robots

Let's consider the robot's body-centric coordinate system.

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$$v_r = \dot{\theta}_r\left(R + \frac{w}{2}\right)$$

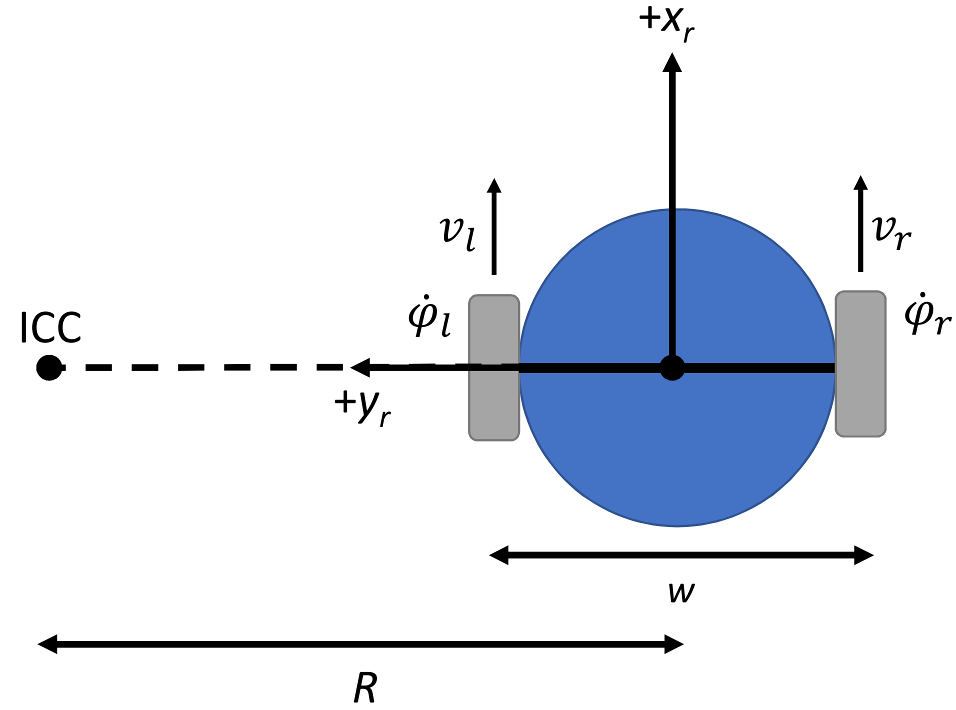
Summing these equations yields:

$$2R\dot{\theta}_r = v_l + v_r$$

But from the diagram, $R\dot{\theta}_r = \dot{x}_r$ is the forward velocity of the robot!

Therefore:

$$\dot{x}_r = \frac{v_l + v_r}{2}$$



Kinematics of differential drive robots

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The forward velocities of the left and right wheels satisfy:

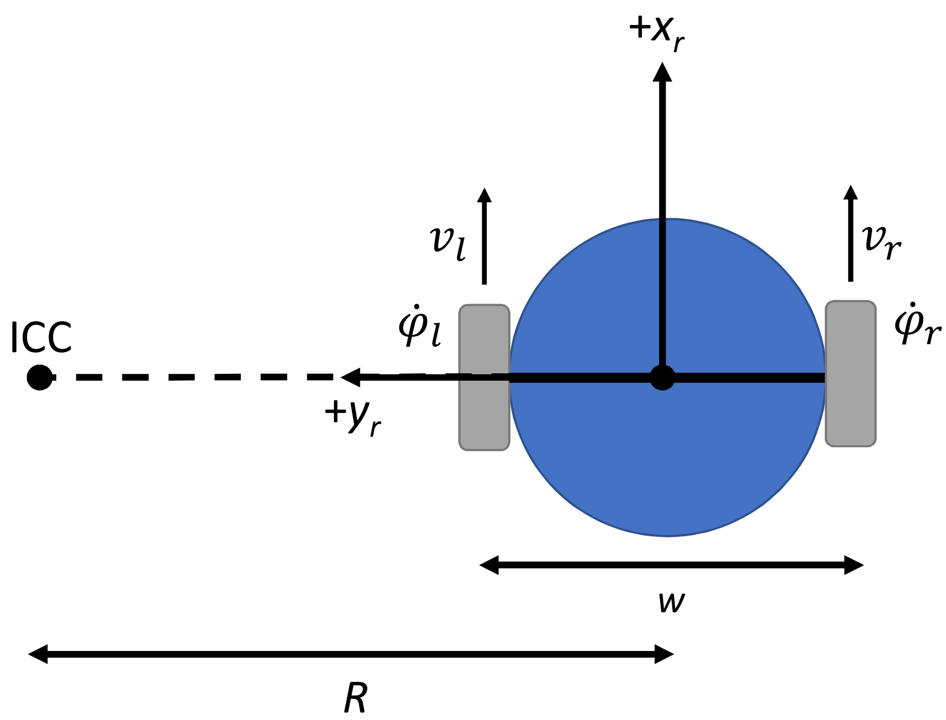
$$v_l = \dot{\theta}_r\left(R - \frac{w}{2}\right)$$
$$v_r = \dot{\theta}_r\left(R + \frac{w}{2}\right)$$

Similarly, subtracting v_l from v_r produces:

$$\dot{\theta}_r = \frac{v_r - v_l}{w}$$

Finally, recall the *no slip assumption* implies that the wheels' *translational* and *angular* velocities satisfy:

$$v_l = r\dot{\varphi}_l \quad \text{and} \quad v_r = r\dot{\varphi}_r$$

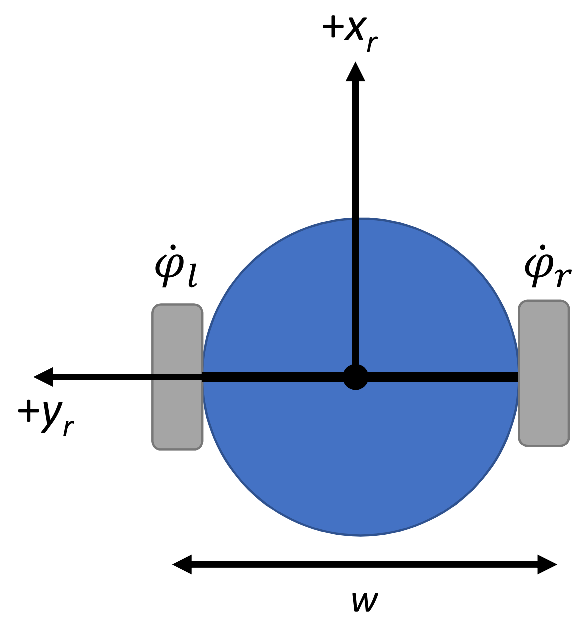


Kinematics of differential drive robots

Let's consider the robot's body-centric coordinate system.

Combining these results, we have:

$$\begin{pmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{pmatrix} = \begin{pmatrix} \frac{r}{2}(\dot{\varphi}_r + \dot{\varphi}_l) \\ 0 \\ \frac{r}{w}(\dot{\varphi}_r - \dot{\varphi}_l) \end{pmatrix}$$



But: These are in the robot's body-centric frame
Q: What about the world reference frame?

Kinematics of differential drive robots

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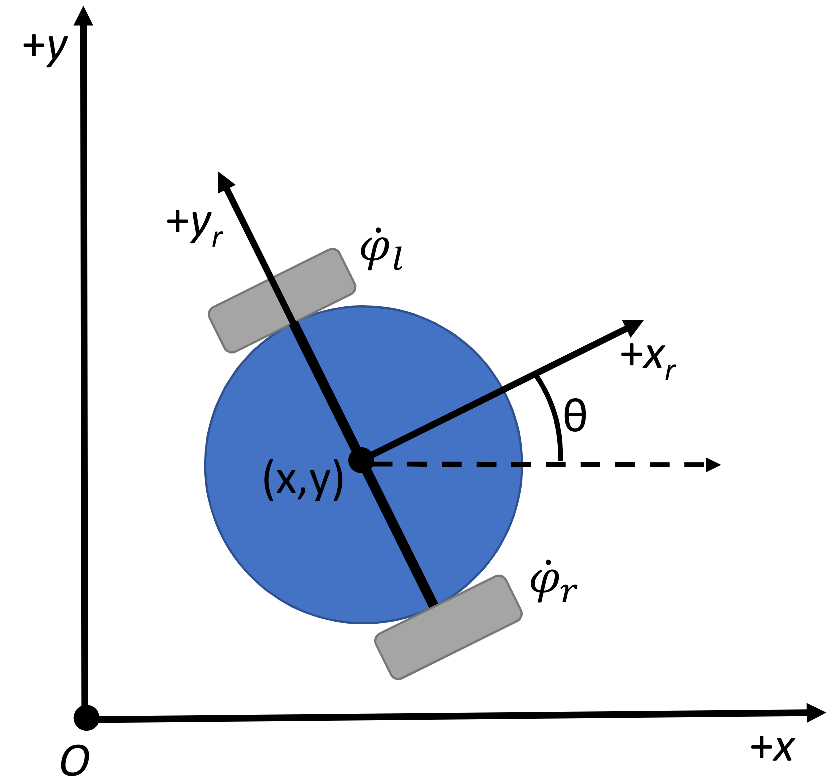
$$\begin{pmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{pmatrix} = \begin{pmatrix} \frac{r}{2}(\dot{\varphi}_r + \dot{\varphi}_l) \\ 0 \\ \frac{r}{w}(\dot{\varphi}_r - \dot{\varphi}_l) \end{pmatrix}$$

Transforming these velocities into the *world frame* O:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} R(\theta) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{pmatrix}$$

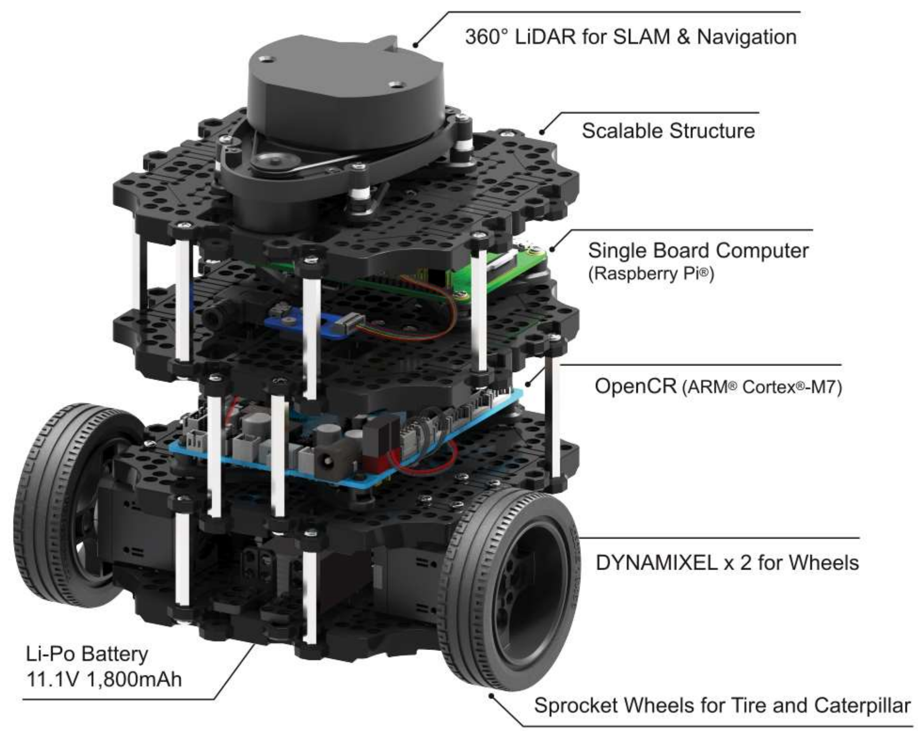
where:

$$R(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$



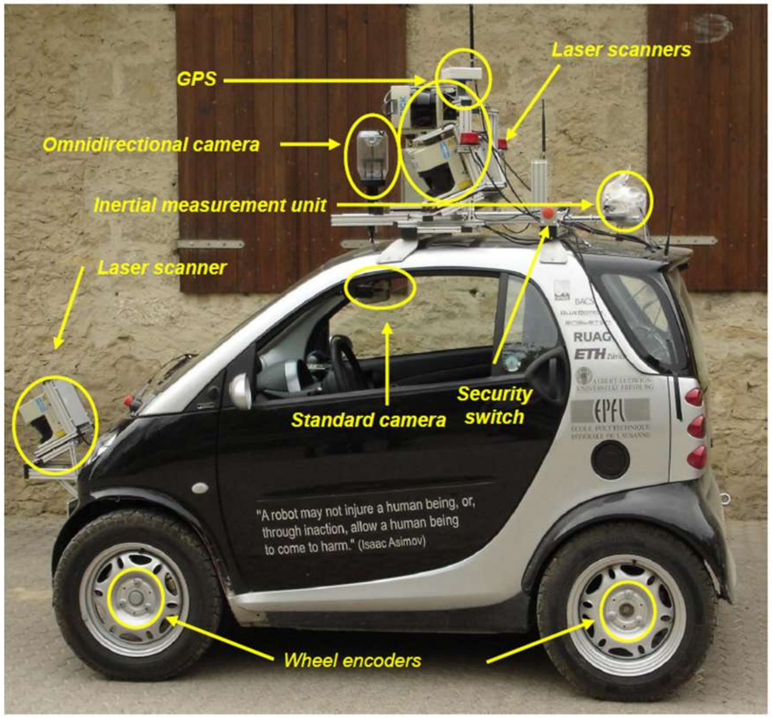
Plan of the day

- Basic kinematics of wheeled vehicles
- Mobile robot sensors



A taxonomy of common robot sensors

- Tactile or “bumper” sensors
  - Detect physical contact with the environment
  - Can be used for simple proximity sensing, e.g. early Roomba
- Rotary (wheel) encoders
  - Measure position / velocity of rotating shafts
  - Useful for dead reckoning (w/ kinematic models)
- Laser scanners
  - Measures range and bearing to points in the environment
  - Captures high-accuracy geometry (although often only in a *plane*)
- Cameras
  - Provides dense field-of-view information about the environment’s visual appearance (color, texture, etc.).
  - A very rich source of information for scene understanding
- Global position system (GPS)
  - Direct measurement of absolute position on Earth’s surface
  - Requires clean line-of-sight to GPS satellites



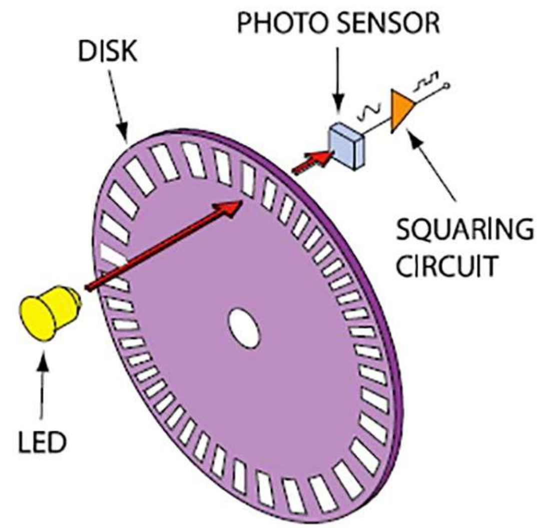
Optical rotary encoders

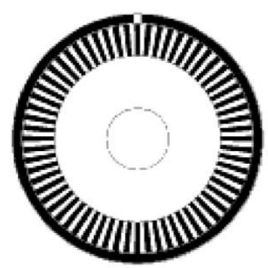
Principle of operation: Measure the rotation of a shaft by "counting" pulses of light that pass through a thin grating

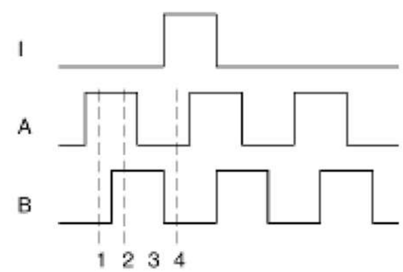
Typical resolution (for robotics) is ~2000 cycles per revolution

Used for measuring:

- Joint angles (for manipulators)
- Wheel speed / odometry
  ⇒Can be used for *dead reckoning* (w/ kinematic models)

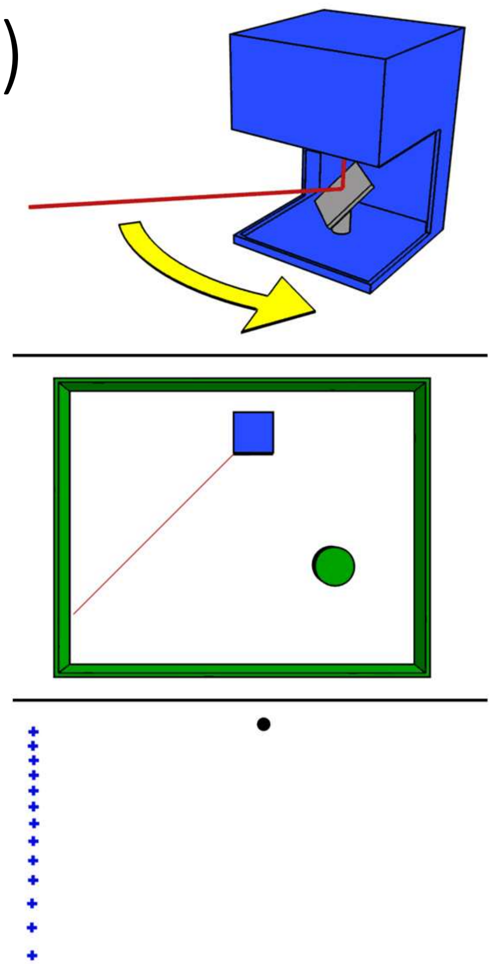






State	Ch A	Ch B
s_1	high	low
s_2	high	high
s_3	low	high
s_4	low	low

Laser scanners (LIDAR)



Principle of operation: **Li**ght **d**etection **a**nd **r**anging. Measure range and bearing to points in the environment by measuring *time-of-flight* of a reflected laser beam

Typically these units will "sweep" a single laser through a planar section of the environment using a rotating mirror

Pro:

- Captures high-precision geometry at long range (cm-level accuracy at distances of tens to hundreds of meters)
- Active sensing: Less sensitive to environmental properties (e.g. texture)

Con:

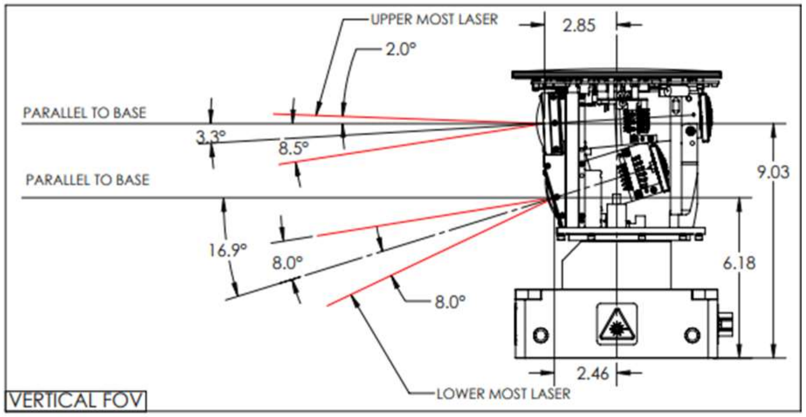
- Standard LIDARs only scan a 2D-cross-section of the environment – can't capture 3D information (e.g. variation w/ height)
- Active sensing: Requires power (can be an issue for resource-limited mobile platforms)

3D LIDAR

Some recently-developed lidars (e.g. Velodyne) can return 3-dimensiona, 360-degree field-of-view pointclouds

Main idea: Stack multiple (e.g. 64) lasers pitched at different elevations in a common housing, and rotate the entire unit

These are commonly used for autonomous vehicles



Example: 3D LIDAR on an autonomous vehicle

Cameras

Principle of operation: Focus ambient light rays onto a photo-sensitive surface (in robotics, a digital sensor)

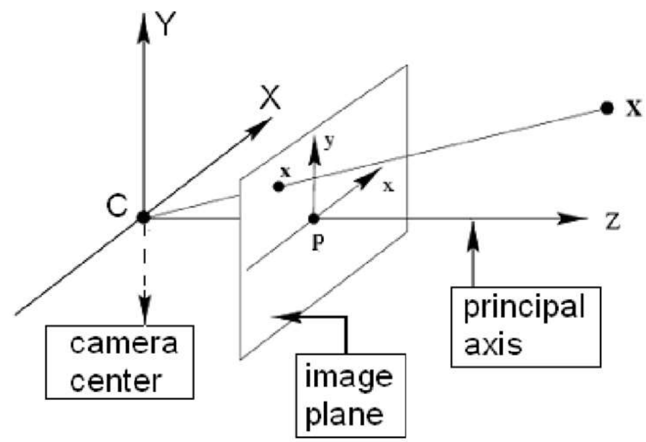
NB: The location at which an incident beam is imaged depends upon its *direction of origin* – cameras are *orientation* sensors

Pro:

- Can capture extremely rich, dense visual information over a large field of view – very useful for scene classification, object recognition
- Passive sensing: Relatively low power (compared to lasers)

Con:

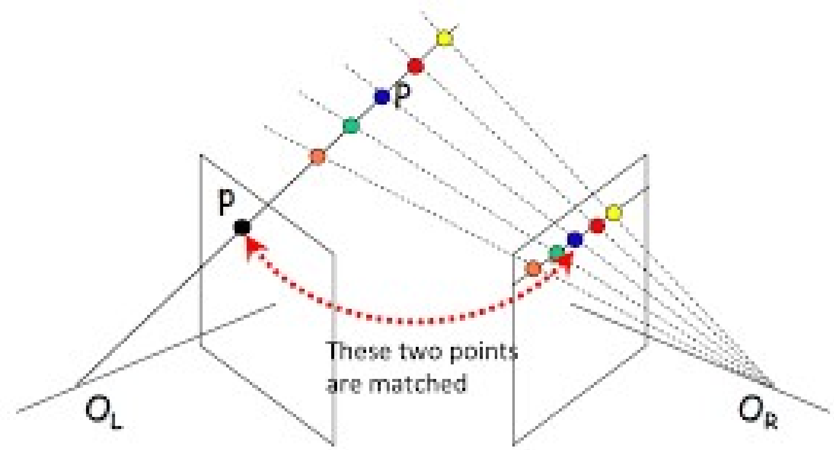
- Passive sensing: Efficacy depends upon external environmental factors (illumination, visual texturing)
- Cameras may not be as geometrically accurate as lasers, esp. at long ranges (more on this later)



Stereo cameras

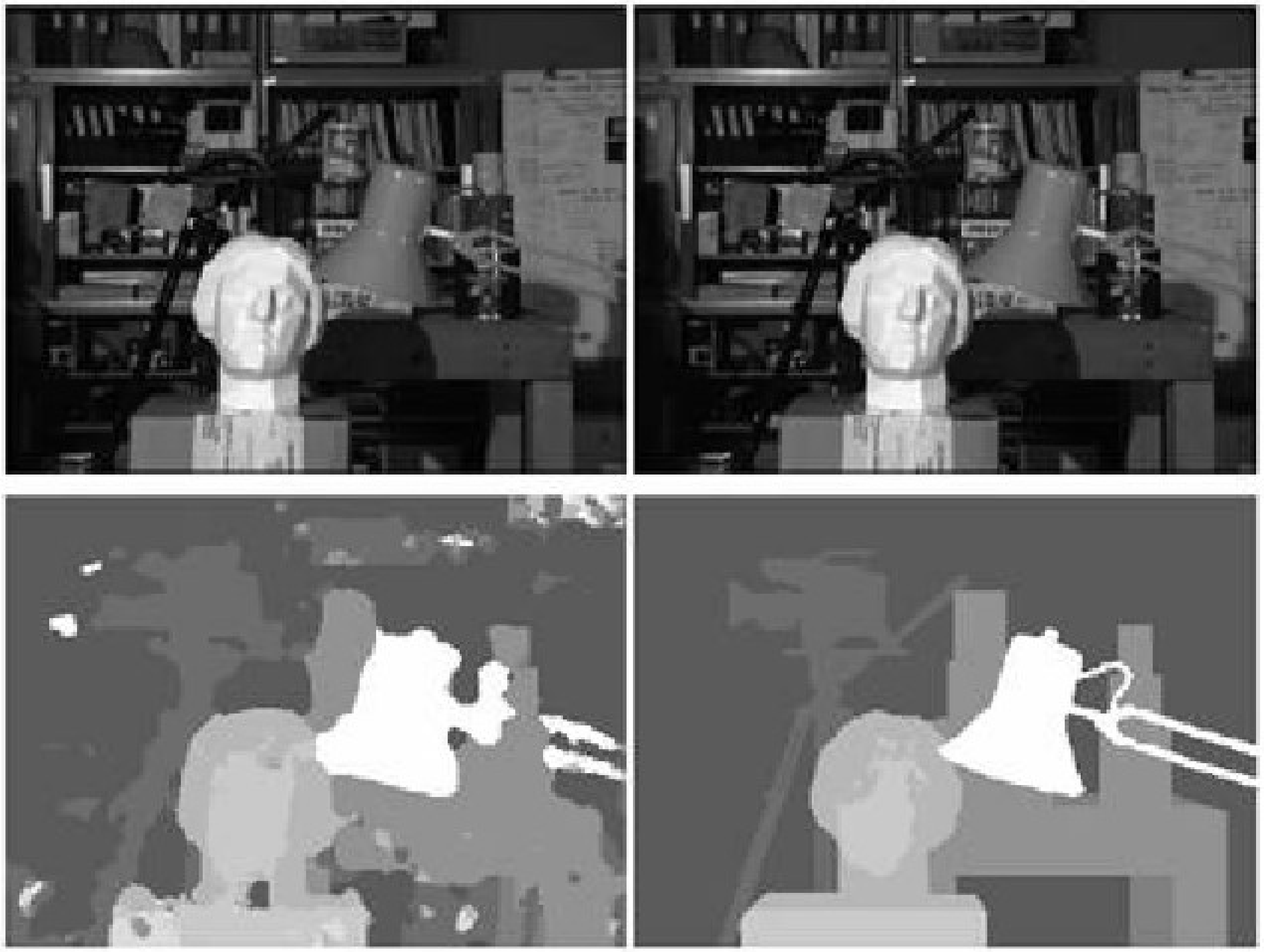
Recall: Cameras are ***bearing*** sensors -- can't determine depth (position) of a point from a single image

Idea: Use *two* cameras! If *relative pose* of the cameras is know (calibrated) the position of a point seen in both images can be triangulated



- Provides information about geometry *and* visual appearance over a field of view
- **But:** This depends upon finding corresponding points in the two images
  ⇒Requires visual texture!

Example stereo depth map

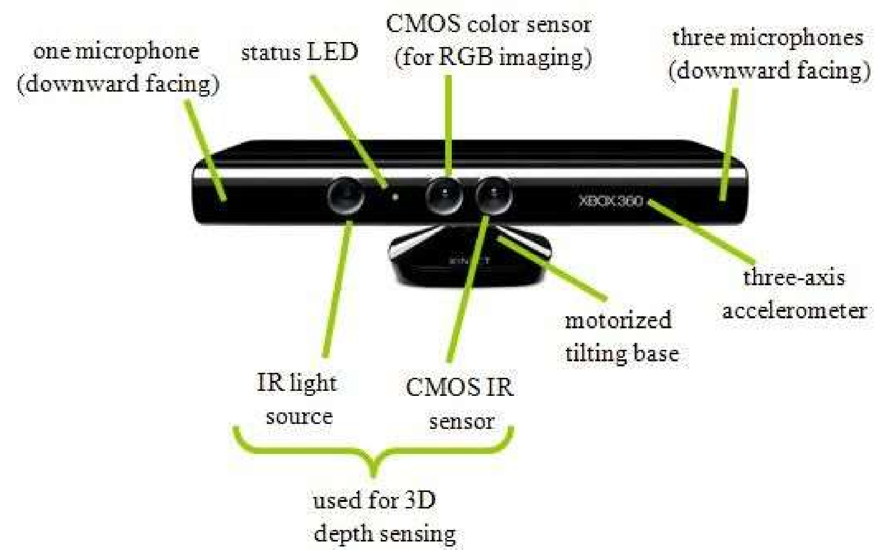


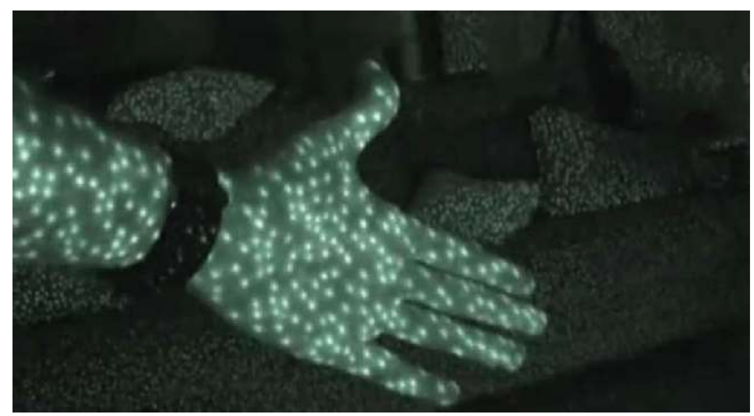
Structured light sensors

Principle of operation: Same as stereo, but one camera is replaced with an *emitter* that projects a *known pattern*

Solves two main challenges in (passive) stereo:

- Visual texture: The camera makes its own
- Feature matching: The projected pattern is designed to make *feature matching* easy





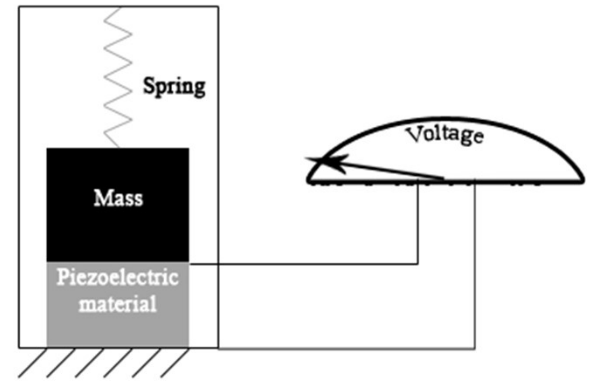
Kinect dot pattern

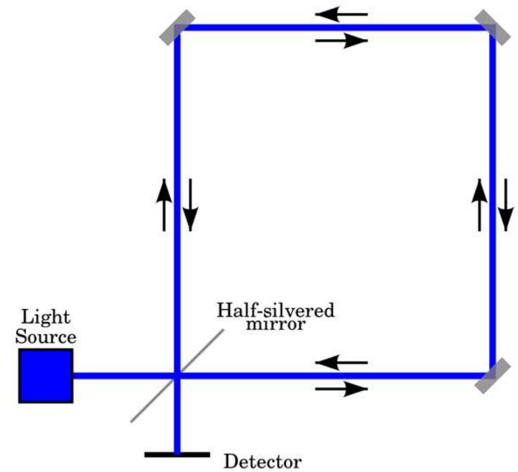
Inertial measurement units

Principle of operation: Measures *linear acceleration* and/or *angular velocity* using a combination of accelerometers and gyroscopes

- **Accelerometer:** Measures deflection of a *proof mass* as the device accelerates

- **Gyroscopes:**
  - **Mechanical:** Track *absolute orientation* of the sensor using conservation of angular momentum
  - **Ring laser:** Measure *angular rate* using Sagnac effect





Global positioning system

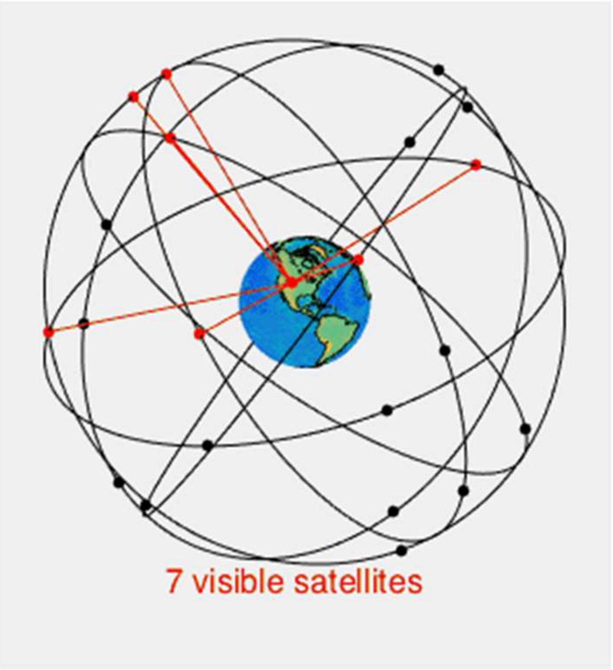
Principle of operation: Measure the arrival times of radio signals from a constellation of satellites.

- Time-of-flight for each signal x speed of light = range to each satellite
- Receiver's position can then be determined via *multilateration*

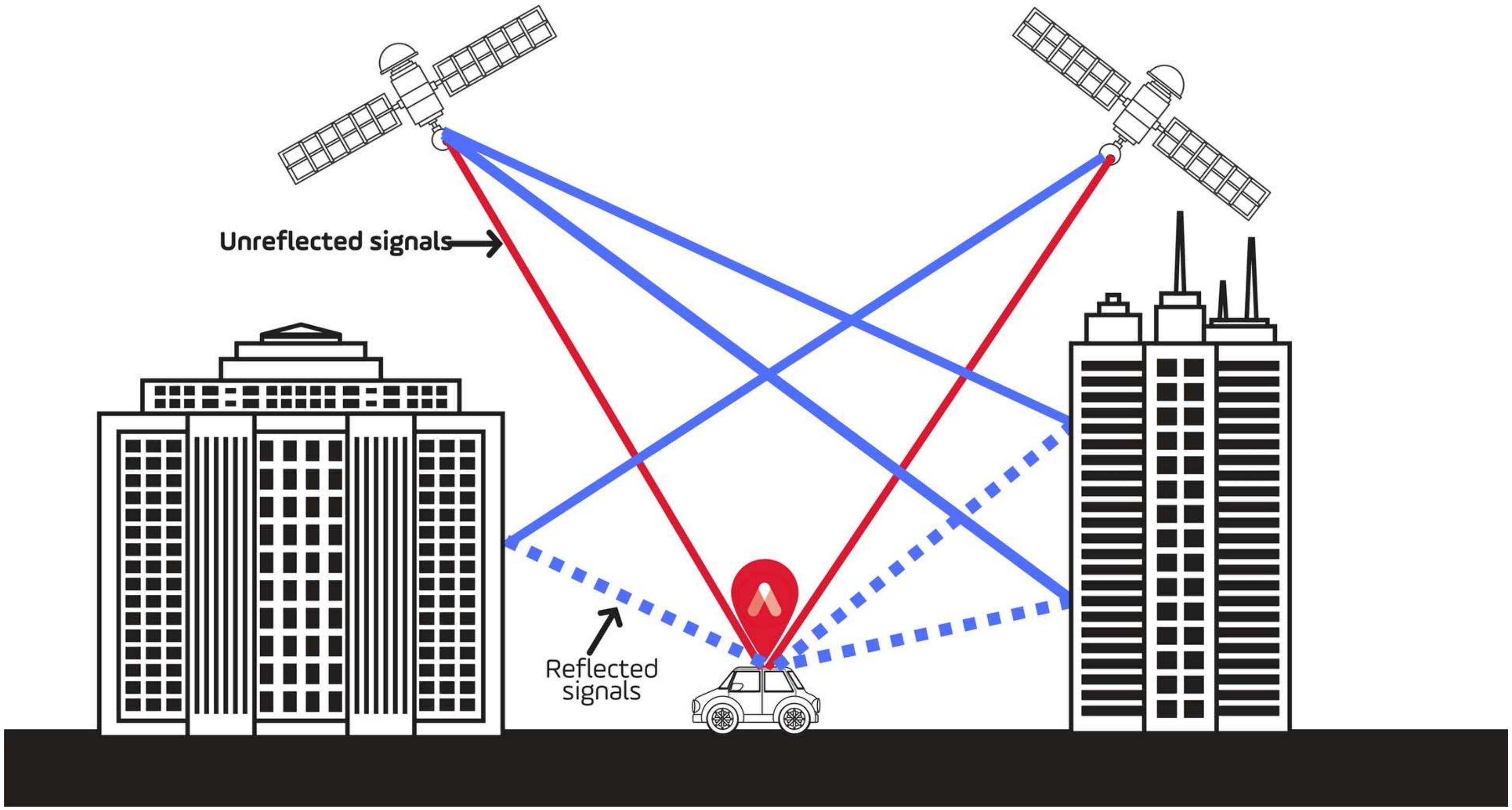
Pro: Enables direct measurement of *absolute position*

Con: Accuracy can be significantly affected by several external factors

- Constellation position (how many satellites are visible?)
- Atmospheric effects (affects EM propagation speed)
- Requires clear line-of-sight to satellites
  - Only works outdoors
- Multipath effects are a significant problem



GPS multipath



Our platform: Turtlebot3 Burger