Dis 2C: Graphs II

Wednesday, 1 July 2020 1:45 AM

1 Short Answers - Graphs

(a) Bob removed a degree 3 node from an n-vertex tree. How many connected components are there in the resulting graph?

(b) Given an n-vertex tree, Bob added 10 edges to it and then Alice removed 5 edges. If the resulting graph has 3 connected components, how many edges must be removed in order to remove all cycles from the resulting graph?

A)

b) I.e. What's the **minimum number of edges** you'd need to remove to remove all cycles from the resulting graph?

FLOW:

Definition of a **planar graph**: can be drawn without edge crosses

Properties of Connected Planar Graphs:

- *Euler's formula*: connected planar graphs obey the formula
  e + 2 = v + f,
  where f is number of faces when drawn without edge crosses
  [*Proof: induction on e*]

- For connected planar graphs with v >= 3,
  e <= 3v – 6
  [*Proof: remove variable f using I(in)equalities,*
  i.e. relationships between number of edges and degree of faces]

  intuitively, this means planar graph edges are **sparse** (there aren't that many)

- For connected bipartite graphs with v >= 3,
  e <= 2v – 4
  [Proof: remove variable f using (in)equalities, and extra fact that the 'degree' of each face must be >=4]

Property of Non-planar graphs:

- G 'contains' K_{3,3} or K_5 <=> G is non-planar
  'contains' K_{3,3} or K_5 = has a subgraph homeomorphic to K_{3,3} or K_5

2 Planarity

(a) Prove that $K_{3,3}$ is nonplanar.

(b) Consider graphs with the property T: For every three distinct vertices v_1, v_2, v_3 of graph G, there are at least two edges among them. Use a proof by contradiction to show that if G is a graph on ≥ 7 vertices, and G has property T, then G is nonplanar.

b) Propositional Logic explanation of proof by contradiction on implications e.g. A => B

3 Graph Coloring

Prove that a graph with maximum degree at most k is $(k+1)$-colorable.

Notation: G = (V, E), let |V| = n and |E| = m

4 Hypercubes

The vertex set of the n-dimensional hypercube $G = (V, E)$ is given by $V = \{0,1\}^n$ (recall that $\{0,1\}^n$ denotes the set of all n-bit strings). There is an edge between two vertices x and y if and only if x and y differ in exactly one bit position. These problems will help you understand hypercubes.

(a) Draw 1-, 2-, and 3-dimensional hypercubes and label the vertices using the corresponding bit strings.

(b) Show that for any $n \geq 1$, the n-dimensional hypercube is bipartite.

Connection: G is bipartite graph <=> G is two-colorable