Dis 4C: Discrete Probability

Thursday, 16 July 2020 5:10 PM

1 Sample Space and Events

Consider the sample space Ω of all outcomes from flipping a coin 3 times.

(a) List all the outcomes in Ω. How many are there?

2^3 — — —

(b) Let A be the event that the first flip is a heads. List all the outcomes in A. How many are there?

H $\underset{}{\overset{2}{\_}}\ \overset{2}{\_}$

(c) Let B be the event that the third flip is a heads. List all the outcomes in B. How many are there?

$\overset{2}{\_}\ \overset{2}{\_}$ H

(d) Let C be the event that the first and third flip are heads. List all outcomes in C. How many are there?

H _ H

(e) Let D be the event that the first or the third flip is heads. List all outcomes in D. How many are there?

$|A \cup B| = |A| + |B| - |A \cap B|$

(f) Are the events A and B disjoint? Express C in terms of A and B. Express D in terms of A and B.

No

1st is H, 3rd is H

$C = A \cap B$ $D = A \cup B$

(g) Suppose now the coin is flipped $n \geq 3$ times instead of 3 flips. Compute $|\Omega|, |A|, |B|, |C|, |D|$.

$|\Omega| = 2^n$ $|A| = 2^{n-1}$ H $\overset{2}{\_}\ \overset{2}{\_} \ldots \overset{2}{\_}$ $|A| = |B|$

(h) Your gambling buddy found a website online where he could buy trick coins that are heads or tails on both sides. He puts three coins into a bag: one coin that is heads on both sides, one coin that is tails on both sides, and one that is heads on one side and tails on the other side. You shake the bag, draw out a coin at random, put it on the table without looking at it, then look at the side that is showing. Suppose you notice that the side that is showing is heads. What is the probability that the other side is heads? Show your work. [*Hint:* The answer is NOT 1/2.]

2H 1st side

2H 2nd

TH 2nd

- Sample space (i.e experiment and possible outcomes):

Ω

(HH, 1) (HH, 2)
(TT, 1) (TT, 2)
(TH, 1) (TH, 2)

- Probability of each outcome / sample point:

$\frac{1}{3} \times \frac{1}{2}$

- Event (i.e subset of sample space) we're interested in:

(HH, 1) (HH, 2) (TH, 2)

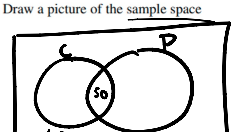
Hint: does our sample space change due to the 'suppose' statement?

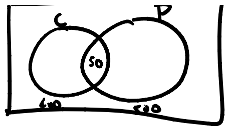
$\frac{2}{3}$

2 Venn Diagram

Out of 1000 computer science students, 400 belong to a club (and may work part time), 500 work part time (and may belong to a club), and 50 belong to a club and work part time.

(a) Suppose we choose a student uniformly at random. Let C be the event that the student belongs to a club and P the event that the student works part time. Draw a picture of the sample space Ω and the events C and P.





(b) What is the probability that the student belongs to a club?

$P(\text{belongs to club}) = \dfrac{\#\text{ in club}}{\text{total}}$

$= \dfrac{500}{1000}$

(c) What is the probability that the student works part time?

$= \dfrac{400}{1000}$

(d) What is the probability that the student belongs to a club AND works part time?

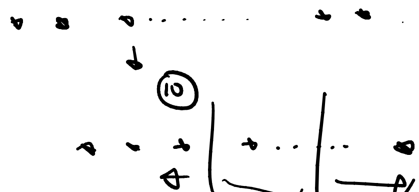
$Pr(C \cap P) = Pr(C) + Pr(P) - Pr(C \cup P)$

$= \dfrac{50}{1000}$

(e) What is the probability that the student belongs to a club OR works part time?

$\dfrac{850}{1000}$

How many ways are there to split 30 lives among Andrew, Angela, and Aren
so that Andrew has >= 10 lives and Angela has >= 10 lives?



$\binom{10+2}{2}$

3 Counting & Probability

Consider the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 70$ where each x_i is a non-negative integer. We choose one of these solutions uniformly at random.

(a) What is the size of the sample space?

$|\Omega| = \binom{70+5}{5}$

(b) What is the probability that both $x_1 \geq 30$ and $x_2 \geq 30$?

$70 - 60 = 10$

A B

$P(A \cap B) = \dfrac{|A \cap B|}{|\Omega|} = \dfrac{\binom{15}{5}}{\binom{75}{5}}$

(c) What it the probability that either $x_1 \geq 30$ or $x_2 \geq 30$?

$P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

40

$= \dfrac{\binom{45}{5}}{\binom{75}{5}} + \dfrac{\binom{45}{5}}{\binom{75}{5}} - P(A \cap B)$

$= \dfrac{2\binom{45}{5} - \binom{15}{5}}{\binom{75}{5}}$

$P(W) = 0.2$

$P(R|W) = 0.3$

1 Rain and Wind

$P(R|W^c) = 0.8$

The local weather channel just released a statistic for the months of November and December. It said that the probability that it would rain on a windy day is 0.3 and the probability that it would rain on a non-windy day is 0.8. The probability of a day being windy is 0.2. As a student in EECS 70, you are curious to play around with these numbers. Find the probability that:

(a) A given day is both windy and rainy.

$P(W \cap R) = P(W|R)P(R) = \boxed{P(R|W)P(W)}$

$= 0.3 \times 0.2 = 0.06$

(b) A given day is rainy. $= 0.70$

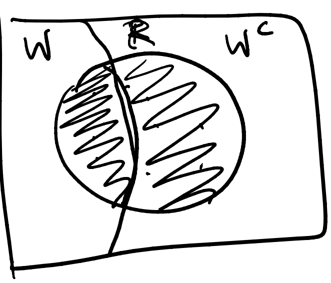
$P(R) = P(R \cap W) + \boxed{P(R \cap W^c)}$

$P(R \cap W^c) = P(R|W^c)P(W^c)$

$= 0.8 \times (1 - 0.2)$

$= 0.64$

(c) For a given pair of days, exactly one of the two days is rainy. (You may assume that the weather on the first day does not affect the weather on the second.)



$P(A|B) = \frac{P(A \cap B)}{P(B)} \Leftrightarrow P(A \cap B) = P(A|B)P(B)$

R_1 R_2

$P((R_1 \cap R_2^c)) + P(R_1^c \cap R_2)$

$= P(R_1)P(R_2^c) + P(R_1^c)P(R_2)$

if A & B are indep: $P(A \cap B) = P(A)P(B)$

$= 2 \times 0.7 \times 0.3$

$= 0.42$