Maths stuff

Kevin W Li

February 18, 2017

This is my notes on some maths that I encountered and found to be interesting

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1 Lampart W function

This function is define as the inverse of the function $f(x) = xe^x$, written as $W(z) = f^{-1}(z)$ where $z \in [-e^{-1}, +\infty]$. It can be used to solve many problems, for example the problem involving propagation of action potential as in Gatsby theoretical neuroscience here. One property is this $x = W(x) \exp W(x)$ by substituting $z = W(x)$ back to $f(x) = xe^x$

1.1 Solve $xa^x = b$

$$\begin{aligned} xa^x &= xe^{x\ln a} = b \\ x\ln(a)e^{x\ln(a)} &= \ln(a)b \\ x &= \frac{1}{\ln(a)}W(b\ln(a)) \end{aligned}$$

1.2 Solve $x^n a^x = c$

$$\begin{aligned} x^n a^x &= c \\ xa^{\frac{x}{n}} &= c^{\frac{1}{n}} \\ \frac{x}{n}a^{\frac{x}{n}} &= \frac{c}{n} \\ x &= \frac{n}{\ln a}W(\frac{c^{\frac{1}{n}}}{n}\ln a) \end{aligned}$$

Special case is when $a = e^b$, then $x = \frac{n}{b}W(c^{\frac{1}{n}}\frac{b}{n})$

1.3 Solve $a^x = cx + d$

$$\begin{aligned} a^x &= cx + d \\ \frac{1}{c}a^x &= x + \frac{d}{c} \\ \frac{1}{c}a^{-\frac{d}{c}}a^{x+\frac{d}{c}} &= x + \frac{d}{c} \\ \left(x + \frac{d}{c}\right)a^{x+\frac{d}{c}} &= ca^{\frac{d}{c}} \\ x + \frac{d}{c} &= -\frac{1}{\ln a}W(-\frac{\ln a}{ca^{\frac{d}{c}}}) \\ x &= -\frac{1}{\ln a}W(-\frac{\ln a}{ca^{\frac{d}{c}}}) - \frac{d}{c} \end{aligned}$$

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1.4 Solve $x \ln x = c$ and $x^x = b$

$$
\begin{aligned}
x \ln x &= c \\
e^{\ln x} \ln x &= c \\
\ln x &= W(\mathrm{c}) \\
x &= \exp W(c) = \frac{c}{W(c)}
\end{aligned}
$$

Taking $c = \log a$ and exponentiate both size gives $xe^x = a$ whose solution is $\exp W(\log a)$

1.5 The limit of (when converges) $h(x) = x^{x^{x^{\cdots}}}$

$$
\begin{aligned}
h(x) &= x^{h(x)} \\
h(x) &= -\frac{1}{\ln x} W(-\ln x)
\end{aligned}
$$

$\delta \dot{v_E} = \frac{\partial \phi_E}{\partial v_E}$