HME: A Hyperbolic Metric Embedding Approach for Next-POI Recommendation

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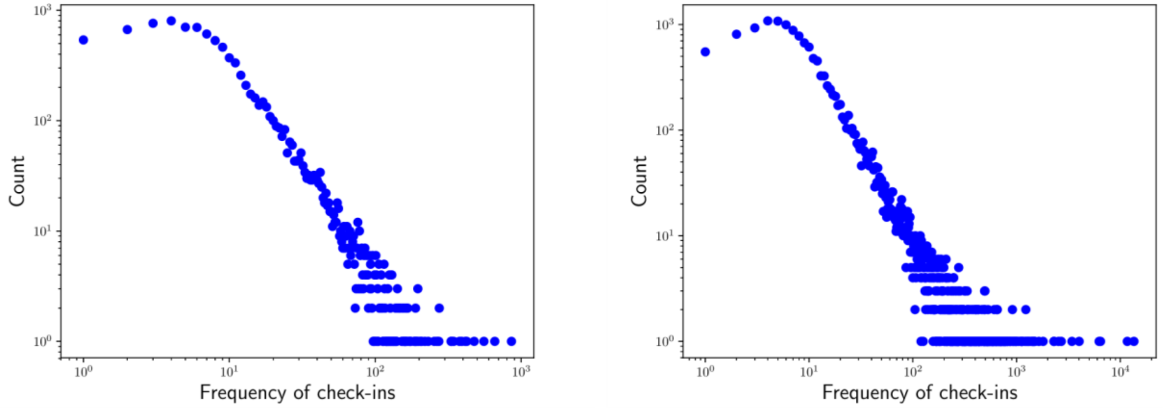
2021-9-27

Observation



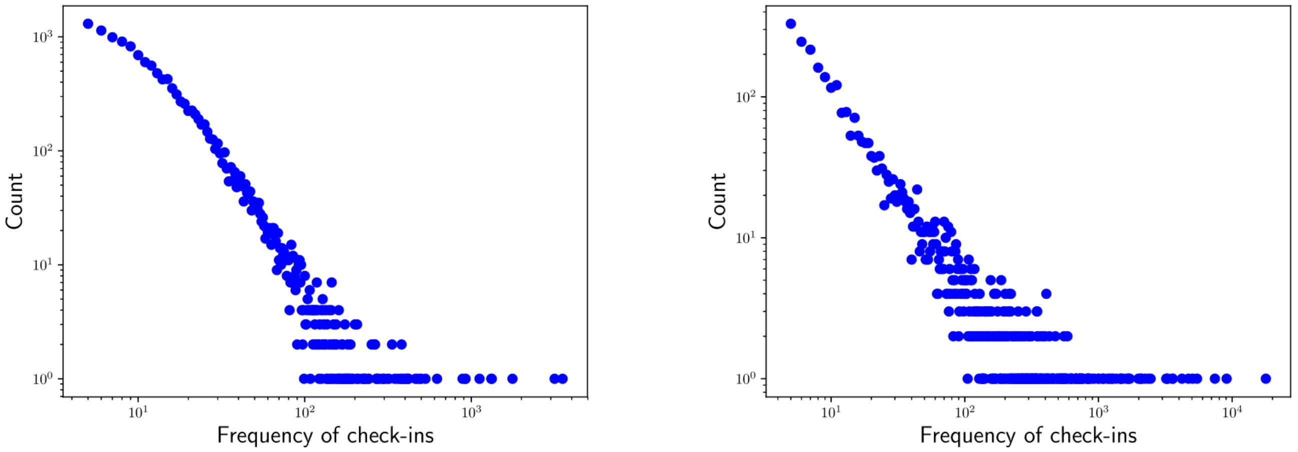
(a) NYC (b) Tokyo

Figure 1: Distributions of POI-POI relations on NYC and Tokyo. The X-axis presents the POIs' number of POI-POI transitions and the Y-axis shows the count of such POIs.

POI-POI relation(sequential transition)

- Some POIs are more likely to be involved in sequential transitions with other POIs, while most POIs are only connected to a small number of POIs.

Observation



(a) Distribution of POI's check-ins (b) Distribution of user's check-ins

Figure 2: Distributions of the frequency of check-ins for users and POIs. The X-axis presents the number of check-ins associated with a POI or user, and the Y-axis shows the count of such POIs or users.

POI-User relation(User preference)

- Some POIs attract many users while the majority of POIs are only visited by a small number of users.
- A small portion of users have many check-ins while most of users only have a few check-ins.

Observation

- POI-POI relation and POI-User relation follow **power-law distributions**: a majority of nodes have very few connections, and a few nodes have a huge number of connections.

- power-law distributions often indicate **implicit hierarchical structures**.

- Based on the category tree and region tree, we can obtain POI-Category relation and POI-Region relation that reflect the **explicit hierarchical structures**.

Innovation

- **Euclidean embedding** models' capability of learning complex patterns is limited by the dimensionality of the Euclidean space.
- The paper aims to learn the representations of check-in activities in a hyperbolic space and proposes a novel **hyperbolic metric embedding** model based on the **Poincaré ball model**.
- Investigate four kinds of relations: POI-POI, POI-User, POI-Category and POI-Region by projecting then in a shared hyperbolic space.

Relations

- **POI-POI Relation**: If the time interval between two consecutive check-ins of a user is smaller than $\tau = 6$ hours, a POI-POI edge exists.
- **POI-User Relation**: If a user u has visited a POI l , there is a POI-User edge.
- **POI-Region Relation**: If a POI is located in a region, there exists an edge between them.
- **POI-Category Relation**: If a POI is associated with a category , there is a POI-Category edge.

Methodology

Hyperbolic Metric Embedding

- The basic idea of hyperbolic metric embedding (HME) is to represent items with the **Poincaré ball model**, such that the related items are close to each other.

- Given an edge < a,b > , the representations **x a** and **x b** in the Poincaré ball should be close to each other.

should be close to each other. Different from the intuitive Euclidean distance, the distance in the Poincaré ball is stated as follows:

$$\mathcal{D}_{ab} = arcosh\left(1 + 2\frac{||\mathbf{x}_a - \mathbf{x}_b||^2}{(1 - ||\mathbf{x}_a||^2)(1 - ||\mathbf{x}_b||^2)}\right), \tag{1}$$

where $arcosh(x) = ln(x + \sqrt{x^2 - 1})$ is an inverse hyperbolic cosine function. One interesting feature is that the distance varies with

Methodology

- **Hyperbolic Metric Embedding**

◆ Regard an edge <a, b> as **positive pair**, randomly sample a small number of negative nodes to construct **negative pairs**.

◆ The distance between a negative pair should be larger than the distance between a positive pair.

$$P(b > n|a) = \sigma(\mathcal{D}_{an} - \mathcal{D}_{ab}), \tag{2}$$

where $\sigma(z) = \frac{1}{1+e^{-z}}$ is a logistic function, and $\mathcal{D}_{an}$ is the distance between $\mathbf{x}_a$ and $\mathbf{x}_n$ in the Poincaré ball model. Equation (2) reflects

$$\begin{aligned}\Theta &= \underset{\Theta}{\operatorname{argmax}} \sum_{(a,b)\in\mathcal{E}} \sum_{n\in\mathcal{N}_{ab}} \log P(b > n|a) \\ &= \underset{\Theta}{\operatorname{argmax}} \sum_{(a,b)\in\mathcal{E}} \sum_{n\in\mathcal{N}_{ab}} \log \sigma(\mathcal{D}_{an} - \mathcal{D}_{ab}).\end{aligned} \tag{3}$$

Here $\mathcal{N}_{ab}$ is the negative nodes sampled for each pair $< a, b >$ in the training dataset; and Θ is the hyperbolic representations of node set $\mathcal{V}$. In this paper, we set $k = |\mathcal{N}_{ab}| = 5$.

Methodology

- **Hyperbolic Metric Embedding**

  - Can not directly use the Stochastic Gradient Descent(SGD) due to the Riemannian manifold structure of the Poincaré ball.

  - First calculate Euclidean gradients and then combine them with the Riemannian gradient to update parameters.

$$
\begin{aligned}
\frac{\partial E}{\partial \mathbf{x}_a} &= (1-\sigma(\mathcal{D}_{an}-\mathcal{D}_{ab}))(\frac{\partial \mathcal{D}_{an}}{\partial \mathbf{x}_a} - \frac{\partial \mathcal{D}_{ab}}{\partial \mathbf{x}_a})\\
\frac{\partial E}{\partial \mathbf{x}_n} &= (1-\sigma(\mathcal{D}_{an}-\mathcal{D}_{ab}))(\frac{\partial \mathcal{D}_{an}}{\partial \mathbf{x}_n})\\
\frac{\partial E}{\partial \mathbf{x}_b} &= (1-\sigma(\mathcal{D}_{an}-\mathcal{D}_{ab}))(-\frac{\partial \mathcal{D}_{ab}}{\partial \mathbf{x}_b}).
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{D}_{ab}}{\partial \mathbf{x}_a} &= \frac{4}{\beta_v\sqrt{\gamma_{ab}^2-1}}\left(\frac{||\mathbf{x}_b||^2-2<\mathbf{x}_a,\mathbf{x}_b>+1}{\alpha^2}\mathbf{x}_a - \frac{\mathbf{x}_b}{\alpha}\right)\\
\frac{\partial \mathcal{D}_{ab}}{\partial \mathbf{x}_b} &= \frac{4}{\alpha\sqrt{\gamma_{ab}^2-1}}\left(\frac{||\mathbf{x}_a||^2-2<\mathbf{x}_a,\mathbf{x}_b>+1}{\beta_b^2}\mathbf{x}_b - \frac{\mathbf{x}_a}{\beta_b}\right)\\
\frac{\partial \mathcal{D}_{an}}{\partial \mathbf{x}_a} &= \frac{4}{\beta_n\sqrt{\gamma_{an}^2-1}}\left(\frac{||\mathbf{x}_n||^2-2<\mathbf{x}_a,\mathbf{x}_n>+1}{\alpha^2}\mathbf{x}_a - \frac{\mathbf{x}_n}{\alpha}\right)\\
\frac{\partial \mathcal{D}_{an}}{\partial \mathbf{x}_n} &= \frac{4}{\alpha\sqrt{\gamma_{an}^2-1}}\left(\frac{||\mathbf{x}_a||^2-2<\mathbf{x}_a,\mathbf{x}_n>+1}{\beta_n^2}\mathbf{x}_n - \frac{\mathbf{x}_a}{\beta_n}\right),
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
\mathbf{x}_a^{t+1} &\leftarrow norm\left(\mathbf{x}_a^t + lr\frac{(1-||\mathbf{x}_a^t||^2)^2}{4}\frac{\partial E}{\partial \mathbf{x}_a}\right)\\
\mathbf{x}_b^{t+1} &\leftarrow norm\left(\mathbf{x}_b^t + lr\frac{(1-||\mathbf{x}_b^t||^2)^2}{4}\frac{\partial E}{\partial \mathbf{x}_b}\right)\\
\mathbf{x}_n^{t+1} &\leftarrow norm\left(\mathbf{x}_n^t + lr\frac{(1-||\mathbf{x}_n^t||^2)^2}{4}\frac{\partial E}{\partial \mathbf{x}_n}\right),
\end{aligned}
\tag{6}
$$

$$
norm(\mathbf{x}) = \begin{cases} \mathbf{x}/||\mathbf{x}|| - \epsilon, & \text{if } ||\mathbf{x}|| \geq 1\\ \mathbf{x}, & \text{otherwise}. \end{cases}
\tag{7}
$$

Methodology

Recommending

- Due to the hyperbolic geometry, we cannot simply combine the user embedding with the POI embedding by linear interpolation in the Poincaré ball model.
- Einstein midpoint aggregation method in the Klein model is used to provides an efficient aggregation operation.
- By sorting the fused distance scores of POI candidates, a list of POIs with the **smallest fused distance scores** are returned as the recommendation result.

$$\mathbf{x}_u^{\mathcal{K}} = \frac{2 \cdot \mathbf{x}_u}{1 + ||\mathbf{x}_u||^2}, \quad \mathbf{x}_{l_c}^{\mathcal{K}} = \frac{2 \cdot \mathbf{x}_{l_c}}{1 + ||\mathbf{x}_{l_c}||^2}. \tag{8}$$

Convert user embedding and poi embedding to Klein model

$$\mathbf{x}_{ag}^{\mathcal{K}} = \frac{w \cdot \psi_u}{w \cdot \psi_u + (1-w) \cdot \psi_{l_c}} \cdot \mathbf{x}_u^{\mathcal{K}} + \frac{(1-w) \cdot \psi_{l_c}}{w \cdot \psi_u + (1-w) \cdot \psi_{l_c}} \cdot \mathbf{x}_{l_c}^{\mathcal{K}}, \tag{9}$$

Einstein midpoint aggregation

$$\mathbf{x}_{ag} = \frac{\mathbf{x}_{ag}^{\mathcal{K}}}{1 + \sqrt{1 - ||\mathbf{x}_{ag}^{\mathcal{K}}||^2}}. \tag{10}$$

converted into the Poincaré ball model

the aggregated point $\mathbf{x}_{ag}$ of the given query (u, l_c). For each POI candidate l, we calculate the distance $\mathcal{D}_{l,ag}$ between $\mathbf{x}_l$ and $\mathbf{x}_{ag}$ using Equation (1). In addition, the geographical distance can be accommodated, which has been demonstrated beneficial for next-POI recommendation. For a fair comparison, we exploit the same strategy as [11]. Specifically, we compute the fused distance score

$$\mathcal{D}_{l,ag}^{Geo} = (1 + d_{l,l_c})^{0.25} * \mathcal{D}_{l,ag}, \tag{11}$$

where d_{l,l_c} is the distance calculated by the geographical coordinates of POI l and current POI l_c. By sorting the fused distance