EECS2020
Signals and Systems
Computer Homework #3:
Experiencing Your First Fourier Analysis and Filter Design Using a Computer

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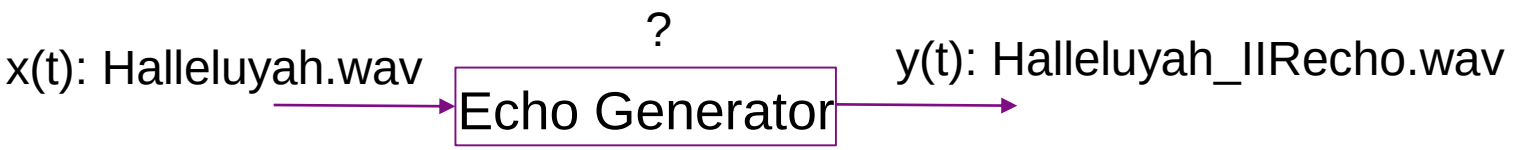
In Computer HW3, You Will
First, Try to Be Familiar with the Properties of the Implemented CTFT
Secondly, Apply the Implemented CTFT for System Design and Analysis

Why Fourier Analysis Using a Computer? (1/2)

Given Arbitrary Input x(t) or x[n]
and Output y(t) or y[n],
Can You Decipher an Unknown LTI System
via
Calculating Its Frequency Response
by Hand Writing?

Decipher the Echo Generator (i.e., Comb Reverberator) in the Computer HW2 If You Only Have “Halleluyah.wav” and “Halleluyah_IIRecho.wav”



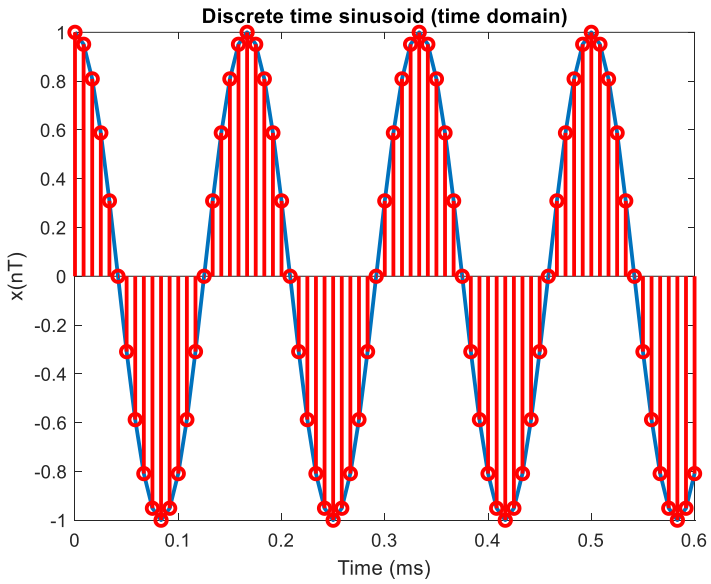
Below Is How I Implement CTFT
via
Two Key Steps –
Sampling in Time Domain
&
Sampling in Frequency Domain

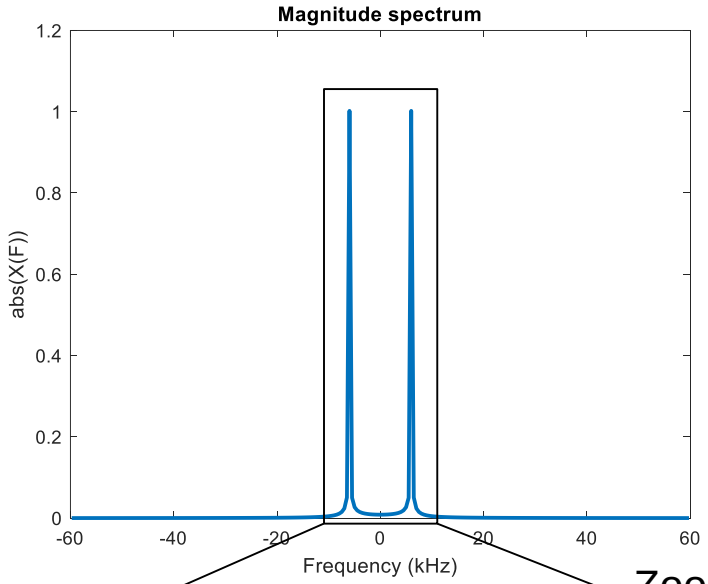
(See ComputerHW3_SampleCodes.m)

```
% Generate sampled consine
F0 = 6; % in kHz
Fs = 120; % sampling rate/sampling frequency, in kHz or ksamples/sec
T = 1/Fs;  % time resolution, i.e., sampling interval in time domain
total_time = 2; % in ms

% !!! Sampling in time
t_axis = (0:T:total_time);  % time axis
x = cos(2*pi*F0*t_axis);  % sampled cosine/discrete time sinusoid ,time domain
Npoint = length(x);   % number of points in sampled cosine
```

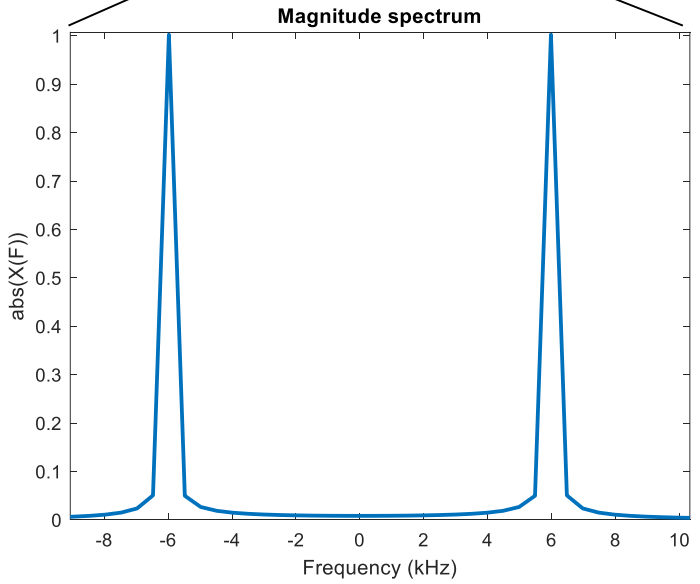
```
%% ---------- Fourier transform - Analysis ----------
% !!! Sampling in frequency
dF = Fs/Npoint; % frequency resolution, i.e., sampling interval in frequency domain
%f_axis = (0:1:(Npoint-1))*dF;   % frequency axis (from 0 to Fs)
f_axis = ((1:1:Npoint)-(Npoint+1)/2)*dF; % frequency axis (from -Fs/2 to Fs/2)
X = zeros(1,length(f_axis)); % spectrum

% implementatoin of X(Fk) = summation x(nT)*exp(-j*2*pi*Fk*(nT))*T
for iFreq = 1:length(f_axis),
    iFreq
   for iTime = 1:length(t_axis),
       X(iFreq) = X(iFreq) + x(iTime)*exp(-sqrt(-1)*2*pi*f_axis(iFreq)*t_axis(iTime))*T;
   end
end
```

Output of the Provided Sample Codes





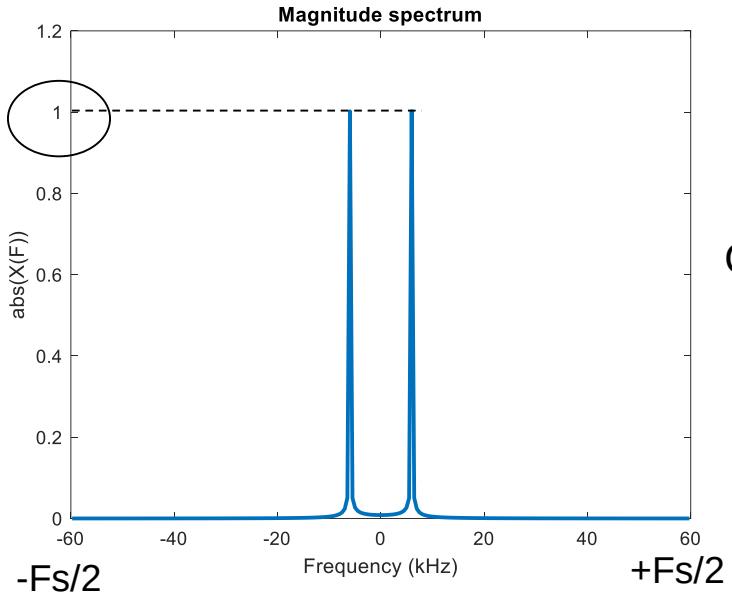
Zoom in



To Which Fourier Representation (CTFS, CTFT, DTFT, DTFS) Are the Sample Codes More Similar?

```
% implementatoin of X(Fk) = summation x(nT)*exp(-j*2*pi*Fk*(nT))*T
for iFreq = 1:length(f_axis),
    iFreq
   for iTime = 1:length(t_axis),
       X(iFreq) = X(iFreq) + x(iTime)*exp(-sqrt(-1)*2*pi*f_axis(iFreq)*t_axis(iTime))*T;
   end
end
```

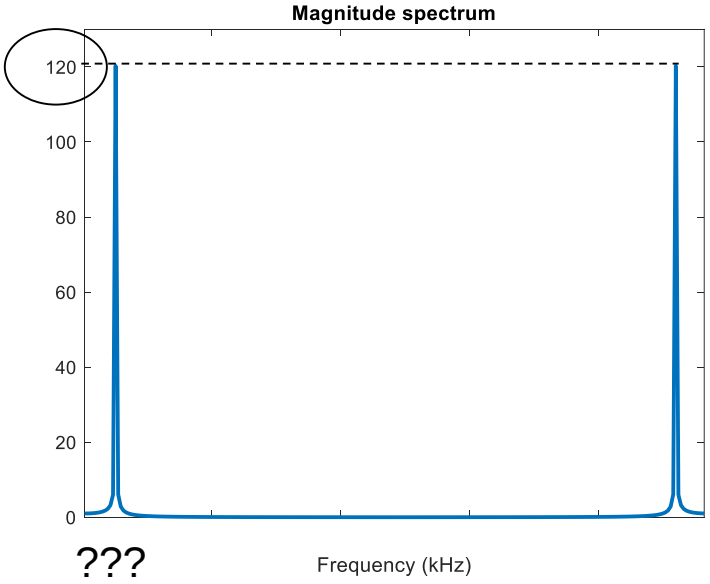
Our CTFT Codes vs. MATLAB fft()



-Fs/2

+Fs/2

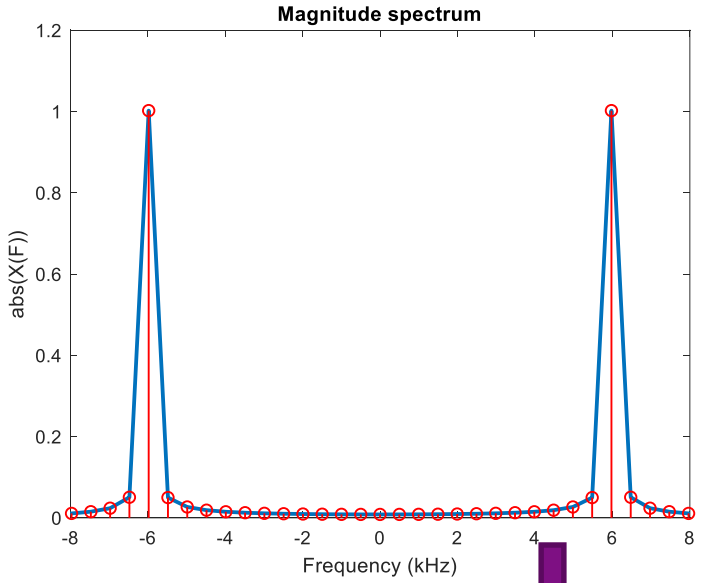
Our own CTFT codes



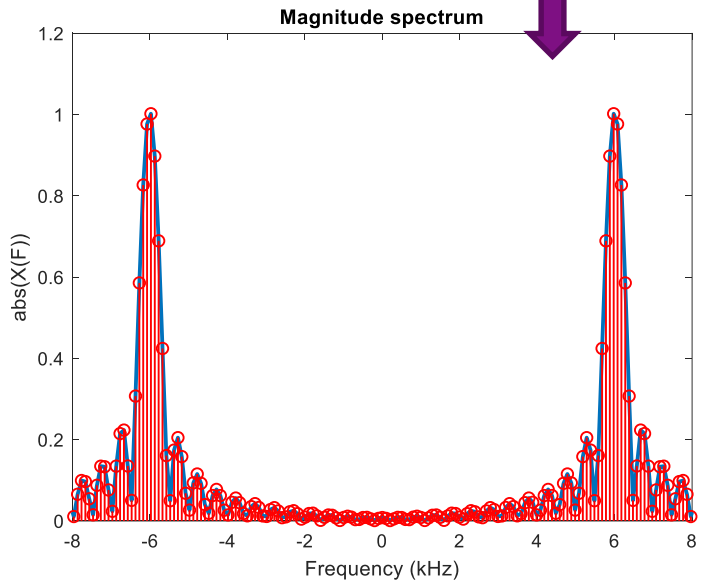
???

MATLAB fft()
(Implement DFT
See “doc fft”)

A Smoother Spectrum

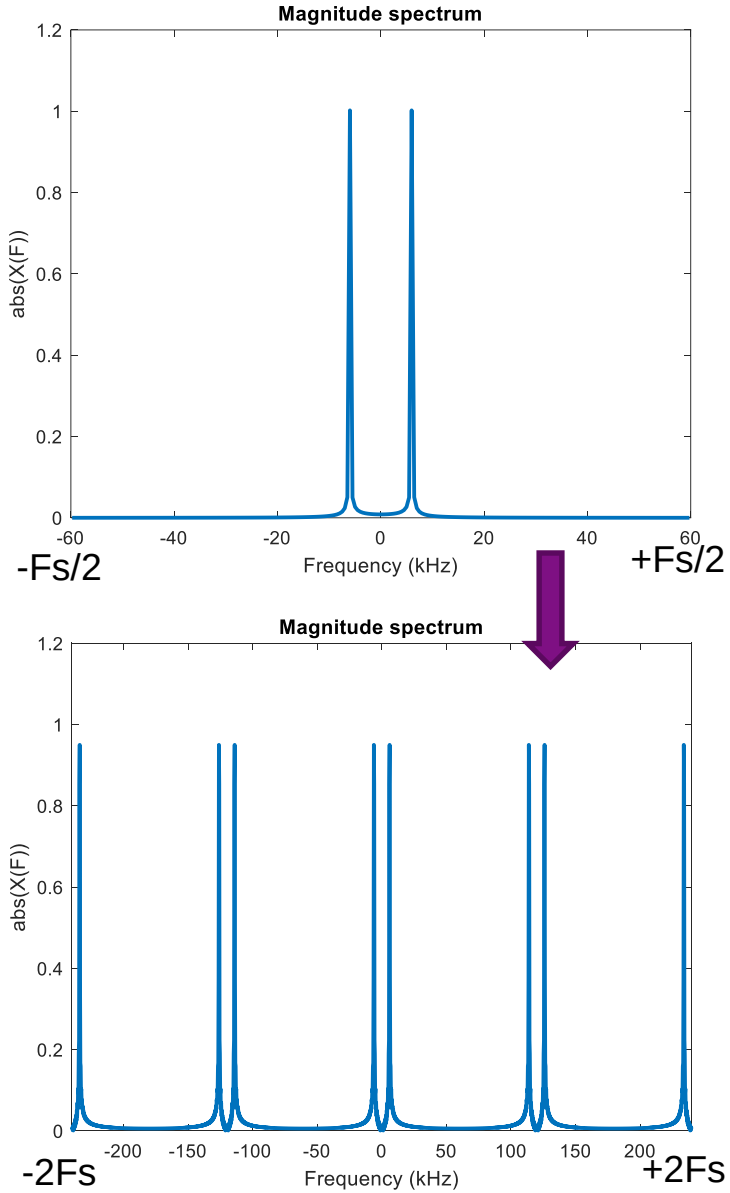


Low frequency resolution



High frequency resolution

Change the Observation Frequency Range



Vectorization of the Sample Codes (FYI)

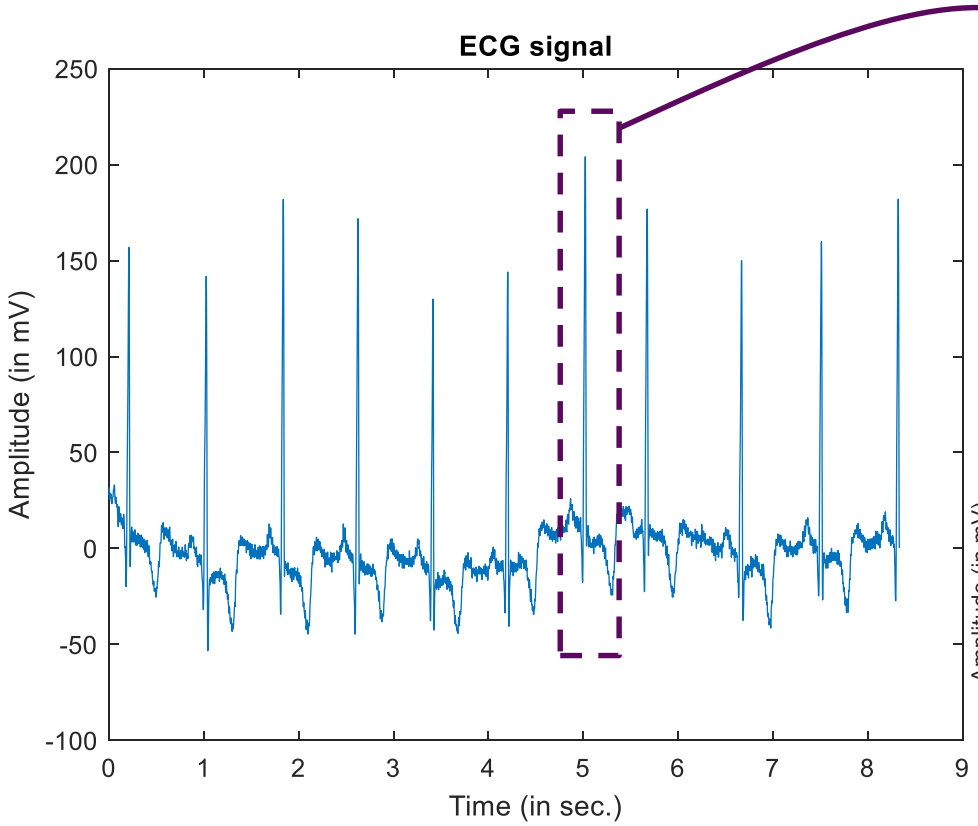
```
% implementatoin of X(Fk) = summation x(nT)*exp(-j*2*pi*Fk*(nT))*T
for iFreq = 1:length(f_axis),
    iFreq
   for iTime = 1:length(t_axis),
       X(iFreq) = X(iFreq) + x(iTime)*exp(-sqrt(-1)*2*pi*f_axis(iFreq)*t_axis(iTime))*T;
   end
end
```

```
% implementatoin of X(Fk) = summation x(nT)*exp(-j*2*pi*Fk*(nT))*T
% !!! Vectorization which removes one looping
for iFreq = 1:length(f_axis),
            iFreq
            X(iFreq) = sum(x.*exp(-sqrt(-1)*2*pi*f_axis(iFreq)*t_axis))*T
end
```

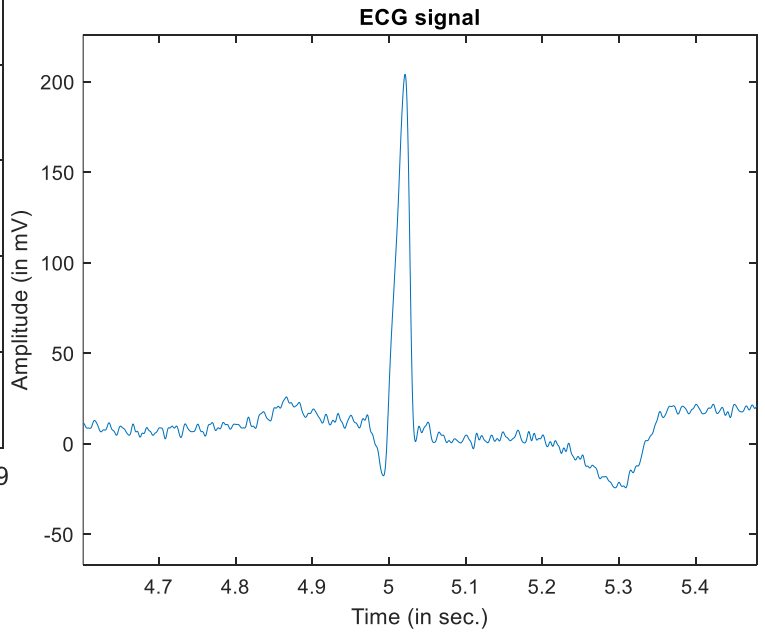
```
% implementatoin of X(Fk) = summation x(nT)*exp(-j*2*pi*Fk*(nT))*T
% !!! Vectorization which removes all the loopings
X = exp(-sqrt(-1)*2*pi*((f_axis.')*t_axis))*(x.')*T;
```

Why Fourier Analysis Using a Computer? (2/2)

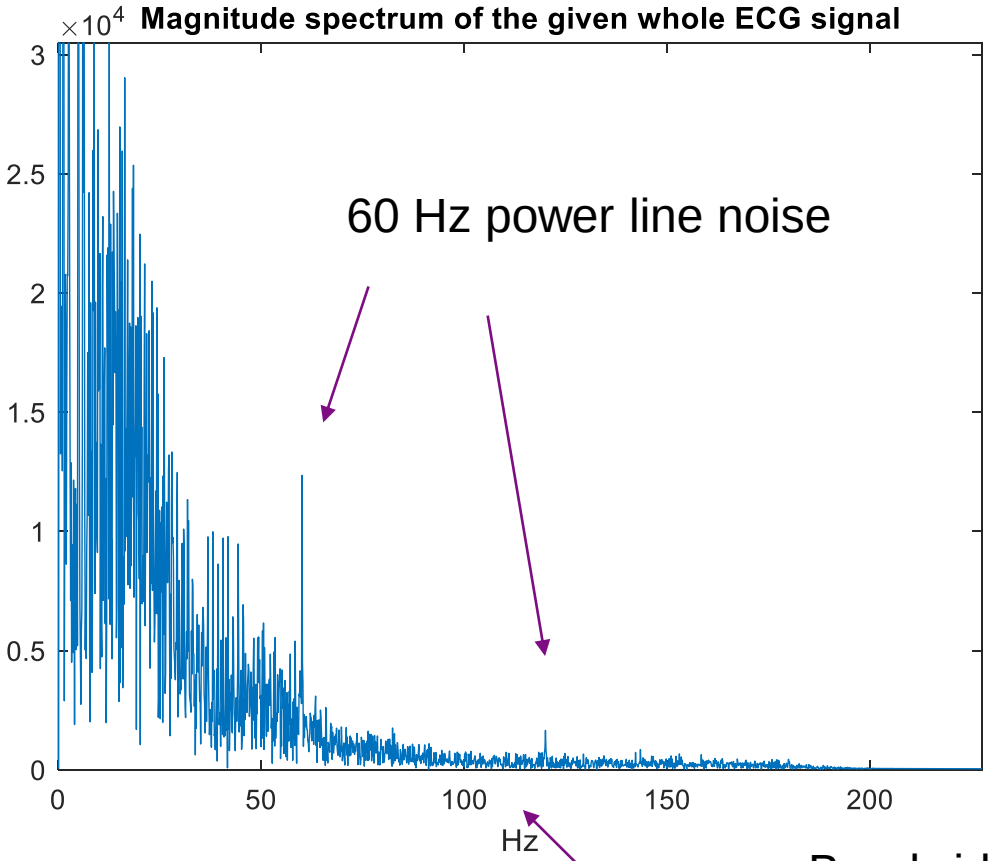
Can You Calculate the Spectrum of the Following Signal by Hand Writing?



One single cycle ECG signal



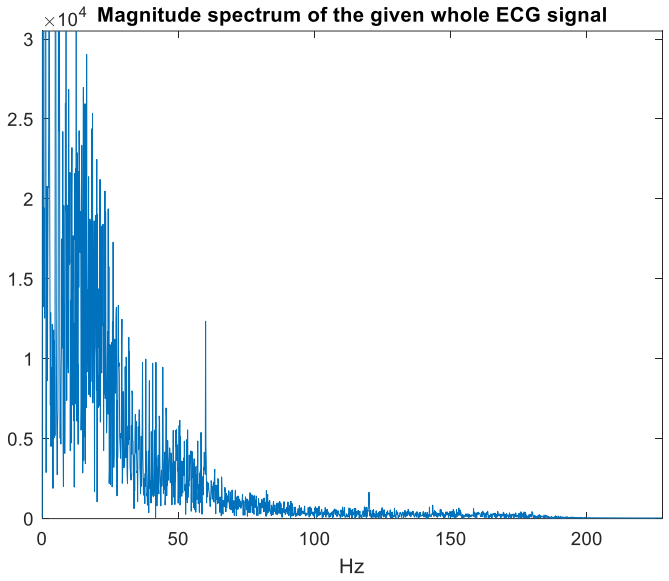
Spectrum Computed by MATLAB fft() (Details Please doc fft()) Design Moving Average Filter to Remove Power Line Noise

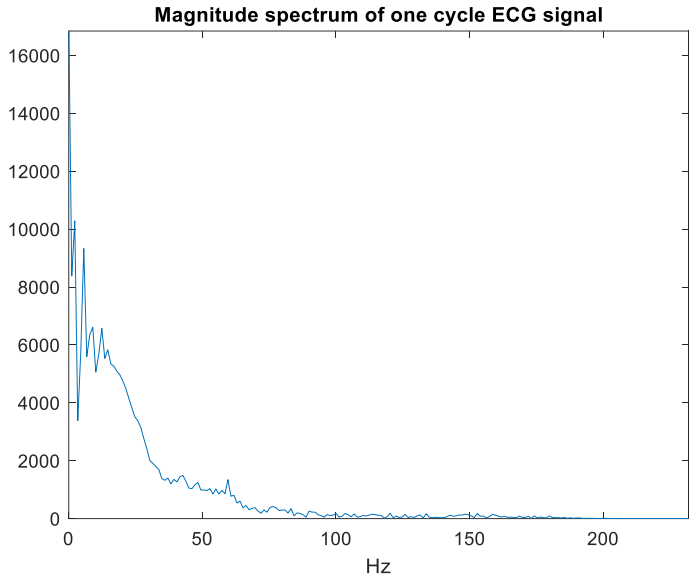


Bandwidth of the signal

Maximum frequency of the signal???

One Cycle ECG Signal vs. Whole ECG Signal Difference? What Causes the Difference? Can You Find Out Heart Rate from the Spectra?





How Do We Use These Spectral Information for System Design?

e.g.,

Analog Front-end Circuit Design,

Pre-amplifier Design,

Analog or Digital Filter Design

… etc.

Generally How Do We Obtain DT or Digital Signals and Reconstruct CT Signals in EE?
Sampling Hardware: Analog to Digital Converter (ADC)
Reconstruction Hardware: Digital to Analog Converter (DAC)
Think about Digital Music You Listen Every Day

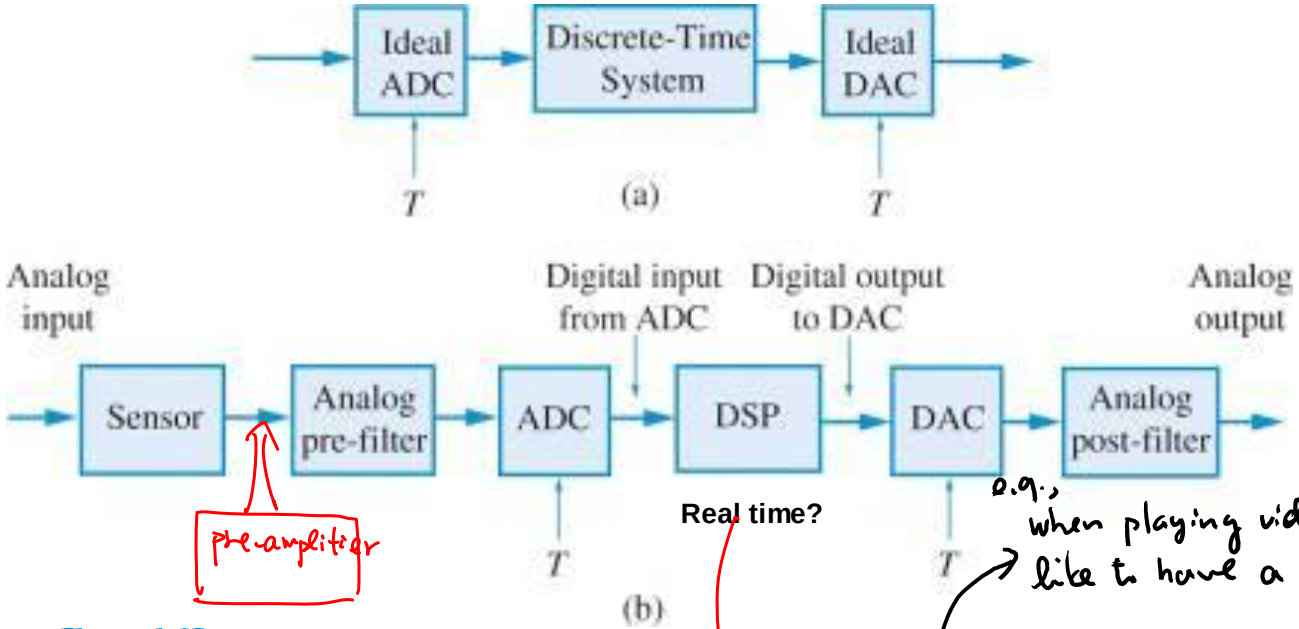


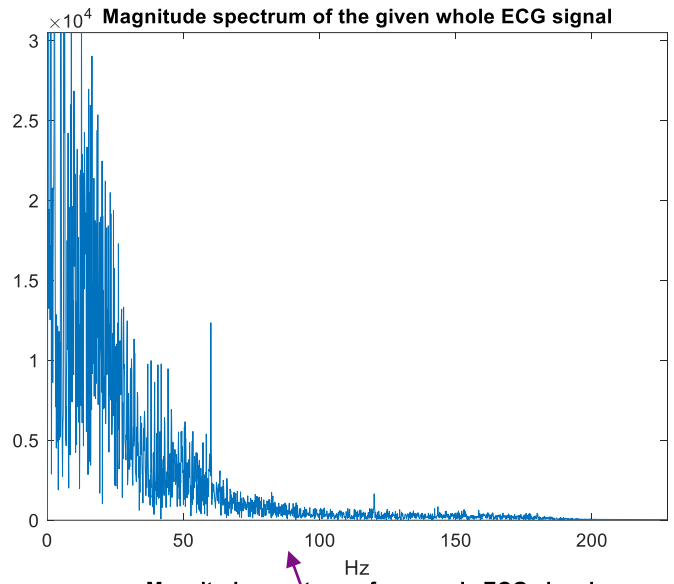
Figure 1.13 Simplified block diagram of idealized system for (a) continuous-time processing of discrete-time signals, and (b) its practical counterpart for digital processing of analog signals.

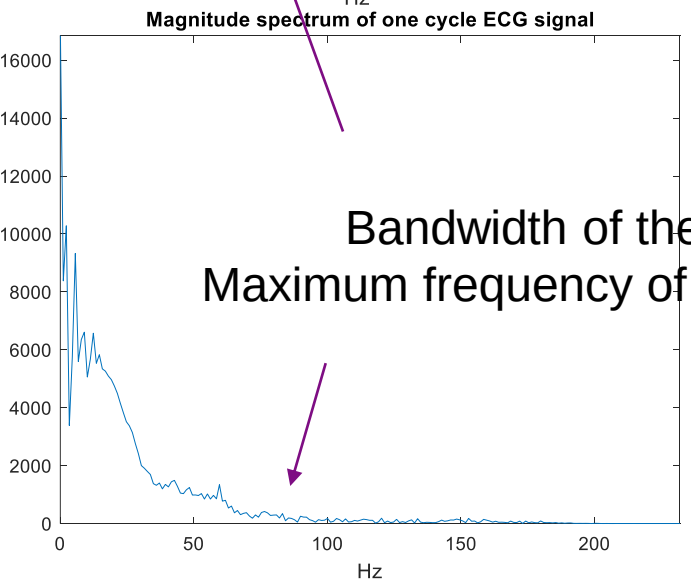
Real time?

Complete processing within allowable available time between samples.

e.g., when playing video games, you would like to have a real time video game system.

Pre-amplifier Design and ADC Sampling Rate





Bandwidth of the signal
Maximum frequency of the signal???

Generally it is a low pass filter
- Cutoff frequency?

Proper sampling rate of ADC?