Signals and Systems

(10920EECS202002)

Computer Homework #3: Experiencing your first Fourier analysis and filter design using a computer

Due: 24:00, 05/20/2021

The goal of this homework is to let you know how to perform Fourier analysis using a computer and design a filter to remove the noises according to the Fourier analysis.

Given ComputerHW3_SampleCodes.m,

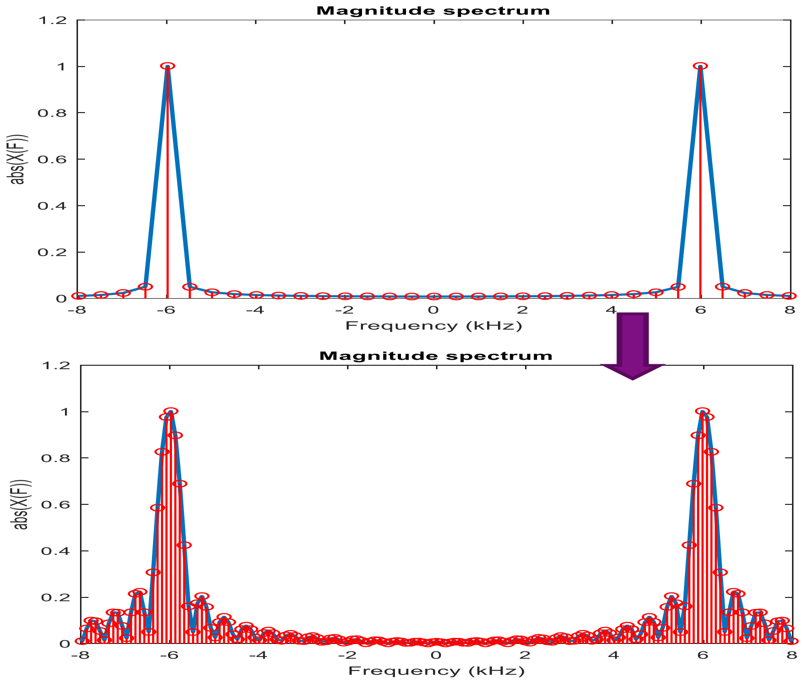
Part 1, Try to be familiar with the properties of the implemented CTFT

(a) (10%) Go through ComputerHW3_SampleCodes.m, run the codes, and tell whether or not the computed magnitude spectrum is the same as what you expect. Please elaborate why you have such a computed magnitude spectrum for a cosine signal.

(b) (5%) To which Fourier representation (CTFS, CTFT, DTFT, DTFS) are the implemented sample codes more similar although I am trying to implement CTFT? Please verify your answer.

(c) (10%) Perform Fourier analysis using MATLAB built function fft(), i.e., **X** = fft(**x**) (see the provided sample codes). Plot the magnitude spectrum obtained by fft() with correct x label and tell the differences between fft() spectrum and that obtained by using the provided sample codes. Once you know the differences, later on, you can use fft() to help you perform Fourier analysis instead of using our slow CTFT codes.

(d) (10%) In the following figure, top panel is the magnitude spectrum generated by the provided sample codes, which is poor sampled, i.e., not so smooth (low frequency resolution), and bottom panel is the smoother magnitude spectrum which is desired in practice because it is more close to the ideal CTFT spectrum. Please provide your strategy, elaborate your idea, and modify the provided sample codes accordingly to obtain a smoother magnitude spectrum as shown in the bottom panel. Remember to show how you modify the codes in your report.



(e) (10%) In the provided sample codes, the magnitude spectrum is computed and observed in the frequency range [–Fs/2 Fs/2] where Fs is the sampling rate used to sample the CT cosine signal. Please modify the codes so that the magnitude spectrum is computed (or observed) in the frequency range [–Fs Fs] and [–2Fs 2Fs], respectively. Please tell the change in the magnitude spectrum as the frequency range changes, and justify what you observe. Then try to show the magnitude spectrum with the normalized frequency axis and elaborate what you observe. Remember to explain why.

(f) (10%) Change the value of **F0** in the sample codes to 0, 6, 30, 60, 90, 114, and 120 kHz, respectively, and plot the discrete-time sinusoid x[n] with different **F0** as a function of time n. Please tell the change in the rate of oscillation of the discrete-time sinusoid as **F0** changes, and elaborate what you observe in time domain as well as in frequency domain. Remember to explain why. In your elaboration, please explain from the point of view of the sampling theorem concept introduced in slides 40-45 of Topic3_FourierRepresentation_Part3_1_HandWriting0505_2021.pdf, and also try to convert **F0** to normalized frequency, and explain from the point of view of normalized frequency as well.

(g) (5%) From the above problems, what is the working frequency range in the continuous-time domain for the provided CTFT codes? Justify your answer. The working frequency range is the frequency range where the CTFT codes can truly show the representative CTFT spectrum of the corresponding CT signal.

Part 2, Apply the implemented CTFT for realistic signal analysis, system design and implementation

(a) (10%) Given a MATLAB data file – ECG.mat where a raw ECG signal and Fs (in Hz) used to acquire the ECG signal are stored, perform Fourier analysis over (1) one single-cycle ECG wavelet (i.e., one heart-beat cycle) and (2) the whole given ECG signal, and then justify which ((1), (2), or both (1) and (2)) magnitude spectrum can provide the spectral information needed for ECG front end circuit design, e.g., determine bandwidth of a pre-amplifier for signal amplification, frequency response of an analog filter for noise reduction, and a proper ADC sampling rate. (Note that the ECG signal was acquired by a digital oscilloscope with sufficiently high sampling rate, and you can use fft() for Fourier analysis instead of my slow CTFT codes if you know what fft() does for you)

(b) (5%) Tell the difference (i.e., spectral difference) between Fourier analysis (1) and (2) in Part 2(a) and elaborate what causes the difference.

(c) (5%) From the Fourier analysis (2) in Part 2(a), can you figure out the heartbeat frequency, i.e., heart rate which is defined as the inverse of the R-R interval? Justify your answer.

(d) (10%) Following Part 2 (a), once you see the magnitude spectrum, you will find that the acquired ECG signal suffers power line noise. The power line noise in Taiwan includes 60-Hz noise and its harmonics (i.e., 120 Hz, 180 Hz, 240 Hz, etc.). Design a proper moving average filter to remove the power line noise, i.e., 60-Hz noise and its harmonics (hint: utilize zero-crossing in the frequency response of the moving average filter), apply the designed moving average filter to the given ECG signal, and then verify whether or not the filter works by comparing the original and filtered ECG signals in both time domain and frequency domain (i.e., check the spectra). Remember to justify your filter design and implementation. (Note that you can use fft() instead of my slow CTFT codes for the filter design and implementation if you know what fft() does for you)

(e) (10%) Derive the frequency response of the comb reverberator in your computer homework 2, and according to the derived frequency response, plot the magnitude response as a function of normalized frequency ranging from 0 to 0.5 using MATLAB. Please use the same values of a and D in Part 2(c) of your computer homework 2 for the frequency response plot. Then based on the implemented CTFT codes (note that you can use fft() if you know what fft() does for you), **y**

(input), and **ye (output)** in the provided **EchoGen.m**, please compute the magnitude response as a function of normalized frequency using the FT convolution property and compare with your derivation and the magnitude response you obtained in Part 2(g) of your computer homework 2 (note that in this case you have to convert the absolute frequency to normalized frequency). The plotted magnitude response will explain why it is called as "Comb" reverberator".

Notice:

1. You can find some ECG details at http://en.wikipedia.org/wiki/Electrocardiography or http://zh.wikipedia.org/wiki/%E5%BF%83%E7%94%B5%E5%9B%BE
2. Please hand in your solution files to the LMS elearning system, including your word or pdf file of the detailed solutions, the associated Matlab codes, and all the related materials. It would be nice that you can put your "KEY" code segment with comments side by side along with your answer in the word or pdf file whenever necessary.
3. Name your solution files "EECS2020_HW3_StudentID.doc" (or "EECS2020_HW3_StudentID.pdf") and "EECS2020_HW3_StudentID.m", and archive them as a single zip file: EECS2020_HW3_StudentID.zip.
4. The first line of your word/pdf or Matlab file should contain your name and some brief description, e.g., % EECS2020 Calvin Li u9612345 HW3 MM/DD/2021