CS542200 Parallel Programming

Homework 3: All-Pairs Shortest Path

Due: Sun. Dec 11 23:59, 2022

1 GOAL

This assignment helps you manage to solve the all-pairs shortest path problem with CPU threads and then further accelerate the program with CUDA accompanied by Blocked Floyd-Warshall algorithm. In this assignment, you will realize how powerful GPUs can be. Finally, we encourage you to optimize your program by exploring different optimizing strategies for performance points.

2 REQUIREMENTS

- In this assignment, you are asked to implement 3 versions of programs that solve the all-pairs shortest path problem.
 - *CPU version (hw3-1)*
 - You are required to use **threading** to parallelize the computation in your program.
 - ◆ You can choose any threading library or framework you like (pthread, std::thread, OpenMP, Intel TBB, etc).
 - You can choose any algorithm to solve the problem.
 - ◆ You must implement the shortest path algorithm yourself. (Do not use libraries to solve the problem. Ask TA if unsure).
 - Single-GPU version (hw3-2)
 - ◆ Should be optimized to get the performance points (20%).
 - Multi-GPU version (hw3-3)
 - Must use 2 GPUs. Single GPU version is not accepted and will get 0 for correctness and performance score in hw3-3 (even if you get AC on scoreboard).

3 BLOCKED FLOYD-WARSHALL ALGORITHM

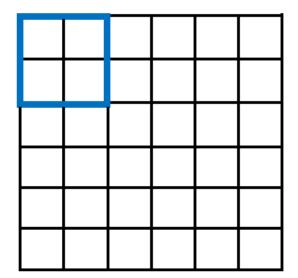
Given an $V \times V$ matrix W = [w(i, j)] where $w(i, j) \ge 0$ represents the distance (weight of the edge) from a vertex i to a vertex j in a directed graph with V vertices. We define an $V \times V$ matrix D = [d(i, j)] where d(i, j) denotes the shortest-path distance from a vertex i

to a vertex j. Let $D^{(k)} = [d^{(k)}(i,j)]$ be the result which all the intermediate vertices are in the set $\{0, 1, 2, ..., k-1\}$.

We define $d^{(k)}(i, j)$ as the following:

$$d^{(k)}(i,j) = \begin{cases} w(i,j) & \text{if } k=0; \\ \min(d^{(k-1)}(i,j),d^{(k-1)}(i,k-1)+d^{(k-1)}(k-1,j)) & \text{if } k \ge 1 \end{cases}$$

The matrix $D^{(V)} = d^{(V)}(i,j)$ gives the answer to the all-pairs shortest path problem. In the blocked all-pairs shortest path algorithm, we partition D into $\lceil V/B \rceil \times \lceil V/B \rceil$ blocks of $B \times B$ submatrices. The number B is called the *blocking factor*. For instance, in figure 1, we divide a 6×6 matrix into 3×3 submatrices (or blocks) by B = 2.



$D_{(0,0)}$	$D_{(0,1)}$	$D_{(0,2)}$
$D_{(1,0)}$	$D_{(1,1)}$	$D_{(1,2)}$
$D_{(2,0)}$	$D_{(2,1)}$	$D_{(2,2)}$

Figure 1: Divide a matrix by B = 2

The blocked version of the Floyd-Warshall algorithm will perform $\lceil V/B \rceil$ rounds, and each round is divided into 3 phases. It performs B iterations in each phase.

Assuming a block is identified by its index (I,J), where $0 \le I,J < \lceil V/B \rceil$. The block with index (I,J) is denoted by $D_{(I,I)}^{(k)}$.

In the following explanation, we assume N=6 and B=2. The execution flow is described step by step as follows:

• **Phase 1**: self-dependent blocks.

In the k-th round, the first phase is to compute the $B \times B$ pivot block $D \stackrel{(k \cdot B)}{(k-1,k-1)}$.

For instance, in the 1st round, $D_{(0,0)}^{(2)}$ is computed as follows:

$$d^{(1)}(0,0) = \min(d^{(0)}(0,0), d^{(0)}(0,0) + d^{(0)}(0,0))$$

$$d^{(1)}(0,1) = \min(d^{(0)}(0,1), d^{(0)}(0,0) + d^{(0)}(0,1))$$

$$d^{(1)}(1,0) = \min(d^{(0)}(1,0), d^{(0)}(1,0) + d^{(0)}(0,0))$$

$$d^{(1)}(1,1) = \min(d^{(0)}(1,1), d^{(0)}(1,0) + d^{(0)}(0,1))$$

$$d^{(2)}(0,0) = \min(d^{(1)}(0,0), d^{(1)}(0,1) + d^{(1)}(1,0))$$

$$d^{(2)}(0,1) = \min(d^{(1)}(0,1), d^{(1)}(0,1) + d^{(1)}(1,1))$$

$$d^{(2)}(1,0) = \min(d^{(1)}(1,0), d^{(1)}(1,1) + d^{(1)}(1,0))$$

$$d^{(2)}(1,1) = \min(d^{(1)}(1,1), d^{(1)}(1,1) + d^{(1)}(1,1))$$

Note that the result of $d^{(2)}$ depends on the result of $d^{(1)}$ and therefore cannot be computed in parallel with the computation of $d^{(1)}$.

• Phase 2: pivot-row and pivot-column blocks.

In the *k*-th round, it computes all $D_{(h,k-1)}^{(k\cdot B)}$ and $D_{(k-1,h)}^{(k\cdot B)}$ where $h\neq k-1$.

The result of pivot-row / pivot-column blocks depend on the result in phase 1 and itself.

For instance, in the 1st round, the result of $D_{(0,2)}^{(2)}$ depends on $D_{(0,0)}^{(2)}$ and $D_{(0,2)}^{(0)}$:

$$d^{(1)}(0,4) = min(d^{(0)}(0,4),d^{(2)}(0,0) + d^{(0)}(0,4))$$

$$d^{(1)}(0,5) = min(d^{(0)}(0,5),d^{(2)}(0,0) + d^{(0)}(0,5))$$

$$d^{(1)}(1,4) = min(d^{(0)}(1,4),d^{(2)}(1,0) + d^{(0)}(0,4))$$

$$d^{(1)}(1,5) = min(d^{(0)}(1,5),d^{(2)}(1,0) + d^{(0)}(0,5))$$

$$d^{(2)}(0,4) = min(d^{(1)}(0,4),d^{(2)}(0,1) + d^{(1)}(1,4))$$

$$d^{(2)}(0,5) = min(d^{(1)}(0,5),d^{(2)}(0,1) + d^{(1)}(1,5))$$

$$d^{(2)}(1,4) = min(d^{(1)}(1,4),d^{(2)}(1,1) + d^{(1)}(1,4))$$

$$d^{(2)}(1,5) = min(d^{(1)}(1,5),d^{(2)}(1,1) + d^{(1)}(1,5))$$

Phase 3: other blocks.

In the k-th round, it computes all $D_{(h_1,h_2)}^{(k\cdot B)}$ where $h_1,h_2\neq k-1$.

The result of these blocks depends on the result from phase 2 and itself.

For instance, in the 1st round, the result of $D_{(1,2)}^{(2)}$ depends on $D_{(1,0)}^{(2)}$ and $D_{(0,2)}^{(2)}$:

$$d^{(1)}(2,4) = min(d^{(0)}(2,4),d^{(2)}(2,0) + d^{(2)}(0,4))$$

$$d^{(1)}(2,5) = min(d^{(0)}(2,5),d^{(2)}(2,0) + d^{(2)}(0,5))$$

$$d^{(1)}(3,4) = min(d^{(0)}(3,4),d^{(2)}(3,0) + d^{(2)}(0,4))$$

$$d^{(1)}(3,5) = min(d^{(0)}(3,5),d^{(2)}(3,0) + d^{(2)}(0,5))$$

$$d^{(2)}(2,4) = min(d^{(1)}(2,4),d^{(2)}(2,1) + d^{(2)}(1,4))$$

$$d^{(2)}(2,5) = min(d^{(1)}(2,5),d^{(2)}(2,1) + d^{(2)}(1,5))$$

$$d^{(2)}(3,4) = min(d^{(1)}(3,4),d^{(2)}(3,1) + d^{(2)}(1,4))$$

$$d^{(2)}(3,5) = min(d^{(1)}(3,5),d^{(2)}(3,1) + d^{(2)}(1,5))$$

	Pivot block	Pivot row	Pivot row	Pivot block	Pivot row	Pivot row
	Pivot column			Pivot column		
	Pivot column			Pivot column		
(a) Phase 1	(b) Phase	2	(0	c) Phase	3

Figure 2: The 3 phases of the blocked FW algorithm in the first round.

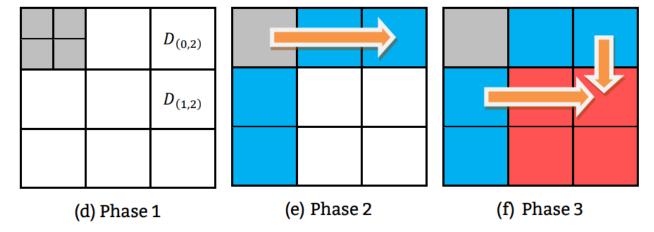


Figure 3: The computations of $D_{(0,2)}^{(2)}$, $D_{(1,2)}^{(2)}$ and their dependencies in the first round.

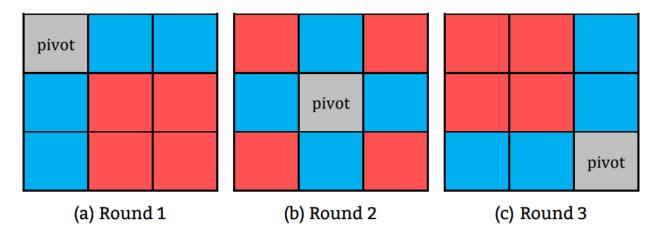


Figure 4: In this particular example where V = 6 and B = 2, we will require $\lceil V/B \rceil = 3$ rounds.

4 Run your programs

• Command line specification

```
# CPU
srun -N1 -n1 -cCPUS ./hw3-1 INPUTFILE OUTPUTFILE
# Single-GPU
srun -N1 -n1 --gres=gpu:1 ./hw3-2 INPUTFILE OUTPUTFILE
# Multi-GPU
srun -N1 -n1 -c2 --gres=gpu:2 ./hw3-3 INPUTFILE OUTPUTFILE
```

- o CPUS: Number of CPUs, specified by TA.
- INPUTFILE: The pathname of the input file. Your program should read the input graph from this file.
- OUTPUTFILE: The pathname of the output file. Your program should output the shortest path distances to this file. CPUS: Number of CPUs, specified by TA.

• Input specification

- The input is a directed graph with non-negative edge distances.
- The input file is a binary file containing 32-bit integers. You can use the int type in C/C++.
- The first two integers are the number of vertices (V) and the number of edges (E).

- Then, there are *E* edges. Each edge consists of 3 integers:
 - 1. source vertex id (src,)
 - 2. destination vertex id (dst_i)
 - 3. $edge\ weight\ (w_{i})$
- The values of vertex indexes & edge indexes start at 0.
- The ranges for the input are:
 - 2≤*V*≤6000 (CPU)
 - $2 \le V \le 40000$ (Single-GPU)
 - $2 \le V \le 60000$ (Multi-GPU)
 - $0 \le E \le V \times (V 1)$
 - $0 \le src_i, dst_i < V$
 - $src_i \neq dst_i$
 - if $src_i = src_j$ then $dst_i \neq dst_j$ (there will not be repeated edges)
 - $0 \le w_i \le 1000$

Here's an example:

offset	type	decimal value	description
0000	32-bit integer	3	# vertices (V)
0004	32-bit integer	6	# edges (E)
0008	32-bit integer	0	src id for edge 0
0012	32-bit integer	1	dst id for edge 0
0016	32-bit integer	3	edge 0's distance
0020	32-bit integer		src id for edge 1
0076	32-bit integer		edge 5's distance

• Output specification

 $\circ\quad$ The output file is also in binary format.

- For an input file with V vertices, you should output an output file containing V^2 integers.
- The first V integers should be the shortest path distances for starting from edge 0: dist(0,0), dist(0,1), dist(0,2), ..., dist(0,V-1); then the following V integers would be the shortest path distances starting from edge 1: dist(1,0), dist(1,1), dist(1,2), ..., dist(1,V-1); and so on, totaling V^2 integers.
- o dist(i, j) = 0 where i = j.
- o If there is no valid path between $i \rightarrow j$, please output with: $dist(i,j) = 2^{30} 1 = 1073741823$.

Example output file:

offset	type	decimal value	description
0000	32-bit integer	0	dist(0, 0)
0004	32-bit integer	?	dist(0, 1)
0008	32-bit integer	?	dist(0, 2)
$4V^2-8$	32-bit integer	?	dist(V-1, V-2)
$4V^2-4$	32-bit integer	0	dist(V-1, V-1)

5 REPORT

Answer the questions below. You are recommended to use the same section numbering as they are listed.

1. Implementation

- a. Which algorithm do you choose in hw3-1?
- b. How do you divide your data in hw3-2, hw3-3?
- c. What's your configuration in hw3-2, hw3-3? And why? (e.g. blocking factor, #blocks, #threads)
- d. How do you implement the communication in hw3-3?

e. Briefly describe your implementations in diagrams, figures or sentences.

2. Profiling Results (hw3-2)

Provide the profiling results of following metrics on the biggest kernel of your program using NVIDIA profiling tools. NVIDIA Profiler Guide.

- occupancy
- o sm efficiency
- shared memory load/store throughput
- o global load/store throughput

3. Experiment & Analysis

a. System Spec

If you didn't use our hades server or NCHC container for the experiments, please show the CPU, RAM, disk of the system.

b. Blocking Factor (hw3-2)

Observe what happened with different blocking factors, and plot the trend in terms of Integer GOPS and global/shared memory bandwidth. (You can get the information from profiling tools or manual) (You might want to check nvprof and Metrics Reference)

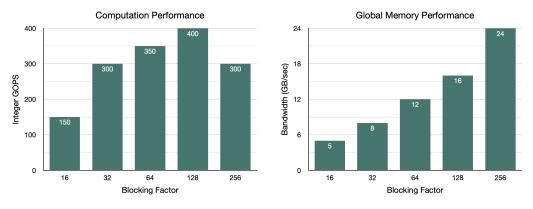


Figure 5: Example chart of performance and global memory bandwidth trend w.r.t. blocking factor

Note:

To run nvprof on hades with flags like --metrics, please run on the slurm partition prof. e.g. srun -p prof -N1 -n1 --gres=gpu:1 nvprof --metrics gld_throughput./hw3-2 /home/pp22/share/hw3-2/cases/c01.1 c01.1.out

Please note that nvprof is only available for the GTX 1080. You need to use nsight-compute (command: ncu) on NCHC for RTX 3070 GPU.

For more details, please refer to the **Nyprof Transition Guide**.

c. Optimization (hw3-2)

Any optimizations after you port the algorithm on GPU, describe them with sentences and charts. Here are some techniques you can implement:

- Coalesced memory access
- Shared memory
- Handle bank conflict
- CUDA 2D alignment
- Occupancy optimization
- Large blocking factor
- Reduce communication

Streaming

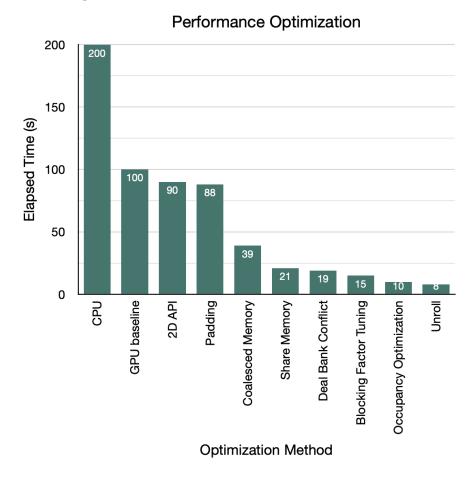


Figure 6: Example chart of performance optimization¶

d. Weak scalability (hw3-3)

Observe weak scalability of the multi-GPU implementations

e. Time Distribution (hw3-2)

Analyze the time spent in:

- computing
- communication
- memory copy (H2D, D2H)
- I/O of your program w.r.t. input size.
- f. Others

Additional charts with explanation and studies. The more, the better.

4. Experience & conclusion

- a. What have you learned from this homework?
- b. Feedback (optional)

6 GRADING

1. **[40%] Correctness**

An unknown number of test cases will be used to test your implementation.

- CPU (15%)
 - You get 15 points if you passed all the test cases, max(0, 15 k) points if there are k failed test cases.
 - Time limit for each case: (960 seconds) / (number of CPU cores).
- Single-GPU (15%)
 - You get 15 points if you passed all the test cases, max(0, 15 k) points if there are k failed test cases.
- Multi-GPU (10%)
 - There are 5 test cases, each case worth 2 points. When judging, we will use hidden test cases similar to /home/pp22/share/hw3-3/cases/c[01-05].1.

2. **[20%] Performance (**Single-GPU version only**)**

- We have 30 performance test cases named pXXk1. $XX = 11 \sim 40$
- Each test case has a 30s time limit.
- Basically, larger XX test cases require longer time.
- You will get XX points if you pass test cases p11k1 \sim pXXk1. Otherwise, zero.
- For example, if you pass test cases p11k1 ~ p23k1, p25k1 and fail other test cases. You will get 23 points.
- If XX > 20, then extra points will still count. (but the max point of this homework is still 100)

3. **[20%] Demo**

• A demo session will be held remotely. You'll be asked questions about the homework.

4. [20%] Report

• Grading is based on your evaluation, discussion and writing. If you want to get more points, design or conduct more experiments to analyze your implementation.

7 Submission

Upload the files below to eeclass. (DO NOT COMPRESS THEM)

- hw3-1.cc
- hw3-2.cu
- hw3-3.cu
- Makefile (optional)
- hw3_{student_ID}.pdf

8 FINAL NOTES

- Type hw3-1-judge(apollo), hw3-2-judge, hw3-3-judge to run the test cases. NCHC also supports the judge commands.
- Scoreboard:
 - https://apollo.cs.nthu.edu.tw/pp22/scoreboard/hw3-1/
 - o https://apollo.cs.nthu.edu.tw/pp22/scoreboard/hw3-2/
 - o https://apollo.cs.nthu.edu.tw/pp22/scoreboard/hw3-3/
- Use the hw3-cat command to view the binary test cases in text format.
- Resources are provided under /home/pp22/share/hw3-*/ on Hades and /tmp/dataset-nthu-pp22/pp22/share/hw3-*/ on NCHC:
 - o Makefile example Makefile
 - o cases/ sample test cases
- We suggest writing your code on NCHC, because it has more resources.
- However, the final result is based on Hades (GTX 1080), so make sure to fine tune your performance on Hades.
- Contact TA via pp@lsalab.cs.nthu.edu.tw or eeclass if you find any problems with the homework specification, judge scripts, example source code or the test cases.
- You are allowed to discuss and exchange ideas with others, but you are required to write the code on your own. You'll get **0 points** if we found you cheating.