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Linear Programming

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1. INTRODUCTION

For this assignment, we are staying on the topic of optimization. This time, however, we will be looking to optimize linear equations which are composed of linear constraints. These linear constraints can be categorized as either equality or inequality constraints. Another portion of linear equations is the objective function. The objective function defines the quantity that is to be maximized or minimized. Equation 1 showcases this below:

$$\xi = c_1x_1 + c_2x_2 + ... + c_nx_n = min \tag{1}$$

Equation 1 is the objective function in which c is the constant, x is a variable, and ζ is the quantity that is to be minimized or maximized. Through the linear programming process, the values of the linear equation which are responsible for optimization of the objective function are found. Equation 2 is also utilized.

$$x_1 \geq 0, x_2 \geq, ..., x_n \geq 0 \tag{2}$$

Equation 2 ensures that the objective function is never negative, meaning it will always reside in the first quadrant of the coordinate plane. From the objective function, the equality and inequality constraint equations are presented below, respectfully:

$$a_{i1}x_1 + ... + a_{in}x_n = b_i \tag{3}$$

$$a_{i1}x_1 + ... + a_{in}x_n \leq b_i \tag{4}$$

The equality constraint indicates that the objective function must lie on the equality constraint line. The inequality constraint shapes/restricts the simplex. Any vector that satisfies these constraints and is positive, is deemed a "feasible vector". An example of this is shown below, (Equation 3):

$$vector \begin{bmatrix} x_1 \\ x_n \end{bmatrix} \tag{5}$$

The simplex resides within this region of the feasible vector. The feasible vector that minimizes the objective function is deemed the "objective feasible vector". If an objective feasible vector exists, there also exists a feasible vector that is optimal.

2. LITTLE ENERGY

A great way to utilize and apply linear programming is when analyzing dietary nutrition. Within this realm, the general goal is to get into a physical healthy state and then be able to maintain it. To be able to achieve this goal, a strict dietary plan must be created and religiously followed (unfortunately). Linear programming can be introduced into this idea as the food intake would be linear, due to the amount of nutrients and proteins needed on a daily basis. As a result, this can be considered as a constraint. Another possible constraint that can be considered are the properties of the body of the person who is on the diet. Properties such as weight, sex, and age are just a few of the properties that can be introduced as parameters. Regardless of the lifestyle, constraints will be present in one form or another.

For this section, we will be selecting 30 food products and minimizing the calorie amount. We will be doing so with a couple of constraints in mind. The averages of the following will be considered and taken into account when minimizing calories: fats, carbs, proteins, calcium, and iron. These averages can be observed in *Table 1*. Additionally, we will also place a constraint on the daily food intake, placing a cap at 2 kg. This constraint will also be in *Table 1*.

Table 1. Constraints Utilized for 30 Foods

Parameter /Constraint	**Min. Value**	**Max. Value**
Calcium (mg)	980	1020
Fats (g)	69	71
Carbohydrates (g)	300	320
Iron (mg)	14	20
Protein (g)	45	55
Weight (g)	0	2001

Utilizing the parameters/constraints from *Table 1*, the results found on *Table 2* were determined. While the problem asks to choose 30 foods the author frequently eats, 30 random foods were selected instead. Please do no think the author eats any of these foods.

Table 2. Resulting Foods and Servings

Category	**Food**	**Servings**
Fruit	Plums	1
Vegetables	Dandelions	1
Nuts & Seeds	Pine nuts	2
Baked Goods	Pretzels	3
Baked Goods	Melba Toast	1
Sweets	Marshmallows	1
Beverages	Grape	5
Mixed Dish	Enchiladas	2
Miscellaneous	Black Pepper	9
Miscellaneous	Chili Powder	1

For *Table 1*, both the minimum and maximum values of the constraints revolved around the daily intake average. Because these results satisfy the constraints applied and minimizes the amount of calories, these 10 results are deemed to be the feasible objective vector values.

3. LITTLE FATS

For this section, we will be building off of the idea from the first section. Now, the question becomes what will become of the results if the calorie constraint shifts to 2000 kcal at the very least, and now focus on minimizing the amount of fats instead of energy? To accommodate these changes, fats will now become the objective function. *Table 3* introduces the sort-of-new list of constraints that will be utilized and applied to determine the resulting foods.

Table 3. New Constraints With Calories

Parameter/Constraint	**Min. Value**	**Max. Value**
Calcium (mg)	980	1020
Calories (kcal)	1985	2010
Carbohydrates (g)	300	320
Iron (mg)	14	20
Proteins (g)	45	55
Weight (g)	0	2001

As previously mentioned, calories has now become a constraint and fats is the objective function and no longer present as it is being minimized.

Table 4 depicts the resulting foods and their servings that satisfy the new list of constraints.

Table 4. Resulting Foods and Serving Sizes

Category	**Food**	**Servings**
Fruits	Cooked Plantains	5
Nuts & Seeds	English Walnuts	1
Fish/Shellfish	Clams	1
Vegetables	Dandelion (Raw)	1
Vegetables	Broccoli (Raw)	3
Vegetables	Carrots (Raw)	3

After making changes to the constraints and the objective function, the number of feasible objective vector values has decreased to 6 from what used to be 10.

4. VARIATION RESTORATION

For the two previous sections, we've been analyzing and optimizing the diet with only 5 constraints in mind. As a result, the resulting foods and servings may not be entirely too realistic of a well-balanced diet. Some servings may be too large whereas others may be too small. A possible solution would be to increase the number of constraints at play so to level the playing field. In addition to the usual 5 constraints we have been utilizing, an additional 4 more constraints have been introduced. *Table 5* lists the updated version of constraints that will be utilized for this section.

Table 5. Longer List of Constraints

Parameter/Constraint	**Min. Value**	**Max. Value**
Calcium (mg)	1000	1500
Calories (kcal)	1985	2010
Carbohydrates (g)	440	655
Iron (mg)	40	45
Protein (g)	100	190
Sodium (mg)	1200	1400
Potassium (g)	3300	3580
Cholesterol (mg)	250	320
Fiber (g)	25	40
Weight (g)	0	2001

In accordance to this new updated list of constraints, *Table 6* displays the new updated list of resulting foods and their respective serving sizes.

Table 6. Updated List of Resulting Foods

Category	**Food**	**Servings**
Fruit	Plantain	3.60
Nuts & Seeds	English Walnuts	2.24
Egg	Raw Whole W No Shell	1.49
Poultry	Roasted Chicken	22.25
Fish/Shellfish	Pacific Oyster	1.40
Vegetables	Mashed Potatoes	5.22
Vegetables	Dandelion (Raw)	6.26

What makes *Table 6* more interesting than the previous tables of results is that this contains decimals. The presence of decimal values is indicative of a lack of an appropriate number of constraints or the constraint value themselves may not be optimal for linear programming. As a result, it can be determined that this list of resulting foods is infeasible. Although, it is not a complete failure as the serving sizes of the foods did become slightly more varied. Only *roasted chicken* resulted in a very high serving size. The range of the minimum and maximum values must be adjusted further to fix this and the other serving sizes as well.

5. COST MINIMIZATION

To further implement a more realistic scenario, we will be taking a look at the prices of the resulting foods. However, the price will now be the objective function and will be minimized. Making the price the objective function is sure to affect the resulting foods. The price is being considered based on price per serving. Bearing this in mind, we expect for the overall number of resulting food to decrease, as well as the overall healthiness and variety of the foods. The constraints utilized in this section of the assignment is the same as *Table 5*, with only the objective function changing to price. *Table 7* includes the updated list of resulting foods after this change.

Table 7. Updated List of Foods by Cost

Category	**Food**	**Servings**
Fruit	Plantain	0.30
Fruit	Banana	2.05
Nuts & Seeds	English Walnuts	0.85
Vegetables	Broccoli (Raw)	0.10
Vegetables	Mashed Potatoes	6.40
Vegetables	Spinach (Raw)	1.84

Analyzing *Table 7*, there is one glaring similarity which should have been expected as well. Due to the usage of the previous table (*Table 5*), the resulting serving sizes of the foods are decimal values. Another thing to note is the actual food themselves. The overall number of food did go down, as expected due to the optimization of cost. However, what is surprising is the overall healthiness of the food themselves. For the most part, the list is very much healthy.

6. MAKE AUTHOR HEALTHY

In this section, there is more freedom available as the 30 food restriction is lifted, and all types of food are now available for consideration. With more freedom, comes a cost. This section will also be implementing more constraints than the previous sections. The diet being implemented in this case will be more realistic as more nutrients are being considered. *Table 8* will list the updated list of constraints being utilized along with the average daily recommended intake values (minimum and maximum values).

Table 8. Updated List of Personal Diet

Parameters/Constraints	Min. Value	Max. Value
Calcium (mg)	1000	1500
Calories (kcal)	1985	2010
Carbohydrates (g)	440	655
Iron (mg)	40	45
Protein (g)	100	190
Sodium (mg)	1200	1400
Potassium (g)	3300	3580
Cholesterol (mg)	250	320
Fiber (g)	25	40
Magnesium (g)	50	380
Zinc (g)	13	20
Riboflavin (g)	1	5
Weight (g)	0	2001

With this list containing our new constraints, the new list of resulting foods and their respective serving sizes can be found on *Table 9*.

Table 9. Resulting Foods and Serving Sizes

Category	Food	Servings
Grain Products	Trix Cereal	2
Grain Products	Cooked Spaghetti	6
Baked Goods	Rye Wafers	2
Baked Goods	Mixed Grain Toasted	1
Poultry	Roasted Whole Turkey	2
Vegetables	Mushrooms (Raw)	1
Vegetables	Dandelion	3
Vegetables	Black Pepper	16

From this resulting list, we can already notice the disappearance of the decimal values, indicating that this list is feasible. Therefore, this list can be deemed the objective feasible vector values. Overall, the number of foods appears to be adequate and the healthiness of the list itself seems to be sufficient enough.

7. CONCLUSION

For this assignment, we stuck with the topic of optimization, but shifted to linear minimization problems and introduced the concept of linear programming. Keeping track of a diet is a great way to apply this process, especially due to the number and variety of constraints that can be placed to optimize a certain aspect. While the overall objective dealt with dietary nutrition, each part of the assignment proposed an addition of new constraints or a shift in the objective function being minimized.