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Pool and Physics

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1. INTRODUCTION

If one is looking for a game that is very rich in math and physics, look no further as billiards (pool) is the answer. In billiards, the objective of the game is to utilize the pool stick to knock the pool balls into the one of any holes that surround the table. There are 16 balls in which 7 are solid colors, 7 have stripes, 1 is the cue ball, and the last one is the black ball. Usually, the first player that makes a ball into one of the holes (excluding the cue and black ball) is responsible for knocking the rest of the balls of the same design. The player does this by using the stick to hit the cue ball into the ball they want to hit. When all of their balls have been made in, the black ball is the last one to be shot in, thus winning the game. *Figure 1* depicts a billiards table along with all of the balls and pool stick.

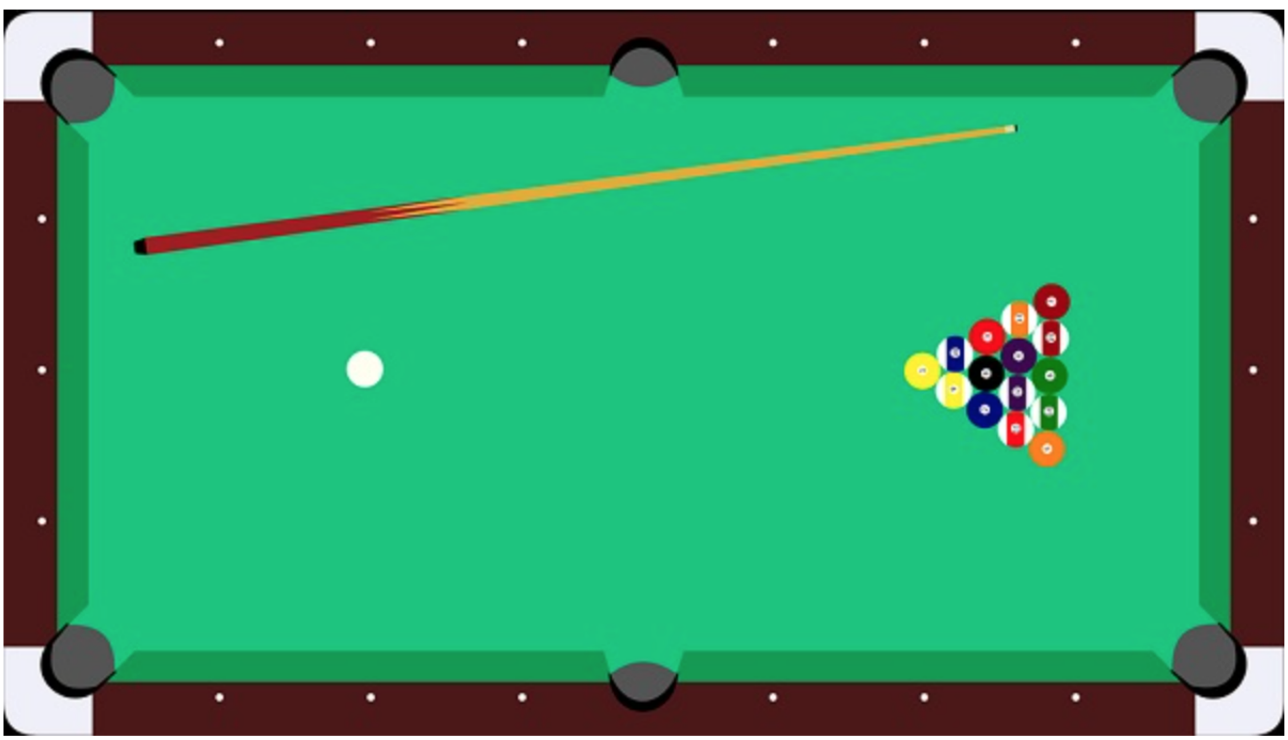

Figure 1. Average looking pool table with accessories.

Due to being able to bounce the balls off the wall of the table, a level of physics is involved. Players can angle their shots with help from the table borders and similarly, they are able to angle their hits on the cue ball itself. Depending on the angle of the strike point, the angular momentum of the ball will dictate its movement. *Figure 2* demonstrates the different movements the cue ball will result in depending on the region the pool stick hits it.

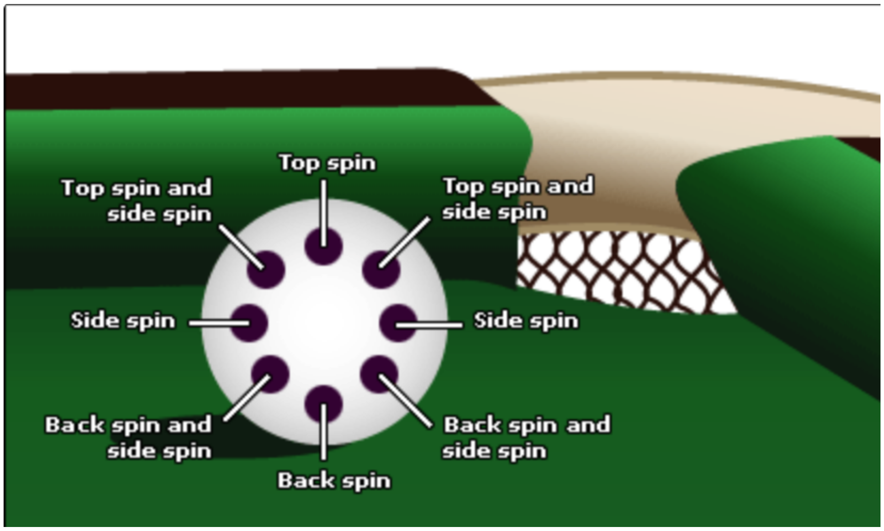

Figure 2. Caption

1.1. *Spin Types*

As mentioned, depending on the region in which the cue ball is struck, its movement can take on many variations. As noted on *Figure 2*, top spin, the center hit, back spin, and side spin are the most notable ones. Though all are different movements, angular momentum is conserved in all three.

1.1.1. *Top Spin*

When the cue ball is struck above its center point, it results in a ”top” spin or a forward one. The higher the cue ball is struck, the more that this spin gets added on. This type of hit is especially useful when looking to bounce the cue ball off the wall as it would take it a greater distance forward after it bounces off it. In addition to its position, it creates a wider angle when hit against the wall.

1.1.2. *Bottom Spin*

Unlike the top spin, striking the cue ball below its center point results in a ”bottom” spin, or a backwards one. Similarly, the lower the cue ball is struck, the more that this spin gets added on.

1.1.3. *Side Spin*

To produce a ”side” spin, the strike point must be either left or right of the center point. Depending on which side it is, it results in two spins that move in the opposite direction. Hitting it on the right side will give the cue ball a counterclockwise spin, while the left side will result in a clockwise spin.

1.2. *Chaos*

Besides spin, another aspect of physics present within billiards is the idea of chaos (which is a tad bit more obvious). This is especially true when a player is ”breaking”, hitting the cue ball towards all the other balls to get the game started. Due to the plethora of directions and positions that each ball can take, they can be considered as chaotic. It is the design of the walls that gives the cue ball its dynamics (aside from friction). To further understand the importance of these walls, lets imagine an elastic collision between it and the cue ball. The equation that describes this collision is Equation (1) below.

$$F(x, y) = 0 \tag{1}$$

The equation that best describes what happens after said collision is Equation (2).

$$F(x + tv_{x,y} + tv_y) = 0 \tag{2}$$

2. FRICTION-LESS SURFACE

Not only is angular momentum conserved, but through the collisions, kinetic energy is conserved as well. Kinetic energy actually gets transferred in from the moment the pool stick hits the cue ball. With the movement from the cue ball, kinetic energy will then be either transferred to another one of the balls or to the wall. Assuming the balls are of the same mass (which is generally true), should they collide, their linear momentum would be conserved. Equation (3) shows this.

$$m_A\vec{V}_{2A} + m_B\vec{V}_{2B} \tag{3}$$

Because of their equal mass, the Equation (4) is the updated one to use.

$$\vec{V}_{1A} = \vec{V}_{2A} + \vec{V}_{2B} \tag{4}$$

$\vec{V}$ is the vector component of both ball A and B. Staying in this situation, Equation (5) demonstrates an elastic collision in which kinetic energy is conserved.

$$\frac{1}{2}m_A(V_{1A})^2 = \frac{1}{2}m_A(V_{2A})^2 + \frac{1}{2}m_b(V_{2B})^2 \tag{5}$$

Equation (6) once again takes on a simpler form due to the equal mass.

$$\frac{1}{2}(V_{1A})^2 = \frac{1}{2}(V_{2A})^2 + \frac{1}{2}(V_{2B})^2 \tag{6}$$

Provided that there is no friction present, angular momentum, linear momentum, and kinetic energy are all conserved.

Another aspect that likened to physics is the fact that when the cue ball is hit, it resembles a rigid body spinning around its center point. Equation (7) describes this further.

$$\sum M_G = I_G\alpha \tag{7}$$

$\sum M_G$ is described as the sum of the moments of the cue ball about its center of mass G, I_G is the moment of inertia of the cue ball again about G, and α is angular acceleration of the cue ball. *Equation 8* further describes this angular acceleration.

$$\alpha = -\frac{\alpha_{Gx}}{r} \tag{8}$$

This is able to occur as a result of no friction being present and thus the ball 'slips' before beginning to roll.

To test these equations, a simulation of a cue ball and 3 other balls was made in which friction was set to 0 and elasticity to 1. All balls were of the same mass. *Figure 3* depicts the resulting plot with kinetic energy and *Figure 4* is the plot of the angular velocity of the cue ball.

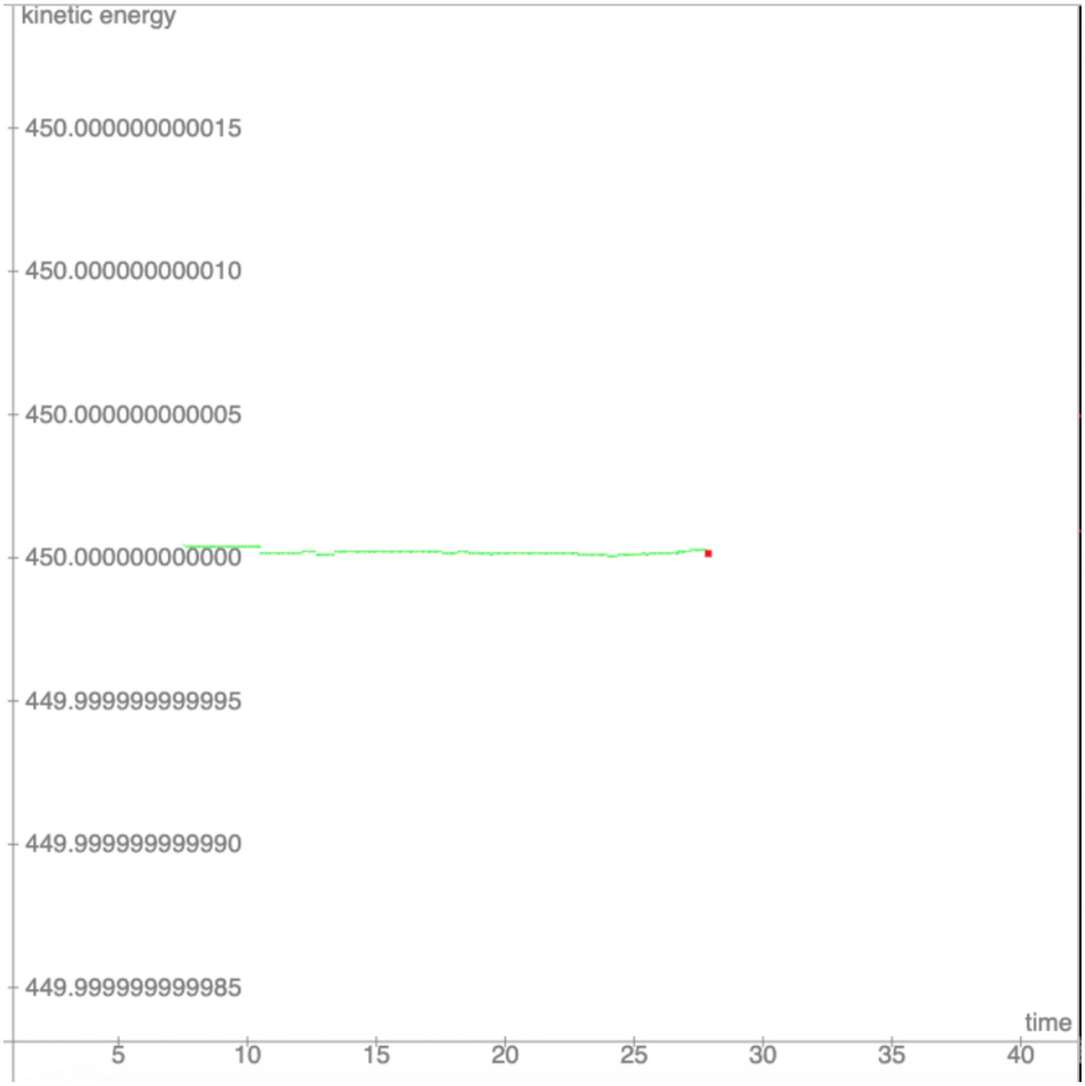

Figure 3. Kinetic Energy vs time plot

As it can be noted, *Figure 3* demonstrates that over time, the kinetic energy of the cue ball is maintained when no friction is present and elasticity is set to 1. The slight bumps within the plot represent the collisions with the balls and the walls.

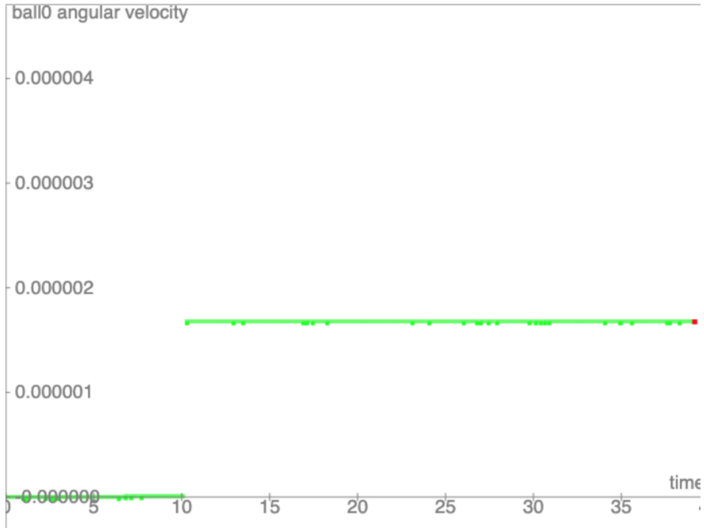

Figure 4. Angular velocity vs time plot

Figure 4 depicts the angular velocity of the cue ball. The sudden jump that occurs at 10 seconds is when it collides with the other 3 balls. Aside from this however, the angular velocity remains constant, with very minuscule fluctuations. Again, this is due to the fact no friction is present.

Figure 5 depicts the plot of the kinetic energy, potential energy, and total energy of the cue ball as a function of time. The blue plot represents total energy, red is potential energy, and green is kinetic energy.

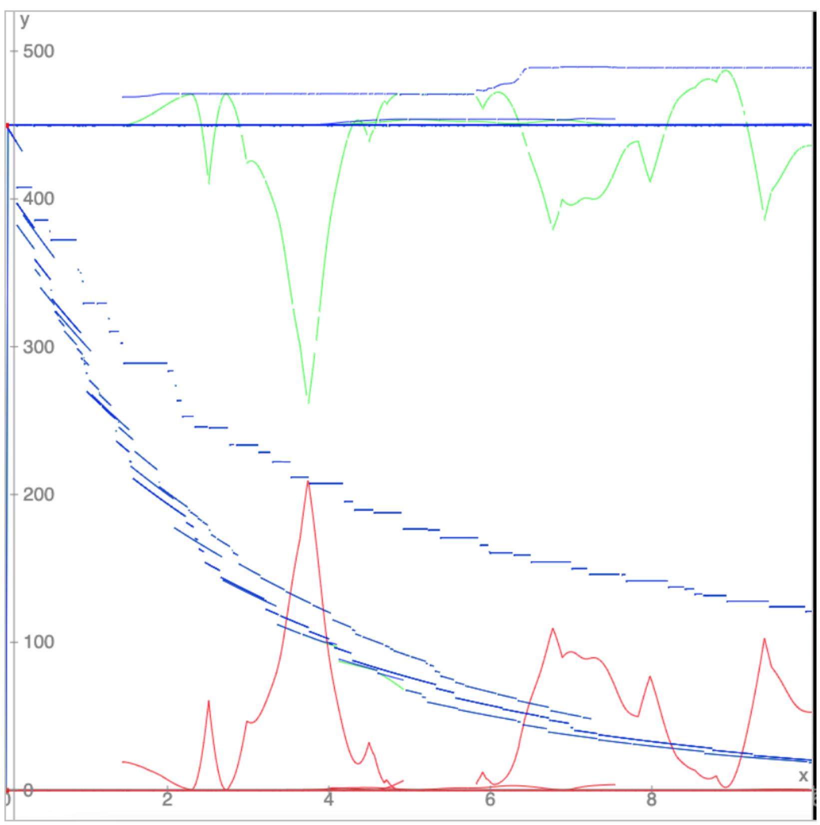

Figure 5. Energies of the cue ball vs time.

Figure 6 is meant to represent the chaotic nature of the ball movement. In this simulation, no friction is present and the large circles are obstacles within the pool table.

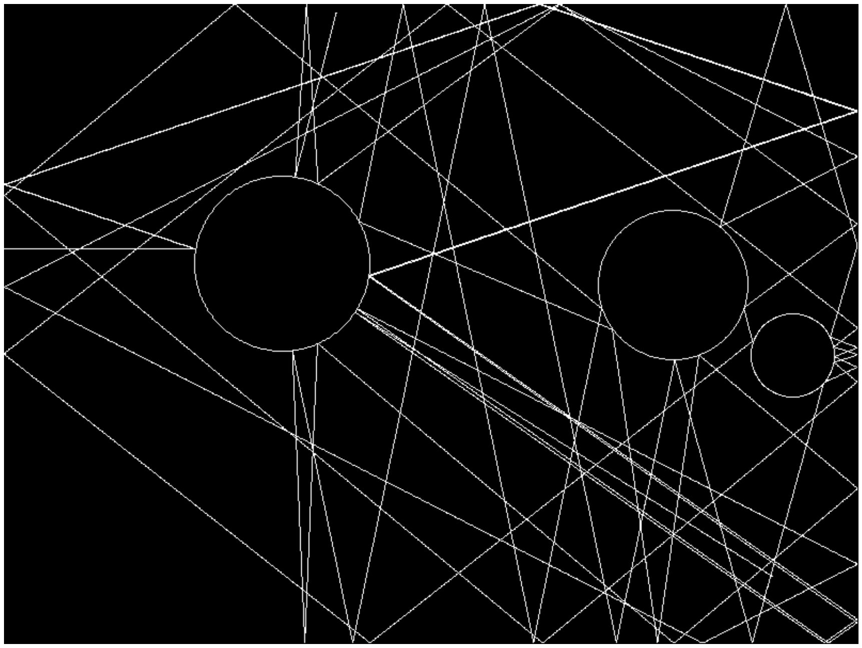

Figure 6. Chaotic behavior without friction

3. SOME FRICTION

Now that we have considered no friction, it is time to make the game more realistic, with the addition of the important parameter itself. Friction is important in this game especially as it prevents a turn from going on forever(that would be annoying). Aside from this, friction is responsible for the ball to stop slipping and begin to roll.

Before friction kicks in, there is only slippage. During this time, Equations (9) and (10) are acting on the cue ball.

$$F = ma \tag{9}$$

$$Fh = I\alpha \tag{10}$$

At this point still, there is no angular velocity is nonexistent and therefore, the equation is:

$$mhv = Iw_0 \tag{11}$$

Once the ball begins to roll, the angular velocity is now:

$$w = w_i + \alpha t \tag{12}$$

The linear velocity of the ball, once rolling has commenced, is:

$$V_G = V_{Gi} + a_{Gx}t \tag{13}$$

$$-V_G = wr \tag{14}$$

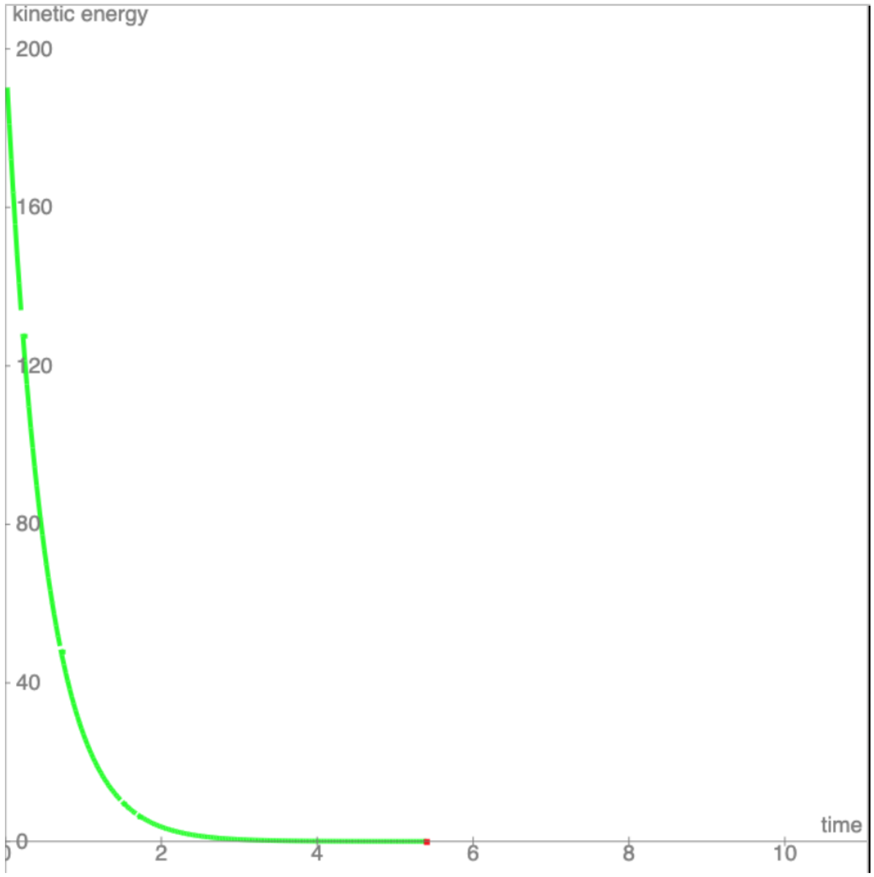

Figure 7. Kinetic Energy vs time plot.

Figure 7 revisits the kinetic energy of the cue ball once friction is added on and given a value of 1. As expected, the cue ball immediately began to slow down until it came to a complete stop.

Table 1 demonstrates the number of wall bounces that a cue ball makes under varying values of the friction coefficient. The same speed was utilized for all 5 trials. The outcome was expected as well.

Table 1. Number of Wall Bounces by Varying Values of Friction

Friction	**Wall Bounces**	**Time (sec)**
0	37	20
0.25	7	20
0.50	3	20
0.75	2	20
1	1	20

4. CONCLUSION

In this assignment, the physics behind the game of billiards is explored. The removal and addition of friction was utilized to provide verification towards the conservation laws mentioned earlier. From different types of spin to the chaos aspect, the brilliant game of billiards truly has it all.