Northern Illinois University

Graphs

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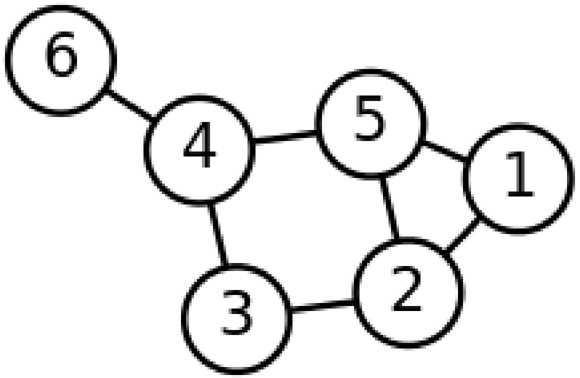
Definitions (1)

- **graph** – a *graph* $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$ consists of a set of *vertices* $\boldsymbol{V}$ and a set of *edges* $\boldsymbol{E}$
- **vertices** – a vertex (plural vertices) or node is the fundamental unit of which graphs are formed*
- **edges** (arcs) – each edge is a pair (v, w) where $v, w \in V$

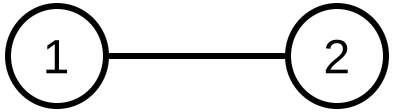


Graph G with 6 vertices (1, 2, 3, 4, 5, 6) and 7 edges ((1, 2), (1,5), (2, 3), (2,5), (3, 4), (4, 5), (4, 6))

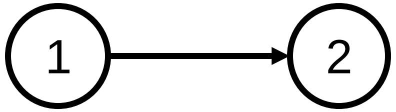
*en.wikipedia.org/wiki/Vertex_(graph_theory)

Definitions (2)

- **directed** (digraphs) – if the pair is ordered then the graph is *directed*



Vertices: 1, 2 Edge: $(1, 2) \equiv (2, 1)$



Vertices: 1, 2 Edge: $(1, 2) \neq (2, 1)$

- **adjacent** – vertices v and w are *adjacent* if they are they are endpoints of the same edge, that is vertex w is adjacent to v if and only if $(v, w) \in E$

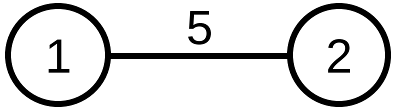
Vertices: 1, 2 are adjacent

Vertices: 1, 2 are **not** adjacent

Definitions (3)

- **weight** (cost) – optional third component to an edge, numerical value assigned as a label to the edge

Vertices: 1, 2 Edge: (1, 2) no weight



Vertices: 1, 2 Edge: (1, 2) weight of 5

- **path** – in a graph is a sequence of vertices $w_1, w_2, w_3, \ldots, w_N$ such that $(w_i, w_{i+1}) \in E$ for $1 \le i \le N$

Vertices: w_1, w_2, w_3, w_4 Edges: (w_1, w_2), (w_2, w_3) with a path from w_1 to w_3, where $N = 3$

Definitions (4)

- **length** – is the number of edges on a path, it is equal to $N - 1$

Vertices: w_1, w_2, w_3, w_4 Edges: (w_1, w_2), (w_2, w_3)
with a path from w_1 to w_3, where $N = 3$ and the length is 2
If a path contains no edges, then its path length is 0

- **loop** – if there is an edge (v, v) from a vertex to itself then this path is known as a *loop*, we will consider graphs in general will be loopless
- **simple path** – is a *path* that all vertices are distinct, except the first and last could be the same

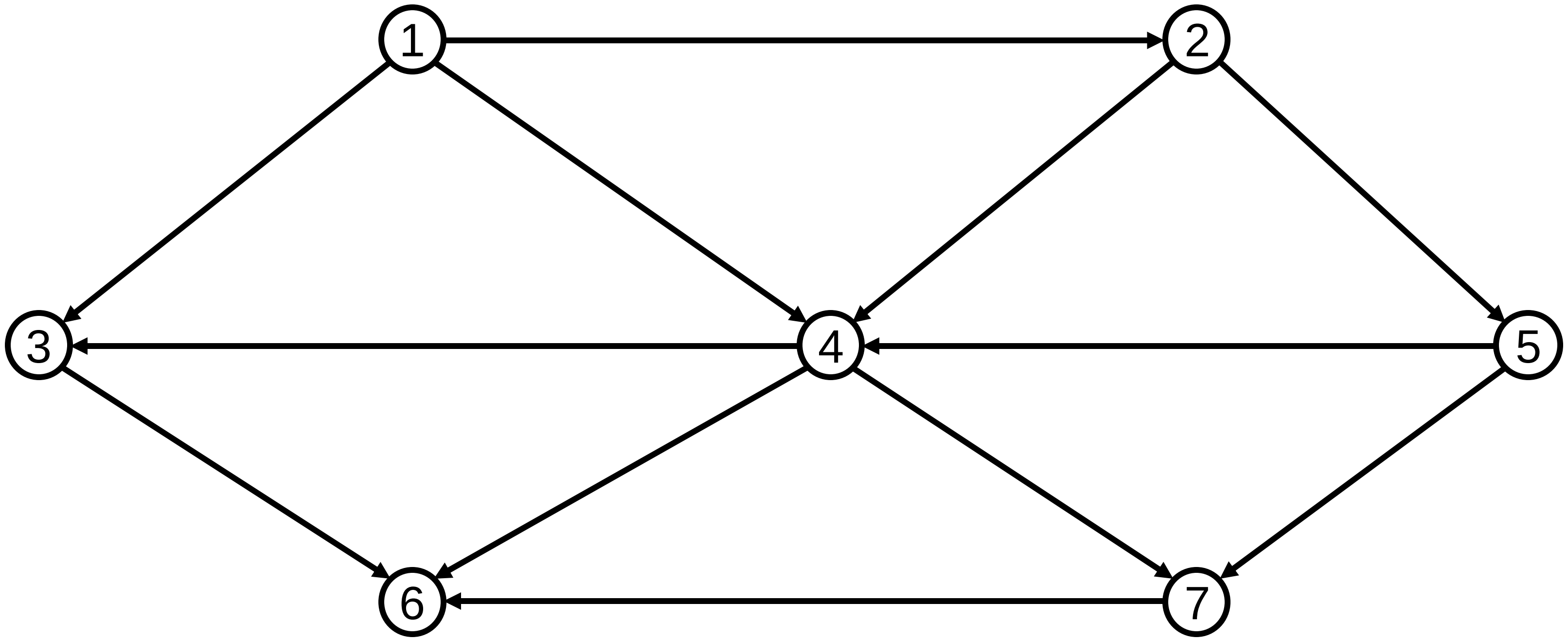
Definitions (5)

- **cycle** – in a directed graph is a path with a length of at least 1, such that $w_1 = w_N$, the *cycle* is simple if the path is simple; in an undirected graph the edges must be distinct
- **acyclic** (DAG) – is a directed graph with no cycles
- **connected** – in an undirected graph, the graph is connected if there is a path from every vertex to every other vertex
- **strongly connected** – a **connected** directed graph is known as a *strongly connected* graph
- **weakly connected** – a graph is *weakly connected* if the directed graph is **connected** when direction of the edges is ignored
- **complete** – is a graph where there is an edge between every pair of vertices
- **Indegree** – the number of incoming edges in directed graph
- **Outdegree** – the number of outgoing edges in directed graph

Topological Sort

- Formally, we use our definition of **indegree** of a vertex v as the number of edges (u, v). Compute the indegree of all vertices in the graph and keep in adjacency list to generate a topological order

1	0
2	1
3	2
4	3
5	1
6	3
7	2



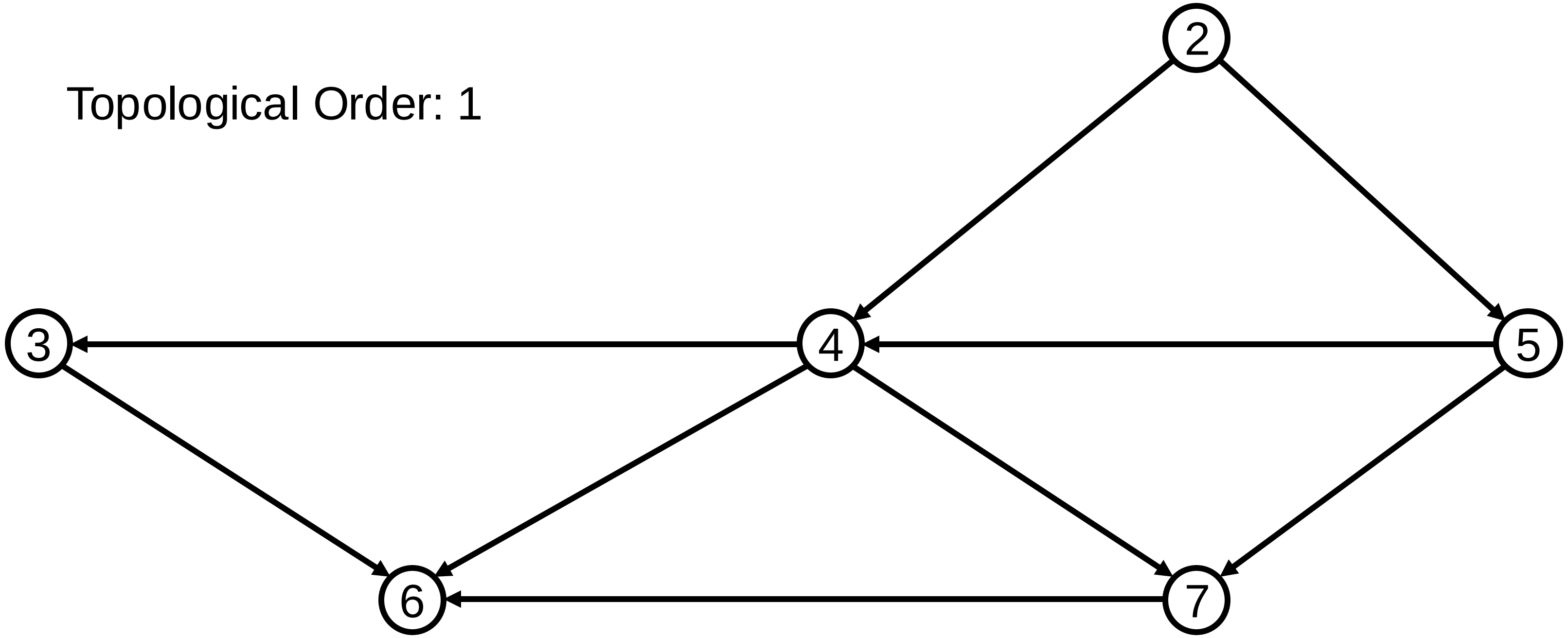
Pick one with Zero (0)

Topological Sort

- Formally, we use our definition of **indegree** of a vertex v as the number of edges (u, v). Compute the indegree of all vertices in the graph and keep in adjacency list to generate a topological order

1	0	X
2	1	0
3	2	1
4	3	2
5	1	1
6	3	3
7	2	2

Topological Order: 1



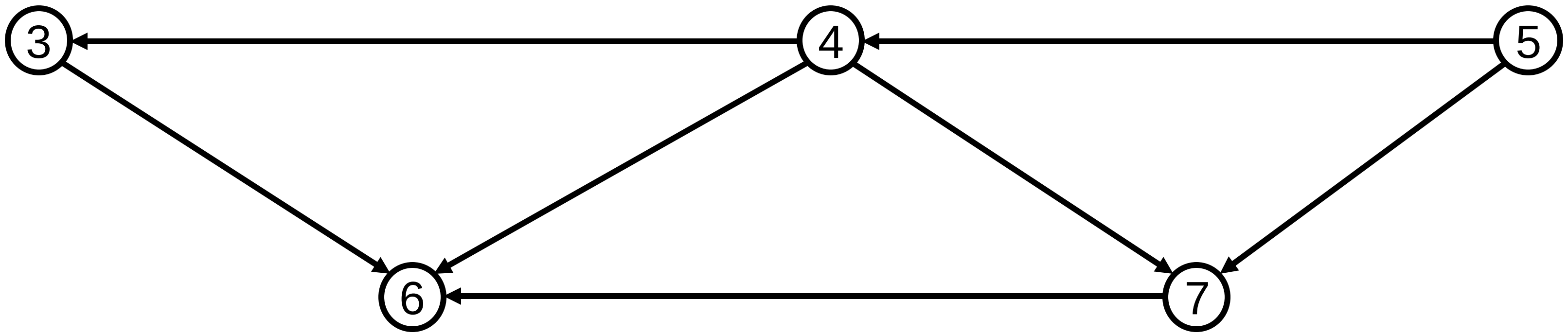
Pick one with Zero (0)

Topological Sort

- Formally, we use our definition of **indegree** of a vertex v as the number of edges (u, v). Compute the indegree of all vertices in the graph and keep in adjacency list to generate a topological order

1	0	X	
2	1	0	X
3	2	1	1
4	3	2	1
5	1	1	0
6	3	3	3
7	2	2	2

Topological Order: 1, 2



Pick one with Zero (0)

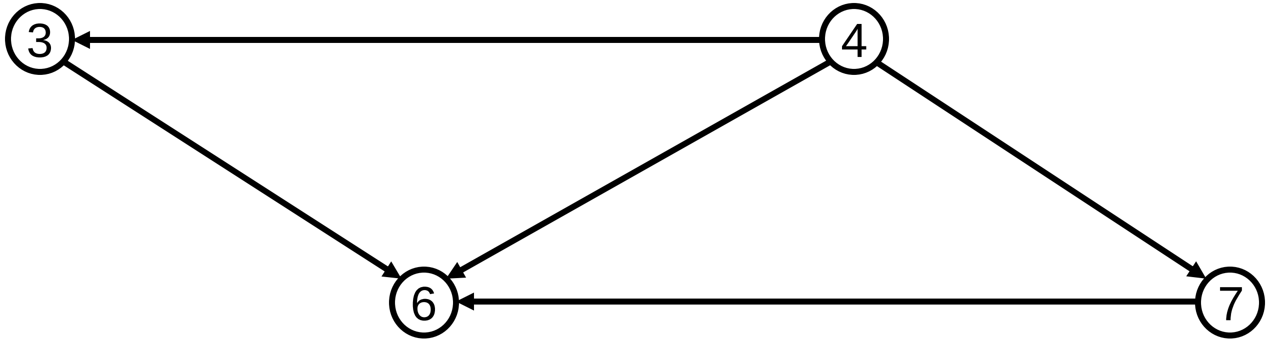
Topological Sort

- Formally, we use our definition of **indegree** of a vertex v as the number of edges (u, v). Compute the indegree of all vertices in the graph and keep in adjacency list to generate a topological order

1	0	X		
2	1	0	X	
3	2	1	1	1
4	3	2	1	0
5	1	1	0	X
6	3	3	3	3
7	2	2	2	1

Pick one with Zero (0)

Topological Order: 1, 2, 5

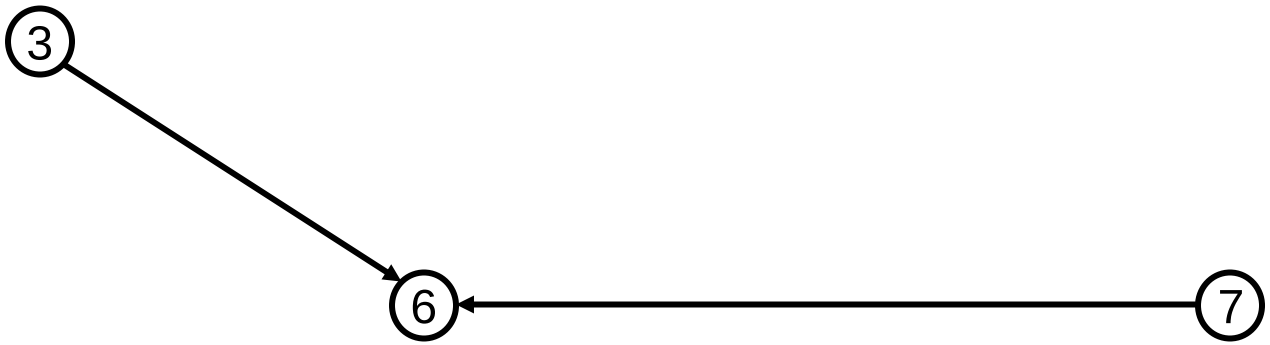


Topological Sort

- Formally, we use our definition of **indegree** of a vertex v as the number of edges (u, v). Compute the indegree of all vertices in the graph and keep in adjacency list to generate a topological order

1	0	X			
2	1	0	X		
3	2	1	1	1	0
4	3	2	1	0	X
5	1	1	0	X	
6	3	3	3	3	2
7	2	2	2	1	0

Topological Order: 1, 2, 5, 4



Pick one with Zero (0)

Topological Sort

- Formally, we use our definition of **indegree** of a vertex v as the number of edges (u, v). Compute the indegree of all vertices in the graph and keep in adjacency list to generate a topological order

1	0	X				
2	1	0	X			
3	2	1	1	1	0	X
4	3	2	1	0	X	
5	1	1	0	X		
6	3	3	3	3	2	0
7	2	2	2	1	0	X

Topological Order: 1, 2, 5, 4, {3, 7}

6

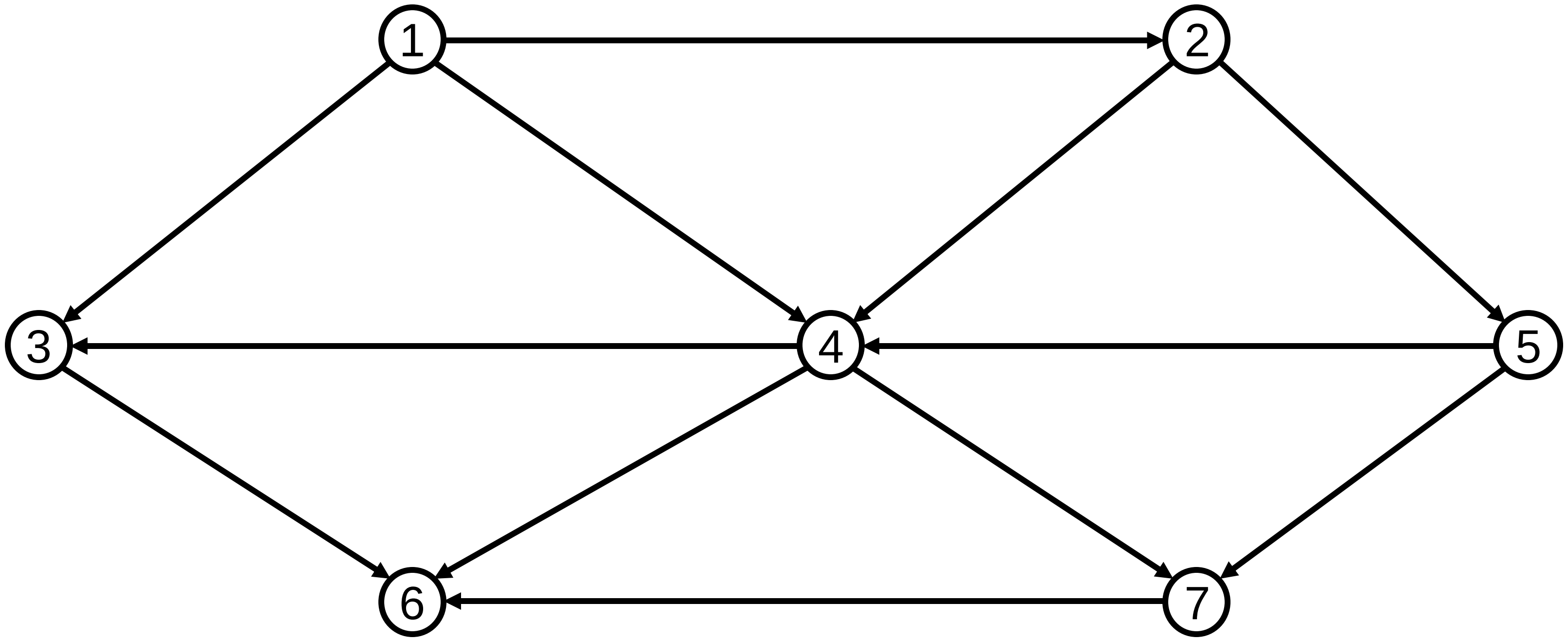
Pick one with Zero (0)

Topological Sort

- Formally, we use our definition of **indegree** of a vertex v as the number of edges (u, v). Compute the indegree of all vertices in the graph and keep in adjacency list to generate a topological order

1	0	X					
2	1	0	X				
3	2	1	1	1	0	X	
4	3	2	1	0	X		
5	1	1	0	X			
6	3	3	3	3	2	0	X
7	2	2	2	1	0	X	

Pick one with Zero (0)



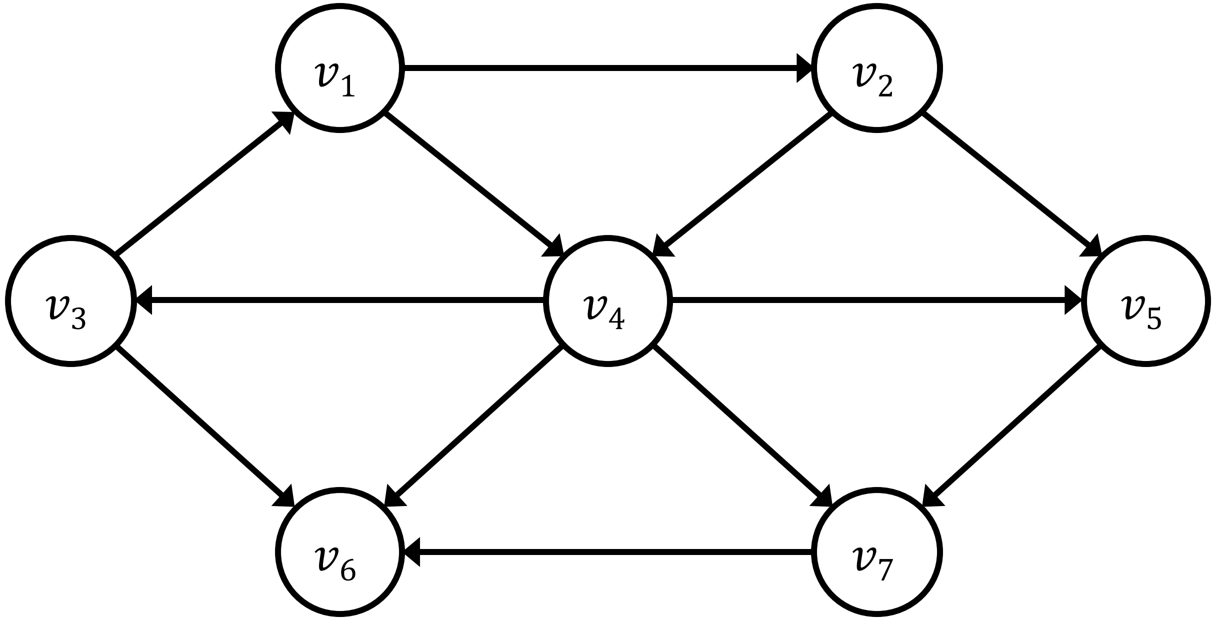
Topological Order: 1, 2, 5, 4, {3, 7}, 6

Shortest Path Algorithms

- The input is a weighted graph
- Associated with each edge (v_i, v_j) is a cost $c_{i,j}$ to traverse the edge
- The cost of a path $v_1, v_2, v_3, \dots v_N$ is $\sum_{i=1}^{N-1} c_{i,i+1}$, this is called the **weighted path length**, the **unweighted path length** is the number of edges on the path, $N - 1$

Unweighted Shortest Path Problem

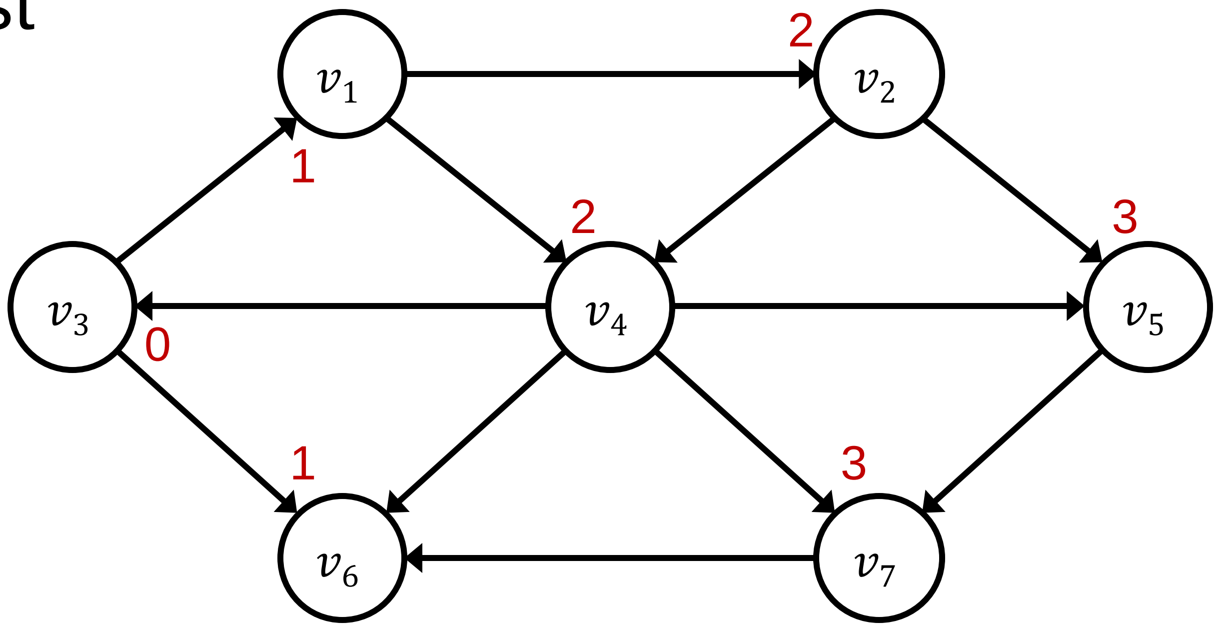
- Given an unweighted graph G, given some vertex s as an input, find the shortest path from s to all other vertices. Given its an unweighted graph we are only interested in the number of edges in the path



- Think of this as a special case of the weighted problem where all the weights are 1

Unweighted Shortest Path Problem

- This should look familiar it is a **breadth-first search.** It operates by processing all the vertices in layers
- The vertices closest to the start are evaluated first, then the next layer and so on until the most distance vertices are evaluated last

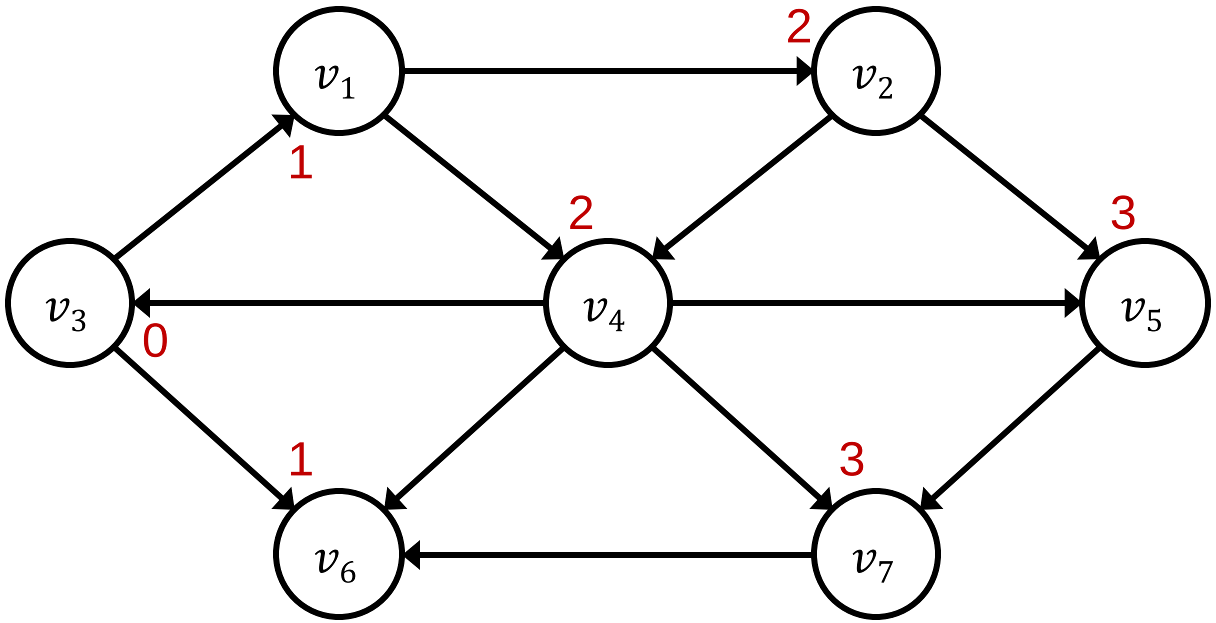


Unweighted Shortest Path Problem

- Construct a table, where d_v is the distance from s (initially unknown and set to ∞, except s itself set to 0)

Q:
v_3

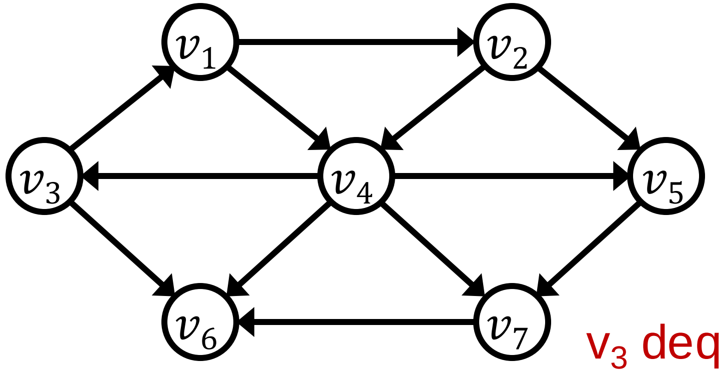
V	known	d_v	p_v
v_1	F	∞	0
v_2	F	∞	0
v_3	F	0	0
v_4	F	∞	0
v_5	F	∞	0
v_6	F	∞	0
v_7	F	∞	0



known is initially set to F, after being visited it will be set to T, $\mathbf{d_v}$ is distance and $\mathbf{p_v}$ is a bookkeeping variable

Unweighted Shortest Path Problem

- Construct a table, where d_v is the distance from s (initially unknown and set to ∞, except s itself set to 0)



Initial State

Q: v_3

V	known	d_v	p_v
v_1	F	∞	0
v_2	F	∞	0
v_3	F	0	0
v_4	F	∞	0
v_5	F	∞	0
v_6	F	∞	0
v_7	F	∞	0

v_3 dequeued

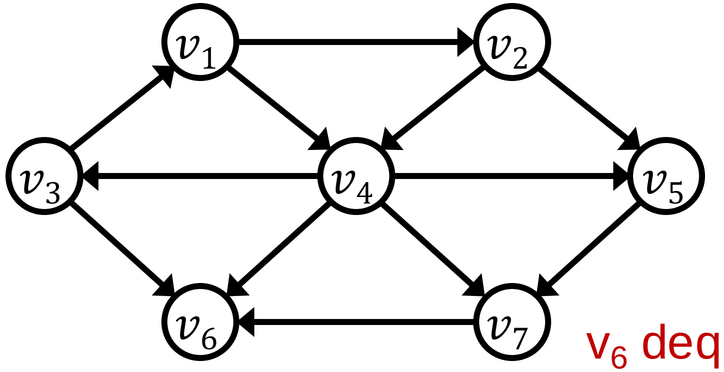
Q: v_1 v_6

V	known	d_v	p_v
v_1	F	1	v_3
v_2	F	∞	0
v_3	T	0	0
v_4	F	∞	0
v_5	F	∞	0
v_6	F	1	v_3
v_7	F	∞	0

known is initially set to F, after being visited it will be set to T, $\mathbf{d_v}$ is distance and $\mathbf{p_v}$ is a bookkeeping variable

Unweighted Shortest Path Problem

- Construct a table, where d_v is the distance from s (initially unknown and set to ∞, except s itself set to 0)



v_1 dequeued

Q: v_6 v_2 v_4

V	known	d_v	p_v
v_1	T	1	v_3
v_2	F	2	v_1
v_3	T	0	0
v_4	F	2	v_1
v_5	F	∞	0
v_6	F	1	v_3
v_7	F	∞	0

v_6 dequeued

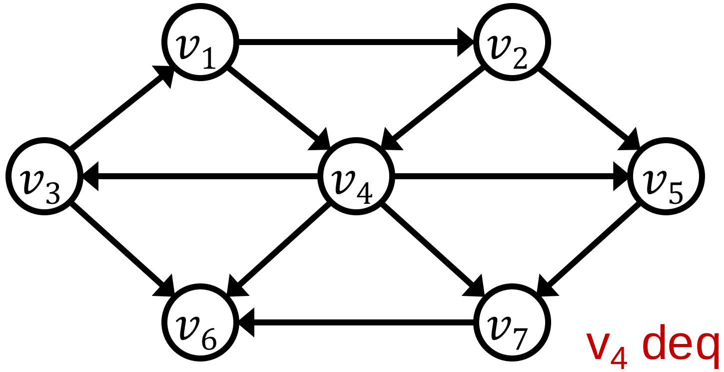
Q: v_2 v_4

V	known	d_v	p_v
v_1	T	1	v_3
v_2	F	2	v_1
v_3	T	0	0
v_4	F	2	v_1
v_5	F	∞	0
v_6	T	1	v_3
v_7	F	∞	0

known is initially set to F, after being visited it will be set to T, $\mathbf{d_v}$ is distance and $\mathbf{p_v}$ is a bookkeeping variable

Unweighted Shortest Path Problem

- Construct a table, where d_v is the distance from s (initially unknown and set to ∞, except s itself set to 0)



v_2 dequeued

Q: v_4 v_5

V	known	d_v	p_v
v_1	T	1	v_3
v_2	T	2	v_1
v_3	T	0	0
v_4	F	2	v_1
v_5	F	3	v_2
v_6	T	1	v_3
v_7	F	∞	0

v_4 dequeued

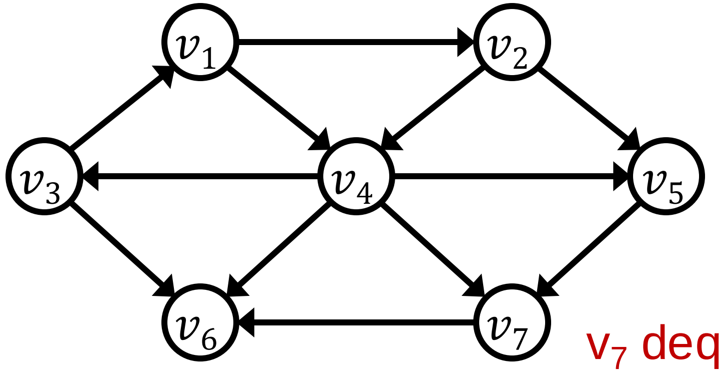
Q: v_5 v_7

V	known	d_v	p_v
v_1	T	1	v_3
v_2	T	2	v_1
v_3	T	0	0
v_4	T	2	v_1
v_5	F	3	v_2
v_6	T	1	v_3
v_7	F	3	v_4

known is initially set to F, after being visited it will be set to T, **d_v** is distance and **p_v** is a bookkeeping variable

Unweighted Shortest Path Problem

- Construct a table, where d_v is the distance from s (initially unknown and set to ∞, except s itself set to 0)



v_5 dequeued

Q: v_7

V	known	d_v	p_v
v_1	T	1	v_3
v_2	T	2	v_1
v_3	T	0	0
v_4	T	2	v_1
v_5	T	3	v_2
v_6	T	1	v_3
v_7	F	3	v_4

v_7 dequeued

Q:

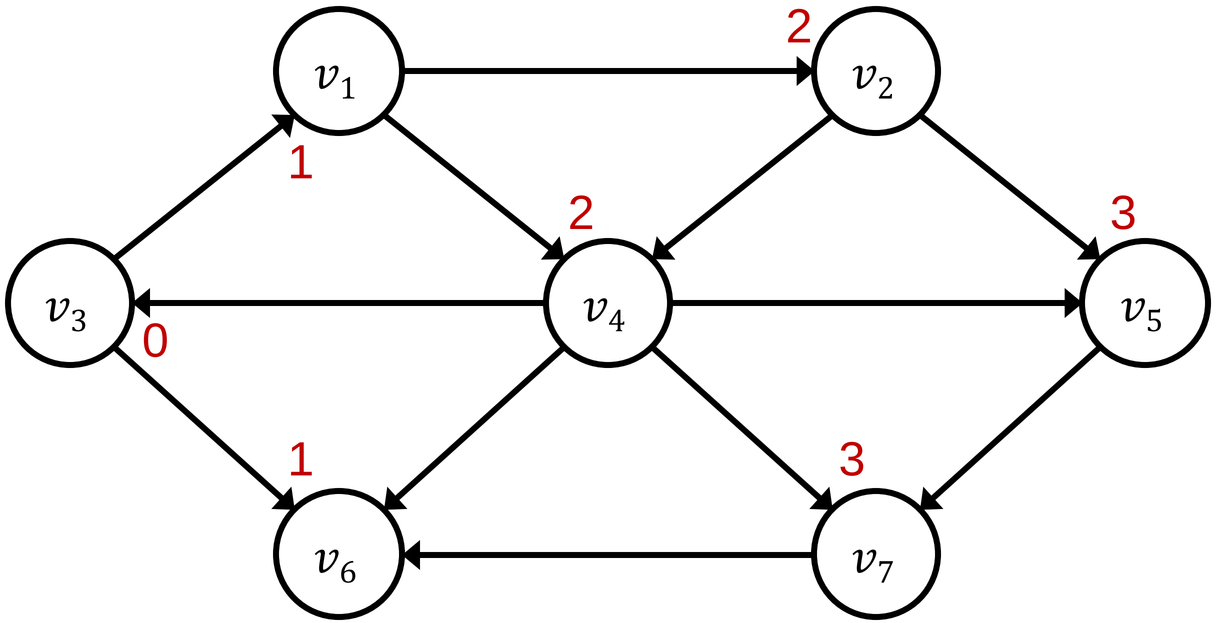
V	known	d_v	p_v
v_1	T	1	v_3
v_2	T	2	v_1
v_3	T	0	0
v_4	T	2	v_1
v_5	T	3	v_2
v_6	T	1	v_3
v_7	T	3	v_4

known is initially set to F, after being visited it will be set to T, $\mathbf{d_v}$ is distance and $\mathbf{p_v}$ is a bookkeeping variable

Unweighted Shortest Path Problem

- Construct a table, where d_v is the distance from s (initially unknown and set to ∞, except s itself set to 0) **Breadth-First Search**

V	known	d_v	p_v
v_1	T	1	v_3
v_2	T	2	v_1
v_3	T	0	0
v_4	T	2	v_1
v_5	T	3	v_2
v_6	T	1	v_3
v_7	T	3	v_4



known is initially set to F, after being visited it will be set to T, $\mathbf{d_v}$ is distance and $\mathbf{p_v}$ is a bookkeeping variable

Weighted Shortest Path Problem

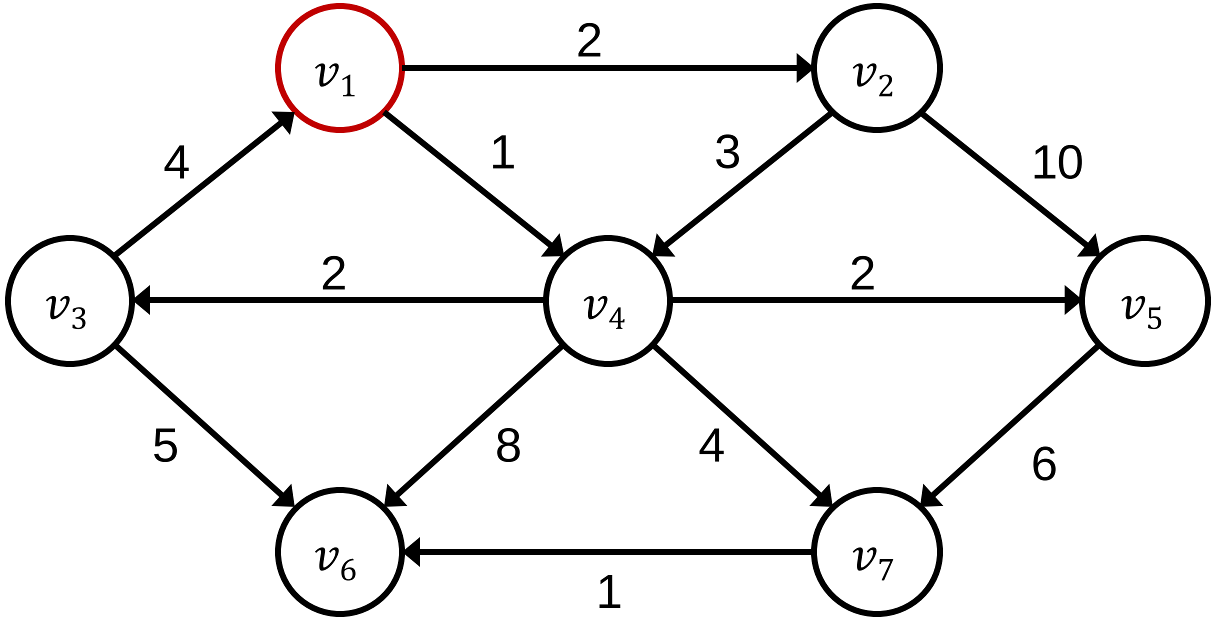
- Using what we know from unweighted, we can now tackle weighted shorted path – p_v is the last vertex to cause a change to d_v, this solution is known as ***Dijkstra's Algorithm*** and it's an example of a **Greedy Algorithm**
- **Greedy algorithms** solve a problem in stages doing what appears to be the best thing at each stage
- Greedy Example: making change at checkout counter – first the checkout person gives quarters, then dimes, then nickels and then pennies
  - $0.79, 3 quarters and 4 pennies
  - $0.87, 3 quarters, 1 dime and two pennies
  - Minimizing the number of coins given …

Dijkstra's Algorithm

- We choose a v that has the smallest d_v from all unknown vertices and is adjacent to s
- This path is declared the shortest path from s to v and marked known
- The remaining step is updating d_w (we didn't track d_w before, as we just were thinking $d_w = dv + 1$ if $d_w = \infty$) and $d_w = dv + c_{v,w}$ if this new value for d_w would be an improvement
- The algorithm decides if it's a good idea or not to use v on path to w given known cost and new cost

Dijkstra's Algorithm

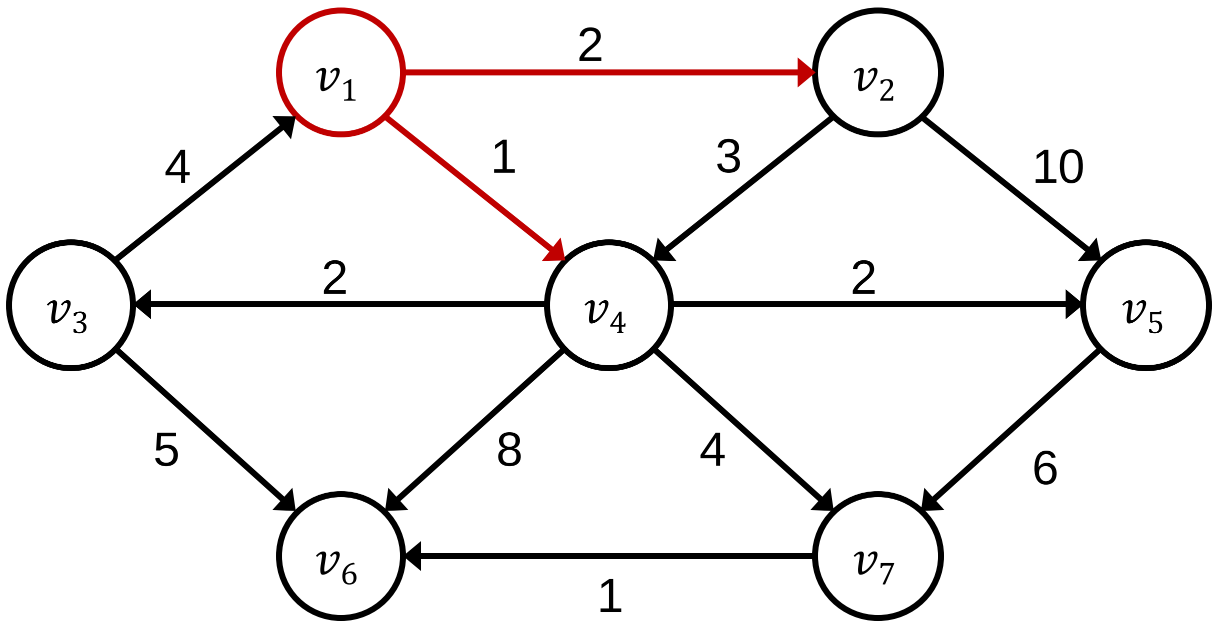
V	known	d_v	p_v
v_1	F	0	0
v_2	F	∞	0
v_3	F	∞	0
v_4	F	∞	0
v_5	F	∞	0
v_6	F	∞	0
v_7	F	∞	0



Pick s to be v_1, the path to v_1 is 0

Dijkstra's Algorithm

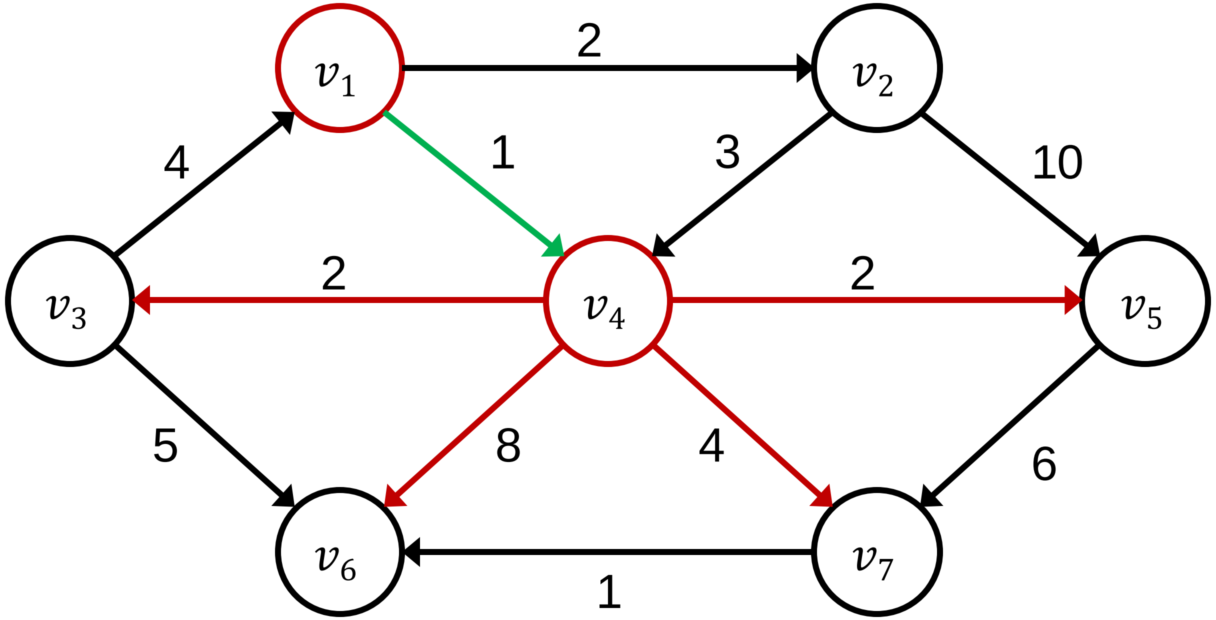
V	known	d_v	p_v
v_1	T	0	0
v_2	F	2	v_1
v_3	F	∞	0
v_4	F	1	v_1
v_5	F	∞	0
v_6	F	∞	0
v_7	F	∞	0



From v_1 we have path to v_2 and v_4, we choose v_4 (why?)

Dijkstra's Algorithm

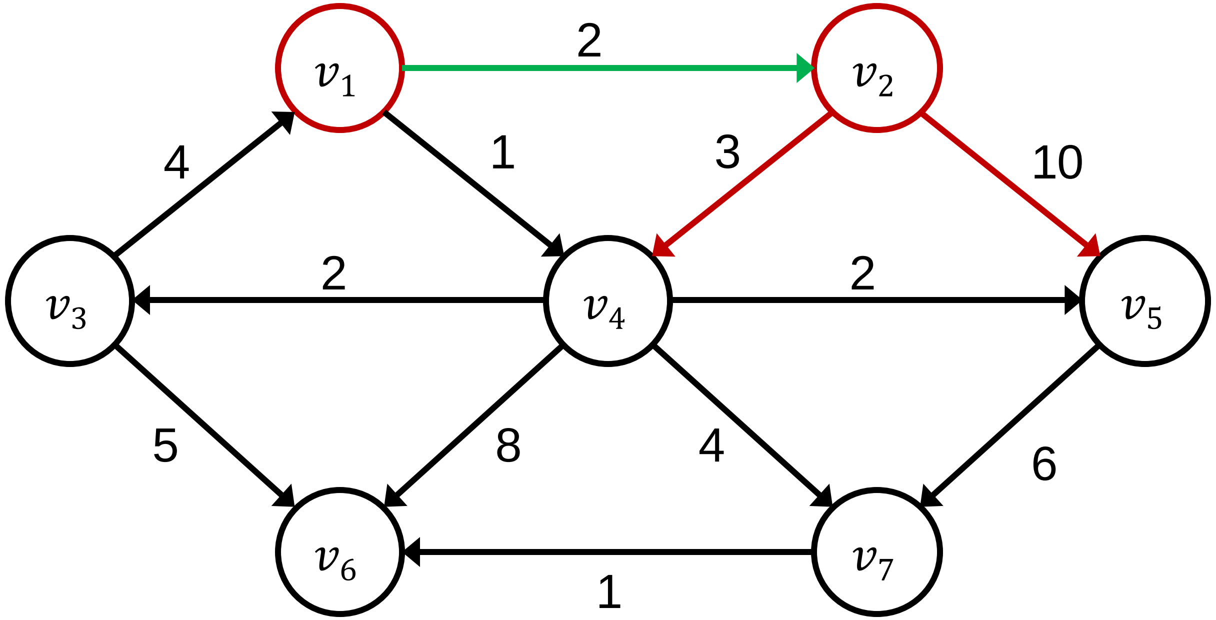
V	known	d_v	p_v
v_1	T	0	0
v_2	F	2	v_1
v_3	F	$3\ (1+2)$	v_4
v_4	T	1	v_1
v_5	F	$3\ (1+2)$	v_4
v_6	F	$9\ (1+8)$	v_4
v_7	F	$5\ (1+4)$	v_4



From v_4 we have path to v_3 , v_5, v_6 , v_7, we choose v_2 (new cheapest)

Dijkstra's Algorithm

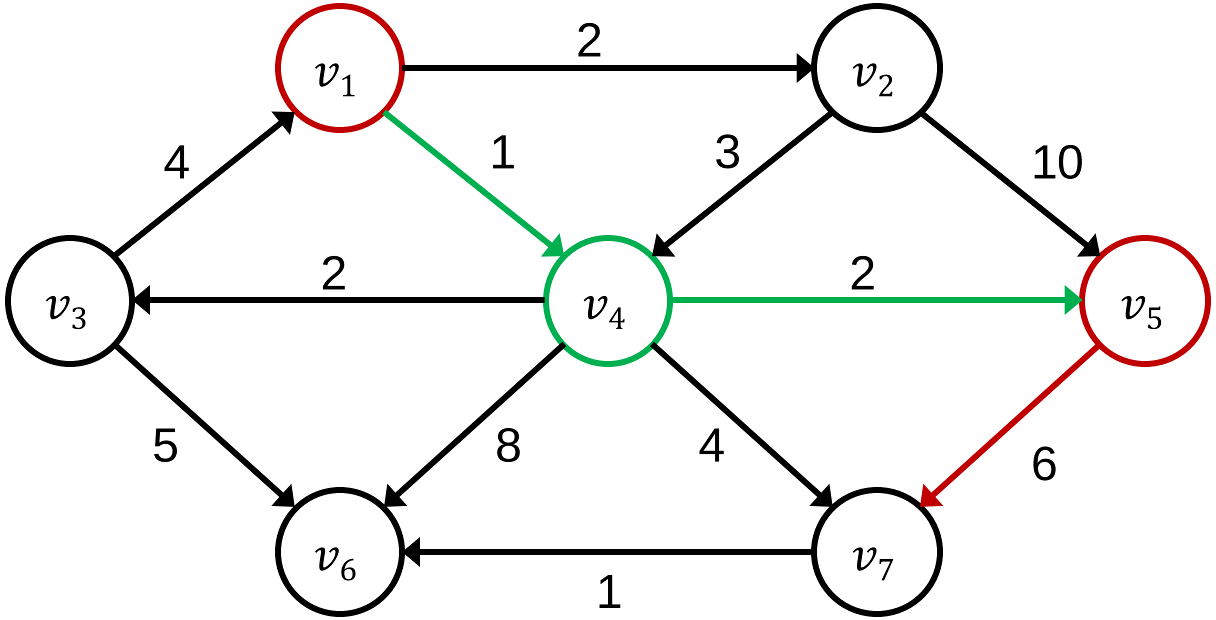
V	known	d_v	p_v
v_1	T	0	0
v_2	T	2	v_1
v_3	F	$3\ (1+2)$	v_4
v_4	T	1	v_1
v_5	F	$3\ (1+2)$	v_4
v_6	F	$9\ (1+8)$	v_4
v_7	F	$5\ (1+4)$	v_4



From v_2 we have path to v_4 , v_5 we look at v_5 (since v_4 is already known) none of the paths are better, v_1 to v_2 to v_5 costs 2 + 10 = 12 > 3

Dijkstra's Algorithm

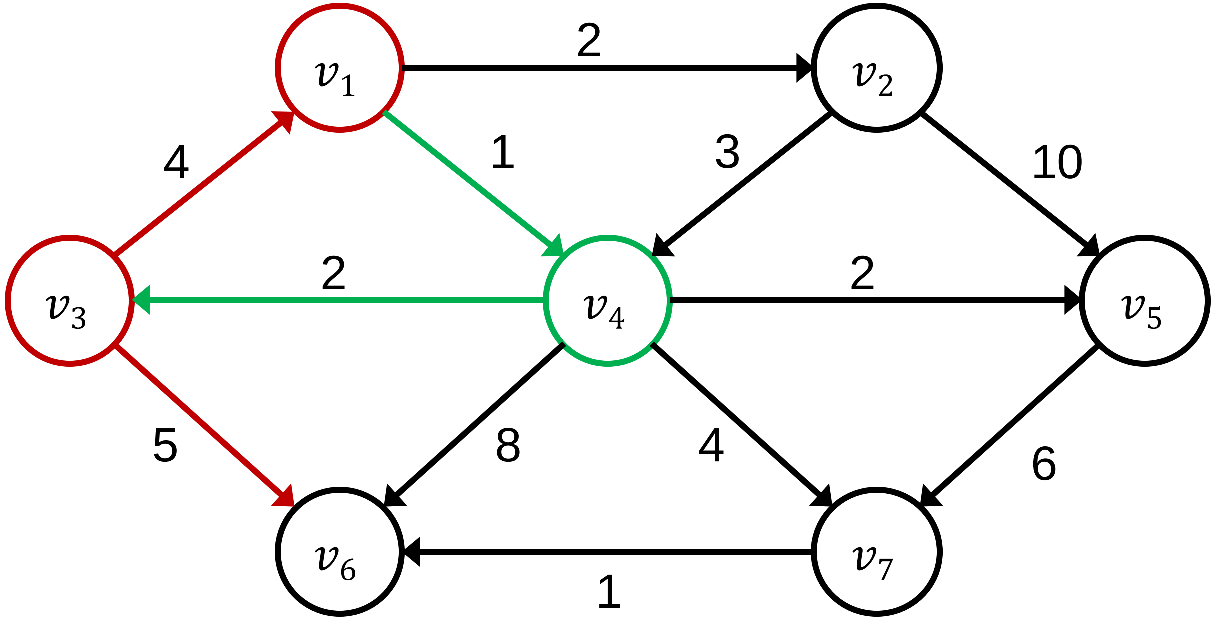
V	known	d_v	p_v
v_1	T	0	0
v_2	T	2	v_1
v_3	F	$3\ (1+2)$	v_4
v_4	T	1	v_1
v_5	T	$3\ (1+2)$	v_4
v_6	F	$9\ (1+8)$	v_4
v_7	F	$5\ (1+4)$	v_4



From v_5 we have path to v_7 none of the paths are better v_1 to v_4 to v_5, 1 + 2 + 6 = 9 > 5 back to selecting the smallest unvisited node which is v_3

Dijkstra’s Algorithm

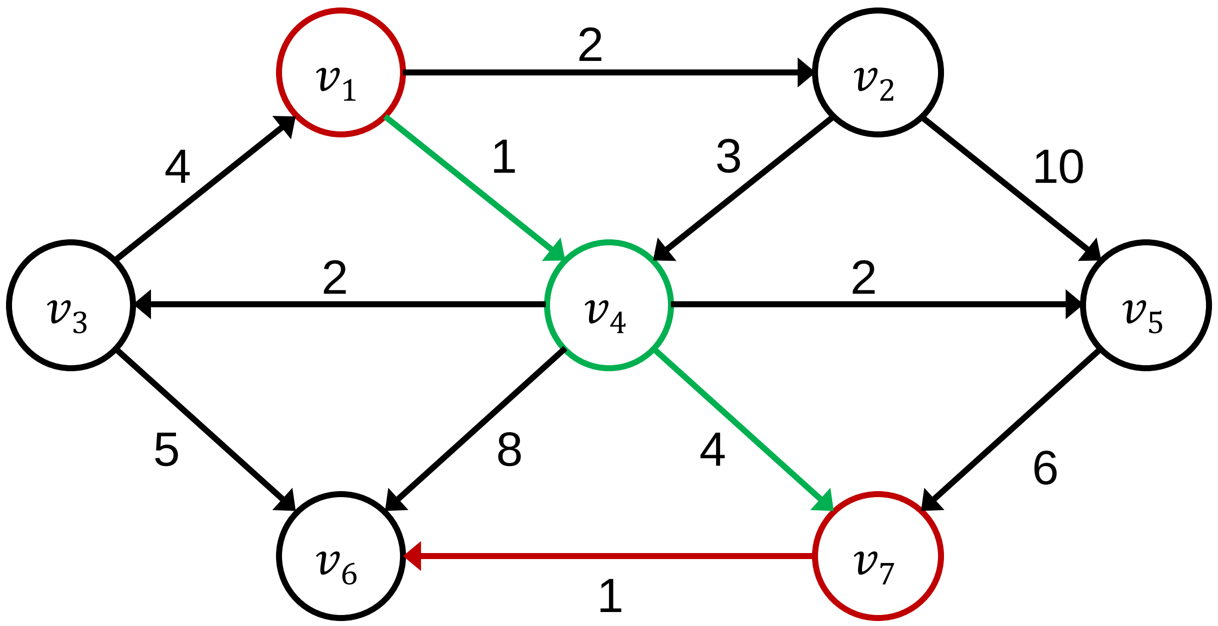
V	known	d_v	p_v
v_1	T	0	0
v_2	T	2	v_1
v_3	T	$3\ (1+2)$	v_4
v_4	T	1	v_1
v_5	T	$3\ (1+2)$	v_4
v_6	F	$8\ (1+2+5)$	v_3
v_7	F	$5\ (1+4)$	v_4



From v_3 we have path to v_1 , v_6 with v_1 cost is 1 + 2 + 4 = 7 > 0 but v_6 is 1 + 2 + 5 = 8 < 9, so we update, v_7 is now selected as the smallest

Dijkstra's Algorithm

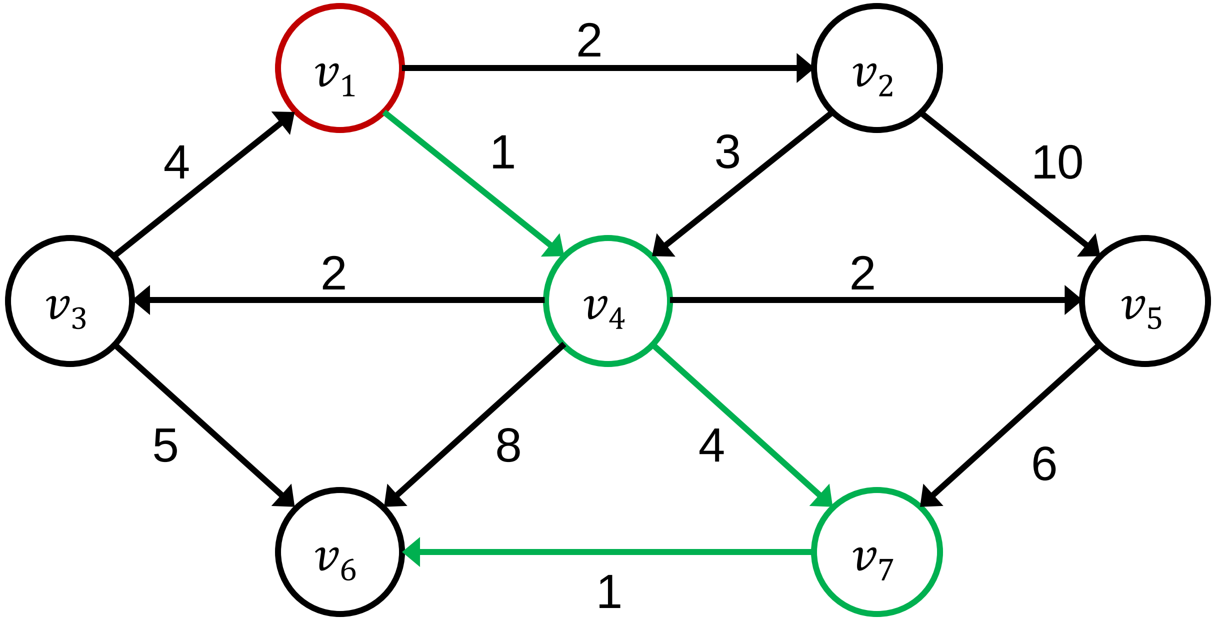
V	known	d_v	p_v
v_1	T	0	0
v_2	T	2	v_1
v_3	T	$3\ (1+2)$	v_4
v_4	T	1	v_1
v_5	T	$3\ (1+2)$	v_4
v_6	F	$6\ (1+4+1)$	v_7
v_7	T	$5\ (1+4)$	v_4



From v_7 we have path to v_6 with cost is 1 + 4+ 1 = 6 < 8, so we update and v_6 is the last one for us to visit

Dijkstra's Algorithm

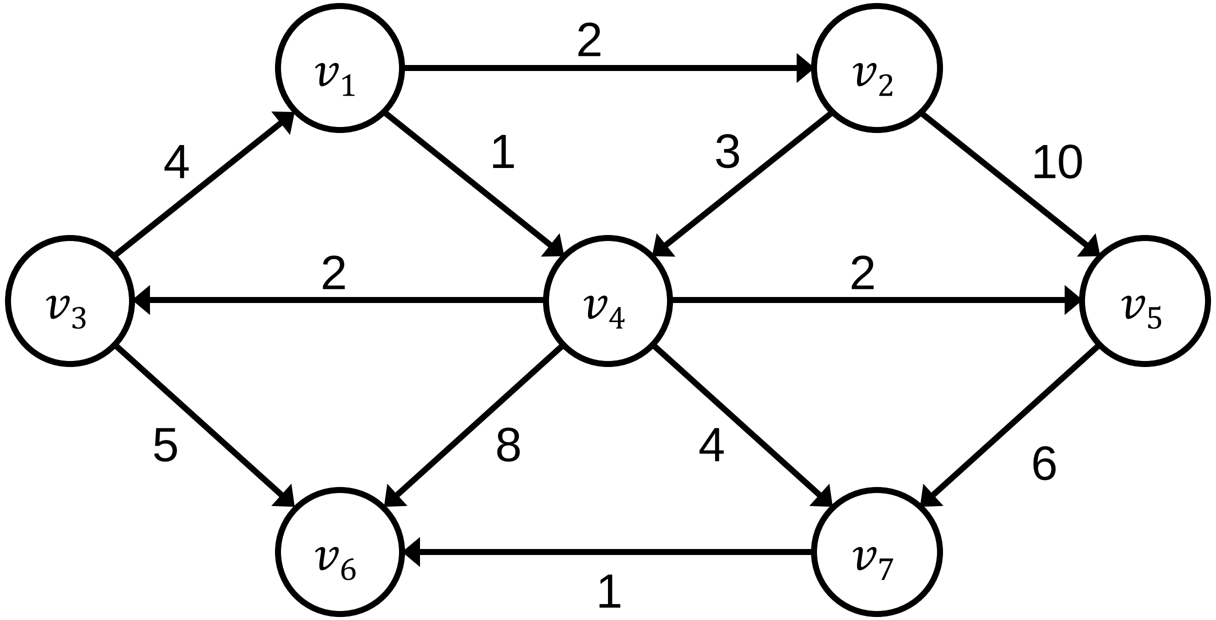
V	known	d_v	p_v
v_1	T	0	0
v_2	T	2	v_1
v_3	T	$3\ (1+2)$	v_4
v_4	T	1	v_1
v_5	T	$3\ (1+2)$	v_4
v_6	T	$6\ (1+4+1)$	v_7
v_7	T	$5\ (1+4)$	v_4



v_6 no where to visit

Dijkstra's Algorithm

V	known	d_v	p_v
v_1	T	0	0
v_2	T	2	v_1
v_3	T	3	v_4
v_4	T	1	v_1
v_5	T	3	v_4
v_6	T	6	v_7
v_7	T	5	v_4



What is the shortest path from:

v_1 to $v_1 \rightarrow v_1$

v_1 to $v_2 \rightarrow v_1 - v_2$

v_1 to $v_3 \rightarrow v_1 - v_4 - v_3$

v_1 to $v_4 \rightarrow v_1 - v_4$

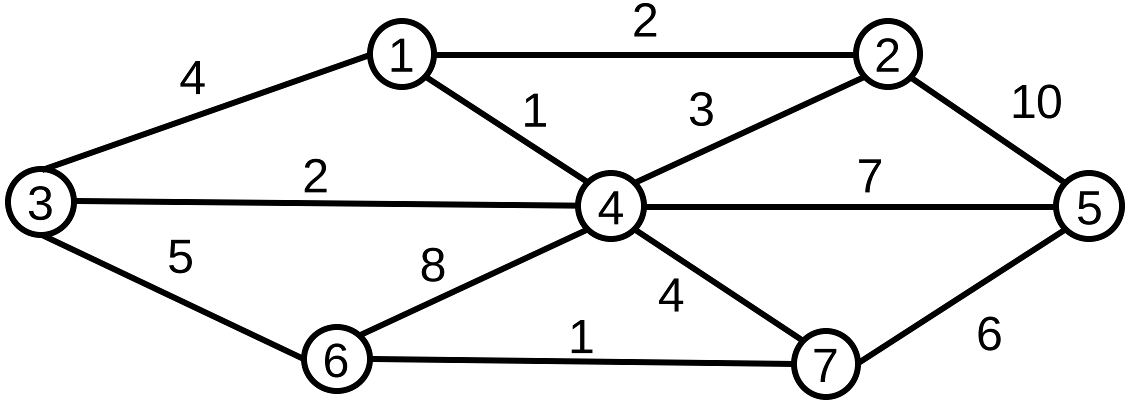
v_1 to $v_5 \rightarrow v_1 - v_4 - v_5$

v_1 to $v_6 \rightarrow v_1 - v_4 - v_7 - v_6$

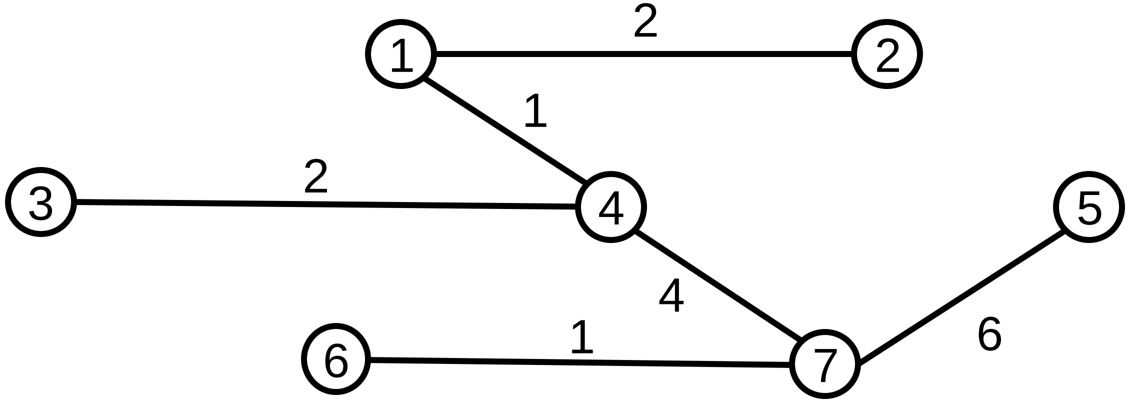
v_1 to $v_7 \rightarrow v_1 - v_4 - v_7$

Minimum Spanning Tree

- A **minimum spanning tree** of an undirected graph G is a formed from the graph edges that connects all the vertices of G at the lowest total cost.
- A minimum spanning tree exists <u>if and only if</u> G is **connected**
- **Reminder**: an undirected graph is **connected** if there is a path from every vertex to every other vertex

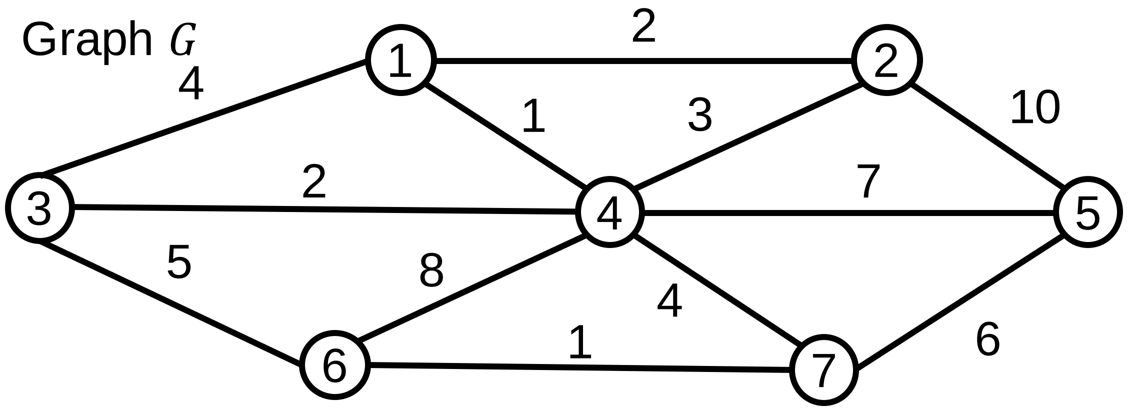


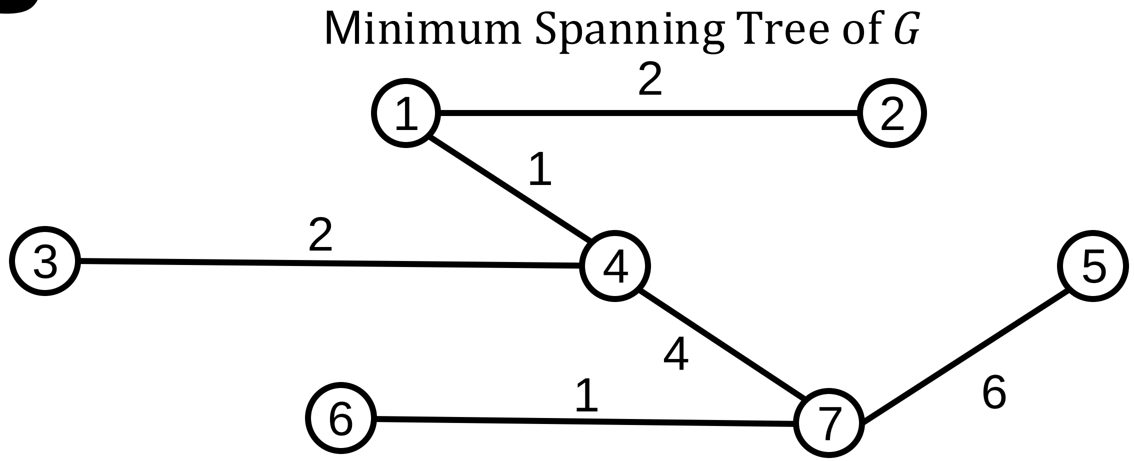
Graph G



Minimum Spanning Tree of G

Minimum Spanning Tree

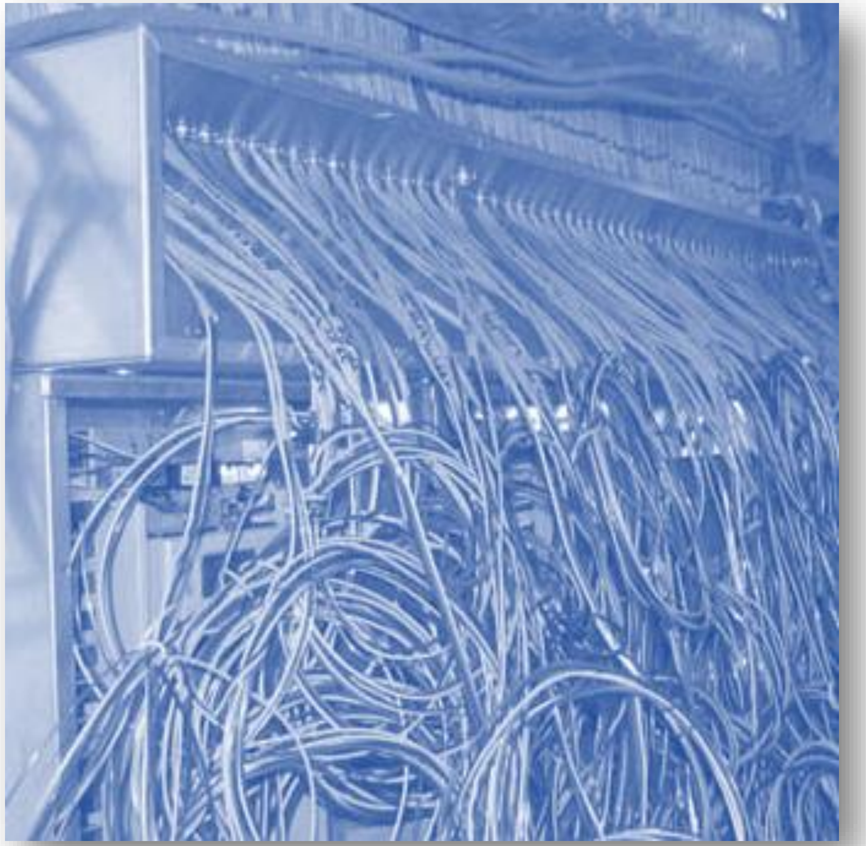




- The number of edges in the minimum spanning tree is the number of **vertices – 1** (|V| - 1)
- It is a **tree** because the graph has become acyclic, **spanning** because it covers all vertices and **minimum** by the defined goal

Minimum Spanning Tree

- Example Use of Minimum Spanning Tree
  - Network design
    - Telephone
    - Electrical
    - TV cable
    - Computer networks
    - Roads
  - Approximation algorithm for NP-hard problems
    - Traveling salesman problem
    - Steiner tree



https://www.geeksforgeeks.org/applications-of-minimum-spanning-tree/

Minimum Spanning Tree

- For any spanning tree T, if an edge e, that is not in T is added a cycle is created,
  - **Reminder**: a **cycle** in an undirected graph is a path where all the edges are distinct and $w_1 = w_N$

  then the removal of any edge on the cycle reinstates the spanning tree property
- The cost of the spanning tree is lowered if *e* has lower cost than the edge removed
- If as a spanning tree is created and the edge that is added is the one of minimum cost – the result is then a spanning tree that can not be improved

What kind of algorithm makes choices at each iteration based on best solution at that step?

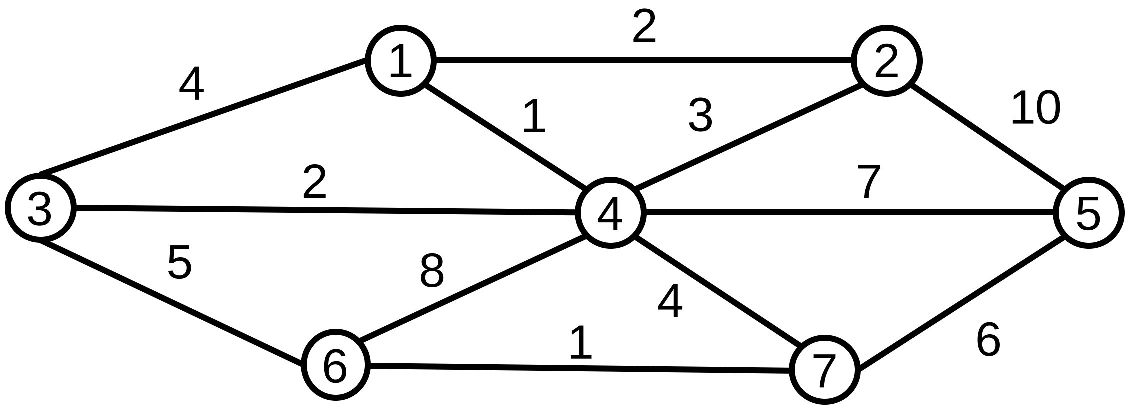
Greedy Algorithm

Minimum Spanning Tree

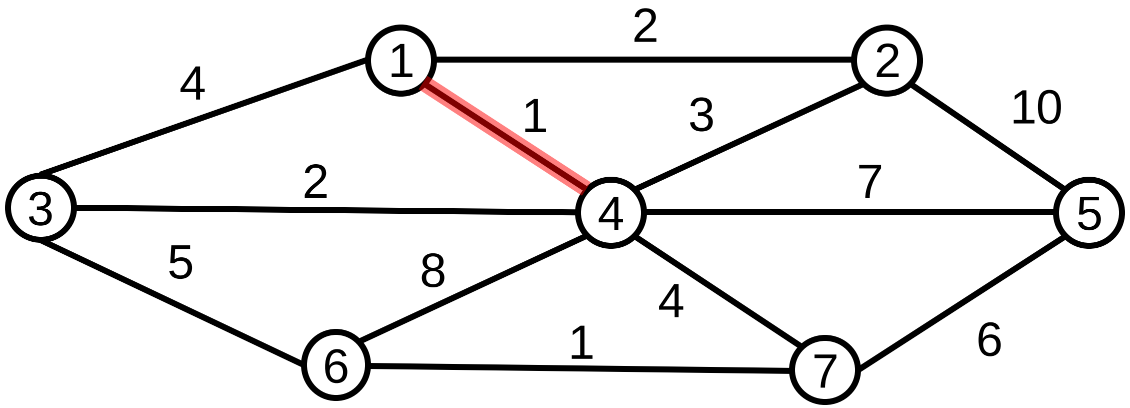
- One way to compute a minimum spanning tree is to grow the tree in successive stages
- In each state one node is picked as the root and an edge is added, along with associated vertex to the tree
- The algorithm keeps track of the vertices that have already been included and those that still have to be visited
- The selection of the new vertex at each stage is done by choosing the edge (u, v) such that the cost of (u, v) is the smallest among all edges where u is in the tree and v is not

Minimum Spanning Tree

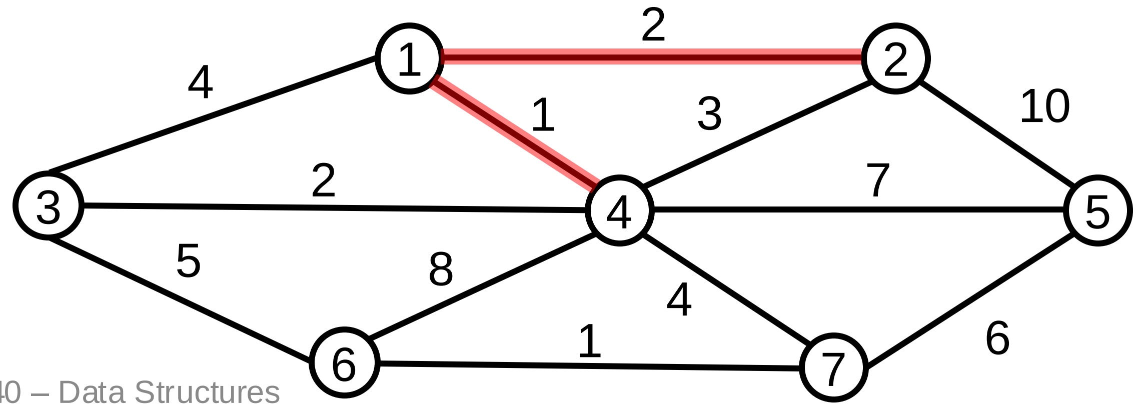
Given this graph:



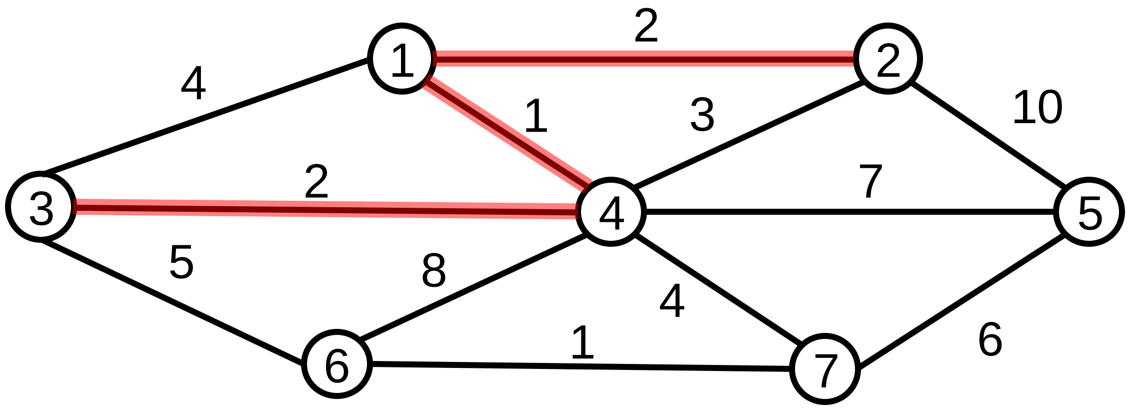
Picking v1, we look for edge that is minimum



Picking v2, we look for edge that is minimum

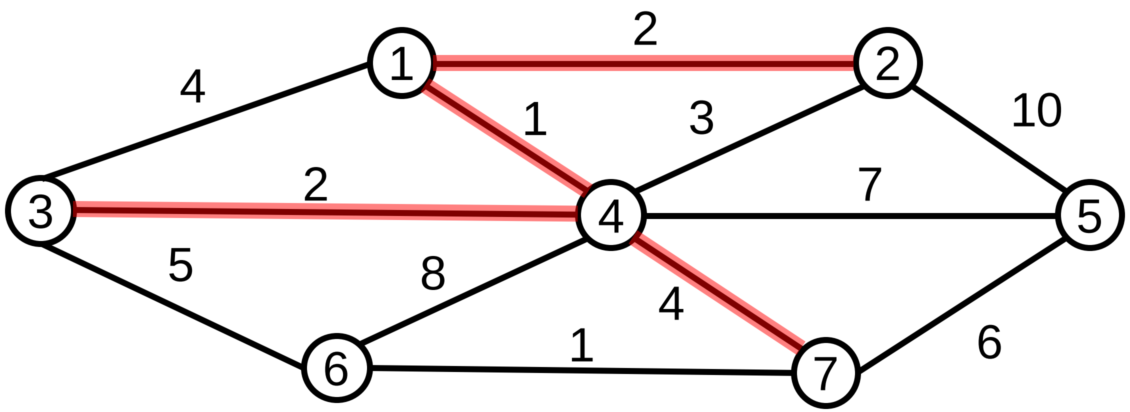


Picking v3, we look for edge that is minimum



Minimum Spanning Tree

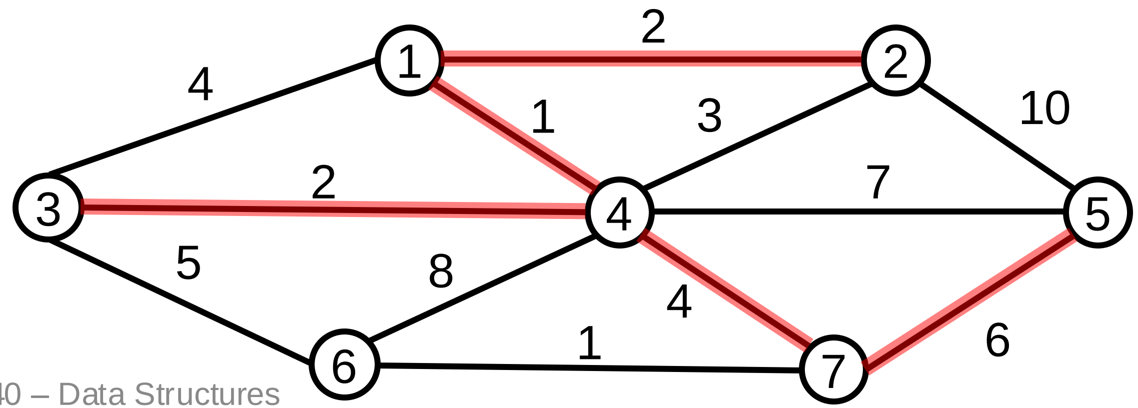
Picking v4, we look for edge that is minimum



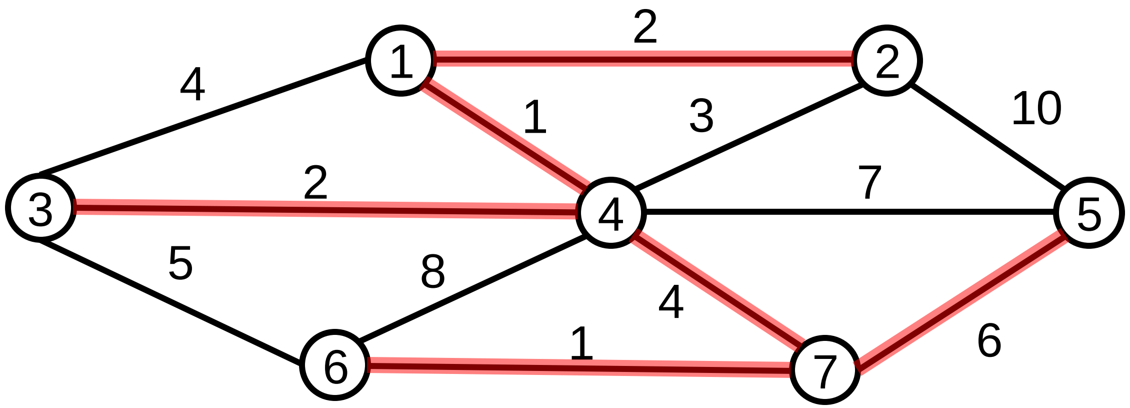
Why 4 to 7 and not 4 to 2?

2 has already been visited

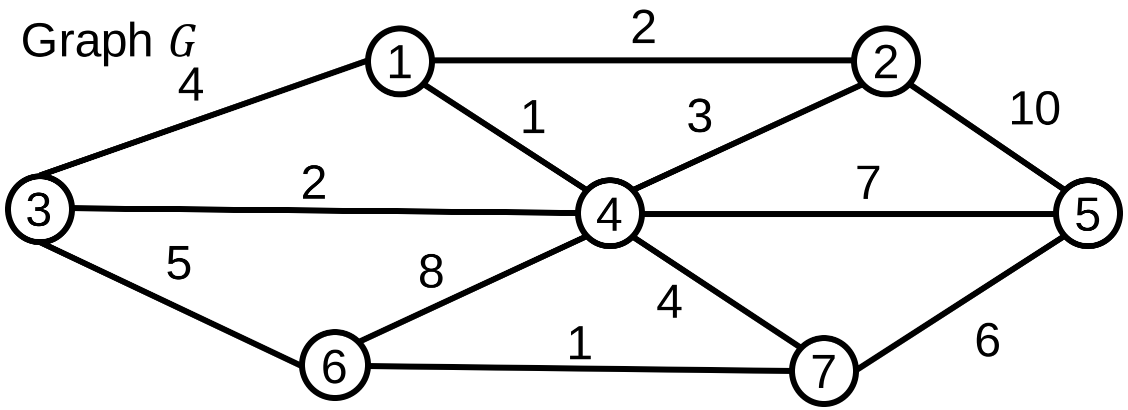
Picking v5, we look for edge that is minimum

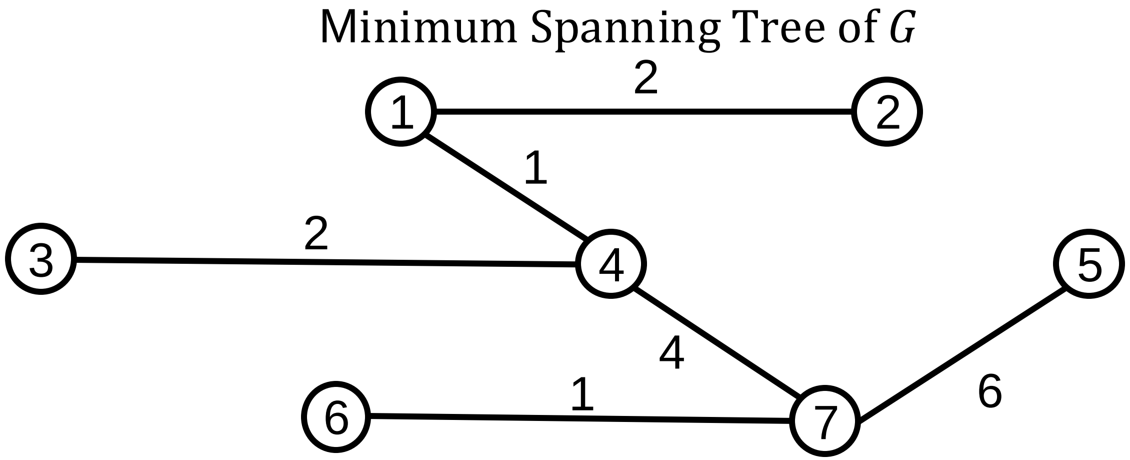


Picking v6, we look for edge that is minimum



Minimum Spanning Tree





As we saw earlier

Minimum Spanning Tree

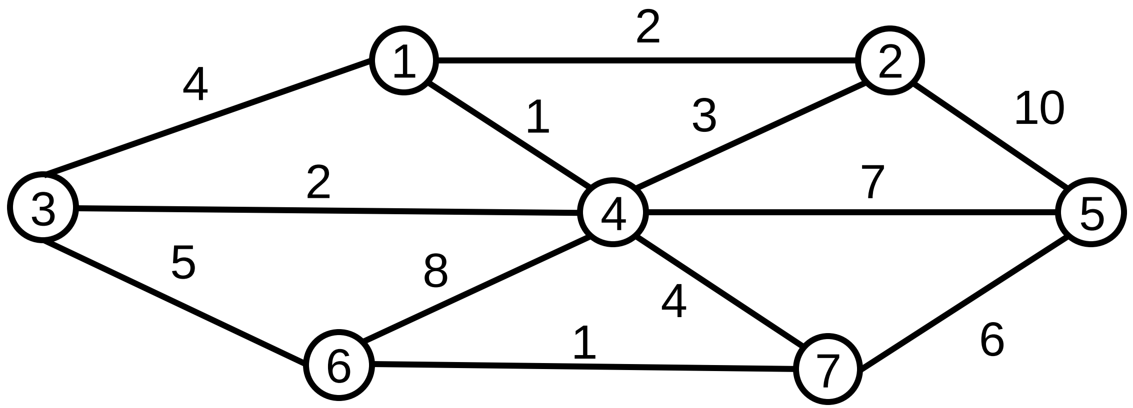
- Programmatically we can leverage the table like approach we saw with *Dijkstra's algorithm* for shortest path
- We will keep track of d_v and p_v for each vertex, will also keep track if the vertex is known or not
- d_v is the weigh of the shortest path connecting v to a known vertex
- p_v is the last vertex that caused a change in d_v
- Algorithm proceeds as it did in the case of the shortest path with an exception of the update (its simpler); a vertex v is selected for each unknown w adjacent to v such that the $d_w = \min(dw, c_{w,v})$

Minimum Spanning Tree

- In *Dijkstra's algorithm* d_v represented the tentative distance, the shortest distance from s (starting point) to v using only known vertices as intermediates
- Here we are only looking at the single edge (not path)

V	known	d_v	p_v
v_1	F	0	0
v_2	F	∞	0
v_3	F	∞	0
v_4	F	∞	0
v_5	F	∞	0
v_6	F	∞	0
v_7	F	∞	0

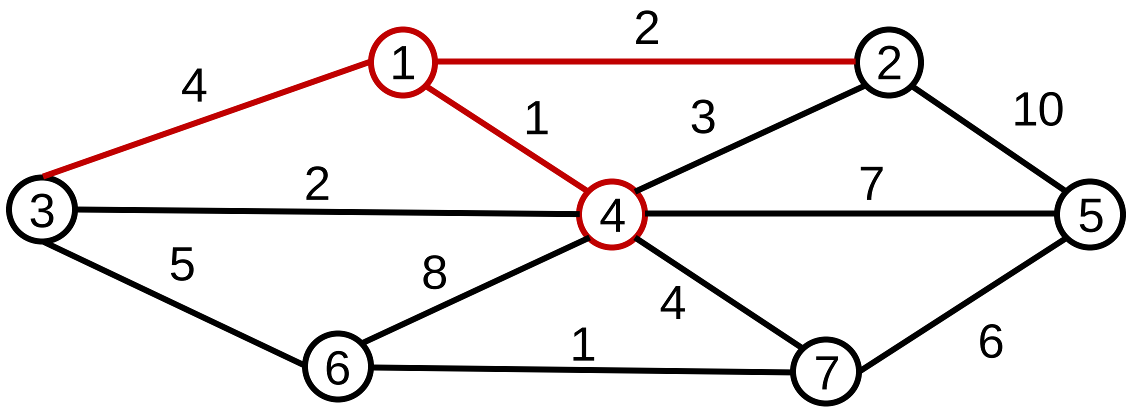
v_1 is the starting point



Minimum Spanning Tree

V	known	d_v	p_v
v_1	T	0	0
v_2	F	2	v_1
v_3	F	4	v_1
v_4	F	1	v_1
v_5	F	∞	0
v_6	F	∞	0
v_7	F	∞	0

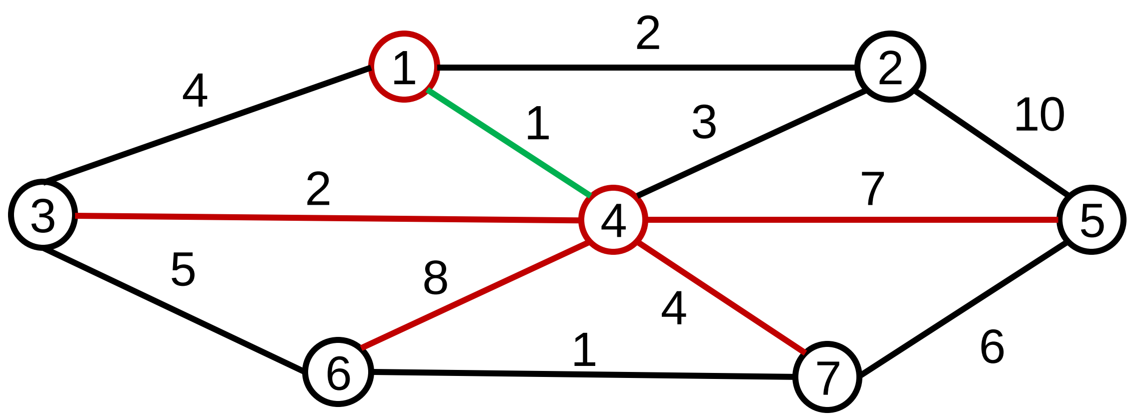
update v_2, v_3, $v_{4,}$ select v_4, lowest cost



Minimum Spanning Tree

V	known	d_v	p_v
v_1	T	0	0
v_2	F	2	v_1
v_3	F	2	v_4
v_4	T	1	v_1
v_5	F	7	v_4
v_6	F	8	v_4
v_7	F	4	v_4

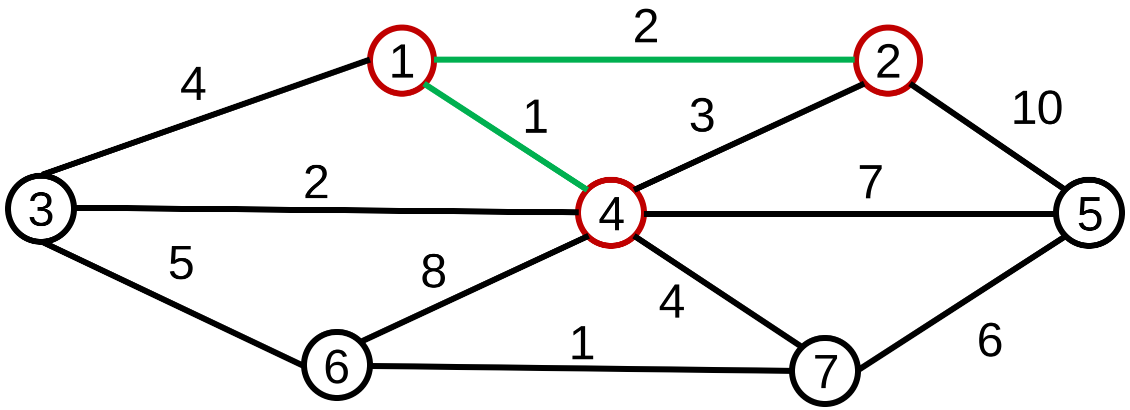
given v_4, lowest cost can be updated to v_3 and fill out the rest
we select v_2 next, could have selected v_2 or v_3 given same cost



Minimum Spanning Tree

V	known	d_v	p_v
v_1	T	0	0
v_2	T	2	v_1
v_3	F	2	v_4
v_4	T	1	v_1
v_5	F	7	v_4
v_6	F	8	v_4
v_7	F	4	v_4

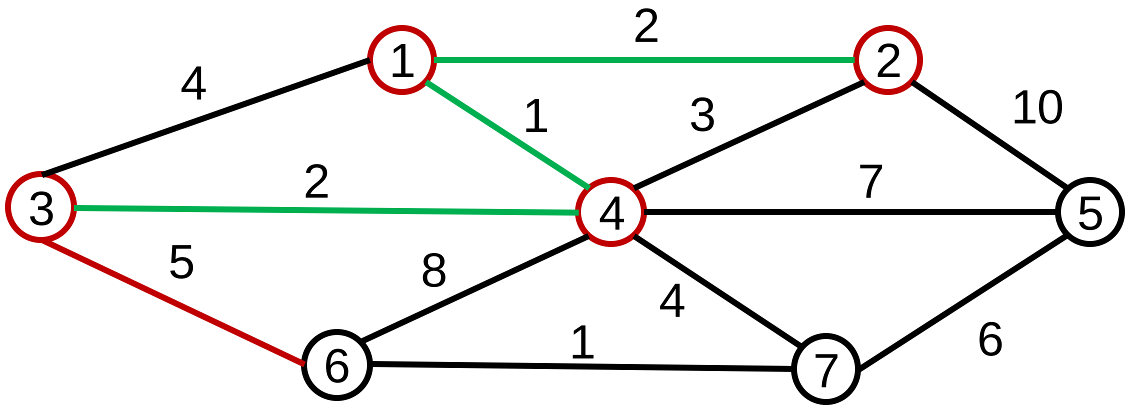
v_2 the only unknown v_2 can reach is v_5 with a cost of 10
now we select v_3



Minimum Spanning Tree

V	known	d_v	p_v
v_1	T	0	0
v_2	T	2	v_1
v_3	T	2	v_4
v_4	T	1	v_1
v_5	F	7	v_4
v_6	F	5	v_3
v_7	F	4	v_4

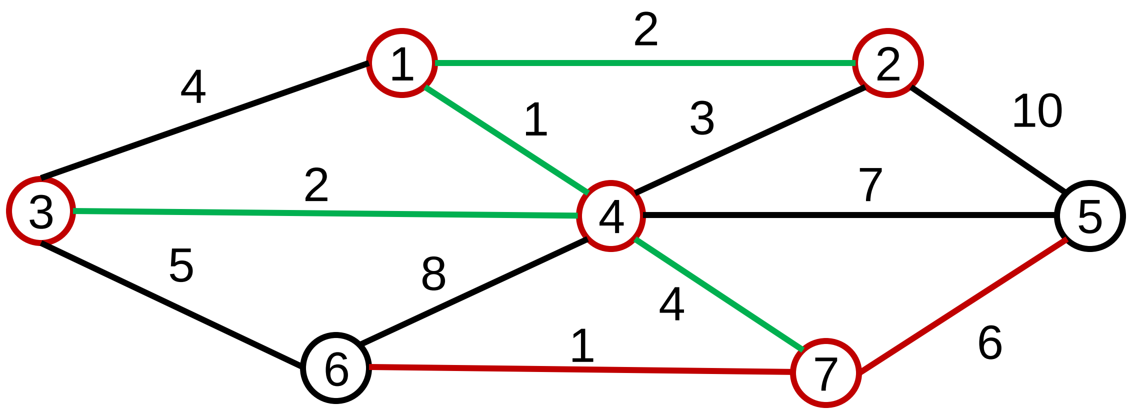
v_3 the only unknown v_3 can reach is v_6 with a better cost, we select v_7 next as lowest cost



Minimum Spanning Tree

V	known	d_v	p_v
v_1	T	0	0
v_2	T	2	v_1
v_3	T	2	v_4
v_4	T	1	v_1
v_5	F	6	v_7
v_6	F	1	v_7
v_7	T	4	v_4

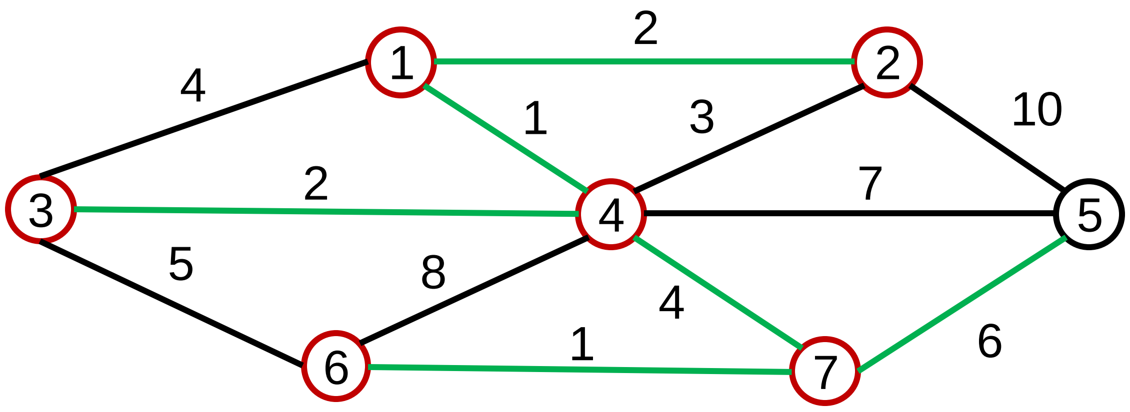
v_7 which can get to v_6 at lower cost and v_5 at lower cost choose v_6 as next unknown to visit



Minimum Spanning Tree

V	known	d_v	p_v
v_1	T	0	0
v_2	T	2	v_1
v_3	T	2	v_4
v_4	T	1	v_1
v_5	F	6	v_7
v_6	T	1	v_7
v_7	T	4	v_4

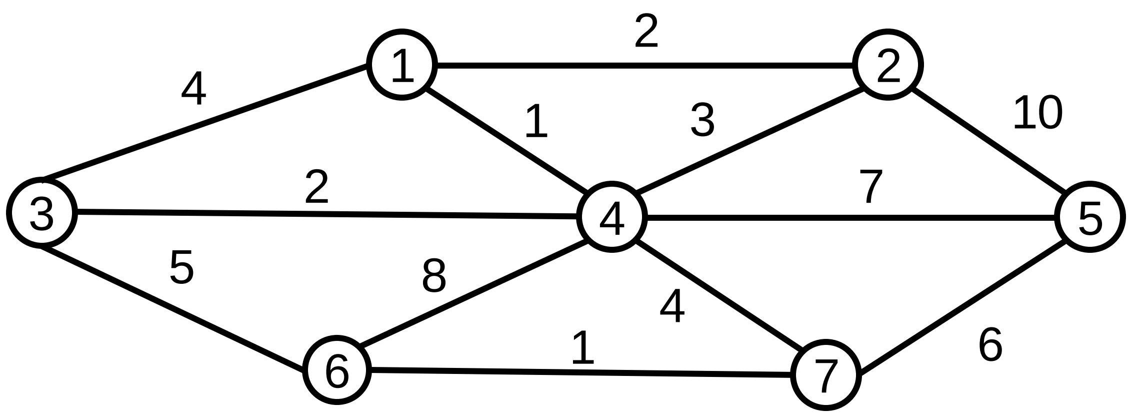
v_6 can not reach any unknowns, v_5 left



Minimum Spanning Tree

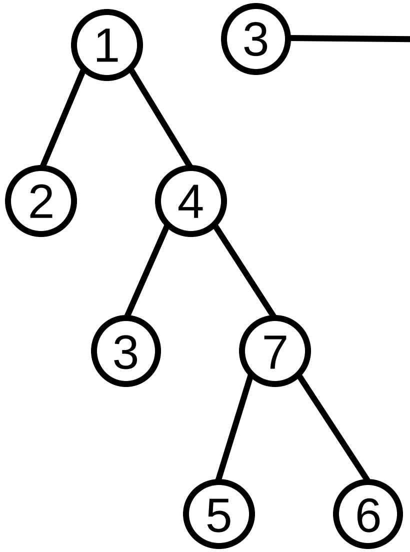
V	known	d_v	p_v
v_1	T	0	0
v_2	T	2	v_1
v_3	T	2	v_4
v_4	T	1	v_1
v_5	T	6	v_7
v_6	T	1	v_7
v_7	T	4	v_4

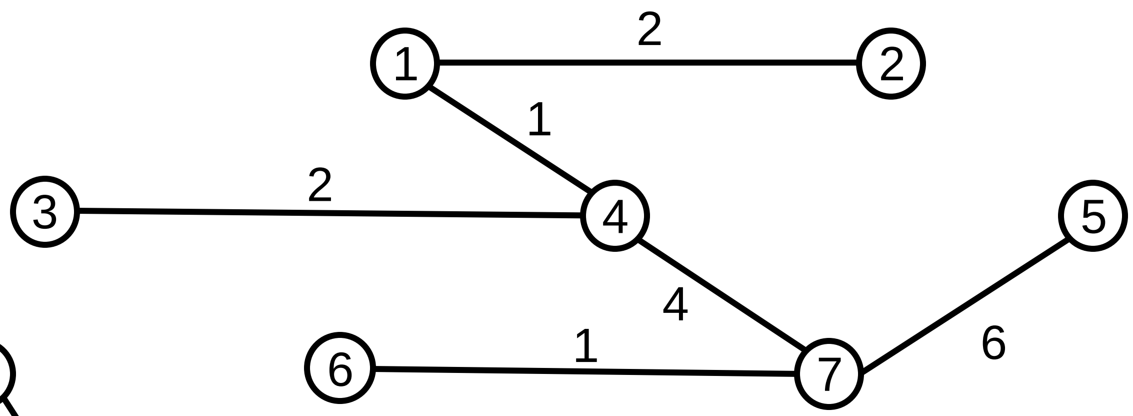
v_5 can get no where new either



Minimum Spanning Tree

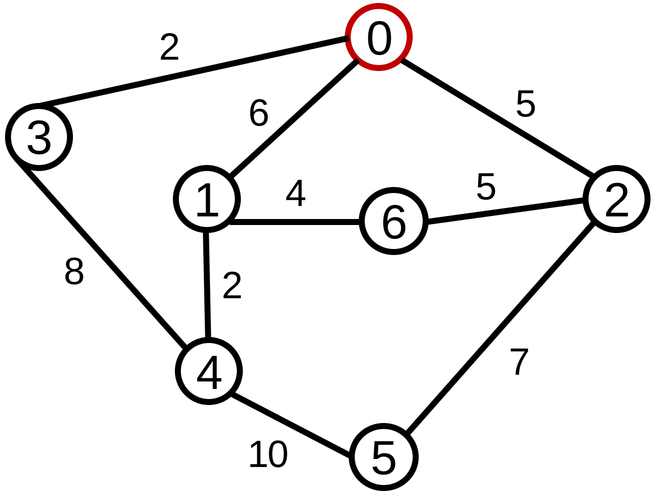
V	known	d_v	p_v
v_1	T	0	0
v_2	T	2	v_1
v_3	T	2	v_4
v_4	T	1	v_1
v_5	T	6	v_7
v_6	T	1	v_7
v_7	T	4	v_4





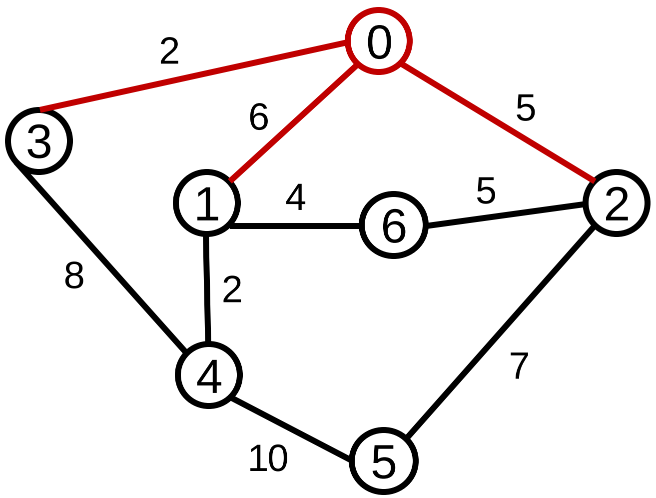
Minimum Spanning Tree

V	known	d_v	p_v
v_0	F	0	0
v_1	F	∞	0
v_2	F	∞	0
v_3	F	∞	0
v_4	F	∞	0
v_5	F	∞	0
v_6	F	∞	0



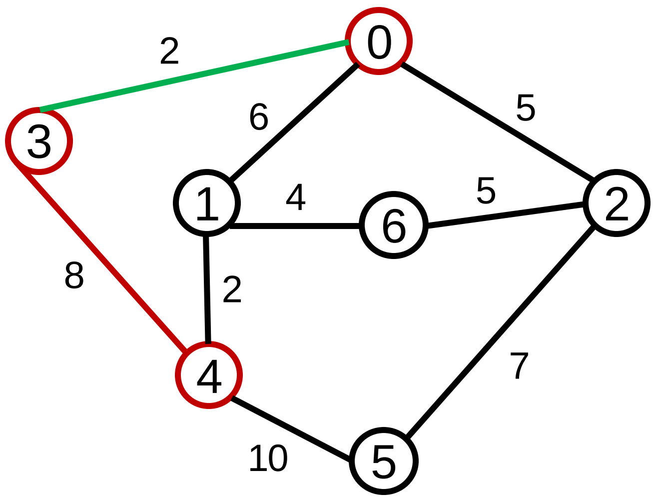
Minimum Spanning Tree

V	known	d_v	p_v
v_0	T	0	0
v_1	F	6	v_0
v_2	F	5	v_0
v_3	F	2	v_0
v_4	F	∞	0
v_5	F	∞	0
v_6	F	∞	0



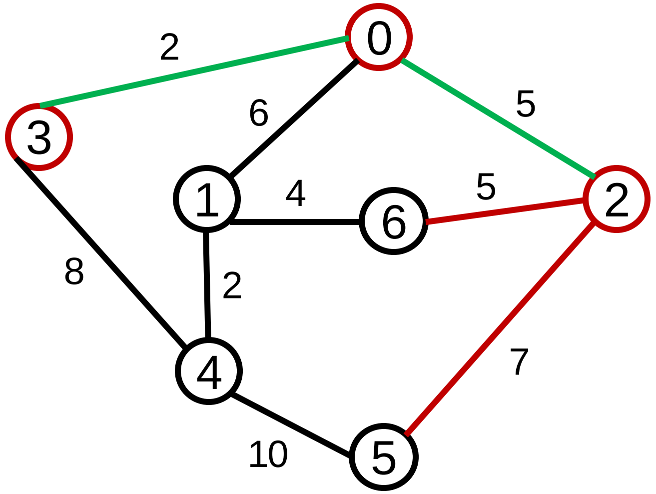
Minimum Spanning Tree

V	known	d_v	p_v
v_0	T	0	0
v_1	F	6	v_0
v_2	F	5	v_0
v_3	T	2	v_0
v_4	F	8	v_3
v_5	F	∞	0
v_6	F	∞	0



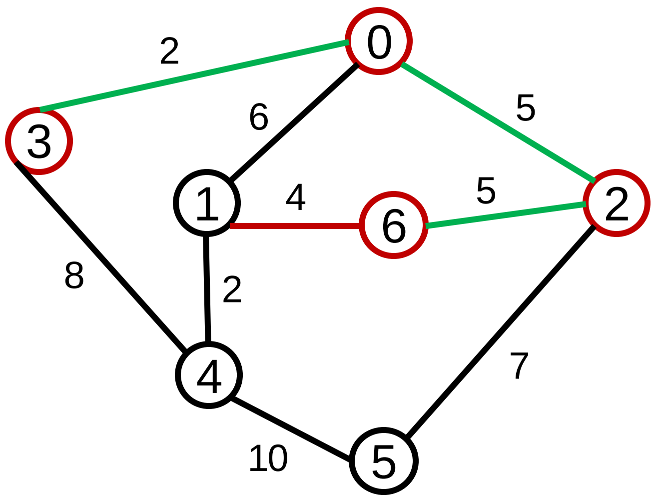
Minimum Spanning Tree

V	known	d_v	p_v
v_0	T	0	0
v_1	F	6	v_0
v_2	T	5	v_0
v_3	T	2	v_0
v_4	F	8	v_3
v_5	F	7	v_2
v_6	F	5	v_2



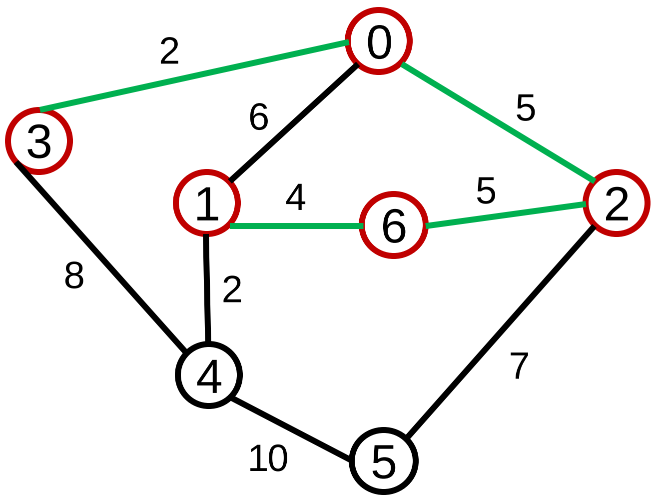
Minimum Spanning Tree

V	known	d_v	p_v
v_0	T	0	0
v_1	F	4	v_6
v_2	T	5	v_0
v_3	T	2	v_0
v_4	F	8	v_3
v_5	F	7	v_2
v_6	T	5	v_2



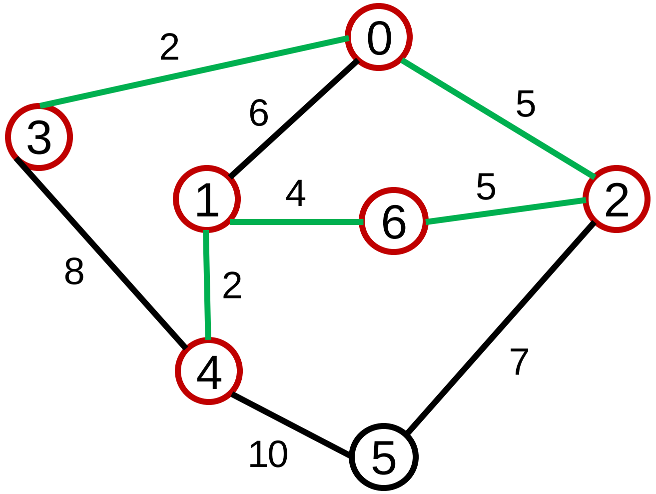
Minimum Spanning Tree

V	known	d_v	p_v
v_0	T	0	0
v_1	T	4	v_6
v_2	T	5	v_0
v_3	T	2	v_0
v_4	F	2	v_1
v_5	F	7	v_2
v_6	T	5	v_2



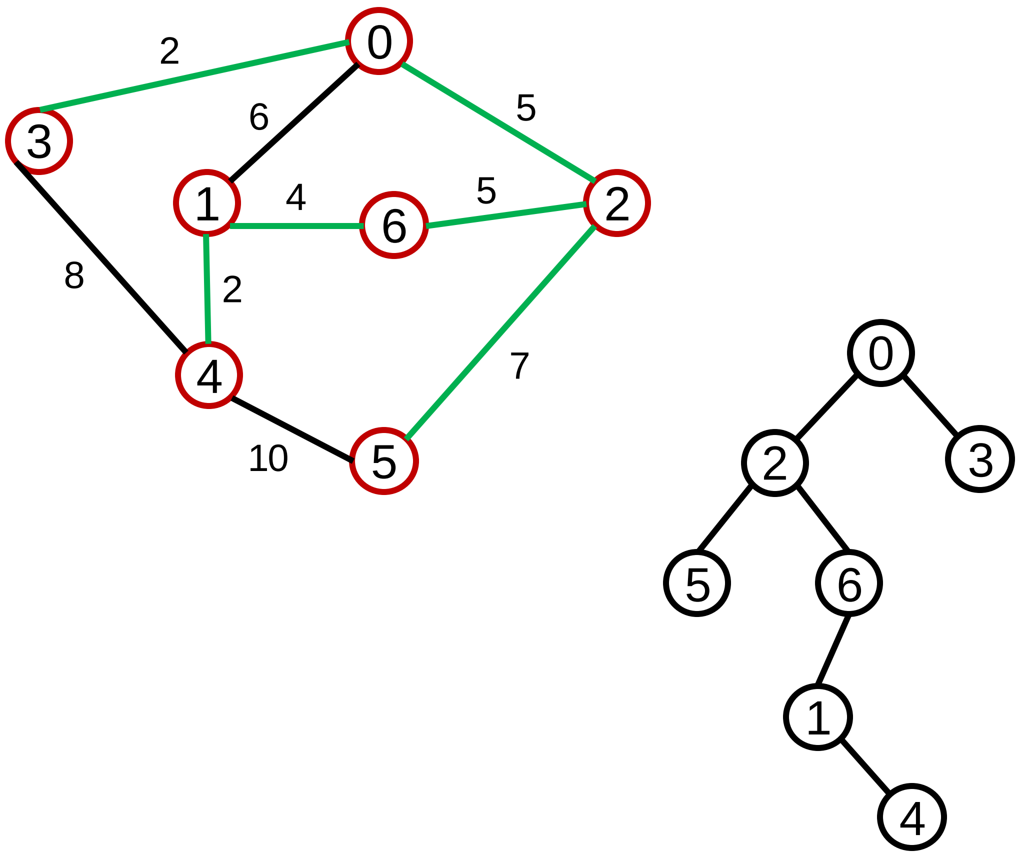
Minimum Spanning Tree

V	known	d_v	p_v
v_0	T	0	0
v_1	T	4	v_6
v_2	T	5	v_0
v_3	T	2	v_0
v_4	T	2	v_1
v_5	F	7	v_2
v_6	T	5	v_2



Minimum Spanning Tree

V	known	d_v	p_v
v_0	T	0	0
v_1	T	4	v_6
v_2	T	5	v_0
v_3	T	2	v_0
v_4	T	2	v_1
v_5	T	7	v_2
v_6	T	5	v_2



Acknowledgement

These slides have been adapted and borrowed from books on the right as well as the CS340 notes of NIU CS department (Professors: Alhoori, Hou, Lehuta, and Winans) and many google searches.