Northern Illinois University

B-Tree

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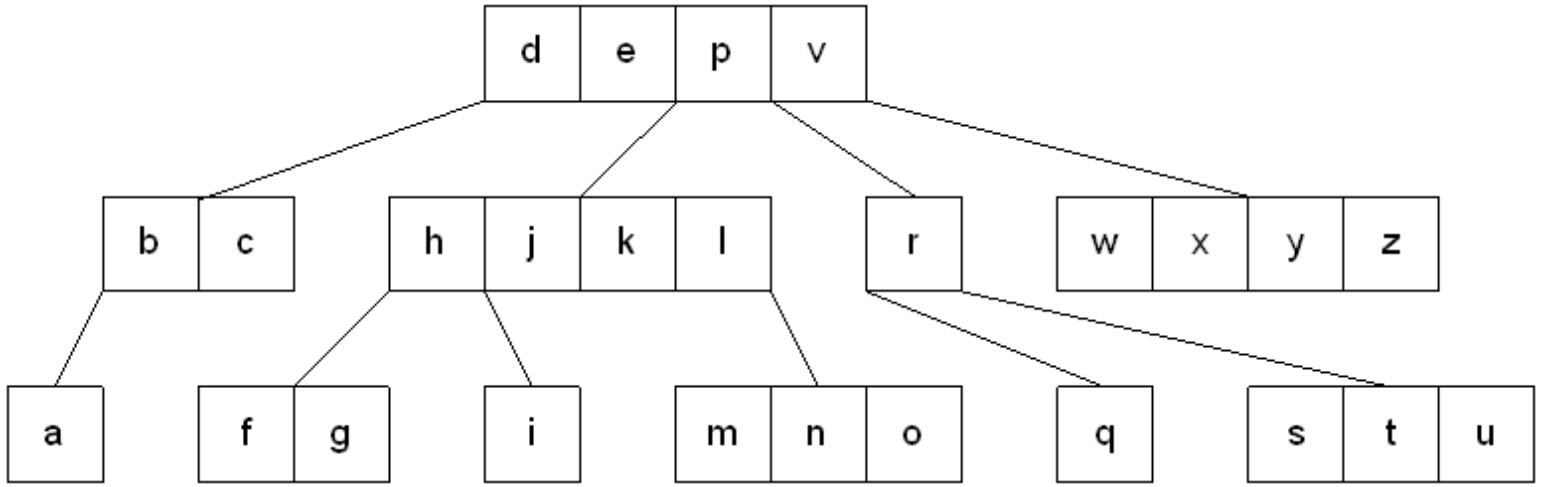
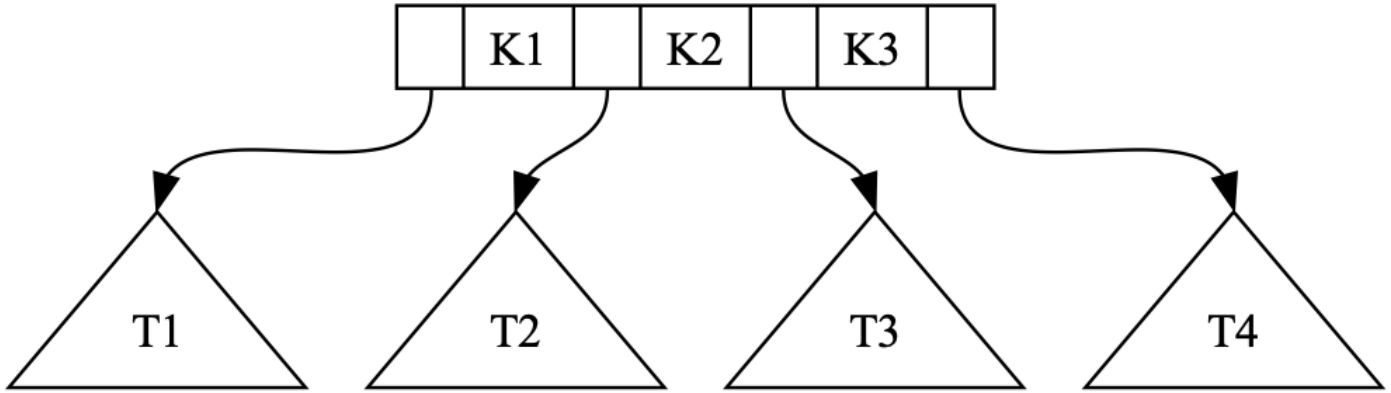
Tree

- Prior to today, we mainly focused on binary tree (at most 2 children)
- Different type of tree that can have many children
- Often called *multi-way tree of order m* or *m-way tree*



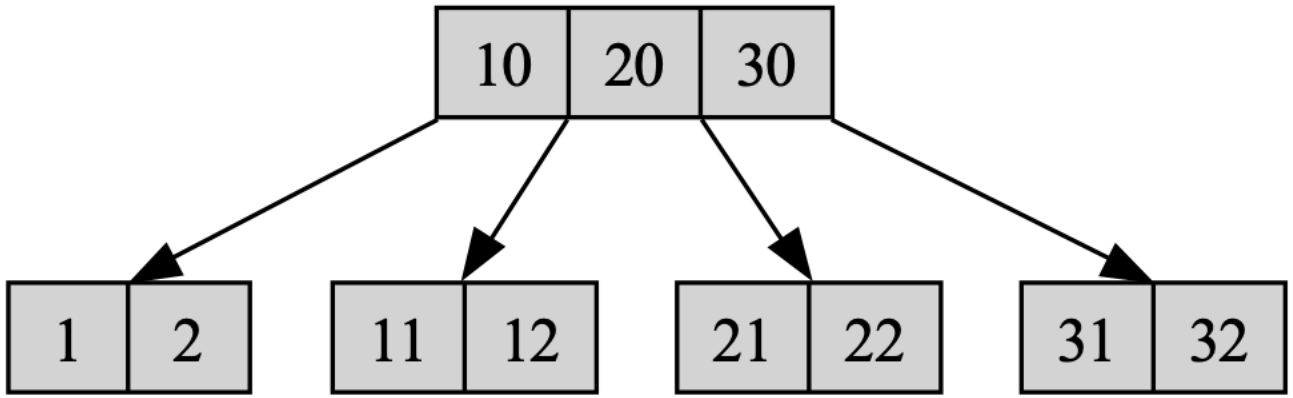
m-ary Tree

- m-ary search tree allows m-way branching (**m children**)
- A node in a m-ary tree stores **m-1 keys** (K1, K2, K3, …) **in order**
- Each piece of data stored is called a **key** (unique, only in one location)
- The keys in a node serve as **dividing points**
- Each node also has **m pointers**



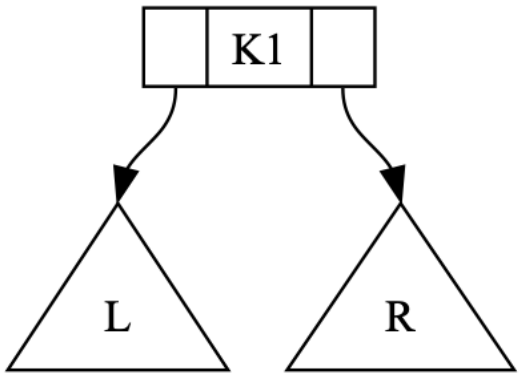
m-ary Tree

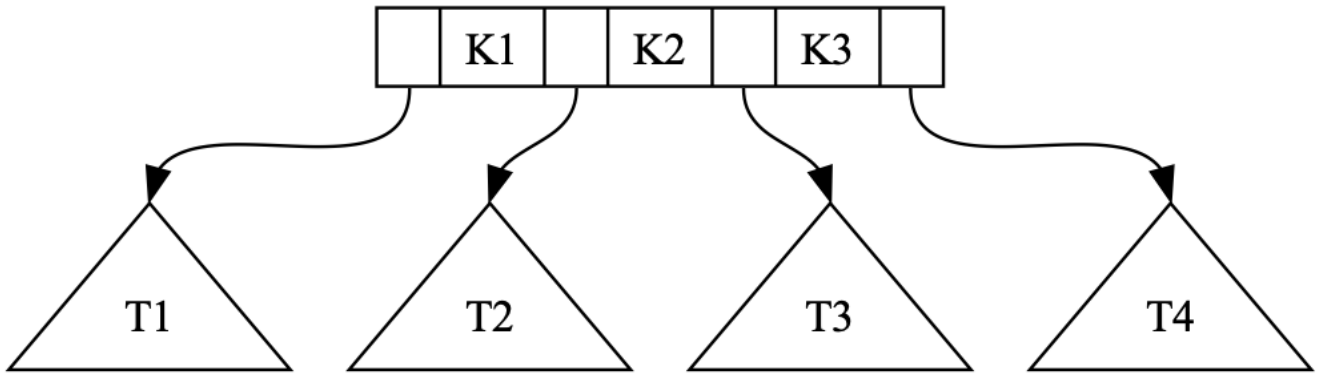
- The keys in the first *i* children are **smaller** than the *i*th key
- The keys in the last *m-i* children are **larger** than the *i*th key
- Order of subtrees is based on parent node keys.



Binary Tree

- An *m*-ary tree has *m* pointers and *m-1* keys
- Binary trees are 2-ary trees (2 pointers, 1 key)





B-tree of Order m Properties

- Developed by Bayer and McCreight in 1972
- Properties of a B-tree:
  1. The root has at least two subtrees unless it is a leaf.
  2. Each nonroot and each nonleaf node holds $k - 1$ keys and k pointers to subtrees where $\left\lceil \frac{m}{2} \right\rceil \leq k \leq m$.
  3. Each leaf node holds $k - 1$ keys where $\left\lceil \frac{m}{2} \right\rceil \leq k \leq m$.
  4. All leaves are on the same level.
- According to these conditions, a B-tree is always at least half full, has a few levels, and is perfectly balanced.

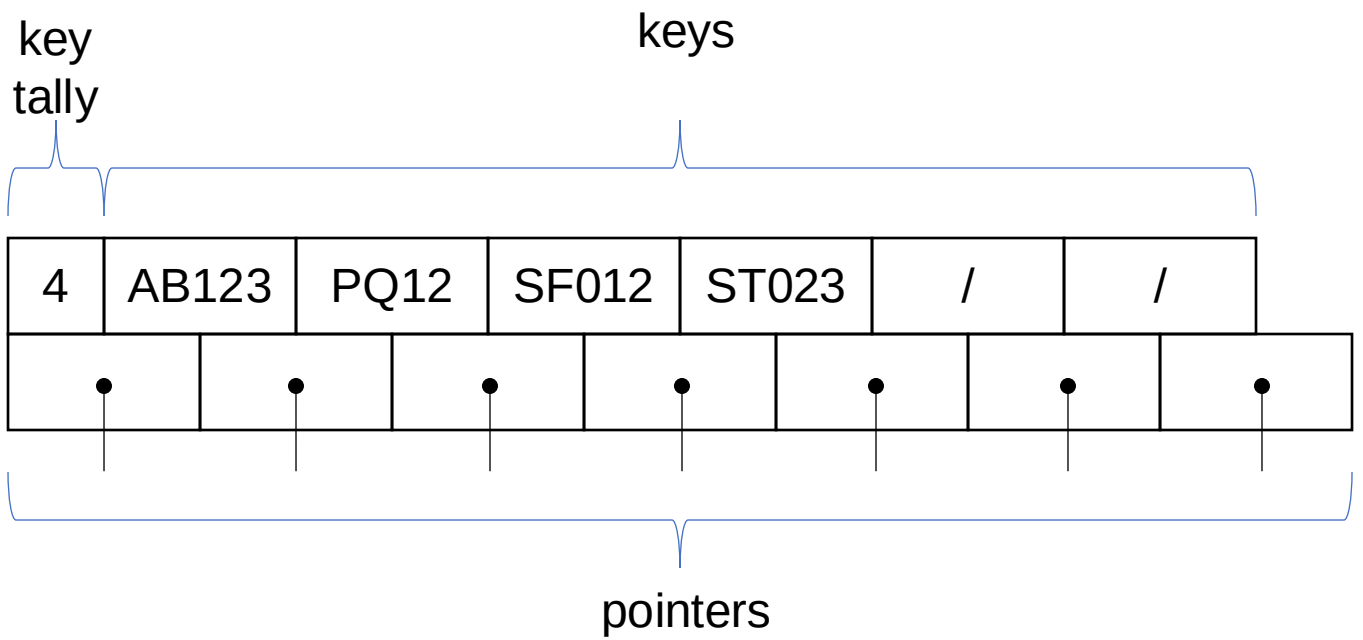
Typical m Size

- m is typically large (50 – 500) with the information stored in one **page/block** of secondary storage fitting into one node

- Most file systems are based on a **block** device, which is a level of abstraction for the hardware responsible for storing and retrieving specified blocks of data, though the block size in file systems may be a multiple of the physical block size

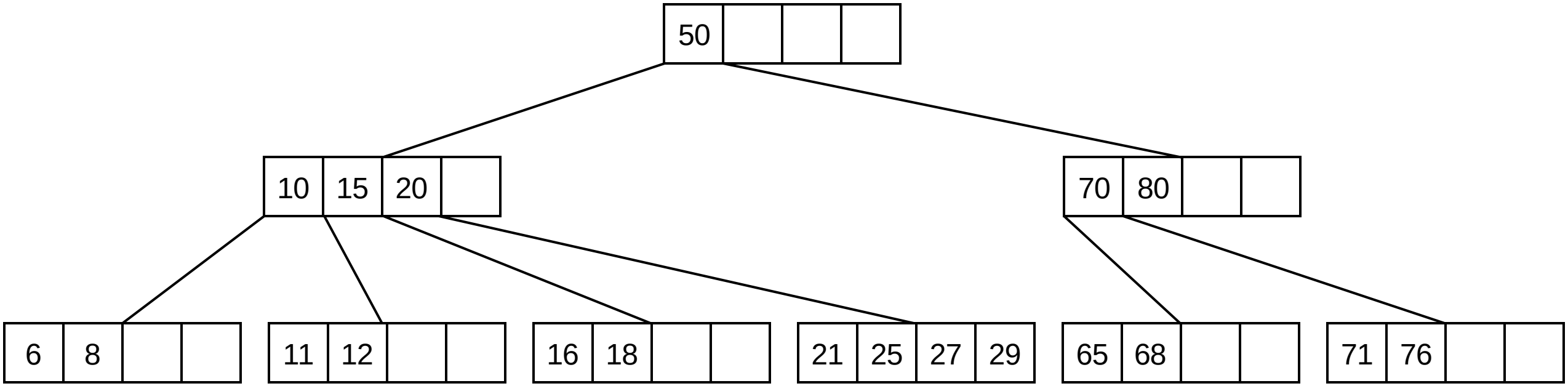
- A **page**, **memory page**, or **virtual page** is a fixed-length contiguous block of virtual memory, described by a single entry in the page table. It is the smallest unit of data for memory management in a virtual memory operating system

Example Node of a B-tree Order 7

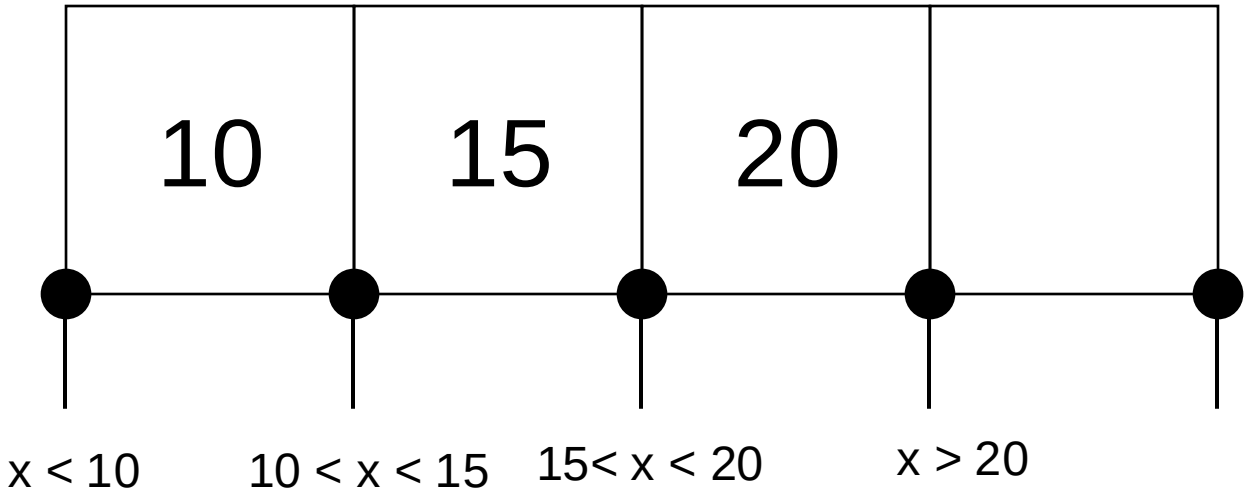


- In general, the keys would have pointers out of them to more data

Example B-tree Order 5

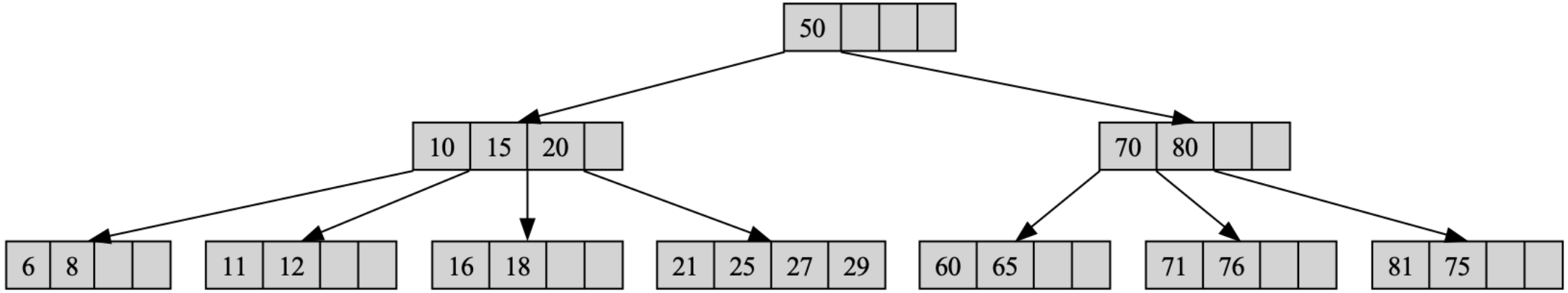


Searching B-tree



Searching B-tree

- Search for 71
- Almost the same process as binary tree



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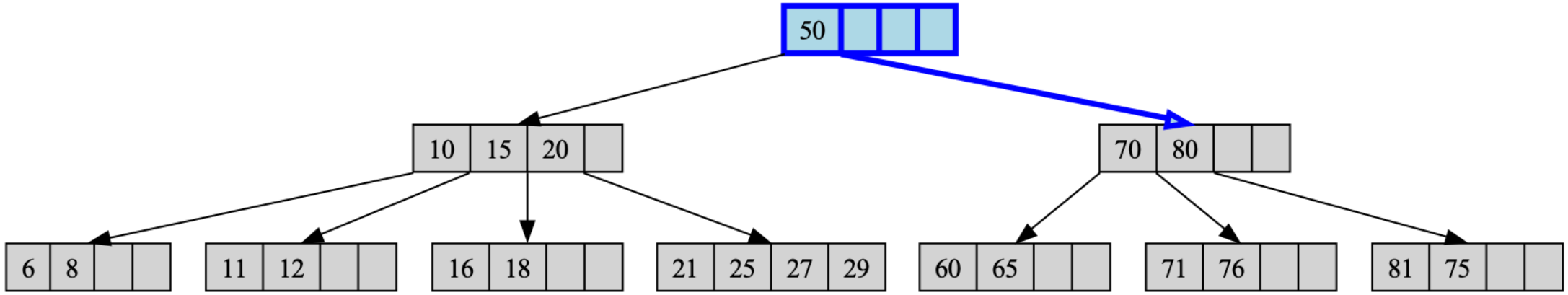

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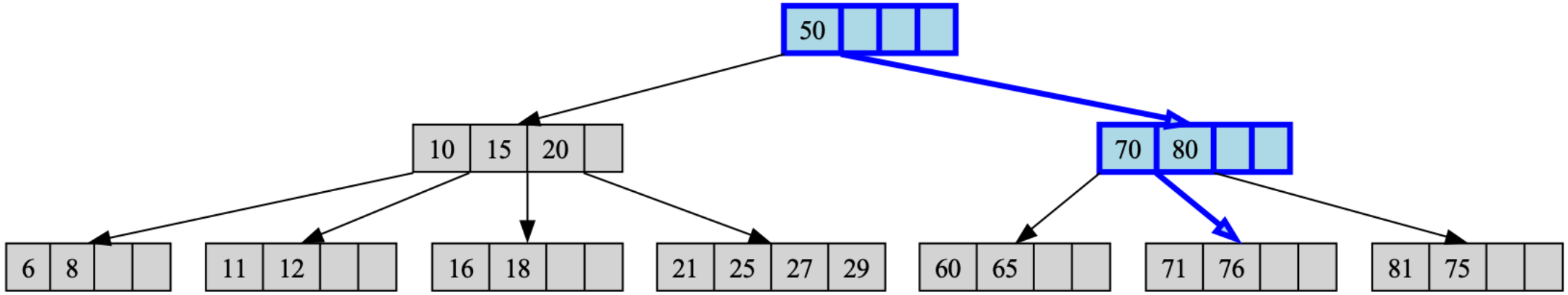

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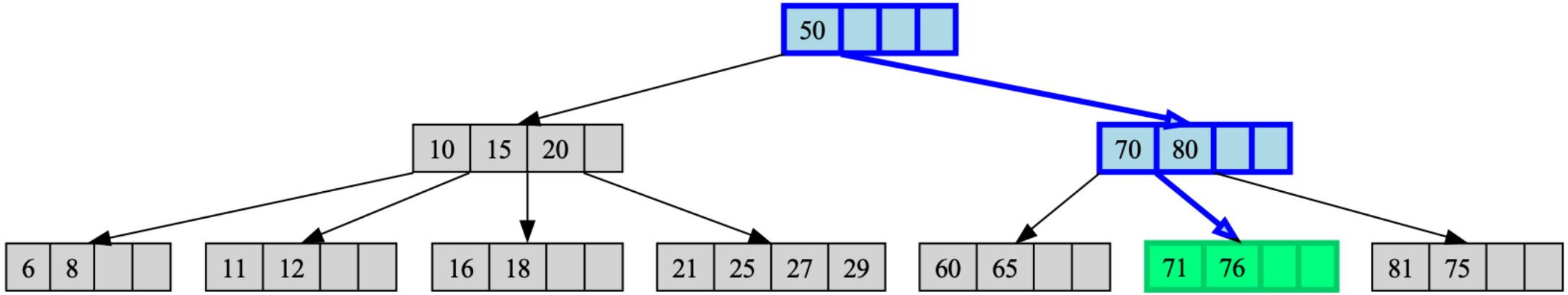
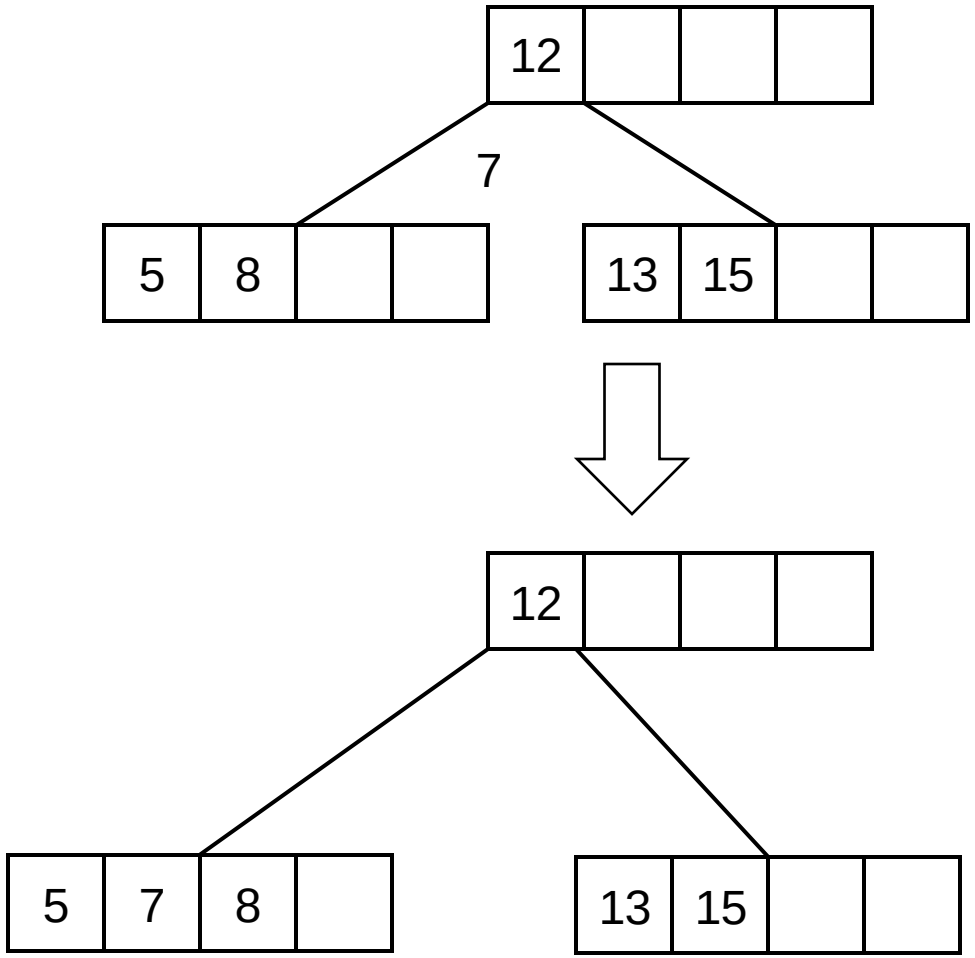


Insertion into a B-tree

- Incoming keys are added directly to a leaf if there is space available.
- When a leaf is full, the keys are divided between the leaves and one key is promoted to the parent.
- If the parent is full the process is repeated until the root is reached and a new root created.

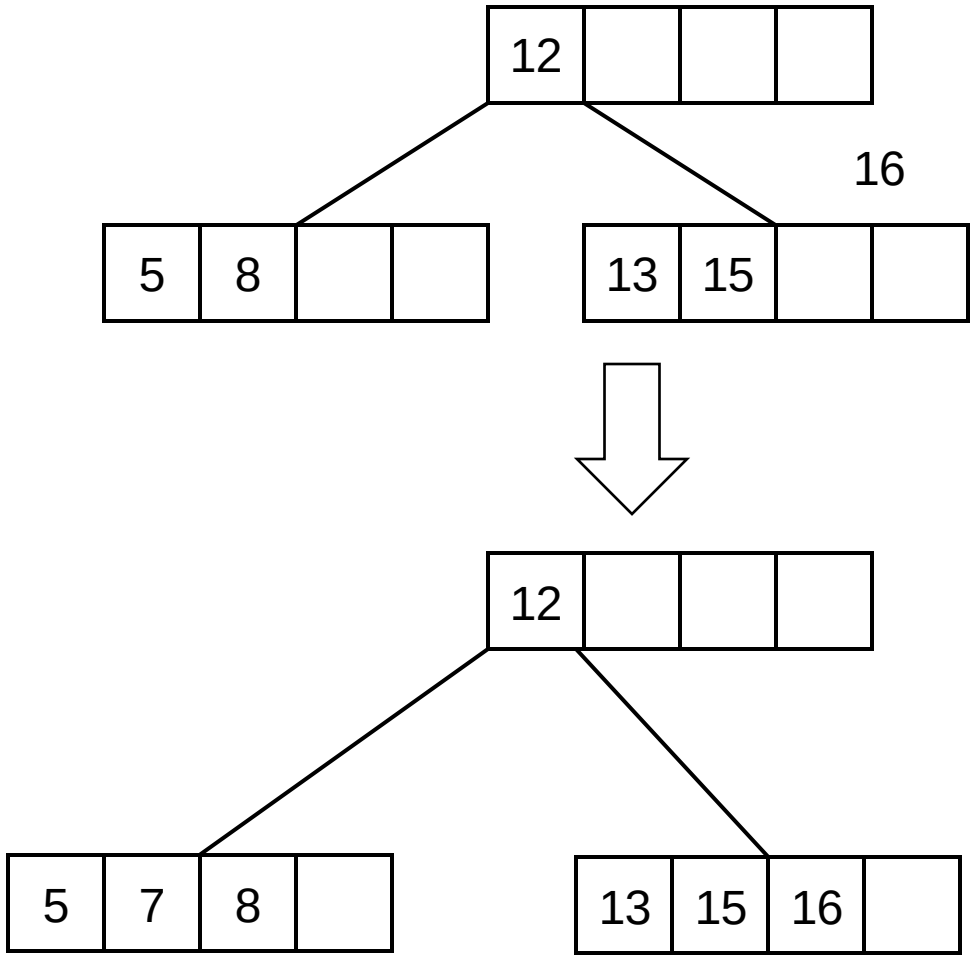
Insertion Example B-tree Order 5

- Insert(7)



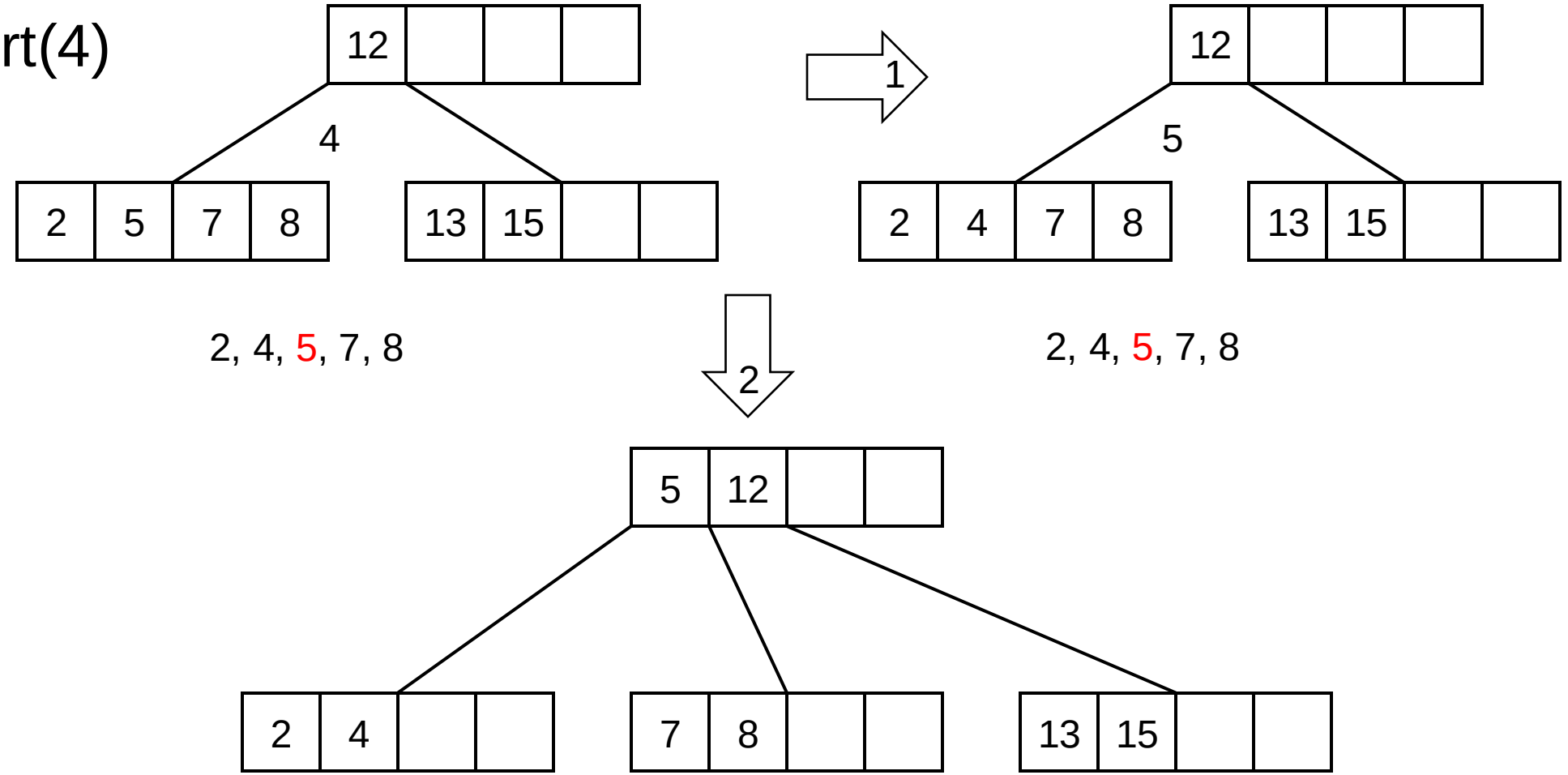
Insertion Example B-tree Order 5

- Insert(16)



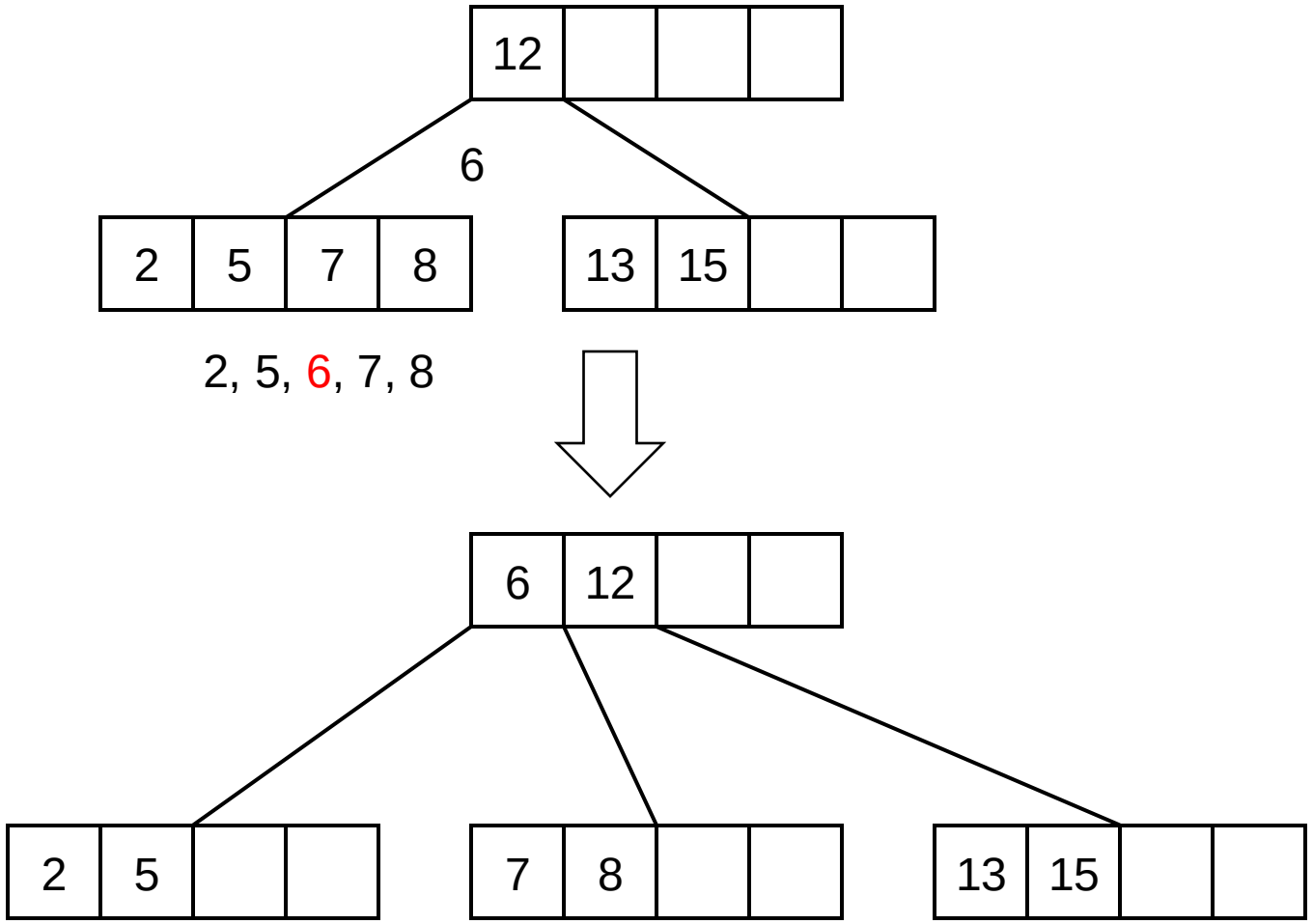
Insertion Example B-tree Order 5

- Insert(4)

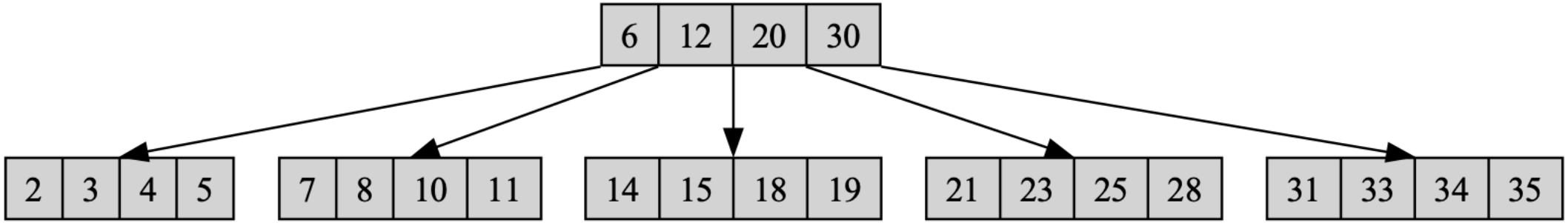


Insertion Example B-tree Order 5

- Insert(6)

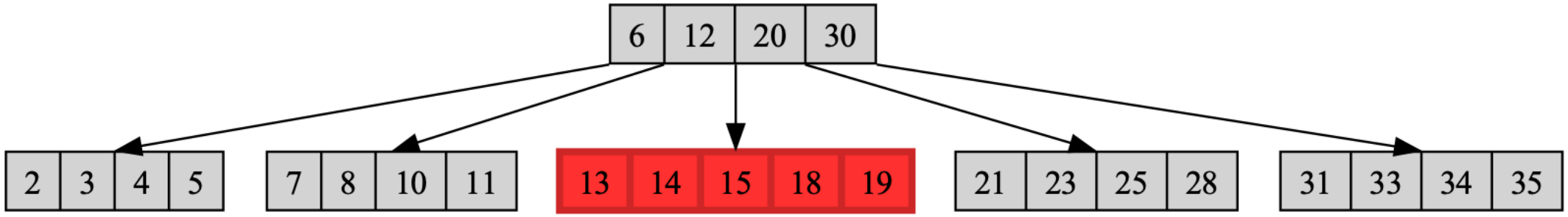


Insertion Example B-tree Order 5



Insertion Example B-tree Order 5

- Insert 13



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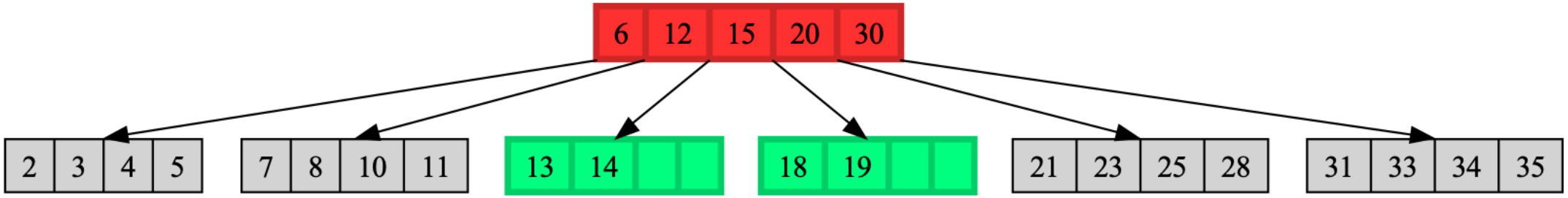
- Insert 13



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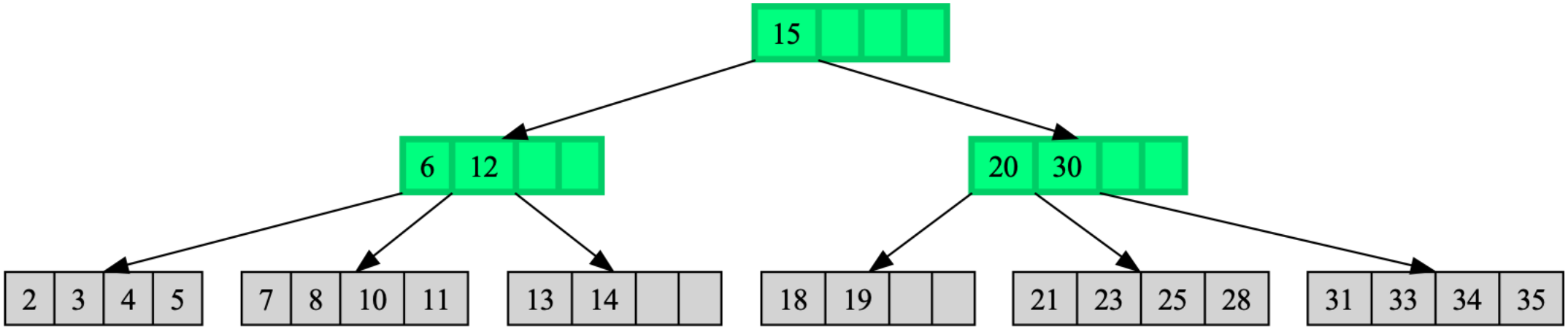
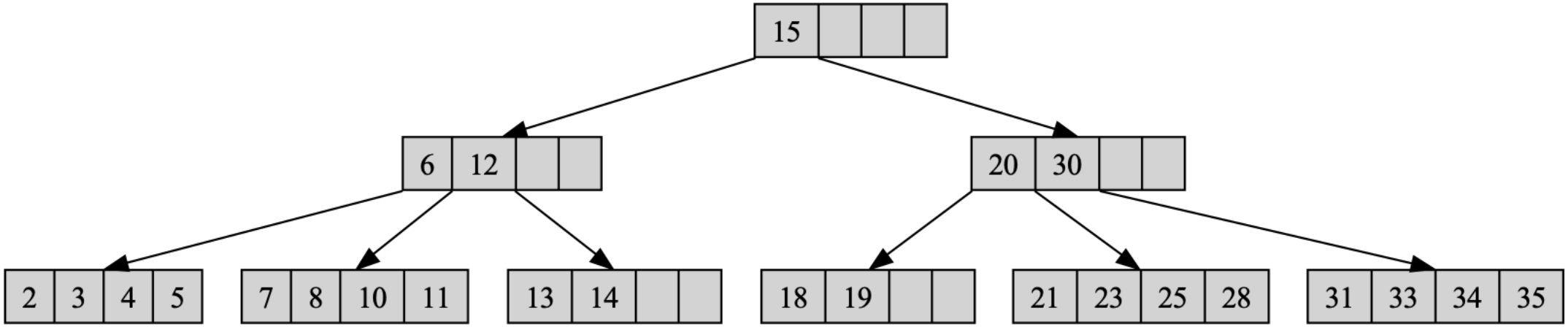


B-tree Deletion

- Deletion is basically the **reverse** of insertion.
- Nodes can not become **less than half full** after a deletion
- If less than half full, nodes need to be **merged**.
- Two cases to look at:
  - Deleting from a leaf: If merging, include the splitting key as it may resplit
  - Deleting from a non-leaf:
    - If either child has more than minimum keys, promote the predecessor/ successor
    - If neither child has more, merge the nodes

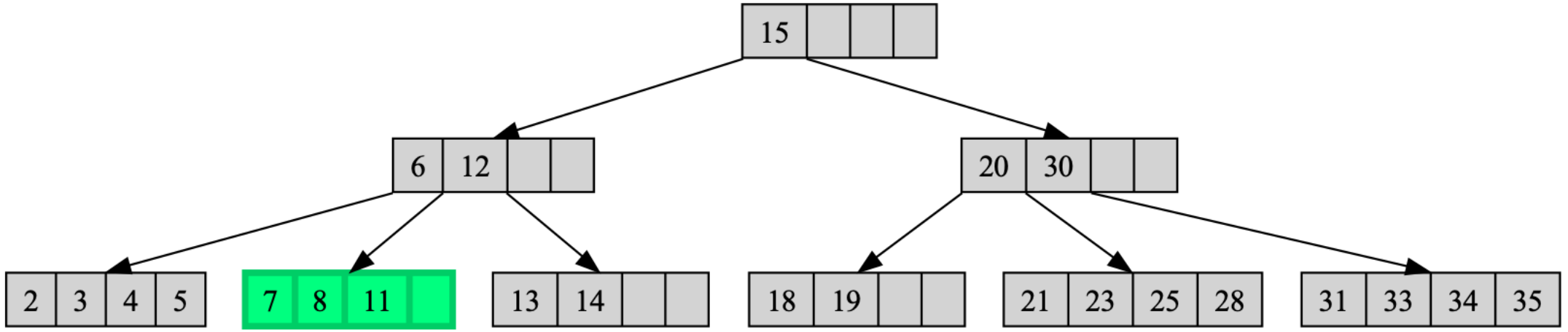
B-tree Delete Example

- Delete 10



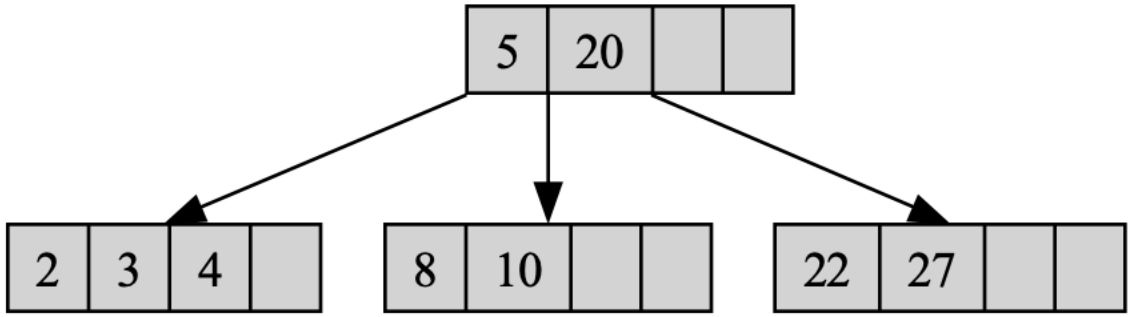
B-tree Delete Example

- Delete 10



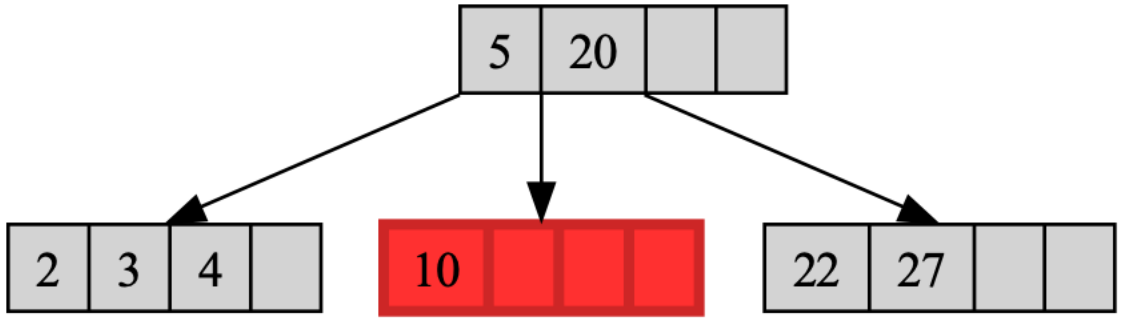
B-tree Delete Example

- Delete 8



B-tree Delete Example

- Delete 8



B-tree Delete Example

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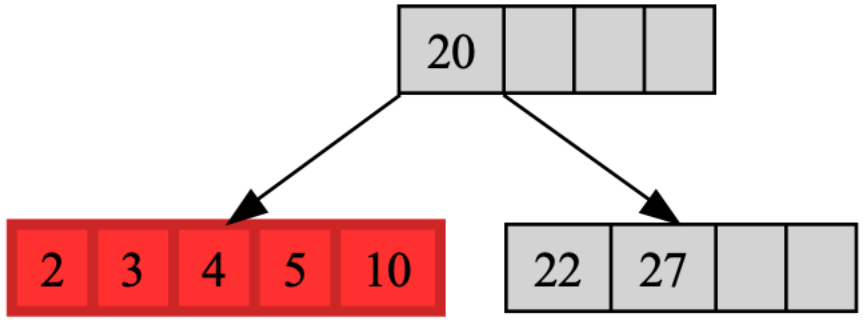

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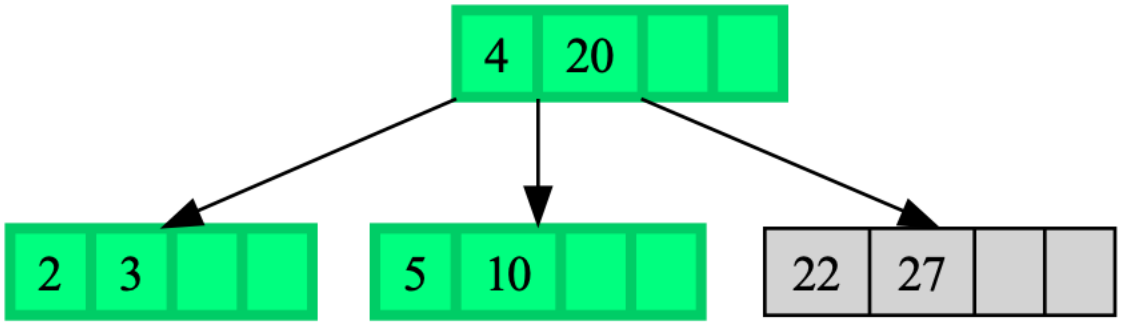

B-tree Delete Example

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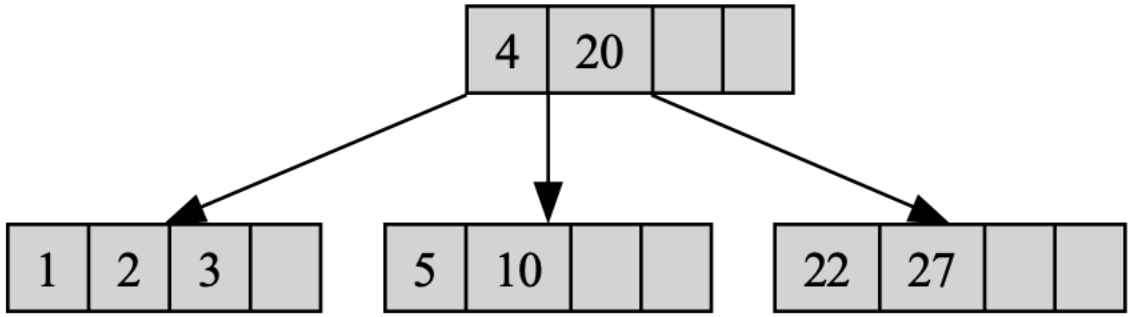

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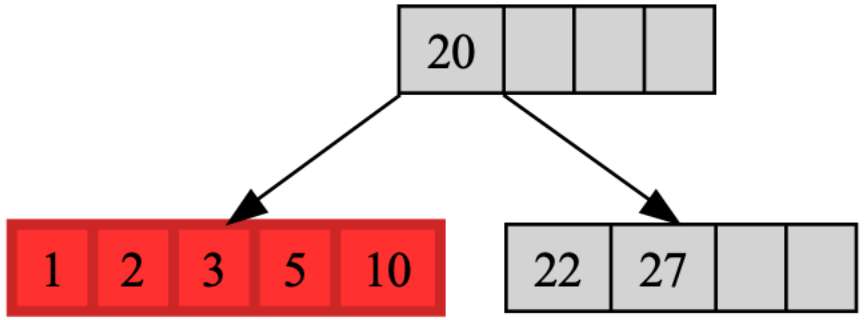

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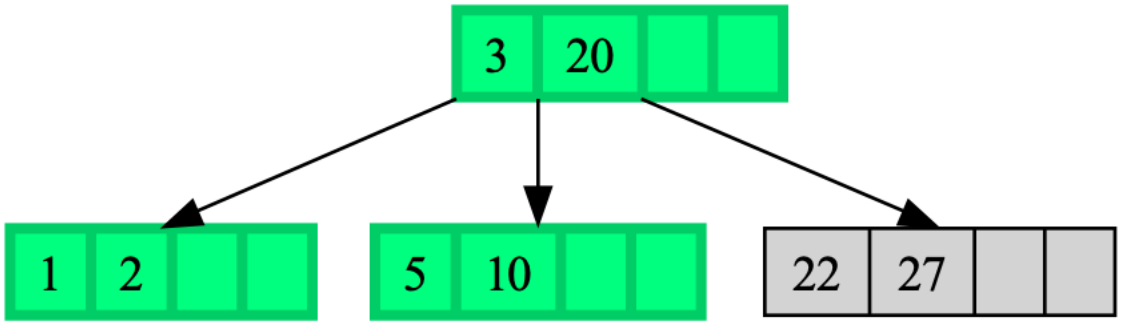

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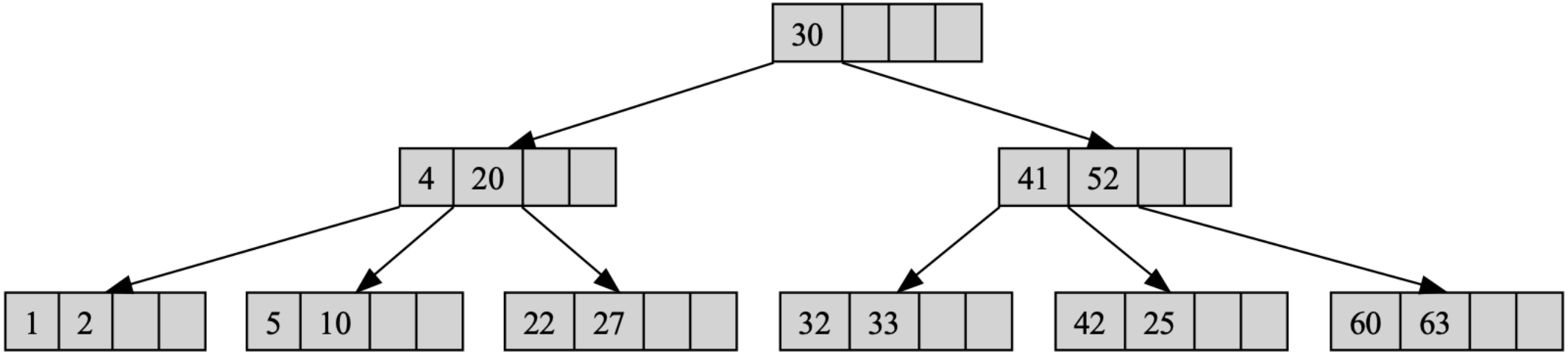

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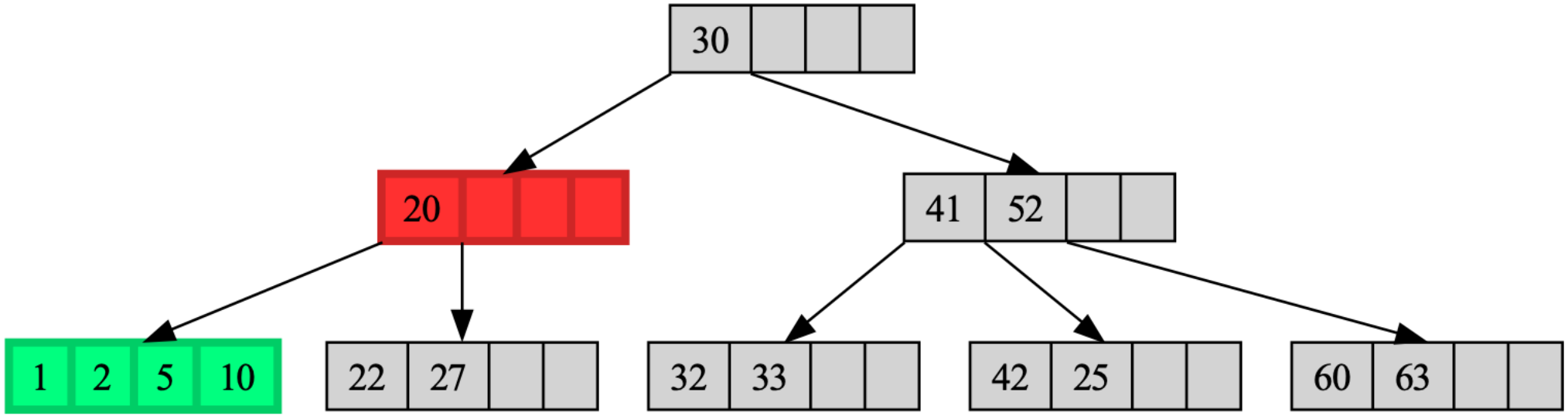

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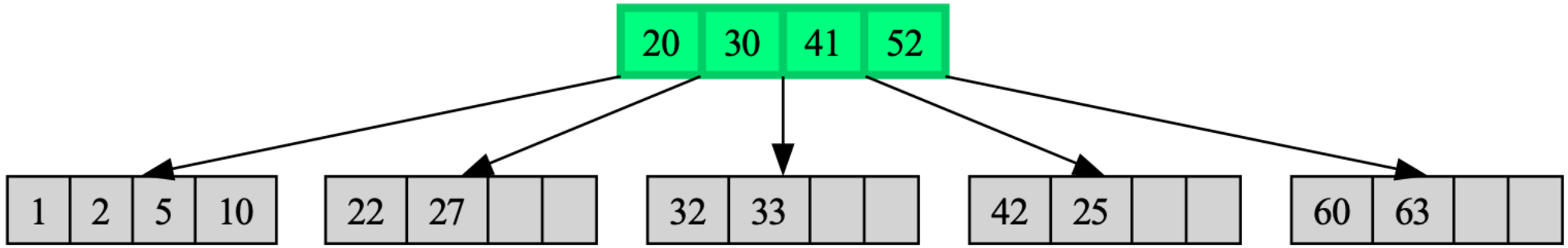

Acknowledgement

These slides have been adapted and borrowed from books on the right as well as the CS340 notes of NIU CS department (Professors: Alhoori, Hou, Lehuta, and Winans) and many google searches.