Homework 3: due 17:00, Friday, 4 November 2016

(1) Consider the following grammar $\mathcal{G} = \langle \Sigma, V, R, S \rangle$:

- $\Sigma = \{a, b\}$.
- $V = \{S\}$, and S is the start variable.
- R consists of the following rules:

$$\begin{array}{lcl} S & \to & SS \mid \epsilon \\ S & \to & aSb \mid bSa \end{array}$$

Determine which of the following words are in $L(\mathcal{G})$.

(i) a^3b^3, i.e., $aaabbb$.
(ii) a^2b^3, i.e., $aabbb$.
(iii) $abba$.
(iv) $a^2b^2a^3b^3$, i.e., $aabbaaabbb$.
(v) $baababba$.

Please substantiate your claim, i.e., if you claim a word is in $L(\mathcal{G})$, you should provide its derivation tree. If you claim it is not, then state your reason why.

(2) Construct the CFG for each of the following languages.

(i) $L_1 = \{a^m b^n \mid m > n\}$.
(ii) $L_2 = \{a^m b^n \mid n > m\}$.
(iii) $L_3 = \{a^{2n} b^n \mid n \geqslant 0\}$.
(iv) $L_4 = \{w\$w^{\mathcal{R}} \mid w \in \{a, b\}^*\}$.
Here L_4 is a language over the alphabet $\{a, b, \$\}$, and $w^{\mathcal{R}}$ denotes the reverse of w. For example, if $w = aabbb$, then $w^{\mathcal{R}} = bbbaa$. If $w = abababa$, then $w^{\mathcal{R}} = abababa$, which is the same as w itself. Likewise, if $w = \epsilon$, then $w^{\mathcal{R}} = \epsilon$.
(v) L_5 is the complement of the language $\{a^n b^n \mid n \geqslant 0\}$ over the alphabet $\{a, b\}$. More formally, $L_5 = \Sigma^* - \{a^n b^n \mid n \geqslant 0\}$, where $\Sigma = \{a, b\}$.

(3) Construct the PDA for each of the languages in (2).

(4) Show that the following languages are not CFL.

(i) $L_1 = \{a^k b^m c^n \mid k \leqslant m \leqslant n\}$.
(ii) $L_2 = \{a^m b^{2m} c^{3m} \mid m \geqslant 0\}$.
(iii) $L_3 = \{a^n \mid n \text{ is a prime number}\}$.

(5) **(bonus point)** Consider the grammar defined in (1). Prove that $w \in L(\mathcal{G})$ if and only if w contains the same number of a's and b's.