Homework 1: due 17:00, 19-September-2016

(1) For an integer $n \neq 0$, define a binary relation $\sim_n$ on $\mathbb{Z}$ as follows. $x \sim_n y$ if and only if $x \equiv y \bmod n$. Show that $\sim_n$ is an equivalence relation. How many equivalence classes are there in $\sim_n$?

(2)
- If $x \equiv 0 \bmod 3$, what is $2x \bmod 3$?
- If $x \equiv 0 \bmod 3$, what is $2x + 1 \bmod 3$?
- If $x \equiv 1 \bmod 3$, what is $2x \bmod 3$?
- If $x \equiv 1 \bmod 3$, what is $2x + 1 \bmod 3$?
- If $x \equiv 2 \bmod 3$, what is $2x \bmod 3$?
- If $x \equiv 2 \bmod 3$, what is $2x + 1 \bmod 3$?

(3) Prove Lemma 1.1 in lecture note 1.

(4) Prove Theorem 1.2 in lecture note 1.