Homework 5: due 17:00, Friday, 6 January 2017

(1) Prove that the following problem is undecidable.

Input: A TM description $\lfloor M \rfloor$.
Task: Output `True`, if M accepts the empty string ϵ. Otherwise, output `False`.

Or equivalently, prove that the language $L = \{\lfloor M \rfloor \mid M \text{ accepts } \epsilon\}$ is undecidable.

(2) Prove that the class **NP** is closed under union and intersection. That is,

- if $L_1, L_2 \in \mathbf{NP}$, then $L_1 \cup L_2 \in \mathbf{NP}$,
- if $L_1, L_2 \in \mathbf{NP}$, then $L_1 \cap L_2 \in \mathbf{NP}$.

(3) Prove that if $\Sigma^* - L \in \mathbf{coNP}$, then $L \in \mathbf{NP}$.

Likewise, if $L \in \mathbf{coNP}$, then $\Sigma^* - L \in \mathbf{NP}$.

(4) Prove that if $\mathbf{NP} \subseteq \mathbf{coNP}$, then $\mathbf{coNP} \subseteq \mathbf{NP}$, and hence, $\mathbf{NP} = \mathbf{coNP}$.

(5) Prove that if $\mathsf{SAT} \in \mathbf{coNP}$, then $\mathbf{NP} \subseteq \mathbf{coNP}$, and hence, $\mathbf{NP} = \mathbf{coNP}$.

Hint: Use the fact that SAT is **NP**-hard.