Homework 4: due 17:00, Monday, 19 December 2016

(1) Describe intuitively the TMs that decide the following languages.

- $L_1 = \Sigma^*$, where Σ is the input alphabet.
- $L_2 = \emptyset$.
- $L_3 = \{w \mid w \text{ is the binary representation of an even number}\}$.
- $L_4 = \{w \mid w \text{ is the binary representation of a prime number}\}$.

(2) Bob defines the following hypothetical language L depending on whether there is a parallel universe.

- If there is a parallel universe, $L = \{0^n \mid n \geqslant 0\}$.
- If there is no parallel universe, $L = \{1^n \mid n \geqslant 0\}$.

Is L decidable? Please explain.

(3) In the class we define that an NTM $\mathcal{M}$ accepts an input word w, if there is an accepting run of $\mathcal{M}$ on w, and it rejects w, if *all* its runs are rejecting.

Now, Bob does not like this definition. He wants to define the acceptance and rejection conditions as follows.

- An ANTM $\mathcal{M}$ accepts w, if there is an accepting run of $\mathcal{M}$ on w and there is no rejecting run.
- Likewise, an NTM $\mathcal{M}$ rejects an input word w, if there is a rejecting run of $\mathcal{M}$ on w and there is no accepting run.

What do you think of Bob's definition? Please explain.

(4) Prove that the following problem is undecidable.

Input: A CFG $\mathcal{G}$ and a DFA $\mathcal{A}$.
Task: Decide whether $L(\mathcal{G}) = L(\mathcal{A})$. That is, return True, if $L(\mathcal{G}) = L(\mathcal{A})$. Otherwise, return False.

(5) Prove that the following problem is undecidable.

Input: A CFG $\mathcal{G}$ with the set of terminals Σ.
Task: Decide whether $\Sigma^* - L(\mathcal{G})$ is CFL. That is, return True, if $\Sigma^* - L(\mathcal{G})$ is CFL. Otherwise, return False.