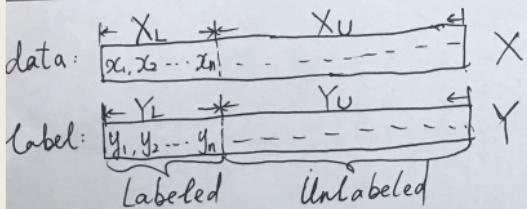


SEMI-SUPERVISED LEARNING

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(1)

SSL (semi-supervised Learning)



Labeled training data $L = \{x_i, y_i\}_{i=1}^L$

Unlabeled data $U = \{x_i\}_{i=L+1}^{L+U}$
(usually $U \gg L$)

Inductive Learning

Our standard classification.

Learn the classifier using (X_L, Y_L)

Classify $X_U \rightarrow Y_U$

Transductive Learning

Model $(X_L, X_U, Y_L) \rightarrow Y_U$

A simple Algorithm: Self-training

steps:

1. Learn classifier using (X_L, Y_L)

2. Classify $X_U \rightarrow Y_U$ (Scores: $S_{L+1}, S_{L+2}, \dots, S_{L+U}$)

3. pick most "confidently predicted labels" $(X_L^{(1)}, Y_L^{(1)})$ ^{(1): first Iteration}

Sort $Y_U = \{y_{U+1}, y_{U+2}, \dots, y_{U+U}\}$ to $\tilde{Y}_U = \{\tilde{y}_{U+1}, \tilde{y}_{U+2}, \dots, \tilde{y}_{U+U}\}$ according to
the scores in descending order

here we assume we that pick up the first 100 data in X_U

4. Enlarge Training data $(X_L + X_L^{(1)}, Y_L + Y_L^{(1)})$. remove those data from (X_U, Y_U)

5. Repeat from step 1

Caution: Prediction mistake can reinforce itself!

(x)

data:	<table border="1"><tr><td>1000</td><td>100</td><td>X_L</td><td>X_U</td></tr><tr><td>X_L</td><td>100</td><td>X_U</td><td></td></tr></table>	1000	100	X_L	X_U	X_L	100	X_U	
1000	100	X_L	X_U						
X_L	100	X_U							
label:	<table border="1"><tr><td>X_L</td><td>100</td><td>X_U</td></tr></table>	X_L	100	X_U					
X_L	100	X_U							

Examples: Pick most confidently predicted labels: build each $X_t \rightarrow Y_t$ with Score

(1) KNN ($K=3$) Score

$X_{t+1} \quad 1, 1, 3 \rightarrow Y_{t+1} = "1"$ with Prob = $\frac{2}{3} \Rightarrow$ Score = $\frac{2}{3}$

$X_{t+2} \quad 1, 1, 1 \rightarrow Y_{t+2} = "1"$ with Prob = 1 \Rightarrow Score = 1 ✓

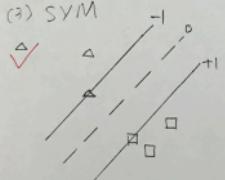
(2) Centroid method score

$X_{t+1} \quad \|X_{t+1} - U_1\| = 3.5 \leftarrow$ prediction X_{t+2} is more
 $\|X_{t+1} - U_2\| = 4.6$ confident compare to X_{t+1} ,

$X_{t+2} \quad \|X_{t+2} - U_1\| = 2.1 \leftarrow$ in this case we can use
 $\|X_{t+2} - U_2\| = 4.7$ some monotonically decreasing
 function to calculate the score

(3) SYM

confident depends on distance from boundary $f=0$
 we can calculate the score according the distance.



$$\text{Solve } \min_{\{\beta, Y_U\}} \| [Y_L, Y_U] - \beta^T [x_L, x_U] \|_F^2$$

initialize Y_U using inductive learning \Rightarrow $\begin{cases} \text{Solve } \min_{\beta} \| Y_L - \beta^T x_L \|_F^2 \\ Y_U = \beta^{*T} x_U \\ Y = [Y_L, Y_U], X = [x_L, x_U] \end{cases}$

M-step : solve $\min_{\beta} \| Y - \beta^T X \|_F^2$

get β^* ($*$: the optimal solution)

E-step : solve $\min_{Y_U} \| [Y_L, Y_U] - \beta^{*T} X \|_F^2$ repeat M,E step until convergence.

$$Y_U^* = \beta^T X_U$$

Transductive learning vs Inductive learning

(4)

$$(\text{Linear Regression}) : \min_B \|Y - B^T X\|_F^2$$

$$\begin{matrix} \text{transductive:} \\ \text{learn} \end{matrix} \min_{\{B, Y_U\}} \| [Y_L, Y_U] - B^T [X_L, X_U] \|_F^2$$

$$\begin{matrix} \text{Inductive:} \\ \text{learn} \end{matrix} \min_{\{B\}} \| Y_L - B^T X_L \|_F^2, \quad Y_U = B^{*\top} X_U$$

Graph Regularization

(6)

Assume the predictions on the entire data $L \cup U$ to be defined by f

Graph regularization assumes that function f is smooth

(Similar examples i and j should have similar prediction f_i and f_j)

$$\text{Objective: } \min_f \sum_{i \in L} (y_i - f_i)^2 + \lambda \sum_{i, j \in L \cup U} w_{ij} (f_i - f_j)^2 \quad \lambda: \text{trade-off parameter.}$$

minimizing the loss on
labeled data

ensures smoothness of labels of labeled
and unlabeled data

(Minimization make f_i and f_j to be very similar if w_{ij} is large)