STAT 5444: Homework #6

Due on December 13, 2016

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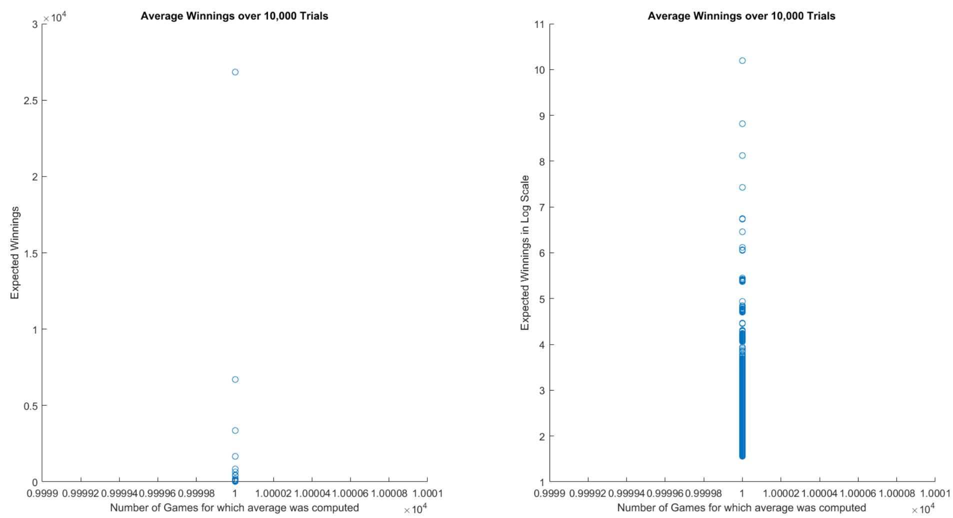
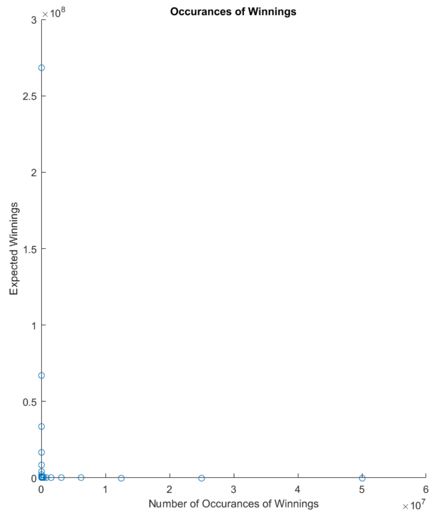
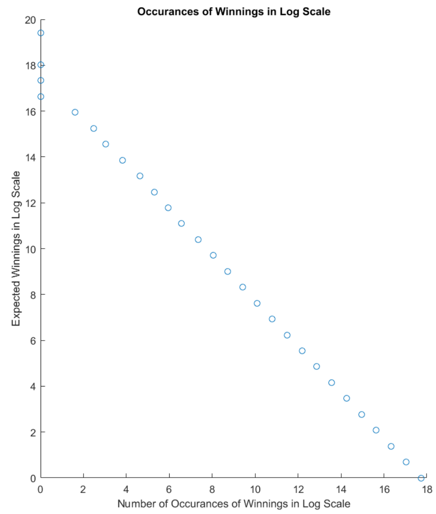
Problem 1

```
K = 10000;
P = 0.5;
T = 10000;
storage = nbinrnd(1, 0.5, K, T);
winnings = 2.^storage;
avgwin = mean(winnings, 2);
tmp = K*ones(T, 1);
subplot(1,2,1)
scatter(tmp, avgwin);
xlabel('Number of Games for which average was computed')
ylabel('Expected Winnings')
title('Average Winnings over 10,000 Trials')
subplot(1,2,2)
scatter(tmp, log(avgwin));
xlabel('Number of Games for which average was computed')
ylabel('Expected Winnings in Log Scale')
title('Average Winnings over 10,000 Trials')

a = unique(storage);
out = [a,histc(storage(:),a)];
out(:, 1) = 2.^out(:, 1);

subplot(1,2,1)
scatter(out(:, 2), out(:, 1));
xlabel('Number of Occurances of Winnings')
ylabel('Expected Winnings')
title('Occurances of Winnings')
subplot(1,2,2)
scatter(log(out(:, 2)), log(out(:, 1)));
xlabel('Number of Occurances of Winnings in Log Scale')
ylabel('Expected Winnings in Log Scale')
title('Occurances of Winnings in Log Scale')
```



Problem 1 continued on next page. . .





I would be willing to spend to pay 1 dollar to play this game because there are not enough outcomes for higher winnings for me to win more than once.

$$E[U] = \sum_{k=1}^{\infty} \frac{log(2^{k-1})}{2^k}$$
$$E[U] = \sum_{k=1}^{\infty} \frac{(k-1)log(2)}{2^k}$$
$$E[U] = log(2) \sum_{k=1}^{\infty} \frac{(k-1)}{2^k}$$
$$E[U] = log(2) \sum_{k=1}^{\infty} \frac{(k-1)}{2^k} = 1 * log(2)$$

The summation converges to 1 by the ratio test. The log utility function was log(2) therefore the utility is 2 dollars.

Problem 2

A Bayes estimator uses the posterior and the loss over the entire parameter space and it is minimized such that the Bayes estimator is better on average. A proper prior has different weights over the parameter space and will sum to one. A improper prior (flat priors) will not always sum to one. Thus, there has to be some part in the parameter space which will yield that the Bayes estimator is better due to the different weights on a proper prior. This will yield a lower loss which can be translated to a lower risk on average, therefore at some point the bayes estimator must be admissible somewhere.

Problem 3

Assume e^{π} is not minmax such that there exist some estimator $R(e^{\pi}, \theta) = C > max_{\theta} R(e, \theta)$. e^{π} is dominating the other risk which means it is inadmissible. Because Bayes estimators are admissible by contradiction e^{π} is minmax.