STAT5444: Homework #3

Due on October 28, 2016 at 3:10pm

Professor Scott Leman 12:20 MWF

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Problem 1

$$p(\mu|X) = \int L(\mu, \Sigma|X)p(\mu)p(\Sigma)d\Sigma$$

$$p(\mu|X) \propto \int \prod_{i=0}^{N} |\Sigma|^{-\frac{1}{2}} e^{\frac{-1}{2}(X-\mu)^T\Sigma^{-1}(X-\mu)}|\Sigma|^{-\frac{k+1}{2}}$$

$$p(\mu|X) \propto \int |\Sigma^{-1}|^{\frac{N+K+1}{2}} e^{\sum_{i=0}^{N} \frac{-1}{2}(X_i-\mu)^T\Sigma^{-1}(X_i-\mu)}$$

$$p(\mu|X) \propto \int |\Sigma^{-1}|^{\frac{N+K+1}{2}} e^{\frac{-1}{2}Tr(\sum_{i=0}^{N}(X_i-\mu)(X_i-\mu)^T\Sigma^{-1})}$$

$$\Sigma^{-1} - InvWishart(\sum_{i=0}^{N}(X_i - \mu)(X_i - \mu)^T, N)$$

$$p(\mu|X) \propto \int |\Sigma^{-1}|^{\frac{N+K+1}{2}} e^{\frac{-1}{2}Tr(\sum_{i=0}^{N}(X_i-\mu)(X_i-\mu)^T\Sigma^{-1})} * \frac{\frac{|\sum_{i=0}^{N}(X_i-\mu)(X_i-\mu)^T|^{N/2}}{2^{\frac{NK}{2}}\Gamma_p(N/2)}}{\frac{|\sum_{i=0}^{N}(X_i-\mu)(X_i-\mu)^T|^{N/2}}{2^{\frac{NK}{2}}\Gamma_p(N/2)}}$$

$$p(\mu|X) \propto \frac{2^{\frac{NK}{2}}\Gamma_p(N/2)}{|\sum_{i=0}^{N}(X_i - \mu)(X_i - \mu)^T|^{N/2}}$$

$$p(\mu|X) \propto |\sum_{i=0}^{N}(X_i - \mu)(X_i - \mu)^T|^{-N/2}$$

$$p(\mu|X) \propto |\sum_{i=0}^{N}[(X_i - \bar{X}) + (\bar{X} - \mu)][(X_i - \bar{X}) + (\bar{X} - \mu)]^T|^{-N/2}$$

$$p(\mu|X) \propto |\sum_{i=0}^{N}(X_i - \bar{X})(X_i - \bar{X})^T + (X_i - \bar{X})(\bar{X} - \mu)^T + (\bar{X} - \mu)(X_i - \bar{X})^T + (\bar{X} - \mu)(\bar{X} - \mu)^T|^{-N/2}$$

$$p(\mu|X) \propto |\sum_{i=0}^{N}(X_i - \bar{X})(X_i - \bar{X})^T + (\bar{X} - \mu)(\bar{X} - \mu)^T|^{-N/2}$$

$$S^2 = \frac{\sum_{i=0}^{N}(X_i - \bar{X})(X_i - \bar{X})^T}{N - K}$$

$$p(\mu|X) \propto |(N - K)S^2 + N(\bar{X} - \mu)(\bar{X} - \mu)^T|^{-N/2}$$

$$Sherman - Morrison - Woodbury |A + XBX^T| = |A||B||B^T + X^TA^{-1}X|$$

$$p(\mu|X) \propto \left(|(N - K)S^2||1 + (\bar{X} - \mu)[(N - K)S^2]^{-1}(\bar{X} - \mu)^T|\right)^{-N/2}$$

$$p(\mu|X) \propto |1 + (\bar{X} - \mu)[(N - K)S^2]^{-1}(\bar{X} - \mu)^T|^{-(N-k+k)/2}$$

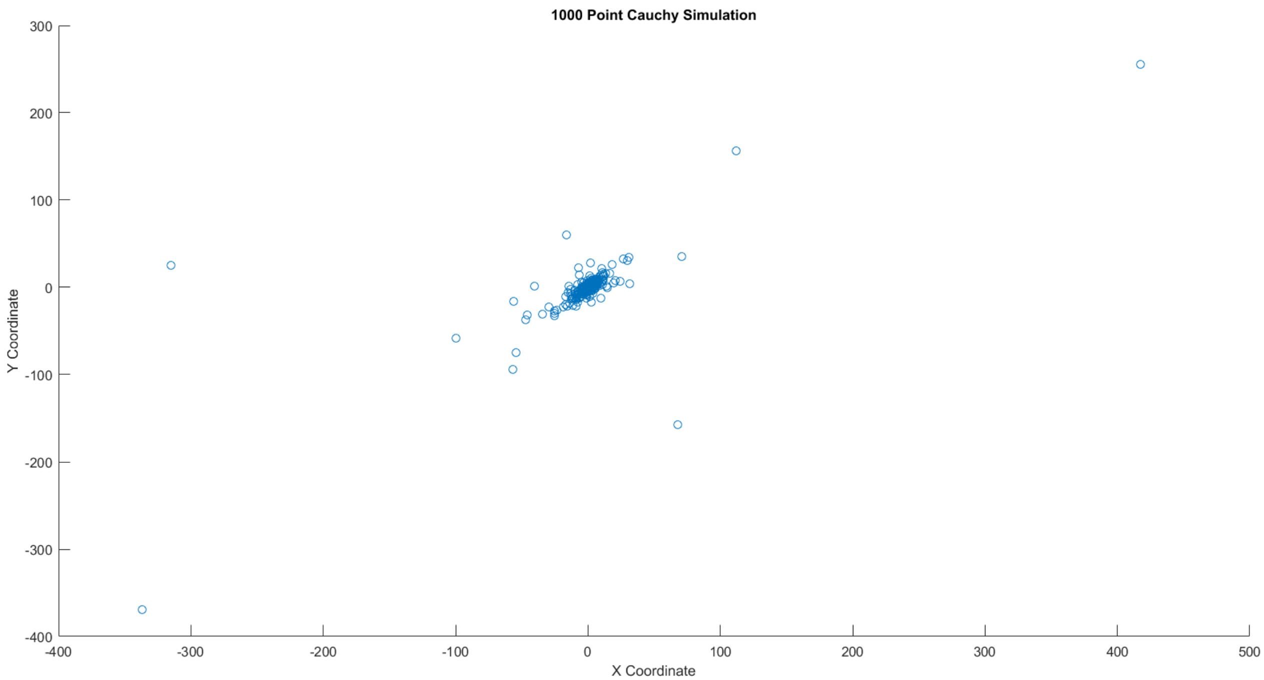
$$p(\mu|X) \propto |1 + \frac{N - k}{N - k}(\bar{X} - \mu)[(N - K)S^2]^{-1}(\bar{X} - \mu)^T|^{-(N-k+k)//2}$$

$$p(\mu|X) \propto |1 + \frac{1}{N - k}(\bar{X} - \mu)[(N - K)S^2]^{-1}(\bar{X} - \mu)^T|^{-(N-k+k)//2}$$

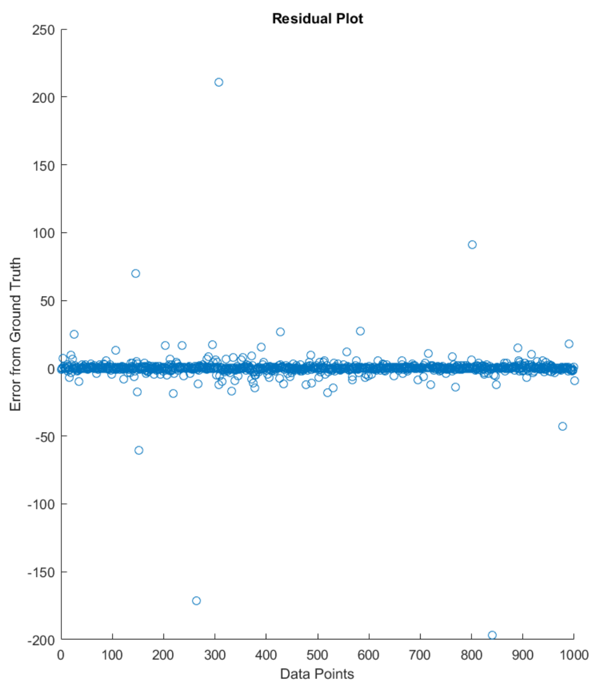
$$\mu - t_{(N-k)}(\bar{X}, [(N - K)S^2]^{-1})$$

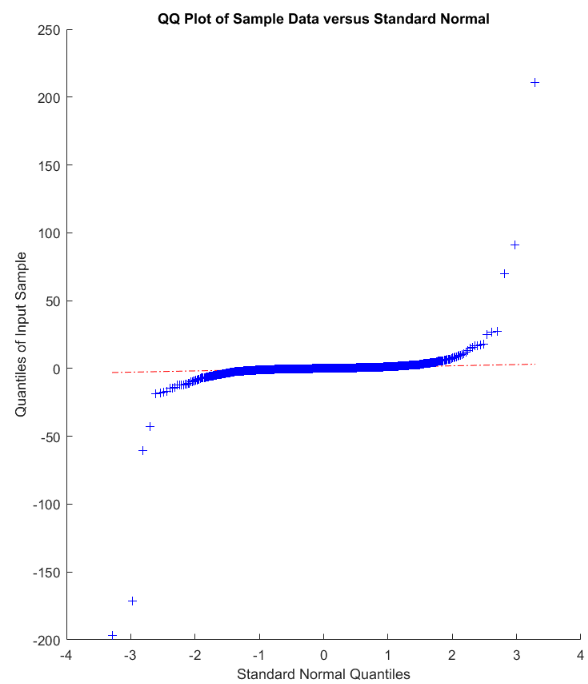
Problem 2

Part 1:



Part 2:





```
%PART 1 %%%%%%%%%%%%%%%%%%%
C = [1 0.8; 0.8 1;];
cauchy = mvtrnd(C, 1, 1000);
figure(1)
scatter(cauchy(:, 1), cauchy(:, 2));
title('1000 Pgoint Cauchy Simulation')
xlabel('X Coordinate')
ylabel('Y Coordinate')

%PART 2 %%%%%%%%%%%%%%%%%%%
x = [ones(size(cauchy, 1), 1) cauchy(:, 1)];
y = cauchy(:, 2);
sigma = 1;
%beta = mvnrnd((x'*x)^(-1)*x'*y, sigma^2 * (x'*x)^(-1));
beta = ((x'*x)^(-1)*x'*y)';
y_plot = x * beta';
resid = y - y_plot;

figure(2);
subplot(1, 2, 1)
scatter(1:size(resid, 1), resid);
title('Residual Plot')
xlabel('Data Points')
ylabel('Error from Ground Truth')

subplot(1, 2, 2)
qqplot(resid);
```

Part 3:

$$\mu = (X^T X)^{-1} Y$$
$$\psi = \sigma^2 (X^T X)^{-1}$$
$$\gamma - Gamma(a, b)$$
$$\beta - N(\mu, \frac{\psi}{\gamma})$$

$$p(\beta|\mu, \psi) = \int L(\beta|\mu, \psi, \gamma) p(\gamma) d\gamma$$

$$p(\beta|\mu, \psi) \propto \int |\frac{\psi}{\gamma}|^{-1/2} e^{\frac{-1}{2}(\beta-\mu)^T |\frac{\psi}{\gamma}|^{-1}(\beta-\mu)} * \gamma^{a-1} e^{-b\gamma} d\gamma$$

$$p(\beta|\mu, \psi) \propto \int |\psi|^{-1/2} \gamma^{\frac{1}{2}+a-1} e^{\frac{-1}{2}(\beta-\mu)^T |\frac{\psi}{\gamma}|^{-1}(\beta-\mu)} * e^{-b\gamma} d\gamma$$

$$p(\beta|\mu, \psi) \propto |\psi|^{-1/2} \int \gamma^{\frac{1}{2}+a-1} e^{-\gamma\left(\left[\frac{1}{2}(\beta-\mu)^T |\psi|^{-1}(\beta-\mu)\right]+b\right)} d\gamma$$

$$\gamma - Gamma(\frac{1}{2} + a, \left(\left[\frac{1}{2}(\beta-\mu)^T |\psi|^{-1}(\beta-\mu)\right] + b\right))$$

$$p(\beta|\mu, \psi) \propto \int \gamma^{\frac{1}{2}+a-1} e^{-\gamma\left(\left[\frac{1}{2}(\beta-\mu)^T |\psi|^{-1}(\beta-\mu)\right]+b\right)} d\gamma * \frac{\frac{\left(\left[\frac{1}{2}(\beta-\mu)^T |\psi|^{-1}(\beta-\mu)\right]+b\right)^{(\frac{1}{2}+a)}}{\Gamma(\frac{1}{2}+a)}}{\frac{\left(\left[\frac{1}{2}(\beta-\mu)^T |\psi|^{-1}(\beta-\mu)\right]+b\right)^{(\frac{1}{2}+a)}}{\Gamma(\frac{1}{2}+a)}}$$

$$p(\beta|\mu, \psi) \propto \frac{\Gamma(\frac{1}{2} + a)}{\left(\left[\frac{1}{2}(\beta-\mu)^T |\psi|^{-1}(\beta-\mu)\right] + b\right)^{(\frac{1}{2}+a)}}$$

$$p(\beta|\mu,\psi) \propto \frac{\Gamma(\frac{1}{2}+a)}{\left(\left[\frac{1}{2}(\beta-\mu)^T|\psi|^{-1}(\beta-\mu)\right]+b\right)^{(\frac{1}{2}+a)}}$$
$$p(\beta|\mu,\psi) \propto \frac{1}{\left(b\left[\left[\frac{1}{2b}(\beta-\mu)^T|\psi|^{-1}(\beta-\mu)\right]+1\right]\right)^{(\frac{1}{2}+a)}}$$
$$p(\beta|\mu,\psi) \propto \frac{1}{\left(\left[\frac{1}{2b}(\beta-\mu)^T|\psi|^{-1}(\beta-\mu)\right]+1\right)^{(\frac{1}{2}+a)}}$$
$$p(\beta|\mu,\psi) \propto \frac{1}{\left(\left[\frac{1}{2\frac{1}{2}}(\beta-\mu)^T|\psi|^{-1}(\beta-\mu)\right]+1\right)^{(\frac{1}{2}+\frac{1}{2})}}, a=\frac{1}{2}, b=\frac{1}{2}$$
$$p(\beta|\mu,\psi) \propto \frac{1}{\left(\left[(\beta-\mu)^T|\psi|^{-1}(\beta-\mu)\right]+1\right)}, a=\frac{1}{2}, b=\frac{1}{2}$$

$$\beta - Cauchy(\mu,\psi), a=\frac{1}{2}, b=\frac{1}{2}$$
$$\beta - Cauchy((X^TX)^{-1}Y, \sigma^2(X^TX)^{-1}), a=\frac{1}{2}, b=\frac{1}{2}$$

Problem 3

Part 1:

$$\beta - N(\mu, \frac{\phi}{\gamma})$$
$$\gamma - Gamma(1/2, 1/2)$$
$$\beta - Cauchy(\mu, \phi)$$
$$L(\beta|\gamma,\mu,\phi) \propto (\frac{\phi}{\gamma})^{1/2} e^{\frac{-\phi}{2\gamma}(\beta-\mu)^2} * \gamma^{a-1} * e^{-b\gamma}$$
$$p(B|-) \rightarrow \beta - N(\mu, \frac{\phi}{\gamma})$$
$$p(\phi|-) \propto (\frac{\phi}{\gamma})^{1/2} e^{\frac{-\phi}{2\gamma}(\beta-\mu)^2} * \gamma^{a-1} * e^{-b\gamma}$$
$$p(\phi|-) \propto (\phi)^{1/2} e^{\frac{-\phi}{2\gamma}(\beta-\mu)^2}$$
$$\phi - N(\mu, 1/\gamma)$$
$$\gamma_i - Gamma(1/2, 1/2)(Notsure)$$

Part 2:
Initialize all the parameters to some values
Pick some Runs:
Apply full conditional of beta
Apply full conditional of phi using previous values of other hyperparameters
Apply full conditional of gamma using previous values of other hyperparameters
end
Account for burn in and plot histograms and trace plots

Problem 4

Part 1:

$$
\begin{aligned}
tr(A+B) &= tr(A) + tr(B) \\
tr(A+B) &= \sum_{i=1}^{N}(a_{i,i} + b_{i,i}) \\
tr(A+B) &= \sum_{i=1}^{N} a_{i,i} + \sum_{i=1}^{N} b_{i,i} \\
tr(A+B) &= tr(A) + tr(B)
\end{aligned}
$$

Part 2:

$$
\begin{aligned}
tr(AB) &= tr(BA) \\
tr(AB) &= \sum_{i=1}^{N}\sum_{j=1}^{M} A_{i,j}B_{j,i} \\
tr(AB) &= \sum_{j=1}^{M}\sum_{i=1}^{N} B_{j,i}A_{i,j} \\
tr(AB) &= tr(BA)
\end{aligned}
$$

Problem 5

$$x_{ij} - N([1,j]\theta_i, \sigma^2 = 1/\phi)$$
$$\theta_i - N(\theta_0, \Sigma)$$

Conjugate Priors:

$$\phi - Gamma(a,b)$$
$$\theta_0 - N(\eta, \Psi)$$
$$\Sigma^{-1} - Wishart((\rho R)^{-1}, \rho)$$
$$p(\Sigma^{-1}) \propto |\Sigma^{-1}|^{(\rho-2-1)/2} e^{\frac{-1}{2} tr(\rho R \Sigma^{-1})}$$

Full Conditional Probabilities:

$$L(\theta_0, \phi, \Sigma^{-1}|\Theta, X) \propto \prod_{i=1}^{I} \Big[\prod_{j=1}^{J} \phi^{1/2} e^{\frac{-\phi}{2}(x_{ij}-[1,j]\theta_i)^2} \Big] |\Sigma|^{-1/2} e^{[\theta_i-\theta_0]^T \Sigma^{-1} [\theta_i - \theta_0]}$$
$$p(\phi|\theta_0, \Sigma^{-1}\Theta, X) \propto L(\theta_0, \phi, \Sigma^{-1}|\Theta, X) * p(\phi)$$
$$p(\phi|\theta_0, \Sigma^{-1}\Theta, X) \propto \prod_{i=1}^{I} \Big[\prod_{j=1}^{J} \phi^{1/2} e^{\frac{-\phi}{2}(x_{ij}-[1,j]\theta_i)^2} \Big] * \phi^{a-1} e^{-b\phi}$$
$$p(\phi|\theta_0, \Sigma^{-1}\Theta, X) \propto \phi^{\frac{IJ}{2}+a-1} e^{-\phi \left(\left[\sum_{i=1}^{I} \sum_{j=1}^{J} \frac{-\phi}{2}(x_{ij}-[1,j]\theta_i)^2 \right] + b \right)}$$
$$\phi - Gamma(\frac{IJ}{2} + a, \big[\sum_{i=1}^{I} \sum_{j=1}^{J} \frac{-\phi}{2}(x_{ij} - [1,j]\theta_i)^2 \big] + b)$$

Note: Trace factoring works only if the dimensionality is the same!

$$L(\theta_0, \phi, \Sigma^{-1}|\Theta, X) \propto \prod_{i=1}^{I} \Big[\prod_{j=1}^{J} \phi^{1/2} e^{\frac{-\phi}{2}(x_{ij}-[1,j]\theta_i)^2} \Big] |\Sigma|^{-1/2} e^{[\theta_i-\theta_0]^T \Sigma^{-1} [\theta_i - \theta_0]}$$
$$p(\Sigma^{-1}|\theta_0, \phi, \Theta, X) \propto L(\theta_0, \phi, \Sigma^{-1}|\Theta, X) * p(\Sigma^{-1})$$
$$p(\Sigma^{-1}|\theta_0, \phi, \Theta, X) \propto \prod_{i=1}^{I} |\Sigma|^{-1/2} e^{[\theta_i-\theta_0]^T \Sigma^{-1} [\theta_i - \theta_0]} * |\Sigma^{-1}|^{(\rho-2-1)/2} e^{\frac{-1}{2} tr(\rho R \Sigma^{-1})}$$
$$p(\Sigma^{-1}|\theta_0, \phi, \Theta, X) \propto |\Sigma^{-1}|^{\frac{I+\rho-2-1}{2}} e^{\sum_{i=1}^{I} [\theta_i-\theta_0]^T \Sigma^{-1} [\theta_i - \theta_0]} * e^{\frac{-1}{2} tr(\rho R \Sigma^{-1})}$$
$$p(\Sigma^{-1}|\theta_0, \phi, \Theta, X) \propto |\Sigma^{-1}|^{\frac{I+\rho-2-1}{2}} e^{tr(\sum_{i=1}^{I} [\theta_i-\theta_0][\theta_i - \theta_0]^T \Sigma^{-1})} * e^{\frac{-1}{2} tr(\rho R \Sigma^{-1})}$$
$$p(\Sigma^{-1}|\theta_0, \phi, \Theta, X) \propto |\Sigma^{-1}|^{\frac{I+\rho-2-1}{2}} e^{tr\left(\left[\sum_{i=1}^{I} [\theta_i-\theta_0][\theta_i - \theta_0]^T + \rho R \right] \Sigma^{-1} \right)}$$
$$\Sigma^{-1} - Wishart_2 \Big(\big[\sum_{i=1}^{I} [\theta_i - \theta_0][\theta_i - \theta_0]^T + \rho R \big]^{-1}, I + \rho \Big)$$

Problem 5 continued on next page. . .

$$L(\theta_0, \phi, \Sigma^{-1}|\Theta, X) \propto \prod_{i=1}^{I}\left[\prod_{j=1}^{J}\phi^{1/2}e^{\frac{-\phi}{2}(x_{ij}-[1,j]\theta_i)^2}\right]|\Sigma|^{-1/2}e^{[\theta_i-\theta_0]^T\Sigma^{-1}[\theta_i-\theta_0]}$$

$$p(\theta_0|\Sigma^{-1}, \phi, \Theta, X) \propto \prod_{i=1}^{I}|\Sigma|^{-1/2}e^{[\theta_i-\theta_0]^T\Sigma^{-1}[\theta_i-\theta_0]}|\Psi|^{-1/2}e^{[\theta_0-\eta]^T\Psi^{-1}[\theta_0-\eta]}$$

$$p(\theta_0|\Sigma^{-1}, \phi, \Theta, X) \propto e^{\sum_{i=0}^{I}[\theta_i-\theta_0]^T\Sigma^{-1}[\theta_i-\theta_0]}e^{[\theta_0-\eta]^T\Psi^{-1}[\theta_0-\eta]}$$

$$p(\theta_0|\Sigma^{-1}, \phi, \Theta, X) \propto e^{Tr(\sum_{i=0}^{I}[\theta_i-\theta_0][\theta_i-\theta_0]^T\Sigma^{-1})}e^{[\theta_0-\eta]^T\Psi^{-1}[\theta_0-\eta]}$$

$$p(\theta_0|\Sigma^{-1}, \phi, \Theta, X) \propto e^{Tr(\sum_{i=0}^{I}[(\theta_i-\bar{\theta_i})+(\bar{\theta_i}-\theta_0)][(\theta_i-\bar{\theta_i})+(\bar{\theta_i}-\theta_0)]^T\Sigma^{-1})}e^{[\theta_0-\eta]^T\Psi^{-1}[\theta_0-\eta]}$$

$$p(\theta_0|\Sigma^{-1}, \phi, \Theta, X) \propto e^{Tr(\sum_{i=0}^{I}[(\theta_i-\bar{\theta_i})(\theta_i-\bar{\theta_i})^T+(\bar{\theta_i}-\theta_0)(\bar{\theta_i}-\theta_0)^T]\Sigma^{-1})}e^{[\theta_0-\eta]^T\Psi^{-1}[\theta_0-\eta]}$$

$$p(\theta_0|\Sigma^{-1}, \phi, \Theta, X) \propto e^{Tr(N(\bar{\theta_i}-\theta_0)(\bar{\theta_i}-\theta_0)^T\Sigma^{-1})}e^{[\theta_0-\eta]^T\Psi^{-1}[\theta_0-\eta]}$$

$$p(\theta_0|\Sigma^{-1}, \phi, \Theta, X) \propto e^{Tr(N(\bar{\theta_i}\bar{\theta_i}^T-\bar{\theta_i}\theta_0^T-\theta_0\bar{\theta_i}^T+\theta_0\theta_0^T)\Sigma^{-1})}e^{Tr([\theta_0\theta_0^T-\theta_0\eta^T-\eta\theta_0^T+\eta^T\eta]\Psi^{-1})}$$

$$p(\theta_0|\Sigma^{-1}, \phi, \Theta, X) \propto e^{Tr(N(\theta_0\theta_0^T-\bar{\theta_i}\theta_0^T-\theta_0\bar{\theta_i}^T)\Sigma^{-1})}e^{Tr([\theta_0\theta_0^T-\theta_0\eta^T-\eta\theta_0^T]\Psi^{-1})}$$

$$p(\theta_0|\Sigma^{-1}, \phi, \Theta, X) \propto e^{Tr(N(\theta_0\theta_0^T-2\theta_0\bar{\theta_i}^T)\Sigma^{-1})}e^{Tr([\theta_0\theta_0^T-2\theta_0\eta^T]\Psi^{-1})}$$

$$p(\theta_0|\Sigma^{-1}, \phi, \Theta, X) \propto e^{Tr(\theta_0\theta_0^T(N\Sigma^{-1}+\Psi^{-1})-2\theta_0(\bar{\theta_i}^T\Sigma^{-1}+\eta^T\Psi^{-1}))}$$

$$\theta_0 - Normal((N\Sigma^{-1}+\Psi^{-1})^{-1} * (\bar{\theta_i}^T\Sigma^{-1}+\eta^T\Psi^{-1}), (N\Sigma^{-1}+\Psi^{-1})^{-1})$$

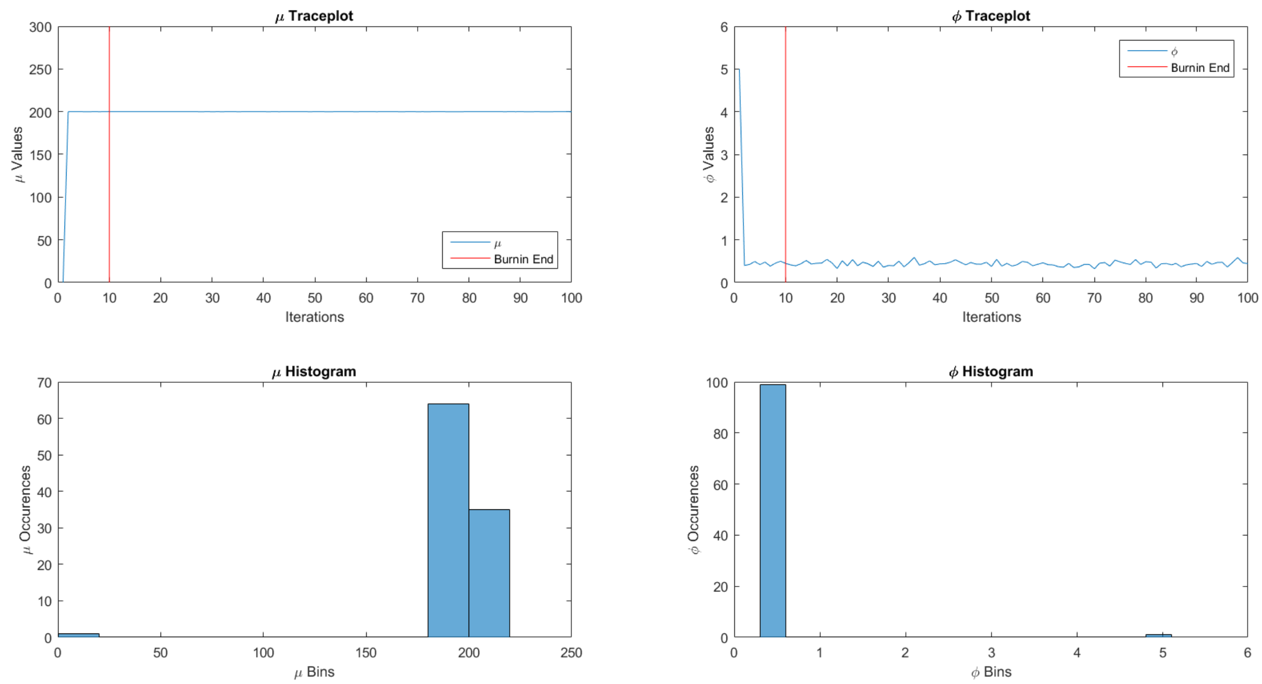
Problem 6

Part 1:

$$\mu - N(\bar{x}, \frac{1}{N\phi})$$

$$\phi - Gamma(\frac{N}{2}, \frac{\sum_i (x_i - \mu)^2}{2}$$

Part 2:
Burn in time = 10
Run time = 100




```
mu = 200;
phi = 0.5;
x = normrnd(mu, 1/sqrt(phi), [100 1]);
xbar = sum(x)/100;
N = 100;
run = 100;
burnin = 10;
mu0 = zeros(run, 1);
phi0 = zeros(run, 1);
mu0(1) = 0;
phi0(1) = 5;
for i=2:run
    mu0(i) = normrnd(xbar, 1/(phi0(i-1)*N));
    beta = sum(bsxfun(@minus, x, mu0(i)).^2)/2;
    phi0(i) = gamrnd(N/2, 1/beta);
end
subplot(2, 2, 1)
```

Problem 6 continued on next page. . .

```
plot(1:run, mu0)
hold on
line([burnin burnin], [0, 300], 'Color', 'r');
hold off
ylabel('\mu Values')
xlabel('Iterations')
title('\mu Traceplot')
legend('\mu', 'Burnin End', 'Location', 'southeast')
subplot(2, 2, 2)
plot(1:run, phi0)
hold on
line([burnin burnin], [0, 6], 'Color', 'r');
hold off
ylabel('\phi Values')
xlabel('Iterations')
title('\phi Traceplot')
legend('\phi', 'Burnin End', 'Location', 'northeast')
subplot(2, 2, 3)
histogram(mu0)
ylabel('\mu Occurences')
xlabel('\mu Bins')
title('\mu Histogram')
subplot(2, 2, 4)
histogram(phi0)
ylabel('\phi Occurences')
xlabel('\phi Bins')
title('\phi Histogram')
```

Problem 7

$$x_i - Bin(N,p)$$
$$I(p) = E([\frac{d}{d\theta}L(p|y)]^2|p)$$
$$I(p) \propto E(\Big[\frac{d}{dp}log(\frac{N}{x}p^x(1-p)^(N-x))\Big]^2|p)$$
$$I(p) \propto E(\Big[\frac{d}{dp}xlog(p) + (N-x)log(1-p)\Big]^2|p)$$
$$I(p) \propto E(\left[\frac{x}{p} - \frac{n-x}{1-p}\right]^2|p)$$
$$I(p) \propto E(\left[\frac{x^2}{p^2} - 2\frac{x}{p}\left(\frac{n-x}{1-p}\right) + \left(\frac{n-x}{1-p}\right)^2\right]|p)$$
$$I(p) \propto E(\left[\frac{x^2}{p^2} - \left(\frac{2nx-2x^2}{p(1-p)}\right) + \left(\frac{n-x}{1-p}\right)^2\right]|p)$$
$$I(p) \propto E(\left[\frac{x^2(1-p)^2 - 2(p(1-p)x(n-x)) + p^2(n-x)^2}{p^2(1-p)^2}\right]|p)$$
$$I(p) \propto E(\left[\frac{p^2x^2 - 2px^2 + x^2 - 2(pxn - px^2 - p^2xn + p^2x^2) + p^2n^2 - 2nxp^2 + p^2x^2}{p^2(1-p)^2}\right]|p)$$
$$I(p) \propto E(\left[\frac{x^2 - 2pxn + p^2n^2}{p^2(1-p)^2}\right]|p)$$
$$I(p) \propto \sum_i \left[\frac{x^2 - 2pxn + p^2n^2}{p^2(1-p)^2}\right]\left[\frac{n}{x}p^x(1-p)^{n-x}\right]$$
$$I(p) \propto \left[\frac{E(x^2) - 2pnE(x) + p^2n^2E(1)}{p^2(1-p)^2}\right]$$
$$Var(x) = E[x^2] - (E[X])^2 \rightarrow E[x^2] = Var(x) + (E[X])^2$$
$$E[x^2] = Var(x) + (E[X])^2 = np(1-p) + n^2p^2$$
$$E[x] = np$$
$$E[1] = 1$$
$$I(p) \propto \left[\frac{E(x^2) - 2pnE(x) + p^2n^2E(1)}{p^2(1-p)^2}\right]$$
$$I(p) \propto \left[\frac{np(1-p) + n^2p^2 - 2p^2n^2 + p^2n^2}{p^2(1-p)^2}\right]$$
$$I(p) \propto \left[\frac{np(1-p)}{p^2(1-p)^2}\right] = \left[\frac{n}{p(1-p)}\right]$$
$$p(p) = I(p)^{1/2} = \left[\frac{n}{p(1-p)}\right]^{1/2} = \left[\frac{\sqrt{n}}{\sqrt{p(1-p)}}\right]$$
$$p - Beta(1/2,1/2) \rightarrow p(p) = (p^{-1/2}(1-p)^{-1/2}) * \frac{\Gamma(1/2+1/2)}{\Gamma(1/2)\Gamma(1/2)} = (p^{-1/2}(1-p)^{-1/2}) * \frac{1}{\pi}$$
$$p(p) = \left[\frac{\sqrt{n}}{\sqrt{p(1-p)}}\right] * \frac{1}{\sqrt{n}\pi}$$

$$p - Beta(\frac{1}{2},\frac{1}{2})$$