

Topic 10

Graph and Circuit

資料結構與程式設計
Data Structure and Programming

11/16/2016

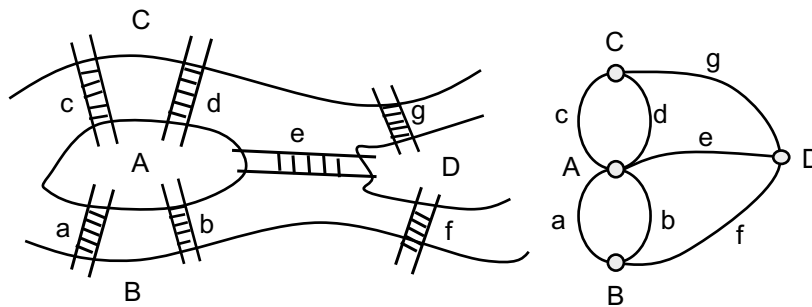
From CS to EE? What does that mean?

- ◆ Most people think that “Data Structure” is a CS class
 - A “must” subject for CS entrance exam
- ◆ In EE area, many problems can be either mapped as graphic problems, or resolved by graphic algorithms
 - e.g. Circuit netlist, network, communication, etc.
 - Understanding graphic data structure and algorithms will be very helpful

The First Use of Graph

◆ Königsberg Bridge Problem

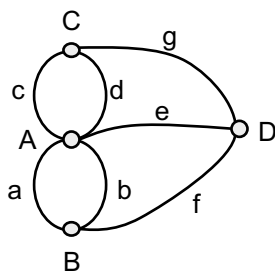
- Leonhard Euler, 1736
- Starting at one land, is it possible to walk across all bridges exactly once and returning to the starting land area?



Eulerian Theorem

- ◆ There is a walk starting at any vertex, going through each edge exactly once and terminating at the starting vertex, iff the degree of each vertex is even.

→ Eulerian walk



No Eulerian walk, since all 4 vertices are of odd degree.

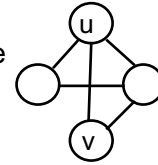
Definition of a Graph

◆ A graph, $G(V, E)$

- V : a finite, nonempty set of vertices $\rightarrow V(G)$
- E : a set of pairs of vertices
these pairs are called edges $\rightarrow E(G)$

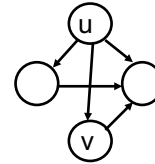
1. Undirected Graph

- If every pair of vertices representing any edge is unordered
- i.e. (u, v) and (v, u) represent the same edge



2. Directed Graph (Digraph)

- Order of the pair of vertices matters
- $\langle u, v \rangle$: 'u' is the tail and 'v' is the head
- e.g. A circuit is a directed graph



Terminologies

◆ Given 2 nodes u, v , and an edge (u, v)

- u and v are called *adjacent*
- The edge (u, v) is *incident* on vertices u and v

➔ If $\langle u, v \rangle$ is a directed edge

- u is *adjacent to* v , and v is *adjacent from* u

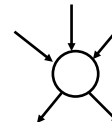


◆ Degree of a vertex

- The number of edges incident to it

➔ If the graph is directed

- *In-degree*
 - The number of edges for which the vertex is the head
- *Out-degree*
 - The number of edges for which the vertex is the tail



Terminologies

◆ *Path*

- A sequence of vertices in which each vertex is adjacent to the next one
 - e.g. $\{ n_1, e_1, n_2, e_2, n_3, e_3, \dots, e_{k-1}, n_k \}$

◆ *Simple path*

- All vertices in a path are distinct

◆ *Length of a path*

- The number of edges in a path

◆ *Loop (self-edge)*

- An edge with 2 identical end-points

◆ *Cycle*

- A path with identical start and end points

Graph Properties

◆ Subgraph $G(V', E')$ of $G(V, E)$

- $V' \subseteq V; E' \subseteq E$

◆ Simple graph

- No loops and no two edges link the same vertex pair

◆ Multigraph

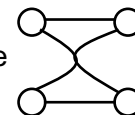
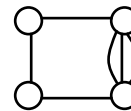
- Not simple graph

◆ Weighted graph

- Each edge is associated with some weight

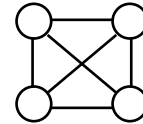
◆ Hypergraph

- An extension of a graph where edges may be incident to any number of vertices



Complete Graph

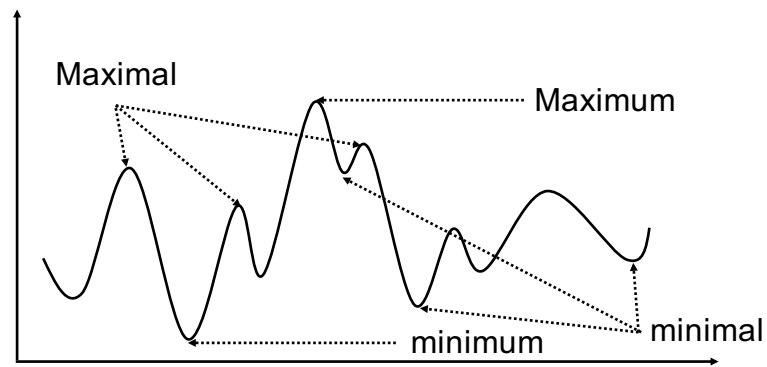
- ◆ Complete graph
 - Each vertex is adjacent to all the other vertices in the graph
 - For complete graph with n vertices
 - $\#edges = n(n - 1) / 2$
- ◆ Clique of a graph
 - Complete subgraph
- ◆ Complement $G(V', E')$ of a graph $G(V, E)$
 - $V' = V; E \cap E' = \emptyset$
 - $G(V, E \cup E')$ is a complete graph



Undirected Graph Properties

- ◆ Two vertices u and v are said to be connected
 - iff there a path from u to v
- ◆ A graph is said to be connected
 - iff every vertex pair is connected
 - ➔ A tree is a connected acyclic graph
- ◆ A connected component (or simply *component*) of a graph
 - A maximal connected subgraph

(FYI) Maximal vs. Maximum

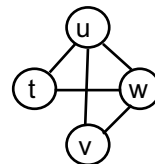


- In many problems, finding maximum/minimum is very hard
 - Finding maximal/minimal is the only possibility
- How to find a better maximal/minimal?

Undirected Graph Properties

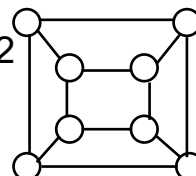
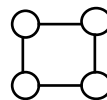
◆ Cutset

- A minimal set of edges whose removal from the graph makes the graph disconnected



◆ Bipartite graph

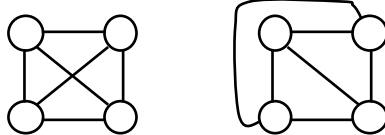
- Vertex set can be partitioned into 2 subsets such that each edge has end-points in different subsets



Undirected Graph Properties

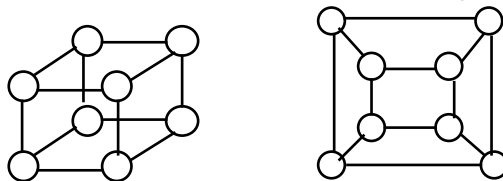
◆ Planar graph

- A diagram on a plane surface such that no two edges cross



◆ Two graphs are isomorphic

- There is a one-to-one correspondence between their vertex sets and preserves adjacency

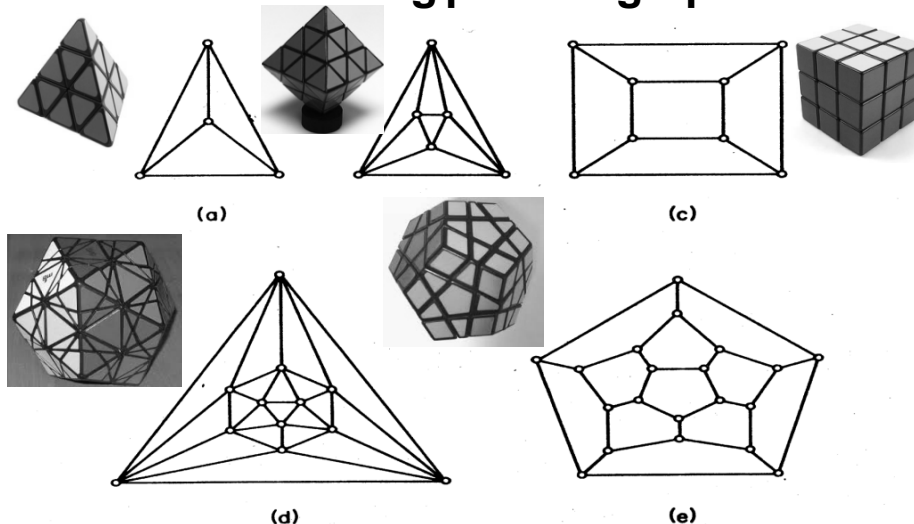


Data Structure and Programming

Prof. Chung-Yang (Ric) Huang

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Some interesting planar graphs



(a) The tetrahedron; (b) the octahedron; (c) the cube; (d) the icosahedron; (e) the dodecahedron

Undirected Graph Properties

- ◆ Each undirected graph can be characterized by four numbers
 1. *Clique number* $\omega(G)$
 - The cardinality of its largest clique, called *clique number*
 2. *Chromatic number* $\chi(G)$
 - The minimum number of colors needed to color the vertices, such that no edge has end-points with the same color
 - e.g. A bipartite graph is a 2-colorable graph

Property: $\omega(G) \leq \chi(G)$

Undirected Graph Properties

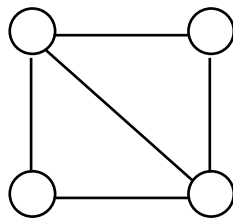
3. *Clique cover number* $\kappa(G)$
 - A graph is said to be *partitioned into cliques* if its vertex set is partitioned into (disjoint) subsets, each one including a clique
 - The cardinality of a minimum clique partition is called *Clique cover number*
4. *Stability number* $\alpha(G)$
 - A *stable set*, or *independent set*, is a subset of vertices with the property that no two vertices in the stable set are adjacent
 - The *stability number* is the cardinality of its largest stable set
 - A *coloring* of a graph is a partition of the vertices into subsets, such that each is a stable set

Property: $\alpha(G) \leq \kappa(G)$

Perfect Graph

◆ A graph is said to be perfect iff

- $\omega(G) = \chi(G)$ (clique = chromatic)
- $\alpha(G) = \kappa(G)$ (stability = clique covering)



$$\omega(G) = \chi(G) = 3$$

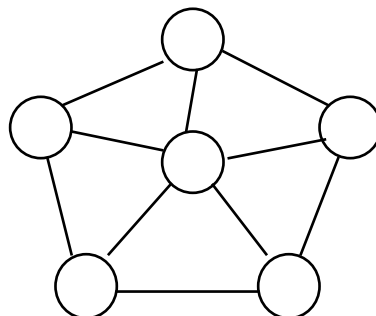
$$\alpha(G) = \kappa(G) = 2$$

Any graph that is NOT perfect?

◆ That is:

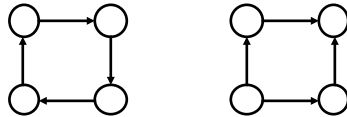
Clique number $\omega(G)$ < Chromatic number $\chi(G)$

Stability number $\alpha(G)$ < Clique cover number $\kappa(G)$



Directed Graph Properties

- ◆ A digraph is said to be strongly connected
 - Iff for every pair of distinct vertices u and v , there is a path from u to v , also from v to u



- ◆ Strongly connected component (SCC)
 - Maximal subgraph that is strongly connected
 - If a graph is strongly connected, it has only one SCC
 - Linear time algorithm for finding SCCs:
Robert E. Tarjan, *Depth-first search and linear graph algorithms*, SIAM Journal on Computing, 1(2):146-160, 1972.

Directed Acyclic graph (DAG)

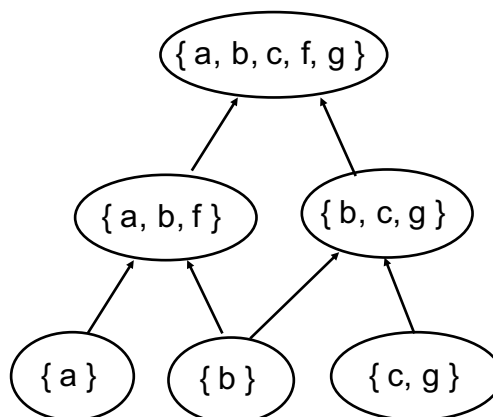
- ◆ A directed graph that has no cycle
- ◆ Can represent *partially ordered set*
 - A vertex v is a successor (descendant) of a vertex u
 - If there is a path from u to v
 - Called direct successor if the path is an edge
 - Predecessor (ancestor)
- ◆ Polar DAG
 - A DAG with 2 distinguished vertices
 - A source and a sink
 - All vertices are reachable from the source
 - Sink is reachable from all the vertices
 - A generic polar DAG may have multiple sources and sinks

Partially vs. Totally Ordered Set

- ◆ A relation “ \leq ” is a partial order on a set S if it has:
 1. Reflexivity: $a \leq a$ for all $a \in S$
 2. Antisymmetry: $a \leq b$ and $b \leq a$ implies $a = b$.
 3. Transitivity: $a \leq b$ and $b \leq c$ implies $a \leq c$
- ◆ A relation “ \leq ” is a total order on a set S if it has the above 3 properties and the following:
 4. Comparability (trichotomy law):
For any $a, b \in S$, either $a \leq b$ or $b \leq a$

A Partial Order Example

- ◆ The “containment” relation among the subsets of a set is a partial order



Graphic Algorithms

- ◆ The importance of learning “graphs” is that many practical problems can be modeled and then solved by standard/well-known graphic algorithms
 1. Breadth-First Search and Depth-First Search
 2. Topological Sort
 3. Strongly Connected Component
 4. Shortest and Longest Path Algorithms
 5. Minimum Spanning Tree
 6. Maximum Flow and Minimum Cut
- ◆ Please refer to “Algorithm” book or class for more information
 - We may cover some of them if we have time...

Graph Traversal

- ◆ In many graph (DAG) applications, it is important to go through every vertex in certain order
 - e.g. checkSum(), simulate(), etc
- ◆ Topological order
 - An order sorted by certain relationship of adjacent vertices
 - e.g.
 - For each vertex, it has higher order than all of its predecessors, and lower order than all of its successors

Depth-First Traversal (Take 1)

```
void
Graph::dfsTraversal(const List<Node*>& srcList)
{
    for_each_source(node, srcList)
        node->dfsTraversal(_dfsList);
}
// post order traversal
void Node::dfsTraversal(List<Node *>& dfsList)
{
    for_each_predecessor(next, _predecessors)
        next->dfsTraversal(dfsList);
    dfsList.push_back(this);
}
```

Any Problem??

Depth-First Traversal (Take 2)

```
void
Graph::dfsTraversal(const List<Node*>& srcList)
{
    for_each_source(node, srcList)
        node->dfsTraversal(_dfsList);
}
// post order traversal
void Node::dfsTraversal(List<Node *>& dfsList)
{
    for_each_predecessor(next, _predecessors)
        if (!next->isMarked()) {
            next->setMarked();
            next->dfsTraversal(dfsList);
        }
    dfsList.push_back(this);
}
```

Any Problem??

Depth-First Traversal (Take 3)

```
void
Graph::dfsTraversal(const List<Node*>& srcList)
{
    for_each_source(node, srcList)
        node->dfsTraversal(_dfsList);
    for_each_node(node, _dfsList)
        node->unsetMarked();
}
// post order traversal
void Node::dfsTraversal(List<Node *>& dfsList)
{
    for_each_predecessor(next, _predecessors)
        if (!next->isMarked()) {
            next->setMarked();
            next->dfsTraversal(dfsList);
        }
    dfsList.push_back(this);
}
```

Any Problem??

Depth-First Traversal (Take 4)

- ◆ Remember: use “static data member in a class”

```
class T
{
    static unsigned    _globalRef;
    unsigned           _ref;

public:
    T() : _ref(0) {}
    bool isGlobalRef() { return (_ref == _GlobalRef); }
    void setToGlobalRef() { _ref = _globalRef; }
    static void setGlobalRef() { _globalRef++; }
};
```

- ◆ Use this method to replace “setMarked()” functions in graph traversal problems

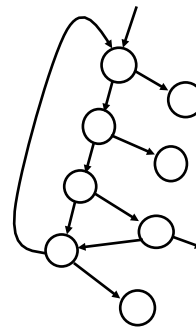
Depth-First Traversal (Take 4)

```
void
Graph::dfsTraversal(const List<Node*>& srcList)
{
    Node::setGlobalRef();
    for_each_source(node, srcList)
        node->dfsTraversal(_dfsList);
}
// post order traversal
void Node::dfsTraversal(List<Node *>& dfsList)
{
    for_each_predecessor(next, _predecessors)
        if (!next->isGlobalRef()) {
            next->setToGlobalRef();
            next->dfsTraversal(dfsList);
        }
    dfsList.push_back(this);
}
```

Any Problem??

Depth-First Traversal (Take 5)

```
void
Graph::dfsTraversal(const List<Node*>& srcList)
{
    Node::setGlobalRef();
    for_each_source(node, srcList)
        node->dfsTraversal(_dfsList, _fbList);
}
// post order traversal
void Node::dfsTraversal
(List<Node *>& dfsList, list<NodePair>& fbList)
{
    for_each_predecessor(next, _predecessors)
        if (!next->isGlobalRef()) {
            next->setToGlobalRef();
            next->setActive();
            next->dfsTraversal(dfsList, fbList);
            next->unsetActive();
        }
        else if (next->isActive())
            fbList.push_back(NodePair(this, next));
    dfsList.push_back(this);    // not push_back(next); why?
}
```



Breath-First Traversal

```
algorithm levelOrder(TreeNode t)
    Input: a tree node (can be considered to be a
           tree)
    Output: None.

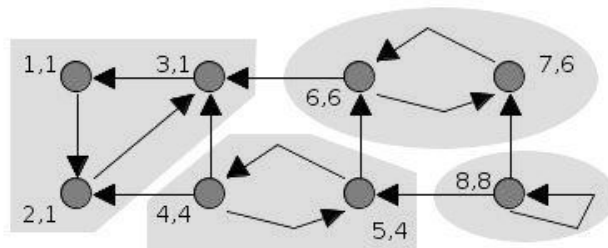
    Let Q be a Queue
    Q.enqueue(t)
    while the Q is not empty
        tree = Q.dequeue()
        Visit node tree
        if tree has a left child
            Q.enqueue(left child of tree)
        if tree has a right child
            Q.enqueue(right child of tree)
```

How about the “marked” and “loop” Issues ??

Tarjan's Strongly Connected Components (SCC) algorithm

```
Global: idx = 0; S = empty; SCCGroup = empty;
for_each_node (v, G)
    if (v is unvisited) SCC(v);
Func SCC(v) {
    v.idx1 = v.idx2 = idx; idx++;
    S.push(v); v.setVisited();
    for_each_predecessor (v, w) {
        if (w is unvisited) {
            SCC(w);
            v.idx2 = min(v.idx2, w.idx2);
        } else if (w is in S) {
            v.idx2 = min(v.idx2, w.idx1);
        }
    }
    if (v.idx1 == v.idx2) {
        SCCGroup.add(S); S.flush();
    }
}
```

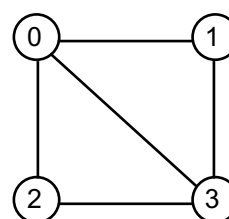

Tarjan's Strongly Connected Components (SCC) algorithm



Graph Implementation (1)

◆ Adjacency Matrix

```
class Graph
{
    bool _adjacency[n][n];
};
```



- For undirected graph \rightarrow upper triangle
- How to perform traversal?
- Difficult to implement various graphic algorithms
- Could be a sparse matrix
- Complexity can be as high as $O(n^2)$

$$\begin{array}{cccc} & 0 & 1 & 2 & 3 \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Graph Implementation (2)

◆ Adjacency List

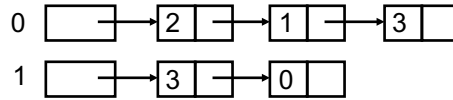
```
class Graph
```

```
{
```

```
    List<int> * _headNodes;
```

```
    int        _numNodes;
```

```
};
```



	0	1	2	3
0	0	1	1	1
1	1	0	0	1
2	1	0	0	1
3	1	1	1	0

- Better for sparse matrix
- Require n `_headNodes` and $2 \cdot e$ `ListNodes`
- (u, v) and (v, u) redefined
- Some operations may still be as expensive as $O(n + e)$

Graph Implementation (3)

◆ Adjacency Multilist

```
class Edge
```

```
{
```

```
    bool        _visited;
```

```
    int         _vertex1, _vertex2;
```

```
    Edge *      _path1, * _path2;
```

```
};
```

```
class Graph
```

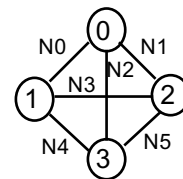
```
{
```

```
    Edge**      _headNodes;
```

```
    int         _numNodes;
```

```
};
```

	_vertex1	_vertex2	_path1	_path2
N0	0	1	N1	N3
N1	0	2	N2	N3
N2	0	3	0	N4
N3	1	2	N4	N5
N4	1	3	0	N5
N5	2	3	0	0

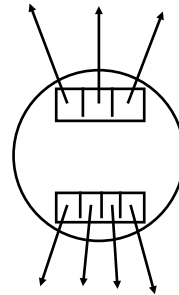


- Same memory requirement as “adjacent list” (except for `_visited` field))
- Not very intuitive to understand

Graph Implementation (4)

- ◆ Two dynamic arrays

```
class Node
{
    Array<Node *> _successors;
    Array<Node *> _predecessors;
};
class Graph
{
    Array<Node *> _nodes;
    // Array<Node *> _sinks;
    // Array<Node *> _sources;
};
```



- Memory usage is about the same ($n + 2 * e$)
- A more intuitive implementation

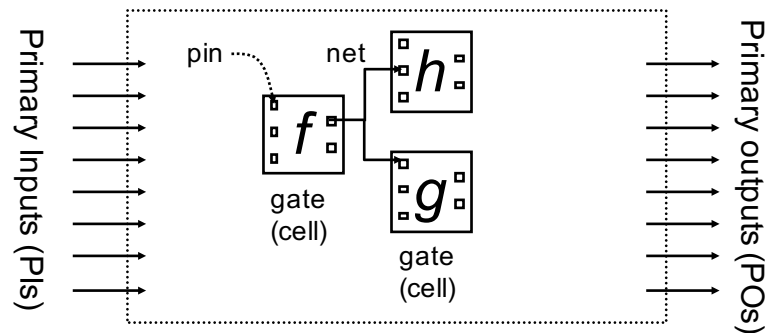
Graph Implementation (5)

- ◆ To contain data in nodes

```
template <class T>
class Node
{
    T _data;
    Array<Node<T> *> _successors;
    Array<Node<T> *> _predecessors;
};
template <class T>
class Graph
{
    Array<Node<T> *> _nodes;
};
```

Circuit

- ◆ A directed diagram for representing the current flow of an electronic design

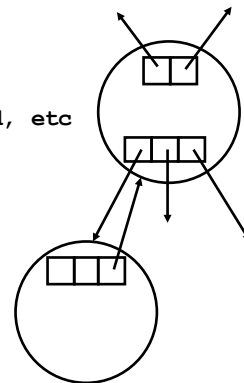


- ◆ h and g are f's *fanouts*
- ◆ f is h's and g's *fanin*

Circuit Implementation (1)

- ◆ Cell-based implementation (1)

```
class Gate
{
    GateType      _type;
    GateFlag      _flag; // visited, etc
    Array<Gate *> _faninList;
    Array<Gate *> _fanoutList;
};
class Circuit
{
    Array<Gate *> _piList;
    Array<Gate *> _poList;
    Array<Gate *> _totalList;
};
```



- Gate::_type is to distinguish different functionalities
 - Drawback: usually need a BIG switch in codes

Circuit Implementation (2)

◆ Cell-based implementation (2)

```
class Gate
{
    GateType _type;
    GateFlag    _flag;
    Array<Gate *> _faninList;
    Array<Gate *> _fanoutList;
};

class And : public Gate
{
};

class Circuit
{
    Array<Gate *> _piList;
    Array<Gate *> _poList;
    Array<Gate *> _totalList;
};
```

Virtual Functions for Different Types of Gates

```
class Gate {
    virtual bool simulate() = 0;
};

class And : public Gate {
    bool simulate() { ... }
};

=====

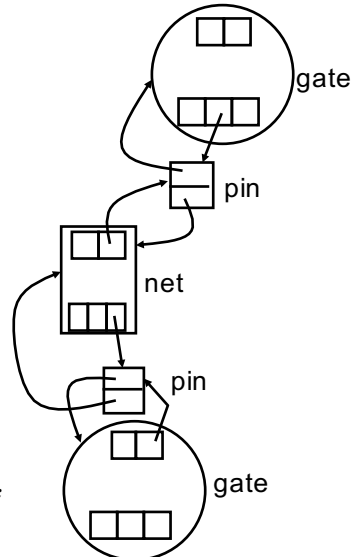
void Circuit::constructNetlist() {
    Gate* newGate = new And;
    ...
}

bool Circuit::simulate() {
    for_each_gate(thisGate, _totalList)
        thisGate->simulate(); // Will call And::simulate()
    ...
}
```

Circuit Implementation (2)

◆ Net-based implementation

```
class Net
{
    Array<Pin *>    _inPinList;
    Array<Pin *>    _outPinList;
};
class Pin
{
    Gate*           _gate;
    Net*            _net;
};
class Gate
{
    GateFlag        _flag;
    Array<Pin *>    _inPinList;
    Array<Pin *>    _outPinList;
};
```



Circuit Implementation (3)

◆ AND-Inverter Graph (AIG)

◆ All the Boolean functions can be represented by “And: \wedge ” and “Inverter: \neg ”

- e.g. $OR(a, b) = \neg(\neg a \wedge \neg b)$

◆ As for circuit implementation, it is better to have simpler data structure

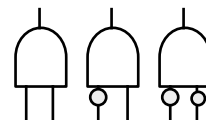
- AIG is enough
- Two classes: AndGate and InvGate?

- InvGate is kind of unnecessary...

- One class: NandGate?

- Still need an object to represent an Inverter

➔ Solution: AndGate with (optional) inverted inputs



AIG Implementation

```
class AigGate {
    Array<AigGateV> _faninList;
    size_t          _ref;
    static size_t   _globalRef_s
};

class AigGateV {
    #define NEG 0x1
    AigGateV(AigGate* g, size_t phase):
        _gateV(size_t(g) + phase) { }
    AigGate* gate() const {
        return (AigGate*) (_gateV & ~size_t(NEG)); }
    bool isInv() const { return (_gateV & NEG); }

    size_t          _gateV;
};
```

Data Structure and Programming

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AIGER Format

- ◆ An simplified, well-accepted AIG format
 - Documents and source codes available at:
<http://fmv.jku.at/aiger/>
- ◆ Two versions
 - ASCII format: text format ← HW #6
 - Binary format: more compact representation
 - ➔ In HW#6 and final project, we will handle ASCII format only

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AIGER ASCII Format

- ◆ ASCII format contains several sections
 - Header
 - Inputs
 - Latches // ignored in HW#6
 - Outputs
 - ANDs
 - Symbols
 - Comments
- ◆ Except for header, any of the above sections can be omitted if it is not necessary
 - However, their relative order cannot be altered

AIGER ASCII Format

- ◆ Header
 - [Syntax] aag M I L O A
 - aag: specify ASCII AIG format
 - [cf] aig: specify binary format
 - M: maximal variable index
 - I, L, O, A: number of inputs, latches, outputs, AND gates
 - [Example] aag 7 2 0 2 3
 - [Note]
 - Exact ONE space before M, I, L, O, A
 - “A” must be immediately followed by a “new line” char.
 - If all variables are used and there are no unused AND gates, then $M = I + L + A$.

AIGER ASCII Format

◆ Variables and Literals

- Each input, latch, and AND gate is assigned with a distinct variable ID (i.e. an unsigned number)
 - Between $[1, M]$
 - Variable 0 means constant FALSE.
 - The input, latch, and AND variable IDs can be arbitrary. No one is necessarily bigger/smaller than the other.
- A “literal” is a positive or negative form of a variable
 - Let v be the ID of a variable, then the literal $(2v)$ and $(2v+1)$ stands for the positive and negative forms of the variable, respectively
 - e.g. Literal 12 is the positive form of variable 6
Literal 1 stands for constant TRUE

AIGER ASCII Format

◆ Inputs

- [Syntax] <inputLiteralID>
- [Example] 2
- [Note]
 - Each line defines exactly one input, which is represented as a literal ID
 - Inputs are non-negative, so the literal IDs must be even numbers

◆ Example

```
aag 3 2 0 1 1 // header
2           // input 0
4           // input 1
```

AIGER ASCII Format

◆ Latches

- [Syntax] <currStateLiteralID>
<nextStateLiteralID>
- [Example] 8 15
- [Note]
 - Each line defines exactly one latch, which contains the current state literal ID followed by the next state ID
 - Current states are non-negative (as inputs), so their literal IDs must be even numbers
 - Next states can be inverted (as outputs), so their literal IDs can be positive or negative

AIGER ASCII Format

◆ Outputs

- [Syntax] <outputLiteralID>
- [Example] 9
- [Note]
 - Each line defines exactly one output, which is represented as a literal ID
 - Outputs can be inverted, so their literal IDs can be even or odd

◆ Example

```
aag 3 2 0 1 1 // header
2           // input 0
4           // input 1
6           // output 0
```

AIGER ASCII Format

◆ AND gates

- [Syntax] <LHS> <RHS1> <RHS2>
- [Example] 12 7 15
- [Note]
 - Each line defines exactly one AND gate, which contains the LHS literal followed by exactly two RHS literals
 - LHS literals must be even, and the RHS literals can be even or odd (i.e. non-inverted or inverted)

◆ Example

```
aag 3 2 0 1 1 // header
2 // input #0 (var id = 1)
4 // input #1 (var id = 2)
6 // output #0 (var id = 3)
6 2 4 // AND gate 3 = 1 & 2
```

AIGER ASCII Format

◆ Symbols

- [Syntax] [ilo]<position> <symbolicName>
- [Example] i0 reset
o1 done
- [Note]
 - Each line defines exactly one symbolic name for inputs, latches, or outputs
 - There is at most ONE symbolic name for each input, latch, or output
 - <position> denotes the position of the corresponding input/latch/output is defined in it section. It counts from 0.
 - Symbolic name can contain any printable character, except for “new line”
 - [Note] White space and numbers are allowed in names

AIGER ASCII Format

◆ Comments

- [Syntax] c
[anything]...
- [Example] c
Game over!!
- [Note]
 - The comment section starts with a *c* character followed by a new line. The following lines are comments.

Notes on AIGER Format

- ◆ No leading or trailing spaces in each line
- ◆ No empty line
- ◆ “New line” character must present at the end of each line
- ◆ All parsed tokens in the same line, except for comments, must be separated by exactly ONE space character
- ◆ Need to identify undefined literals and floating signals in parser

AIGER Examples

1. Empty circuit
aag 0 0 0 0 // header
2. And gate
aag 3 2 0 1 1 // header
2 // input #0 (var id = 1)
4 // input #1 (var id = 2)
6 // output #0 (var id = 3)
6 2 4 // AND gate 3 = 1 & 2
3. Or gate
aag 3 2 0 1 1 // header
2 // input #0 (var id = 1)
4 // input #1 (var id = 2)
7 // output #0 (var id = 3)
6 3 5 // AND gate 3 = !1 & !2

AIGER Examples

4. Half Adder
aag 7 2 0 2 3 // headerline
2 // input #0 1st addend bit 'x'
4 // input #1 2nd addend bit 'y'
6 // output #0 sum bit 's'
12 // output #1 carry 'c'
6 13 15 // AND gate #0 $x \wedge y$
12 2 4 // AND gate #1 $x \& y$
14 3 5 // AND gate #2 $!x \& !y$
i0 x // symbol
i1 y // symbol
o0 s // symbol
o1 c // symbol
c // comment header
half adder // comment

Some notes about HW#6

◆ Topic: An AIGER parser

- Parse an AIGER netlist file into a circuit data structure (a DAG)
 - Note: Error handling can be VERY complicated...
Try to work on “good” circuits first!!
- Check for floating/undefined variables
- ~~Check for cyclic conditions~~
- Report circuit statistics
- Report gate connections
- ~~Perform logic simulations~~
- Output AIG file