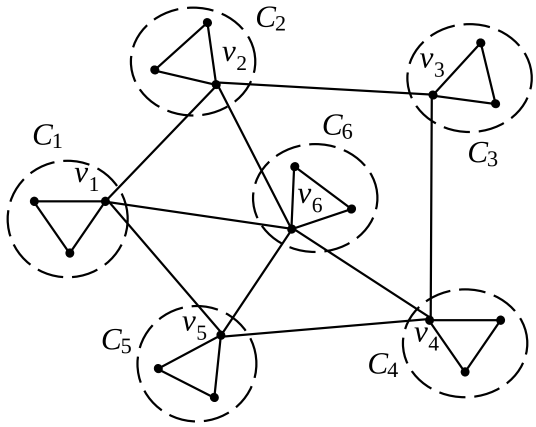
Homework #4 (due in class, December 20, 2018)

1. (a) Exercise 21.2-2 (page 567). (b) Exercise 21.3-1 (page 572).
2. Exercise 23.2-7 (page 637).
3. Problem 23-3 (pages 640).
4. We are given a telephone network (a weighted graph) $N = (V, E)$ as shown below. The set V of vertices consists of t clusters C_1, C_2, ..., C_t. Each cluster is a complete graph K_l on l vertices. Each cluster C_i also has a unique communication center v_i, and the clusters are joined via the communication centers (inter-cluster connections join the respective communication centers). Let m be the number of inter-cluster connections. The figure below shows an example in which $t = 6$, $l = 3$, and $m = 9$. In general, t and l may be **large** and the inter-cluster connections tend to be **sparse**. Suppose we have only two software packages available for solving the Minimum Spanning Tree problem: (1) an $O(E \lg V)$ implementation of Kruskal's algorithm, and (2) an $O(V^2)$ implementation of Prim's algorithm. Design an efficient algorithm to construct a minimum spanning tree of N. What is the worst case complexity of your algorithm (in terms of l, t, and m)?



5. Exercise 24.2-3 (page 657).
6. Exercise 24.4-1 (page 669).
7. Problem 24-3 (page 679).
8. Exercise 25.2-1 (page 699). **(Please work on the subgraph of Figure 25.1 in Page 690 with the edges incident on only the vertices v_2, v_3, v_4, and v_5.)**
9. Exercise 25.3-4 (page 705).
10. In the graph $G = (V, E)$ we associate a reliability $0 < \mu_{ij} \leq 1$ with every edge $(i, j) \in E$; the reliability measures the probability that the edge will be operational. We define the reliability of a directed path P as the product of the reliability of edges in the path (i.e., $\mu(P) = \Pi_{(i,j)\in P}\mu_{ij}$). The maximum reliability path problem is to identify a directed path of maximum reliability from the source node s to every other node in the graph.

(a) Suppose you are not allowed to take logarithms because they might yield irrational data. Specify an efficient algorithm for solving the maximum reliability path problem by modifying Dijkstra's algorithm. **You only need to give the modified code with the line numbers in Dijkstra's algorithm listed in the textbook/lecture notes.** What is the time complexity of your algorithm?

(b) If we permit μ_{ij} to be arbitrary positive numbers, then the maximum reliability path problem becomes the maximum multiplier path problem. Modify the Floyd-Warshall all-pairs shortest path algorithm to determine maximum multiplier paths between all pairs of nodes.

11. Let $G = (V, E)$ be a directed graph with edge costs modelled by the corresponding weights. The *bottleneck* of a path is defined as the **minimum** edge cost among all the edges on the path. Suppose that we want to find a **maximum** bottleneck path between each pair of vertices. Show how to modify Floyd-Warshall's all-pair shortest-path algorithm to solve this problem in $O(V^3)$ time.

12. (DIY Problem) For this problem, you are asked to design a problem *set* related to Chapter(s) 21, 23, 24, and/or 25 and give a sample solution to your problem set. Grading on this problem will be based upon the *quality* of the designed problem as well as the *correctness* of your sample solution.