Assignment 3:

Projective Geometry

Computer Vision

National Taiwan University

Fall 2018

Part 1: Estimating Homography

Recap of Homography

- Matrix form:

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} \sim \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

- Equations:

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

Recap of Homography

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} \sim \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

- Degree of freedom
  - There are 9 numbers in H. Are there 9 DoF?
  - No. Note that we can multiply all h_{ij} by nonzero k without changing the equations:

$$v_x = \frac{kh_{11}u_x + kh_{12}u_y + kh_{13}}{kh_{31}u_x + kh_{32}u_y + kh_{33}} \qquad \Rightarrow \qquad v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$v_y = \frac{kh_{21}u_x + kh_{22}u_y + kh_{23}}{kh_{31}u_x + kh_{32}u_y + kh_{33}} \qquad \qquad v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

Enforcing 8 DoF

- **Solution 1:** set $h_{33} = 1$

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + 1}$$
$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + 1}$$

- **Solution 2:** impose unit vector constraint

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$
$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

Subject to

$$h_{11}^2 + ... + h_{33}^2 = 1$$

Solution 1

- Set $h_{33} = 1$

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + 1}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + 1}$$

- Multiply by denominator

$$(h_{31}u_x + h_{32}u_y + 1)v_x = h_{11}u_x + h_{12}u_y + h_{13}$$

$$(h_{31}u_x + h_{32}u_y + 1)v_y = h_{21}u_x + h_{22}u_y + h_{23}$$

- Rearrange

$$h_{11}u_x + h_{12}u_y + h_{13} - h_{31}u_xv_x - h_{32}u_yv_x = v_x$$

$$h_{21}u_x + h_{22}u_y + h_{23} - h_{31}u_xv_y - h_{32}u_yv_y = v_y$$

Solution 1 (cont.)

- Solve linear system

$$
\begin{array}{rcccc}
 & 2N \times 8 & 8 \times 1 & & 2N \times 1 \\
\begin{array}{r} \textbf{Point 1} \\ \\ \textbf{Point 2} \\ \\ \textbf{Point 3} \\ \\ \textbf{Point 4} \\ \\ \end{array} &
\begin{bmatrix}
u_{x,1} & u_{y,1} & 1 & 0 & 0 & 0 & -u_{x,1}v_{x,1} & -u_{y,1}v_{x,1} \\
0 & 0 & 0 & u_{x,1} & u_{y,1} & 1 & -u_{x,1}v_{y,1} & -u_{y,1}v_{y,1} \\
u_{x,2} & u_{y,2} & 1 & 0 & 0 & 0 & -u_{x,2}v_{x,2} & -u_{y,2}v_{x,2} \\
0 & 0 & 0 & u_{x,2} & u_{y,2} & 1 & -u_{x,2}v_{y,2} & -u_{y,2}v_{y,2} \\
u_{x,3} & u_{y,3} & 1 & 0 & 0 & 0 & -u_{x,3}v_{x,3} & -u_{y,3}v_{x,3} \\
0 & 0 & 0 & u_{x,3} & u_{y,3} & 1 & -u_{x,3}v_{y,3} & -u_{y,3}v_{y,3} \\
u_{x,4} & u_{y,4} & 1 & 0 & 0 & 0 & -u_{x,4}v_{x,4} & -u_{y,4}v_{x,4} \\
0 & 0 & 0 & u_{x,4} & u_{y,4} & 1 & -u_{x,4}v_{y,4} & -u_{y,4}v_{y,4}
\end{bmatrix} &
\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix} & = &
\begin{bmatrix} v_{x,1} \\ v_{y,1} \\ v_{x,2} \\ v_{y,2} \\ v_{x,3} \\ v_{y,3} \\ v_{x,4} \\ v_{y,4} \end{bmatrix} \\
\begin{array}{c} \textbf{Additional} \\ \textbf{points} \end{array} & \vdots & & & \vdots
\end{array}
$$

Solution 1 (cont.)

- What might be wrong with solution 1?
- If h_{33} is actually 0, we can not get the right answer

Solution 2

- A more general solution by confining $h_{11}^2 + ... + h_{33}^2 = 1$

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

- Multiply by denominator

$$(h_{31}u_x + h_{32}u_y + h_{33})v_x = h_{11}u_x + h_{12}u_y + h_{13}$$

$$(h_{31}u_x + h_{32}u_y + h_{33})v_y = h_{21}u_x + h_{22}u_y + h_{23}$$

- Rearrange

$$h_{11}u_x + h_{12}u_y + h_{13} - h_{31}u_xv_x - h_{32}u_yv_x - h_{33}v_x = 0$$

$$h_{21}u_x + h_{22}u_y + h_{23} - h_{31}u_xv_y - h_{32}u_yv_y - h_{33}v_y = 0$$

Solution 2

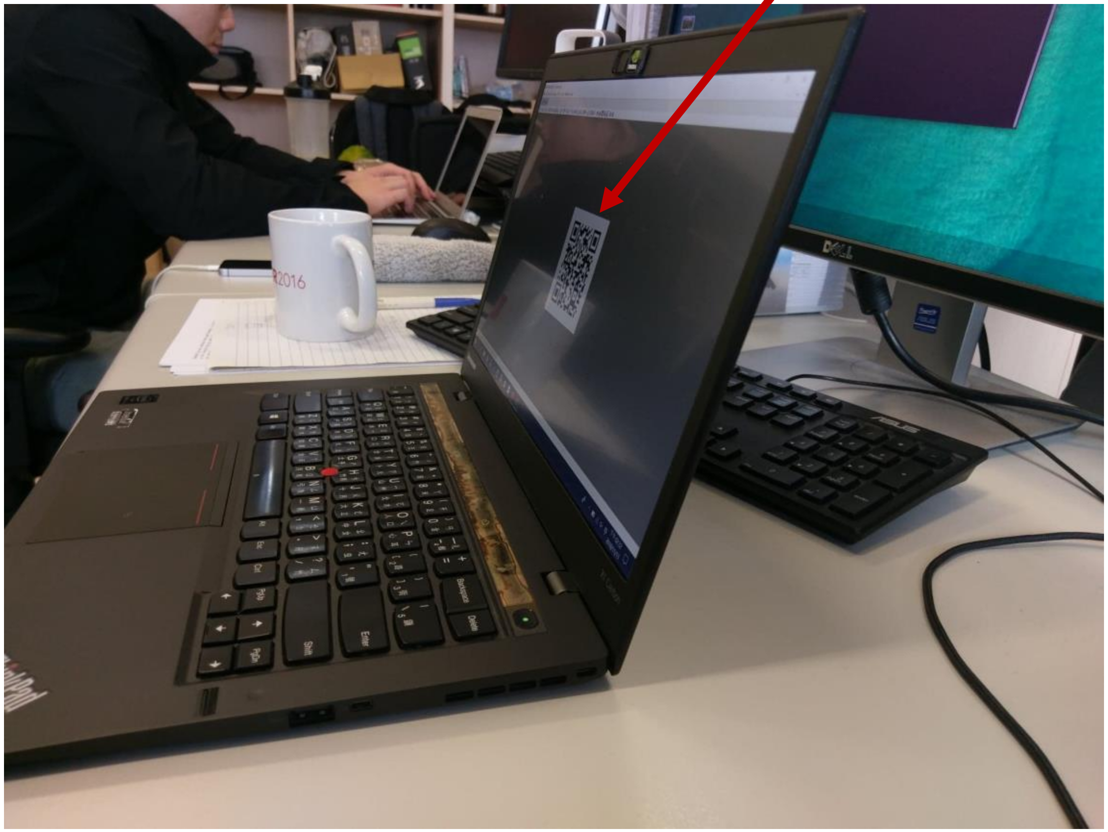
- Similarly, we have a linear system like this:

$$\underset{2N\times 9}{\mathbf{A}}\ \underset{9\times 1}{\mathbf{h}} = \underset{2N\times 1}{\mathbf{b}}$$

- Here, **b** is all zero, so above equation is a homogeneous system
- Solve:
  - $Ah = 0$
  - $A^T A h = A^T 0 = 0$
  - SVD of $A^T A = U \Sigma V^T$
  - Let h be the column of U (unit eigenvector) associated with the smallest eigenvalue in Σ

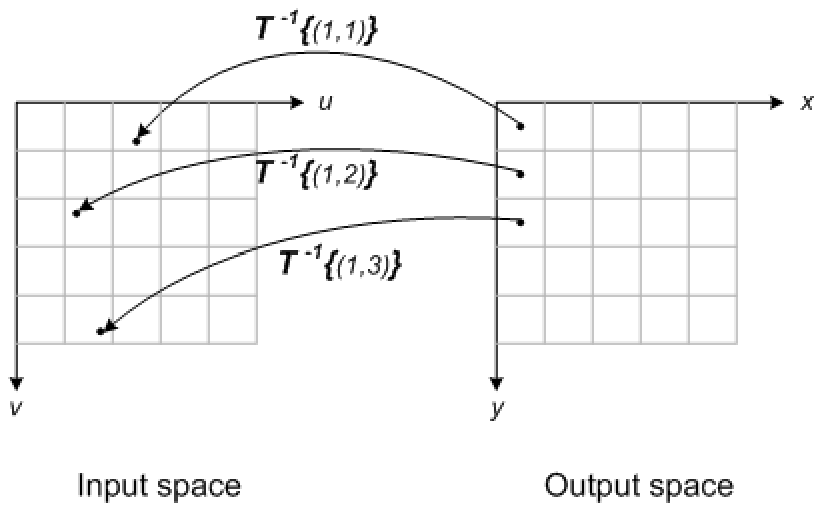
Part 2: Unwarp the Screen

Make the QR code frontal parallel



Backward Warping

- Why?
  - Prevent holes in output space
- Pixel value at sub-pixel location like (30.21, 22.74)?
  - Bilinear interpolation
  - Nearest neighbor

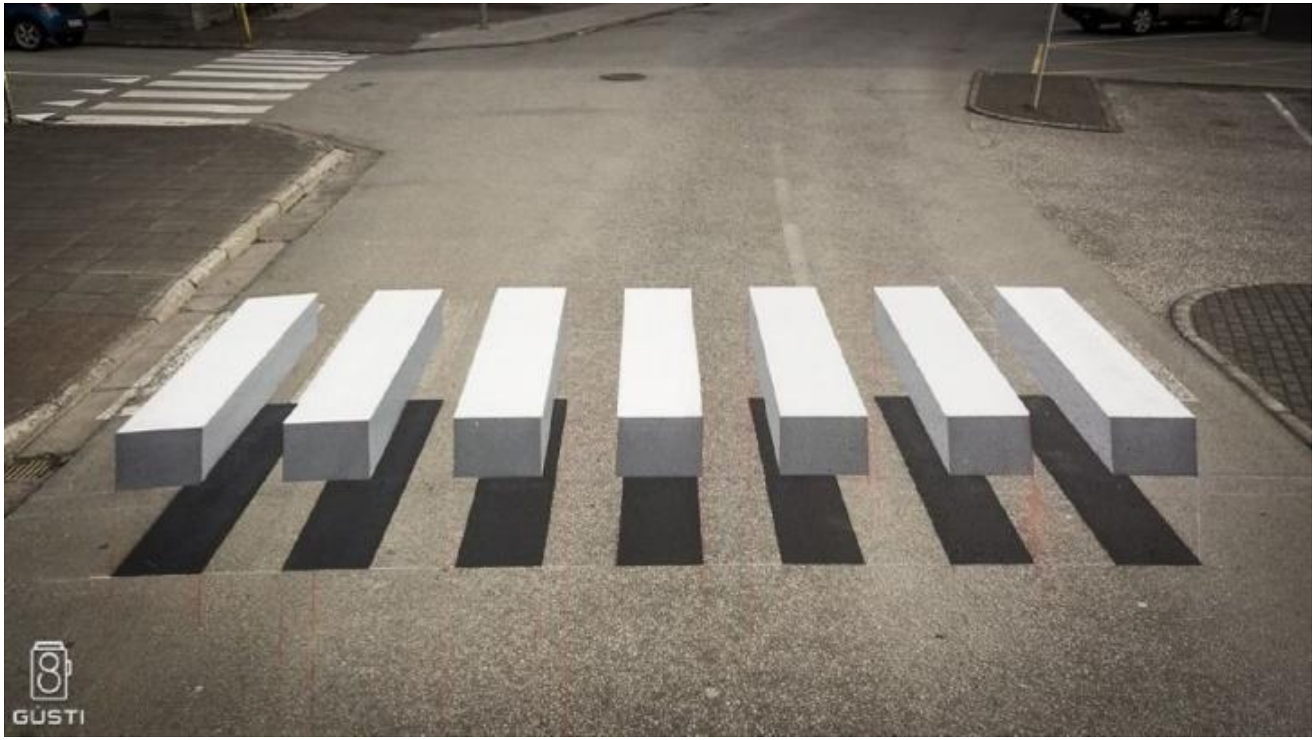


Part 3: Unwarp the 3D Illusion

- 3D illusion art

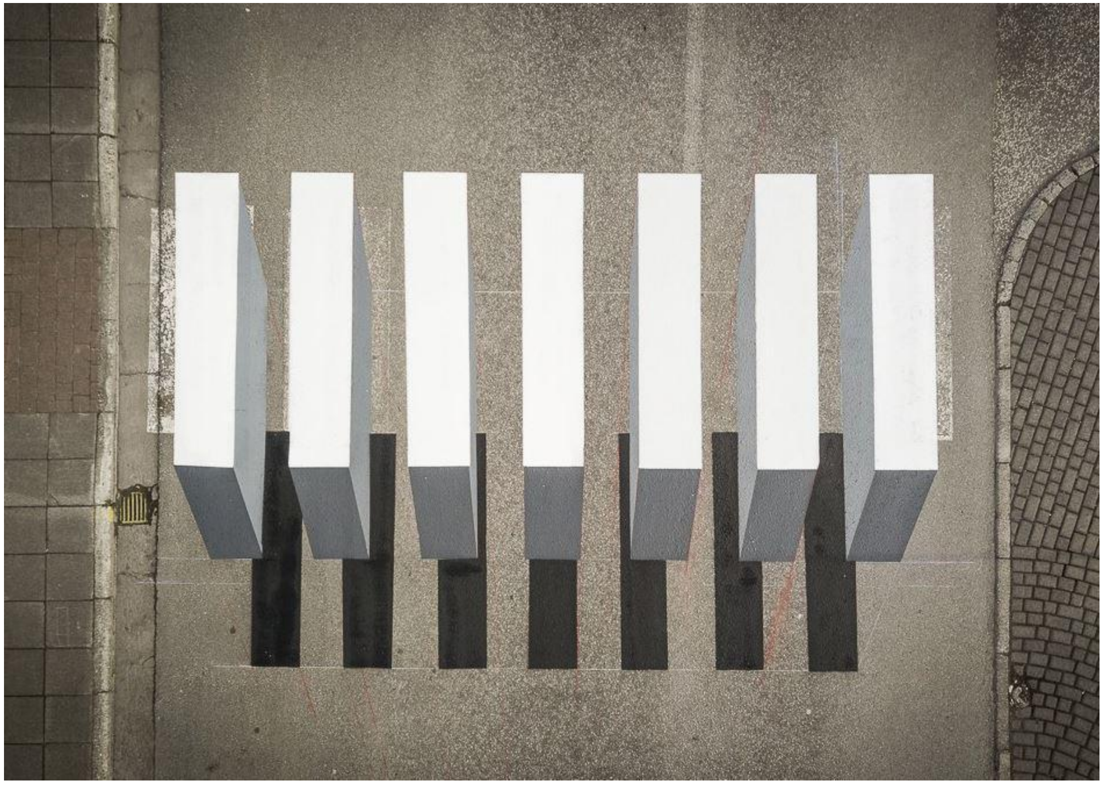
Part 3: Unwarp the 3D Illusion

- Input:



Part 3: Unwarp the 3D Illusion

- Ground-truth top view:



Can you unwarp the input image to match the ground-truth top view?

Assignment Description

- Part 1
  - Implement solution 1 or 2 for estimating homography.
  - Map 5 images of different people to the target surfaces (given in main.py). You can use whatever images you like. Include these images in your submission.
  - Include the function `solve_homography(u, v)` in your report.
- Part 2
  - Choose the unwarp region yourself.
  - The output image should contain the detectable QR code.
  - Include the QR code and the decoded link in your report.
- Part 3
  - Unwarp the image to the top view.
  - Can you get the parallel bars from the top view?
  - If not, why? Discuss in your report.

Bonus (Optional)

- Simple AR
  - Given a short video (~6 sec) and a template
  - Paste an image (it's up to you) on the surface to stick to the marker
  - Include your algorithm in your report

Submission

- Code: main.py (Python 3.5+)
- Input images for part 1
- Output images
  - part1.png, part2.png, part3.png
- A PDF report, containing
  - Your student ID, name
  - Your answers to each part
  - (Optional) algorithm to the simple AR
- (Optional) Input image and output video of the bonus part
- Compress all above files in a zip file named StudentID.zip
  - e.g. R07654321.zip
- Submit to CEIBA
- Deadline: 12/4 11:00 pm