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Modeling of Volumetric Parameterization for Lattice Structure Based on Skeletons

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Dear Editor,

Here within enclosed is our paper for consideration to be published on "Computers & Graphics". The further information about the paper is in the following:

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In recent years, the extensive application of additive manufacturing technology (AM) in manufacturing companies makes it possible to manufacture complex parts, such as lattice structure, pipeline structure, organic structure after topology optimization, parts with curved flow channel, etc. Lattice structure is an ordered ultralight porous structure constructed by simulating the lattice configuration of micro crystal, with the characteristics of small volume density, large specific surface area, high specific mechanical properties. And it is widely used in aerospace, petrochemical, metallurgy, machinery, environmental protection, construction and other industries. However, the drawbacks of the existing design methods and tools hinder the comprehensive development and use of this technology. It is necessary to develop specific models, methods and tools to support the definition and processing paradigm transformation of complex shapes in production development.

As for the modeling analysis of lattice structure, finite element analysis is a common method, which requires transforming the B-rep model into a physical simulation representation based on discrete linear primitive approximation. And tetrahedral and hexahedral mesh are commonly used in model representations. The disadvantage of this discrete representation is the lack of numerical stability and accuracy of analysis results. At the same time, generating discrete approximate meshes from a given three-dimensional B-rep data model is the most critical and time-consuming step in finite element analysis, which consumes 80\% of the work of the whole design and analysis process. Therefore, using a single geometric representation for design and analysis in the whole modeling is of great advantage. Isogeometric analysis (IGA) assumes that the same tensor product B-spline curve in modeling design is used in physical analysis to establish a closer relationship between geometric modeling design and performance analysis, so that there is no need to generate the finite element mesh for analysis. Besides, there is no need to feed back the analysis result model to the geometric modeling based on B-spline.

In this paper, a volumetric parameterization modeling method of lattice structure is proposed, which realize the integration of modeling and simulation. First of all, the unit cell curve skeleton model for lattice structure is designed. It is divided into three categories in order to meet the basic requirements of IGA method, including branch nodes, joint nodes and end nodes according to the skeletons model. The branch nodes use the orientation-subdivision-mapping mechanism of hexahedron box based on curve skeletons to generate the volumetric parameterization model. The end node and joint node generate the volumetric parameterization model by sweeping and lofting the section along the skeleton line, and then form the unit cell model by volumetric parameterization splicing according to the connection relationship of nodes. Finally, the final lattice structure parameterized model is generated by the cell model’s series operations.

The main contributions of this method are as follows:

1) An integrated design method of lattice structure is proposed to support the modeling

and simulation of various typical lattice structures.

2) The methods of modeling complex node model and cell model are realized, and the volumetric parameterization model has a good quality and it is suitable for IGA.

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ABSTRACT

Lattice structure has been widely used due to the development of additive manufacturing technology, but the imperfect design methods and tools hinder the further development of its design and manufacturing technology. A design method about three-dimensional lattice structures' volumetric parameterization modeling is proposed for the integrated model and simulation design in this paper. The curve skeletons model of lattice structure is designed by means of interactive modeling or skeletons extraction. It is divided into branch nodes, end nodes and joint nodes according to the topology of curve skeletons model. The branch nodes use the hexahedral box orientation-subdivision-mapping mechanism based on curve skeletons to generate the volumetric parameterization model. The end nodes are swept along the skeleton line through a single cross profile to generate the volumetric parameterization model. The joint nodes are lofting along the skeleton line through multi cross profile to generate the volumetric parameterization modeling. Then, a unit cell model is formed by splicing the node connection relationship. Finally, volumetric parameterization lattice structure model is generated through the unit cell model's series of operations. The results show that the volumetric parameterization model obtained by this method has good model quality and can be used for isogeometric analysis(IGA). It provides a good support for modeling and simulation integration of complex lattice structure.

1. Introduction

In recent years, the extensive application of additive manufacturing technology (AM) in manufacturing companies makes it possible to manufacture complex parts, such as lattice structure[1], pipeline structure, organic structure after topology optimization[2], parts with curved flow channel[3], etc. Lattice structure is an ordered ultralight porous structure constructed by simulating the lattice configuration of micro crystal, with the characteristics of small volume density, large specific surface area, high specific mechanical properties[4, 5, 6, 7]. And it is widely used in aerospace, petrochemical, metallurgy, machinery, environmental protection, construction and other industries[8]. However, the drawbacks of the existing design methods and tools hinder the comprehensive development and use of this technology. It is necessary to develop specific models, methods and tools to support the definition and processing paradigm transformation of complex shapes in production development.

As for the modeling analysis of lattice structure, finite element analysis is a common method, which requires transforming the B-rep model into a physical simulation representation based on discrete linear primitive approximation. And tetrahedral and hexahedral mesh are commonly used in model representations[9]. The disadvantage of this discrete representation is the lack of numerical stability and accuracy of analysis results [10]. At the same time, generating discrete approximate meshes from a given three-dimensional B-rep data model is the most critical and time-consuming step in finite element analysis, which consumes 80% of the work of the whole design and analysis process [11]. Therefore, using a single geometric representation for design and analysis in the whole modeling is of great advantage. Isogeometric analysis (IGA) assumes that the same tensor product B-spline curve in modeling design is used in physical analysis to establish a closer relationship between geometric modeling design and performance analysis, so that there is no need to generate the finite element mesh for analysis. Besides, there is no need to feed back the analysis result model to the geometric modeling based on B-spline.

In this paper, a volumetric parameterization modeling method of lattice structure is proposed, which realize the integration of modeling and simulation. First of all, the unit cell curve skeleton model for lattice structure is designed. It is divided into three categories in order to meet the basic requirements of IGA method, including branch nodes, joint nodes and end nodes according to the skeletons model. The branch nodes use the orientation-subdivision-mapping mechanism of hexahedron box based on curve skeletons to generate the volumetric parameterization model. The end node and joint node generate the volumetric parameterization model by sweeping and lofting the section along the skeleton line, and then form the unit cell model by volumetric parameterization splicing according to the connection relationship of nodes. Finally, the final lattice structure parameterized model is generated by the cell model's series operations. The main contributions of this method are as follows:

1). An integrated design method of lattice structure is proposed to support the modeling and simulation of various typical lattice structures.
2). The methods of modeling complex node model and cell model are realized, and the volumetric parameterization model has a good quality and it is suitable for IGA.

The content of this paper is as follows. In Section 2, the related work is introduced. The surface modeling method used in this paper is briefly described in Section 3. In Section 4, the node modeling in lattice structure model is described in detail. Section 5 introduces the method of constructing cell model and lattice structure. In Section 6, several modeling examples are given to verify the method.

2. Related works

There are many kinds of micro structures in lattice structure. The geometric parameters of these micro structures need to be accurately described due to the engineering requirements of structure manufacturing, accurate simulation and defect location. Therefore, the key of the lattice structure's modeling method is how to accurately describe and analyze various micro structures.

At present, there are three kinds of modeling methods that can accurately describe microstructure: structural design based on unit statistical information, structural design based on density distribution representation, and structural design based on accurate geometric description. Compared with the former two methods, the structural design method of precise geometric description can more accurately describe the geometric features of microstructure. The description methods are mainly as follows: image-based reverse modeling, feature modeling based on CAD software, modeling based on implicit surface [12], and modeling based on Voronoi diagram[13].

The analysis of microstructure is mainly in material aspect. The microstructure in lattice structure can be regarded as a composite composed of solid phase and gas phase. In addition, the homogenization method[14], heterogeneous multiscale method[15], multi-scale finite element method[16] and variational multi-scale method[17] are mainly used to analyze various properties of this composite. In particular, the homogenization method is widely used because of its strict mathematical theory and high calculation accuracy. In [18], the lattice structure based on surface is designed. The elastic modulus of the lattice structure is determined by finite element analysis, and the surface equation of the lattice structure is given to control the volume proportion of the lattice structure, so that it can act on the additive manufacturing field.

For designing lattice structure, some scholars have put forward their opinions. For example, literature[19, 20] proposed the topology optimization design theory of composite lattice structure, and used the inverse homogenization technology to realize the design, numerical simulation and experimental verification of lattice structure. In [21], the design method of lattice structure with integral flexible mechanism is proposed, and the deformation shape and energy distribution of the structure are analyzed. In [22], two-dimensional honeycomb lattice sandwich structure and sandwich structure are designed. Compared with solid structure, lattice structure has better mechanical properties. Reference[23] proposes a modeling primitive based on generalized cuboid shape, which is called block. Complex objects are carried out by layout blocks, and then the blocks are connected to form the basic shape of the model object. The two-stage method of micro and macro structure design are not allowed in this method. In [24], it is proposed to add multi void structure to the geometric model of tissue surface in the form of polygon, which is composed of interconnected linear channel network, and its size can be changed according to the required porosity and pore size. However, these methods are only suitable for the design of regular and irregular structures, as well as various geometric deformations, and do not support the parametric design of heterogeneous structures and lattice structures.

The traditional additive manufacturing method is only suitable for the common surface model. Lattice structure brings a lot of difficulties to the traditional additive manufacturing method duce to its complex microstructure. Therefore, many scholars have proposed different methods for additive manufacturing of lattice structure. For example, the honeycomb structure design for additive manufacturing is proposed in [25]. The unit cell structure is realized by adaptive triangulation of solid interior, which has more refined tetrahedron along the solid boundary. But this method can't be extended to free form models. Reference[26] has proposed a framework for modeling heterogeneous objects using three variable Bezier slices. It allows the use of various materials to construct various structures, but only allows a single level of detail and is limited to Bezier trivariate. A microstructure design framework for additive manufacturing is proposed in [27]. Their method based on the process-structure-attribute-behavior model, constructs the octagonal lattice structure with parameterized elements.

In summary, most of the existing lattice structures are represented by three-dimensional B-rep data models, and it is difficult to accurately represent the nodes in the lattice structure due to the complexity of the model. At the same time, the physical performance analysis needs to be transformed into discrete approximation mesh model, so the data conversion process is time-consuming, poor stability and accuracy. Therefore, this paper proposes a lattice structure modeling and a design method based on volumetric parameterization. In this method, the nodes of lattice structure are divided into three types, and then the volumetric parameterization modeling is implemented for the three types of nodes respectively. The cell modeling of lattice structure is realized by splicing the nodes. Finally, the complex lattice structure model is obtained through various operations of the cell.

3. Basic theories and overview of algorithm

3.1. Volumetric parameterization model representation

In the NURBS-based IGA, the physical domain is defined as the concatenated set of face pieces in R^3. A face slice is denoted by Ω and represents the image under NURBS mapping in the parameter domain $(0,1)^3$, as shown in equation(1)[28, 29].

$$\Omega = \{x = \{x, y, z\} \in R^3 \mid x = V(u, v, w), 0 \leq u, v, w \leq 1\} \quad (1)$$

The NURBS volumetric parameterization modeling is denoted by $V(u, v, w)$, as is shown in equation (2).

$$\begin{gathered} V(u,v,w) = \sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{l} R_{\mathrm{i,j,k}}^{\mathrm{p,q,r}}(u,v,w) P_{\mathrm{i,j,k}} \\ R_{\mathrm{i,j,k}}^{\mathrm{p,q,r}}(u,v,w) = \frac{N_{i,p}(u)N_{j,q}(v)N_{k,r}(w)\omega_{i,j,k}}{\sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{l} N_{i,p}(u)N_{j,q}(v)N_{k,r}(w)\omega_{i,j,k}} \end{gathered} \quad (2)$$

$R_{\mathrm{i,j,k}}^{\mathrm{p,q,r}}(u,v,w)$ is a rational basis function, $P_{\mathrm{i,j,k}}(i \in [0,n], j \in [0,m], k \in [0,l])$ is a control point, and in the definition of rational basis functions, $N_{\mathrm{i,p}}(u), N_{\mathrm{j,q}}(v)$ and $N_{\mathrm{k,r}}(w)$ are univariate B-spline basis functions defined on the node vectors U, V, W. p,q,r are the degrees of the direction of parameters u,v,w. $\omega_{\mathrm{i,j,k}}$ is the corresponding weight of the control point.

3.2. Modeling definition

Skeletons model: the curve skeletons of the model is an intuitive, compact and easy-to-operate shape descriptor that can maintain the topology of the original model, and can reflect both topological and shape information of the original model, given by multiple interconnected NURBS curves. As is shown in Fig.1, the curve skeletons of several unit cell model are schematically shown, with straight lines representing NURBS curves and black rectangles indicating the curve start and end points. Nodes model: specifies that the degree k of a point in the curve skeletons model is the maximum number of branches extending from that point. Depending on the degree of the nodes, the nodes can be divided into three categories, $k = 1$ for the end nodes E_n, $k = 2$ for the joint nodes J_n and $k \geq 3$ for the branch node B_n, as shown in Figure 2. Unit cell model: the lattice structure model is generated by periodic arrays of one or more type cells stitched together, and each cell of the constructed lattice

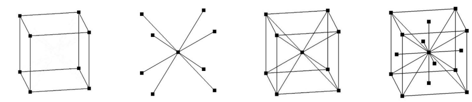


Fig. 1. The lattice monocellular skeleton modeling.

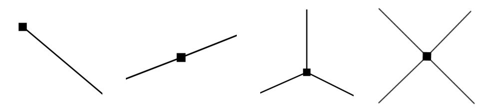


Fig. 2. The skeleton model of nodes modeling.

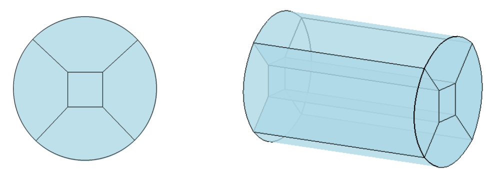


Fig. 3. The lattice structural foundation unit.

structure is called a unit cell mode. The lattice structure model constructed in this paper consists of volume cylinder model. At the request of IGA method, the circular end surface is represented by dividing the circular surface into five NURBS surface patches with the middle rectangle as the base, as is shown in Fig.3(a). The skeleton line passes vertically from the central rectangular surface patches, and the end nodes shape is generated along the skeleton line by a sweeping algorithm, which is shown in Fig 3(b).

3.3. Overview of algorithm

The input of the proposed lattice structure modeling algorithm is unit cell curve skeletons model S_c and nodes profile radius r. The output is volumetric parameterization model V_s of the lattice structure represented by trivariate B-spline volume. The algorithm flow is shown in Fig.4.

1). The curve skeletons model is obtained by skeletons extraction or interaction design for lattice structure unit cells.
2). The model is partitioned into three types of nodes, including joint nodes, end nodes and branch nodes. What's more, the modeling is constructed separately.
3). The branch nodes modeling is divided into two stages because of the complexity. The first stage is to use the optimization method to form a set of inner hexahedral groups matching branch's curve skeletons. The second stage is to obtain an intermediate surface by interpolating the inner hexahedral groups, and then generate a volumetric parameterization model based on this intermediate surface.
4). The joint nodes and end nodes are relatively simple to model. The end node is constructed using the sweeping algorithm with single profile and single skeleton line based on the branch node section information. The joint nodes are constructed by a multi-profile single skeleton line release algorithm.

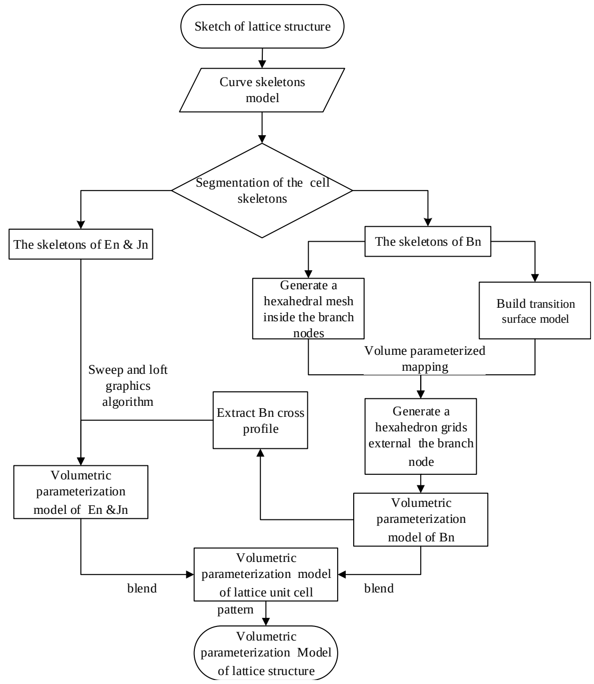


Fig. 4. Algorithmic flow of lattice structure modeling.

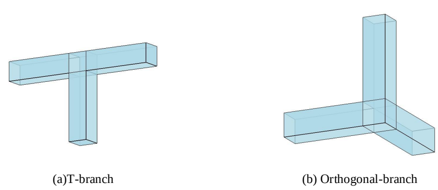


Fig. 5. Typical branch nodes hexahedral mesh.

5). The different node models are matched based on the unit cell skeletons model and then stitched together to generate a unit cell volumetric parameterization model.
6). Unit cell modeling by periodic arrays to obtain a volumetric parameterization model of the lattice structure.

4. Modeling of the node

4.1. Hexahedral box construction of branch nodes

Based on the method[30], a hexahedral meshing method is proposed in this paper. Each face can be "extruded" into the hexahedral structure along the corresponding incident arc direction after subdivision by placing a "skeleton-aware" hexahedral box at each B_n and subdividing the box according to the direction and number of incident arcs d_k on each face. Typical examples of branch nodes are "T-shaped" and "orthogonal", and such can be easily implemented by using a hexahedral box, as is shown in Fig.5.

As for the complex branch nodes with higher degree k, the inner hexahedron combination becomes complex. The central hexahedral box of is "oriented" so that each face is as orthogonal as possible to the incoming branches to obtain the most suitable hexahedral combination. Therefore, the following optimization problems need to be solved on each B_n.

The given branch nodes N and the set of K directions of its incident arc $\{d_1, ..., d_k\}$, solve for an orthogonal basis U, V and W, which makes the function $f(U, V, W)$ to get the minimum value. As shown in equation (3).

$$f_\varepsilon(U,V,W) = \sum_{j=1}^{k} \sqrt{(d_jU)^2+\varepsilon} + \sqrt{(d_jV)^2+\varepsilon} + \sqrt{(d_jW)^2+\varepsilon} \tag{3}$$

Each B_n is a hexahedral box aligned with U, V, W, and each term of the summation reaches its minimum when one of the three axes is aligned with a branch, and the summation reaches its maximum when the branch is passed from any corner of the hexahedral box, so that the summation is an alignment trade-off between different branches and orthogonal bases.

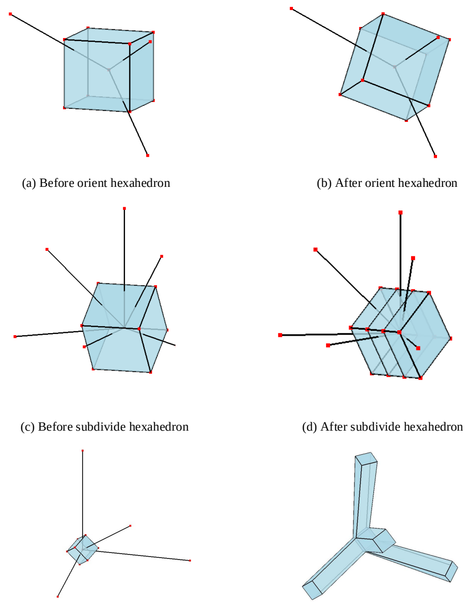
As is shown in Fig.6(a), in this type of branch nodes case, the hexahedron needs to be subdivided into at least three sub hexahedra in order to satisfy the separate "extrusion" of each face into the branching hexahedron structure without the "oriented" operation. After the orientation operation, as is shown in Fig.6(b), the attitude of the hexahedral box is adjusted so that no further subdivision is required and the hexahedron can meet the condition.

After the above steps, there is a set of hexahedral boxes associated with each on B_n, setting to be a hexahedral box with k incident arcs on B_n. Each incident arc at B_n is $a_{\mathrm{i}}(i \in 1, ..., k)$ the face is partitioned into a series of subfaces when more than one a_i matches the same face of B. Each subface satisfies the requirement that "the branches contained in the subplane can be independently extended to connect the tubular structure". Therefore, each face f on the hexahedron B is calculated for all intersections with the incident arc a_{i}. If face f has b_{f} incident arcs, subdivide face f into b_{f} parallel rectangular subfaces. The direction of subdivision selects the direction with the largest projection distance from the intersection point projection to the two orthogonal base directions parallel to face f. The subdivided face f is matched with the corresponding incident arc according to the above rule, and the subdivision operation is shown in Fig.6(c) and (d).

The branch hexahedral structure is generated along the path defined by the curve skeletons branches, and each face f of the subdivided B_n is modeled as a swept volume along the matching branch curve skeletons, thus completing the generation of the inner hexahedral group, as is shown in Fig.6(e) and (f).

4.2. The branch node modeling

The branch node surface shape is given by the multi-patches toric surface. And then the outer surface information topology of the inner hexahedral group samples, fits, and interpolates the toric transition surface to generate a parametric mapping of the constructs to the outer hexahedral group. Toric surface is a polygonal parametric surface using Coons interpolation of polygonal domains[31], a polygonal extension of the classical Bezier surface with corner point interpolatory property, boundary tangent vector, affine invariance, etc. [32, 33].After the toric surface model is obtained by interpolating the boundary



(a) Before orient hexahedron (b) After orient hexahedron

(c) Before subdivide hexahedron (d) After subdivide hexahedron

(e) Hexahedron and branching skeletons (f) Hexahedron of Bn

Fig. 6. Branch nodes orientation and subdivision.

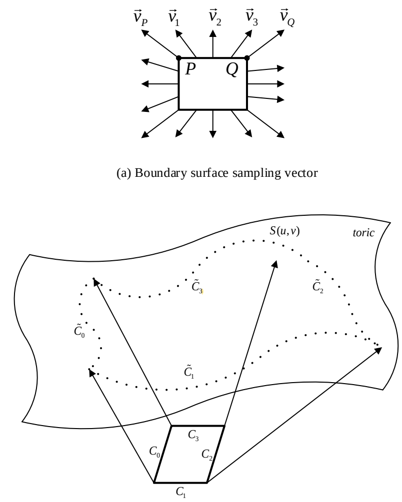
curve with Coons in the multilateral domain, it needs to be partitioned. In other words, it needs to be sampled and binned and fitted to obtain the binned B-sample surface model.

Surface sampling is a key step in boundary hexahedron group generation, which directly affects the modeling quality of the final volumetric parameterization model. The sampling vector is defined by the inner hexahedron group generated in Section 4.2, and the sampling vector is intersected with the toric surface to obtain the sampling points.

The boundary surfaces in the inner hexahedron group that do not produce face-to-face contact are called the outer boundary surfaces. The outer boundary surfaces, which are in the same plane as the end points of the branch nodes are called the cutoff boundary surfaces. Except for the cutoff boundary surface, each outer boundary surface needs to be partitioned into its corresponding surface pieces on the toric surface model. The research proposes the feature point sampling method samples based on the method [34]. It fits and interpolates the toric surface model with one outer edge interface as the processing unit. At the same time, the corner point sampling vectors are defined at the corner points of the outer edge interface of the hexahedral structure in order to reduce the computational effort. The sampling vectors to meet the fitting requirements are obtained by interpolating the corner point sampling vectors at the prism edges of the outer edge interface. As is shown in Fig.7(a), the rectangle in the figure represents an outer edge interface. Take one of the prism edges PQ as an example, $\vec{v}_P, \vec{v}_Q$ are sampling vector on the corner points P, Q. $\vec{v}_1, \vec{v}_2, \vec{v}_3$ is the sampling vector obtained by interpolation along the prism edges PQ.

The sampling vector on the corner points of the outer edge interface of the inner hexahedral group is calculated, which is shown in (4).

$$\vec{v}_P = -\sum_{i=0}^{m} \vec{v}_i \tag{4}$$



(a) Boundary surface sampling vector

(b) Surface sampling and fitting

Fig. 7. Surface sampling method and fitting operation.

The next step is to interpolate the sampling vector at the prismatic edge of the outer interface to meet the fitting requirements, and the sampling vector is shown in equation (5) after the two endpoints of the prismatic curve of the outer interface are defined as above to obtain the sampling vector of corner points.

$$\begin{gathered} \vec{v}_k = C_i(u_k) + Rotate(\vec{v}_R, \vec{v}_P, \alpha)(k = 0, ..., K-1) \\ u_k = k/(K-1) \\ \vec{v}_R = \vec{v}_P \times \vec{v}_Q \\ \alpha = \theta u_k \end{gathered} \tag{5}$$

$C_i(u_k)$ is the uniform K points on the curve $C_i(u)$; $Rotate(\vec{v}_R, \vec{v}_P, \alpha)$ is the rotation of $\vec{v}_P$ counterclockwise by an angle α with $\vec{v}_R$ as the rotation axis. θ is the angle of $\vec{v}_P, \vec{v}_Q$.

By the above way, K sampling vectors are uniformly distributed on the prismatic curve of the outer interface, and the intersection of the sampling vectors and the toric surface is solved for the sampling point by using Newton's iterative method. The sampling points are then fitted to the B-sample curve $\tilde{C}_i(u)(i = 0, 1, 2, 3)$ by using the least squares method which is followed by Coons interpolation with $\{\tilde{C}_i(u)\}$ as the boundary to obtain the B-sample surface slice. As is shown in Fig.7(b), the four prongs of C_0, C_1, C_2, C_3 sample the toric surface separately to obtain the sampling points represented by the black dots in the figure. fitting these sampling points to obtain the four curves of $\tilde{C}_0, \tilde{C}_1, \tilde{C}_2, \tilde{C}_3$, and finally $\tilde{C}_0, \tilde{C}_1, \tilde{C}_2, \tilde{C}_3$ interpolate to generate the B spline surface $S(u, v)$.

After the above operations, the final volumetric parameterization model of the branch nodes is obtained. As is shown in Fig. 8, the inner hexahedral structure shown in Fig.8(a), the toric surface model shown in Fig.8(b), and Fig.8(c) shows the split into a multi-patches B-spline surface model. In the process of sampling the toric surface segmentation and fitting it to a multi-patches B-sample surface model, the relationship between the B-spline surface patches and the outer edge interface of the internal hexahedral structure is also established. The

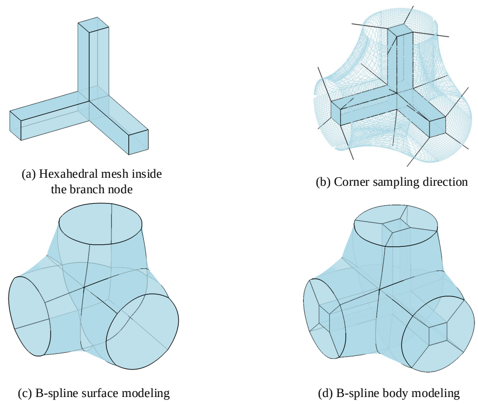


Fig. 8. Branch node modeling generation.

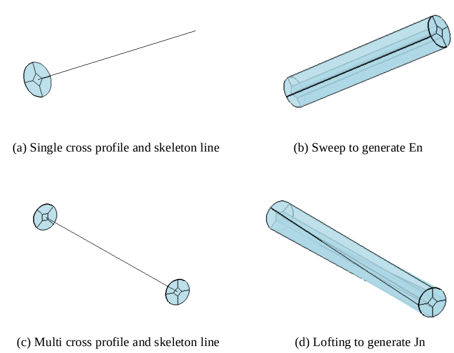


Fig. 9. Join node and end node models generation.

remaining sections can be interpolated between the outer interface of the inner hexahedron group and the corresponding B-spline surface patches, which is constructed as a volumetric parameterization model by using the Coons volumetric interpolation mapping method[35]. Fig.8(d) shows the parametric modeling of a B-spline volume with orthogonal 3-degrees node, consisting of 19 patches of B-spline volume.

4.3. The Joint node and the end node modeling

After the construction of nodes modeling in section 4.2 is completed, the end section group of nodes with the same geometric model is obtained, and then the basic data can be provided for the other two types of nodes modeling. The cross-profile model of each branch is extracted from the branch nodes. According to the corresponding curves skeletons, the single cross profile uses the sweep algorithm to generate end nodes along the skeleton line, the multi cross profile use the lofting algorithm to generate joint nodes along the skeleton line, as shown in Fig.9.

5. Modeling of the unit cell and lattice structure

5.1. The Unit Cell modeling

The unit cell model is the least repeated unit in the large-scale lattice structure, which describes the general characteristics of the lattice structure. According to the different nodes segmented by the unit cell curve skeletons modeling, a set of node volumetric parameterization modeling is obtained by nodes modeling method in Section 4. According to the topological relationship among nodes, the volumetric parameterization model of nodes is carried out by translation, rotation, mirror image and other basic graphic operations, and the volumetric parameterization model of unit cell structure matching curves skeletons model is obtained.

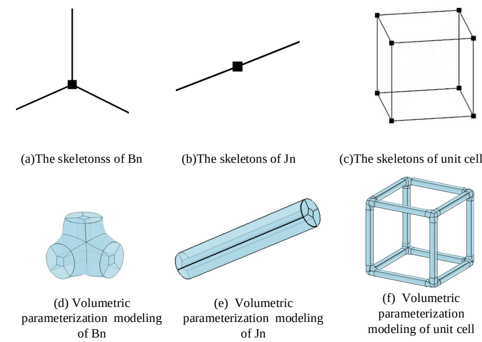


Fig. 10. Cells model generation.

Taking a simple cube cell as an example, the skeletons model of unit cell is shown in Fig.10(c). The cell is divided into two types of nodes through nodes model segmentation, which are shown in Fig.10(a) and (b) for construction. The nodes model is constructed by the method described in Section 4. Between the two types, one is orthogonal 3-degree node volumetric parameterization model, consisting of 19 trivariate B-spline volume, which is shown in Fig.10(d). The other is the model of simple joint nodes is composed of five trivariate B-spline volume, which is shown in Fig.10(e). The model of orthogonal third-order nodes and joint nodes is obtained with 8 branch nodes and 12 joint nodes matching with the skeletons model through a series of graphic operations. The corresponding splices are used to obtain the simple cubic volumetric parameterization unit cell model, which was composed of 212 trivariate B-spline volume as is shown in Fig.10(f).

5.2. Lattice structure modeling

The unit cell structure in uniform lattice structure is generally parallelepiped, which is formed by three translational basis vectors intersecting one vertex in the hexahedron. The length of the three translational basis vectors is defined as the size of the unit cell. The three translational basis vectors are orthogonal to each other, and the lattice structure is called cuboid lattice structure. It is one of the most widely used homogeneous lattice structures because of unique symmetric properties. Therefore, it is taken as a typical example for modeling in this paper. This method is similar to texture tiling. The modeling process mainly consists of two steps. First, preparing the basic object, namely the unit cell structure. These cells are then replicated in space to form lattice structures. In the case of volumetric parameterization model of unit cell and given lattice structure curve skeletons model, the center of each unit cell model is taken as the core point to get the position of unit cell. The curve skeletons model is also divided into discrete points to determine the translation position of unit cell, and then the array can be carried out along

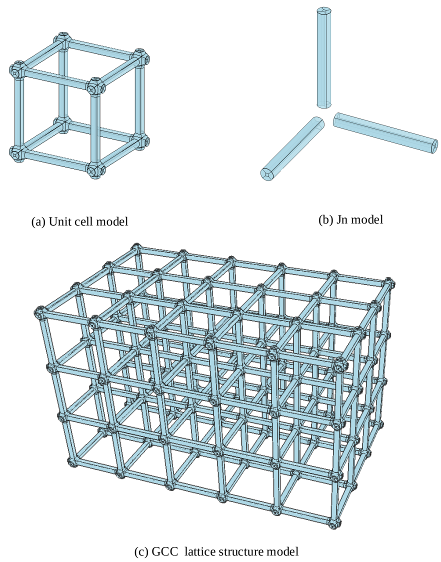


Fig. 11. Lattice structure model generation.

the basis vector of translation. For lattice structure requiring geometrical connectivity, adjacent cells should be connected in a similar way to the joint nodes after arraying, so as to generate the final lattice structure's volumetric parameterization model.

Taking a cubic lattice that satisfies geometric connectivity as an example, the cube cell modeling composed of 308 B-spline volume is shown in Fig. 11(a). Paralleling to the X, Y and Z axes are the translational basis vectors. The unit cell is operated as a translational array, and the connectivity of adjacent unit cells is guaranteed to be composed of connecting rods consisting of 5 B-spline volume in each group, which is shown in Fig.11(b). During splicing, it is necessary to ensure that there is no overlap and intersection among unit cells, and that the number of control points of the connected profile is consistent in the connected area, and each profile is shared with each other. After the tie-array operation, a simple cubic lattice structure is obtained, which is shown in Fig.11(c), consisting of 4096 B-spline volume in total. The lattice structure model is reserved with a connected interface, which can make an infinite array along the basis vector in light of the need.

6. Examples and discussions

In this section, we build a multi branch nodes model from 3 degree to 12 degree. These examples prove the effectiveness and stability of the branch nodes modeling method. Fig.12(a) shows the posture of the hexahedral box after the "orientation" and "subdivision" operations of the incident branch; Fig.12(b) shows the inner hexahedron group generated along the branch curve skeletons; Fig.12(c) shows branch node modeling of multi-patch toric surface constructed by Coons interpolation in the multilateral domain after constructing a closed boundary; Fig.12(d) shows that the boundary surface information of the inner hexahedron group is the datum plane, and the final trivariate B-spline volumetric parameterization nodes model is generated by sampling-fitting-mapping method. Taking all these examples as consideration, this method can generate hexahedron group models, which can meet the requirements of IGA.

Table 1. Parameters in the process of branch node

Bn's name	Bn's Box	Inner patches	Surface	Bn's patches
3 degree	1	4	8	19
6 degree (Ort)	1	7	8	31
4 degree	1	5	10	23
6 degree	1	7	8	31
8 degree	2	10	10	44
10 degree	2	12	12	52
12 degree	4	20	14	76

The quantitative data of all branch nodes instances are shown in Table 1. Including the number of B-spline patches contained in the hexahedral box initially generated. Number of B-spline volume of inner hexahedron set generated along the curve skeletons of branch nodes. In order to construct the modeling design of branch nodes, the number of surfaces of transition model is given and the total number of B-spline volume of the final generated branch node modeling. It can be concluded from the number of patches the final branch nodes model can be represented by a small number of B-spline volume through the construction of this method. Because the cross-profile structure between different branch nodes is the same, it is greatly convenient to construct the model of joint nodes and end nodes in unit cell and lattice structure.

Based on the active design of the unit cell curve skeletons model, the above branch nodes models are translated and rotated to the corresponding position of the curve skeletons model to construct joint nodes and end nodes, and then the multiple unit cell model is formed by splices. A simple cube cell in Fig.13(a) is composed of 8 orthogonal 3-degree branch nodes and 12 joint nodes, each joint node is constructed by 5 B-spline volume. A total of 212 B-spline volume are constructed. The body centered cell in Fig.13(b) is composed of eight 4-degree branch nodes, one 8-degree branch nodes and 20 joint nodes, each joint node is constructed by 5 B-spline volume, and a total of 328 B-spline volume are constructed. The same body-face cube cell in Fig.13(c) is constructed from a total of 387 patches. 3D-Kagome cell in Fig.13(d) consists of one 6-degree branch node and 6 end-nodes, which are constructed from 61 B-spline volume in total. Pyramid cell in Fig.13(e) consists of one 8-degree branch node and eight end-nodes, which are constructed from 84 B-spline volume in total. Body-plus cell in Fig. 13(f) is constructed from 102 B-spline volume.

The Jacobian values of model are calculated in order to show the mesh quality of model and verify the applicability of IGA method. All models of volumetric parameterization are represented by 3×3×3 control points, and the hexahedral scaled Jacobian values composed of hexahedral isoperimetric points is calculated. The scaled Jacobian color diagram of the unit cell model is shown in Fig.14.

Periodic array is made for unit cell model or nodes model according to the needs of curve skeletons model. Thus, the final lattice structure model is formed, which is elaborately shown in

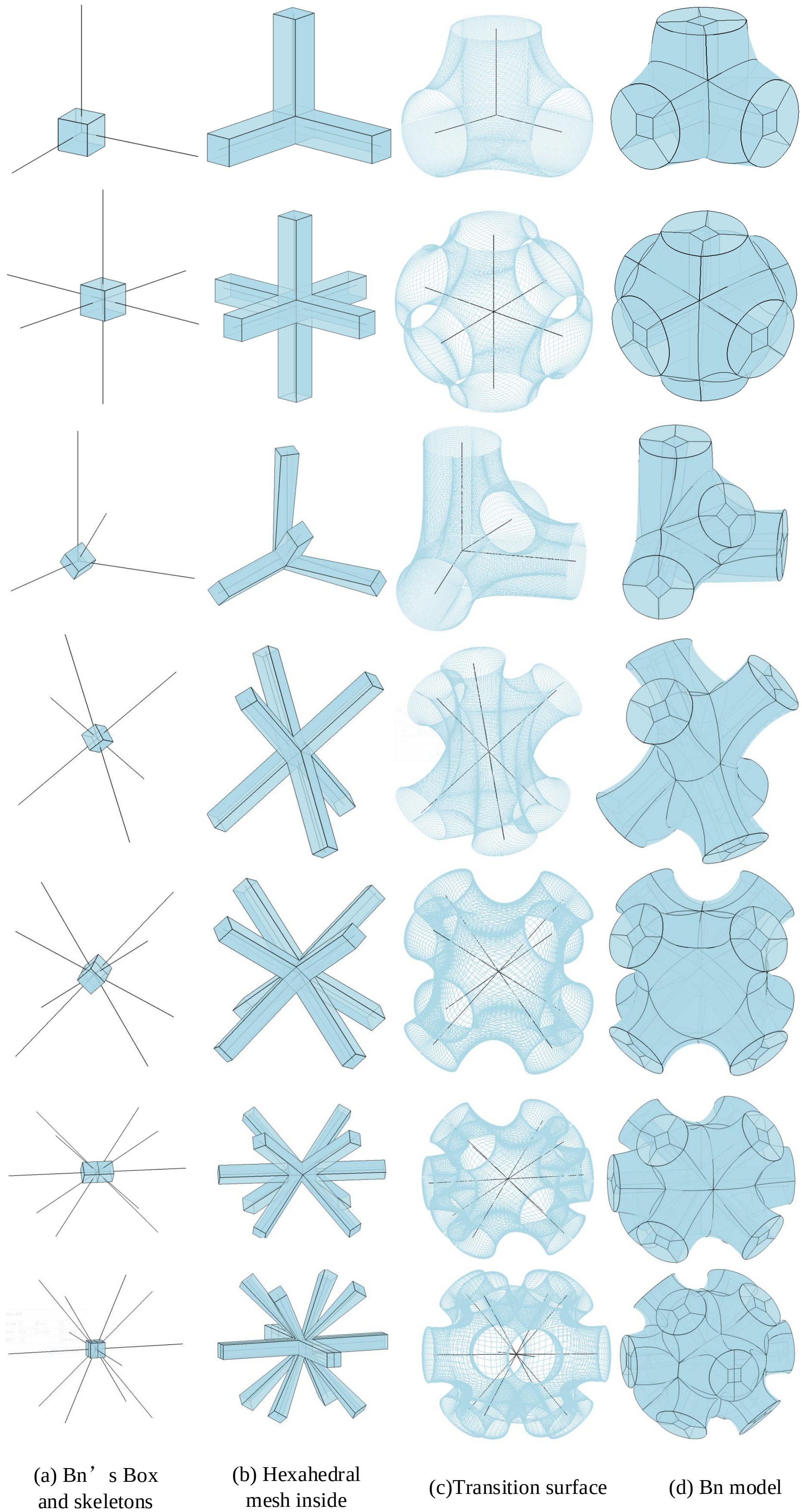


Fig. 12. Modelling of Branch node.

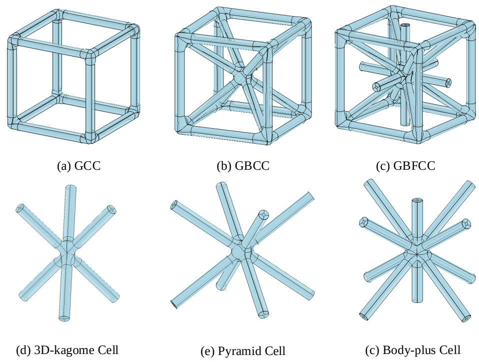


Fig. 13. Parameterized model of cell.

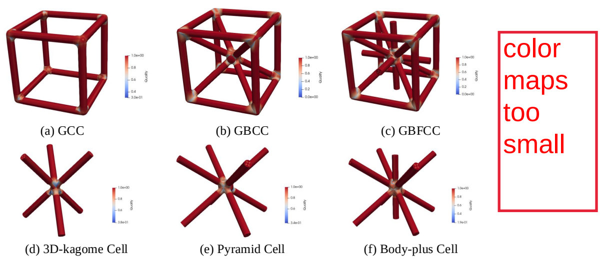


Fig. 14. The Jacobi value of cell.

Fig.15.

The simple cubic lattice structure is spliced by simple cube cell array, with a total of 4096 B-spline volume. The body-centered cubic lattice structure is obtained by body-centered cubic cell array, with a total of 14760 B-spline volume. And the body-face centered cubic lattice structure is obtained by body-face centered cubic cell array, with a total of 17415 B-spline volume. The 3D Kagome sandwich lattice structure is obtained by 3D Kagome cell model and splint arraying, with a total of 3664 B-spline volume. The pyramid sandwich lattice structure is obtained by pyramid cell model and splint arraying, with a total of 5044 B-spline volume. The body-centered plus lattice structure is obtained by body-centered plus cell model and splint arraying, with a total of 6124 B-spline volume. The above models verify the applicability of the modeling method proposed in this paper to the volumetric parameterization modeling of three-dimensional lattice structures. The above examples are transformed into a discrete hexahedral mesh model, and the quality of models is measured by Jacobian values.

As can be seen in Figure 15, the Jacobian values distribution of lattice structure are relatively uniform, especially around joint nodes and end nodes. The region with Jacobian values less than 0.5 is generally generated on some patches of more complex branch nodes, which is followed by the region near the singularity of the model. The reason is that there are sharp angles on the boundary surfaces of the branch nodes. It is a feasible direction to improve the sampling method of the inner hexahedron group and reduce the number of singularities. The results show that the volumetric parameterization models obtained by this method have good model quality and can be used for IGA. It provides a good support for the modeling and simulation integration of complex lattice structures.

7. Conclusions

We present a design method about three-dimensional lattice structure' volumetric parameterization modeling based on curve skeletons. The input curve skeletons model is divided into three types of nodes model, including branch nodes, end nodes and joint nodes. The unit cell model is respectively modeled for the three types of nodes. Finally, the final three-dimensional lattice structure' volumetric parameterization model is obtained through the unit cell model's series of operations. The effectiveness and stability of this modeling method are proved by constructing typical cell types of lattice materials and obtaining complex lattice structure modeling. Because of a good Jacobian values of the model, the IGA method can be applied directly to the model, the proposed method can provide a good model support port for complex lattice structures' integration of modeling and simulation.

In the future, according to the requirements of IGA of complex lattice structure, we will continue to improve the quality of volumetric parameterization model, such as reducing the number of singularities and singular curve of model. In addition, the next research direction is using this method to obtain a more complex shape of the volumetric parameterization model, and using the model obtained in this paper to achieve high degree of freedom 3D printing directly.

Acknowledgments

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Supplementary Material

Currently there is no other material.

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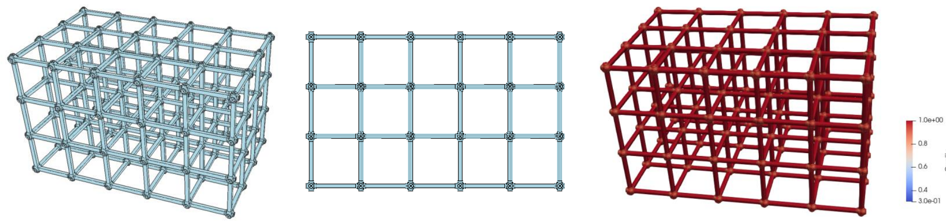
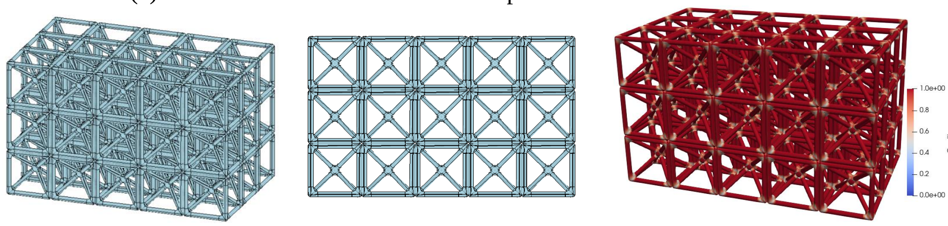
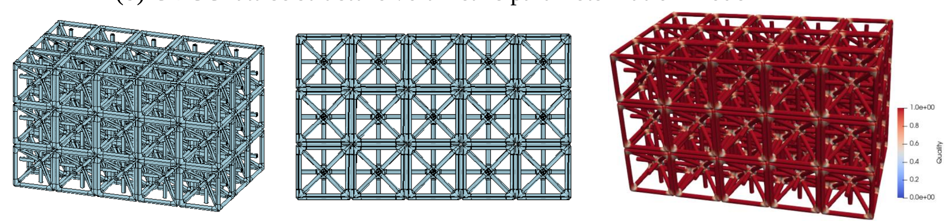


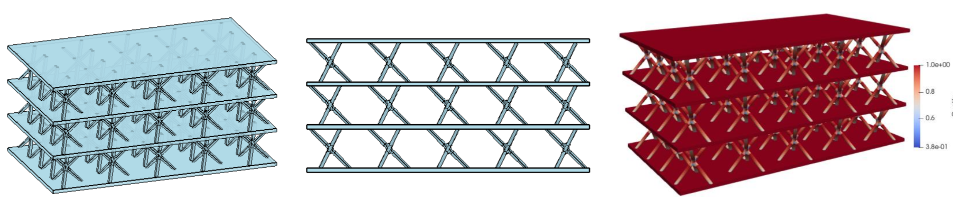
(a) GCC lattice structure volumetric parameterization model



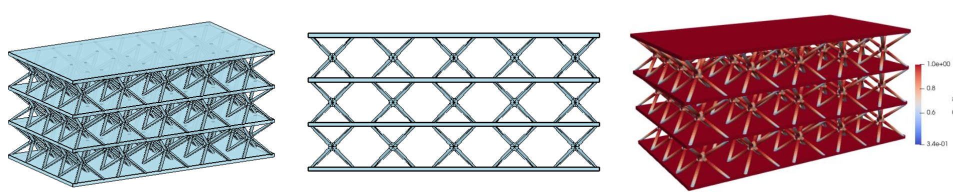
(b) GBCC lattice structure volumetric parameterization model



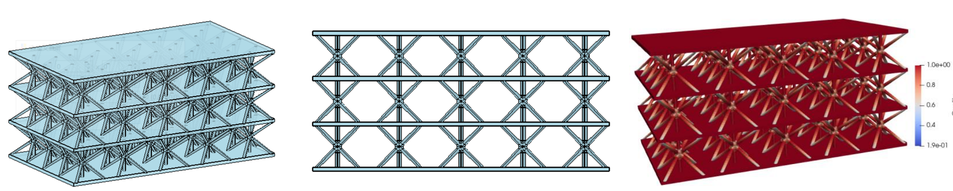
(c) GBFCC lattice structure volumetric parameterization model



(d) 3Dkagome lattice structure volumetric parameterization model



(e) Pyramid lattice structure volumetric parameterization model



(f) Body plus lattice structure volumetric parameterization model

Fig. 15. Lattice structure model and Jacobi value.

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Research Highlights

The main contributions of this method are as follows:

1) An integrated design method of lattice structure is proposed to support the modeling and simulation of various typical lattice structures.

2) The methods of modeling complex node model and cell model are realized, and the volumetric parameterization model has a good quality and it is suitable for IGA.