Homework2

2023-03-10

Data Input

```
time <- c(0, 100, 300, 500, 700, 1000, 2000, 4000, 100, 300, 500, 700, 1000,
          2000, 4000, 10^10)
time <- matrix(time, ncol = 2)
n2000 <- c(292, 494, 332, 236, 261, 308, 73, 4)
n200 <- c(41, 44, 24, 32, 29, 21, 9, 0)
n20 <- c(3, 7, 4, 1, 3, 2, 0, 0)
```

Likelihood Function

```
# for n(200) and n(20)
like <- function(theta, time, dj){
  lj <- rep(NA, nrow(time))
  for(j in 1:nrow(time)){
    lj[j] <- dj[j]*log(pexp(time[j,2], rate = 1/theta)-pexp(time[j,1], rate = 1/theta))
  }
  return(exp(sum(lj)))
}
# for n(2000)
like.2000 <- function(theta, time, dj){
  lj <- rep(NA, nrow(time))
  for(j in 1:nrow(time)){
    lj[j] <- dj[j]*log(pexp(time[j,2], rate = 1/theta)-pexp(time[j,1], rate = 1/theta))
  }
  return(sum(lj))
}
```

Ngative Log-Likelihood Function

```
# for n(200) and n(20)
negLike <- function(theta, time, dj){
  logLike <- log(like(theta, time, dj))
  return(-logLike)
}
# for n(2000)
negLike.2000 <- function(theta, time, dj){
  logLike <- like.2000(theta, time, dj)
  return(-logLike)
}
```

Find Initial

```
theta.x <- seq(200, 1000, 0.01)
# n(2000)
like.y.2000 <- c()
for(i in 1:length(theta.x)){
  like.y.2000 <- c(like.y.2000, like.2000(theta.x[i], time, n2000))
}
indx.max.2000 <- which(like.y.2000 == max(like.y.2000))
max.y.2000 <- theta.x[indx.max.2000]
max.y.2000
```

```
## [1] 612.77
```

```
# n(200)
like.y.200 <- c()
for(i in 1:length(theta.x)){
  like.y.200 <- c(like.y.200, like(theta.x[i], time, n200))
}
indx.max.200 <- which(like.y.200 == max(like.y.200))
max.y.200 <- theta.x[indx.max.200]
max.y.200
```

```
## [1] 572.27
```

```
# n(20)
like.y.20 <- c()
for(i in 1:length(theta.x)){
  like.y.20 <- c(like.y.20, like(theta.x[i], time, n20))
}
indx.max.20 <- which(like.y.20 == max(like.y.20))
max.y.20 <- theta.x[indx.max.20]
max.y.20
```

```
## [1] 440.17
```

Optim

```
# n(2000)
initial.2000 <- 620
theta.hat.2000 <- optim(initial.2000, negLike.2000, time = time, dj = n2000,
                        hessian = TRUE, method = "Brent", lower = 400, upper = 900)
theta.h.2000 <- theta.hat.2000$par
theta.h.2000
```

```
## [1] 612.7727
```

```
# n(200)
initial.200 <- 550
theta.hat.200 <- optim(initial.200, negLike, time = time, dj = n200,
                       hessian = TRUE, method = "Brent", lower = 300, upper = 800)
theta.h.200 <- theta.hat.200$par
theta.h.200
```

```
## [1] 572.2742
```

```r
# n(20)
initial.20 <- 450
theta.hat.20 <- optim(initial.20, negLike, time = time, dj = n20,
                      hessian = TRUE, method = "Brent", lower = 200, upper = 700)
theta.h.20 <- theta.hat.20$par
theta.h.20
```

```
## [1] 440.1711
```

Confidence Interval by Likelihood Ratio

```r
LR <- function(theta, theta.hat, time, dj){
  like(theta, time, dj)/like(theta.hat, time, dj)
}
LR.2000 <- function(theta, theta.hat, time, dj){
  exp(like.2000(theta, time, dj) - like.2000(theta.hat, time, dj))
}
LR.CI <- function(theta, theta.hat, time, dj, alpha = 0.05){
  (LR(theta, theta.hat, time, dj) - exp(-qchisq(1-alpha, 1)/2))^2
}
LR.CI.2000 <- function(theta, theta.hat, time, dj, alpha = 0.05){
  (LR.2000(theta, theta.hat, time, dj) - exp(-qchisq(1-alpha, 1)/2))^2
}
```

Lower

```r
# n(2000)
L.2000 <- optim(550, LR.CI.2000, lower = 450, upper = 650, method = "Brent",
                theta.hat = theta.hat.2000$par, time = time, dj = n2000, alpha = 0.05)
L.2000$par
```

```
## [1] 585.8676
```

```r
# n(200)
L.200 <- optim(500, LR.CI, lower = 400, upper = 550, method = "Brent",
               theta.hat = theta.hat.200$par, time = time, dj = n200, alpha = 0.05)
L.200$par
```

```
## [1] 497.5785
```

```r
# n(20)
L.20 <- optim(300, LR.CI, lower = 200, upper = 400, method = "Brent",
              theta.hat = theta.hat.20$par, time = time, dj = n20, alpha = 0.05)
L.20$par
```

```
## [1] 288.8806
```

Upper

```r
# n(2000)
U.2000 <- optim(700, LR.CI.2000, lower = 600, upper = 800, method = "Brent",
              theta.hat = theta.hat.2000$par, time = time, dj = n2000, alpha = 0.05)
U.2000$par
```

```
## [1] 641.3072
```

```r
# n(200)
U.200 <- optim(600, LR.CI, lower = 600, upper = 700, method = "Brent",
              theta.hat = theta.hat.200$par, time = time, dj = n200, alpha = 0.05)
U.200$par
```

```
## [1] 662.2594
```

```r
# n(20)
U.20 <- optim(700, LR.CI, lower = 600, upper = 800, method = "Brent",
              theta.hat = theta.hat.20$par, time = time, dj = n20, alpha = 0.05)
U.20$par
```

```
## [1] 713.3757
```

Summary

```r
LR.interval.2000 <- paste("[", round(L.2000$par), ",", round(U.2000$par), "]", sep = "")
LR.interval.200 <- paste("[", round(L.200$par), ",", round(U.200$par), "]", sep = "")
LR.interval.20 <- paste("[", round(L.20$par), ",", round(U.20$par), "]", sep = "")
```

Confidence interval by normal approximation

Standard Error

```r
se.theta.2000 <- sqrt(1/theta.hat.2000$hessian)
se.theta.200 <- sqrt(1/theta.hat.200$hessian)
se.theta.20 <- sqrt(1/theta.hat.20$hessian)
```

Normal-Approximate CI

```r
N.interval.2000 <- paste("[", round(theta.h.2000 - qnorm(0.975)*se.theta.2000),
                         ",", round(theta.h.2000 + qnorm(0.975)*se.theta.2000),
                         "]", sep = "")
N.interval.200 <- paste("[", round(theta.h.200 - qnorm(0.975)*se.theta.200),
                        ",", round(theta.h.200 + qnorm(0.975)*se.theta.200),
                        "]", sep = "")
N.interval.20 <- paste("[", round(theta.h.20 - qnorm(0.975)*se.theta.20),
                       ",", round(theta.h.20 + qnorm(0.975)*se.theta.20),
                       "]", sep = "")
```

Homework(a)

```r
results <- data.frame("Inference" = c("ML Estimate", "Standard Error",
                                      "CI by Likelihood", "CI by Normal"),
                      "n.2000" = c(round(theta.h.2000), round(se.theta.2000, digits=2),
                                   LR.interval.2000, N.interval.2000),
                      "n.200" = c(round(theta.h.200), round(se.theta.200, digits=2),
                                  LR.interval.200, N.interval.200),
                      "n.20" =c(round(theta.h.20), round(se.theta.20, digits=2),
```

```r
                                LR.interval.20, N.interval.20))
results
```

```
##          Inference    n.2000     n.200      n.20
## 1      ML Estimate       613       572       440
## 2   Standard Error     14.13     41.72       101
## 3 CI by Likelihood [586,641] [498,662] [289,713]
## 4     CI by Normal [585,640] [491,654] [242,638]
```

Homework(b)

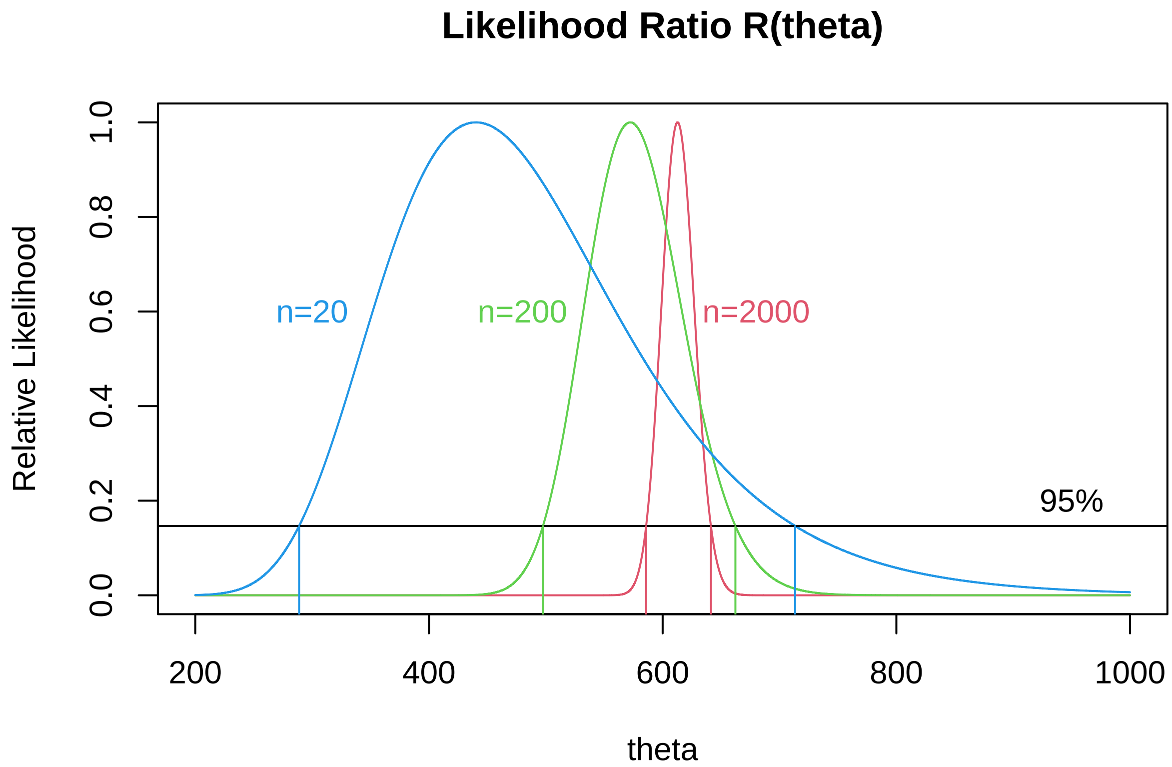
```r
LR.theta.2000 <- c()
for(i in 1:length(theta.x)){
  LR.theta.2000 <- c(LR.theta.2000,
                     LR.2000(theta.x[i],theta.hat.2000$par, time, n2000))
}
LR.theta.200 <- c()
for(i in 1:length(theta.x)){
  LR.theta.200 <- c(LR.theta.200,
                    LR(theta.x[i], theta.hat.200$par, time, n200))
}
LR.theta.20 <- c()
for(i in 1:length(theta.x)){
  LR.theta.20 <- c(LR.theta.20,
                   LR(theta.x[i], theta.hat.20$par, time, n20))
}

plot(theta.x, LR.theta.2000, type = "l", col = "2",
     xlab = "theta", ylab = "Relative Likelihood", main = "Likelihood Ratio R(theta)")
lines(theta.x, LR.theta.200, col = "3")
lines(theta.x, LR.theta.20, col = "4")

lines(c(0, 1200), c(exp(-qchisq(0.95, 1)/2), exp(-qchisq(0.95, 1)/2)))

lines(c(L.2000$par, L.2000$par),
      c(-1, exp(-qchisq(0.95, 1)/2)), col = 2)
lines(c(U.2000$par, U.2000$par),
      c(-1, exp(-qchisq(0.95, 1)/2)), col = 2)
lines(c(L.200$par, L.200$par),
      c(-1, exp(-qchisq(0.95, 1)/2)), col = 3)
lines(c(U.200$par, U.200$par),
      c(-1, exp(-qchisq(0.95, 1)/2)), col = 3)
lines(c(L.20$par, L.20$par),
      c(-1, exp(-qchisq(0.95, 1)/2)), col = 4)
lines(c(U.20$par, U.20$par),
      c(-1, exp(-qchisq(0.95, 1)/2)), col = 4)

text(950, 0.2, "95%")
text(680, 0.6, "n=2000", col = 2)
text(480, 0.6, "n=200", col = 3)
text(300, 0.6, "n=20", col = 4)
```



Homework(c)

When the confidence level is the same at 95%, as the sample size increases, the standard error becomes smaller and the width of the confidence interval also becomes smaller.