A city is divided into n districts. The time (in minutes) it takes an ambulance to travel from District i to District j is denoted as d_{ij}. The population of District i (in thousands) is p_i. An example is shown in Table 2 and Table 3. The distances between districts are shown in Table 2, and the population information is shown in Table 3. In this instance, we have $n = 8$ districts. We may see that, e.g., it takes 5 minutes to travel from District 2 to District 3, and there are 40,000 citizens.

District	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	0	3	4	6	8	9	8	10
2	3	0	5	4	8	6	12	9
3	4	5	0	2	2	3	5	7
4	6	4	2	0	3	2	5	4
5	8	8	2	3	0	2	2	4
6	9	6	3	2	2	0	3	2
7	8	12	5	5	2	3	0	2
8	10	9	7	4	4	2	2	0

District	**Population**
1	40
2	30
3	35
4	20
5	15
6	50
7	45
8	60

The city has m ambulances and wants to locate them to m of the districts. For each district, the *population-weighted firefighting time* is defined as the product of the district population times the amount of time it takes for the closest ambulance to travel to it. The decision maker aims to locate the m ambulances to minimize the *maximum* population-weighted firefighting time among all districts.

As an example, suppose that $m = 2, n = 8, d_{ij}$ and p_i are provided in Table 2, and the two ambulances are located in District 1 and 8. We then know that for Districts 1, 2, and 3 the closest ambulance is in District 1 and for the remaining five districts the closet ambulance is in District 8. The firefighting time for the eight districts are thus 0, 3, 4, 4, 4, 2, 2, and 0 minutes, respectively. The population-weighted firefighting times may then be calculated as 0, 90, 140, 80, 60, 100, 90, and 0. The maximum among the eight districts is therefore 140.

For this problem, formulate an integer program that can minimize the maximum population-weighted firefighting time among all districts. Then write a program to invoke a solver (e.g., write a Python program to invoke Gurobi Optimizer) to solve the above instance and find an optimal solution for each problem. Write down the minimized maximum population-weighted firefighting times among all districts of the two districts that ambulances should be located in (i.e., the objective value of an optimal solution). Do not have any symbol other than numeric values in your answer.

Hint. To formulate a linear integer program for this problem, you may try to define the following set of decision variables: x_j is 1 if an ambulance is located in District j and 0 otherwise, y_{ij} is 1 if for District i the closest ambulance is located in District j and 0 otherwise, and w_i as the distance between District i and its closest ambulance. You may then want to consider the following IP

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{n} w_i \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_j = m \\
& y_{ij} \leq x_j \quad \forall i = 1, \ldots, n, j = 1, \ldots, n \\
& \sum_{j=1}^{n} y_{ij} = 1 \quad \forall i = 1, \ldots, n \\
& w_i \geq \sum_{j=1}^{n} d_{ij} y_{ij} \quad \forall i = 1, \ldots, n. \\
& x_i, y_{ij} \in \{0, 1\} \quad \forall i = 1, \ldots, n, j = 1, \ldots, n \\
& w_i \geq 0 \quad \forall i = 1, \ldots, n
\end{aligned}
$$

about what it will do. You may then try to modify this IP to form your IP for this problem.