Simulated Annealing

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January 31, 2017

1 Introduction

This work is mainly about the simulated annealing process. There are two parts in total. The first part mainly considered the iterative improvement. For both ferro-magnetic and frustrated situations, we will choose to flip 1, 2 or more spins to make the cost lower than before. The algorithm terminates when no improvement for any state.

2 Problem statement

1. In both ferro-magnetic and frustrated situations, how many restarts are needed for reproducible results?
2. In frustrated situation, investigate the influence of neighbourhood size.

3 Results

3.1 Exercise 1.1

Firstly, we did some experiments in the ferro-magnetic situation with $n = 100$. Firstly, in the situation neighbourhood size is 1, we need 436 restarts to get the minimal energy -2512. When we changed the neighbourhood size to 2, it just need 207 restarts. Finally, we tried with neighbourhood size 3, it just need 138 restarts to get the minimal energy.
Then, we did some experiments in the frustrated situation with $n = 100$. When neighbourhood size is 1, we did 1000000 restarts found minimal energy is -602. Then we did with neighbourhood size 2, and found minimal energy is -736 with 1000000 restarts. So for the frustrated situation, it is not easy to convergence. Especially with less neighbourhood size.

3.2 Exercise 1.2

In frustrated situation, it is not easy to get the minimal energy. We firstly did experiments with restart number 100000, it will take on average 10 seconds. The minimal energy is more or less -736, but we got it ranges from -736 to -586 in 5 times experiments.
Then we changes the restart number to 1000000. It costed 95 seconds, and got results ranged from -704 to -644 . It ranges less, but also didn't yield nice results.
When increase the restarts to 10000000, it will take 998 seconds and still didn't give a good result.

3.3 Exercise 2.1

3.3.1 Plot reconstruction for $n = 50$

In figure 1, the resulting plots are shown when simulated annealing is applied on a problem with $n = 50$ in the case that the system is frustrated.

In figure 2, the resulting plots are shown when simulated annealing is applied on a problem with $n = 50$ in the case that the system is ferromagnetic.

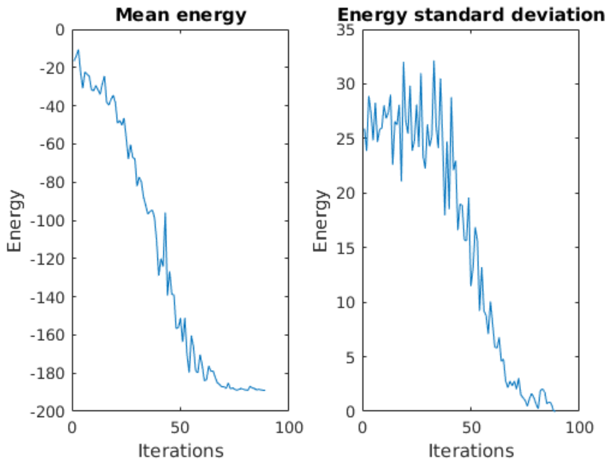

Figure 1: Simulated Annealing in which 1000 samples are generated at each temperature using Metropolis-Hasting with $\beta_0 = \frac{1}{\max(dE)}$ and $\beta_{t+1} = \beta_t \cdot 1.05$ for a frustrated system with $n = 50$.

3.3.2 Choice of β and `factor`

$$p(x) = \frac{\exp(-\beta E(x))}{Z} \tag{1}$$

In equation 1, the definition of the probability distribution of a state x is shown. If you take $\beta \to \infty$, then $p(x) \to 0$ and all states are assigned approximately equal probability. By that, the sampling procedure can get stuck in any state if there are no neighbours with higher probability. Since the probabilities are approximately equal, it is very hard to find any better neighbours. When we ran an experiment with $\beta \approx 1000$ we found indeed that there were only a few number of iterations (< 10) which is caused by the fact that it gets stuck in a particular state. Also, the probability that the final state is a global optimum is extremely low.

The lower the choice for β, the more samples are accepted during the sampling procedure. This will result in a better final state and a better approximation for the minimum energy. Because more samples are accepted, it will take a longer time to finish the simulation.

In conclusion, smaller *beta* (*beta* $\to 0$) will result in better results but it takes a longer time to finish the simulation.

The `factor` variable defines how β is increased at each timestep (since $\beta_{t+1} = \beta_t \cdot \texttt{factor}$). The more `factor` $\to 1$, the more slowly β grows and therefore the more time it takes for the sampling procedure. The larger `factor`, the less time it takes, but the results get worse.

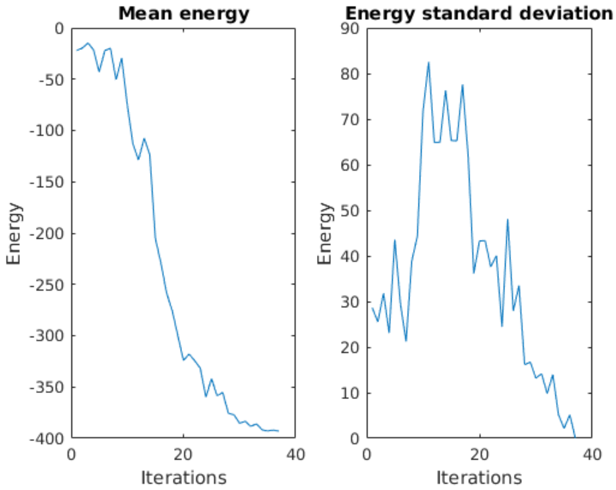

Figure 2: Simulated Annealing in which 1000 samples are generated at each temperature using Metropolis-Hasting with $\beta_0 = \frac{1}{\max(dE)}$ and $\beta_{t+1} = \beta_t \cdot 1.05$ for a ferromagnetic system with $n = 50$.

3.3.3 Choice of `factor`

4 Discussion

5 Conclusion

6 Appendix