Homework 2

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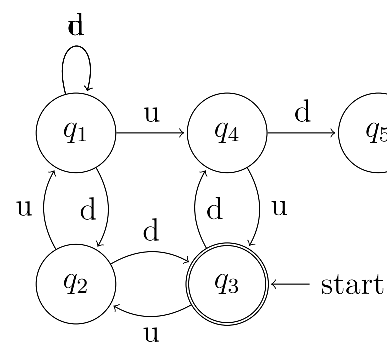
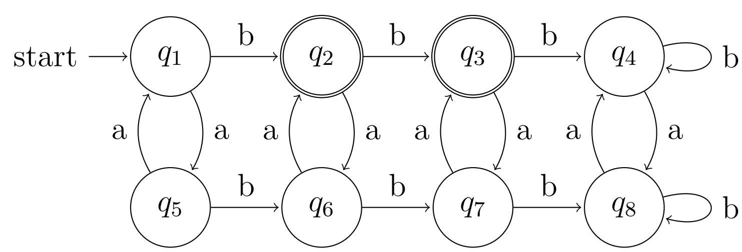
March 15, 2015

Problem 1.

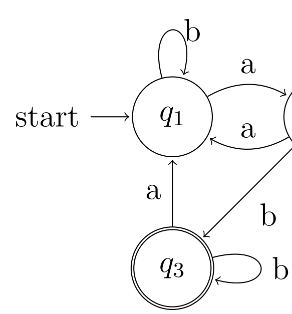


Problem 2.

a.

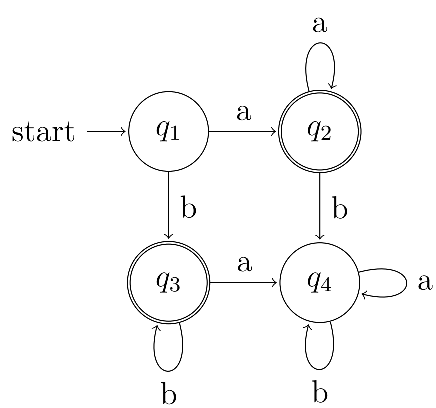


b.

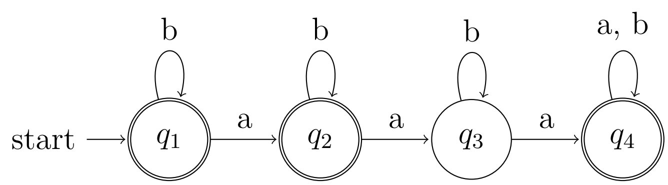


Problem 3.

a.

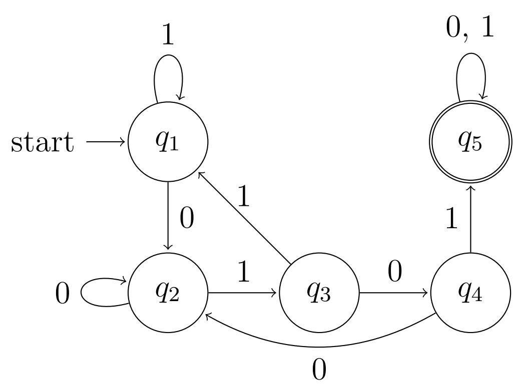


b.

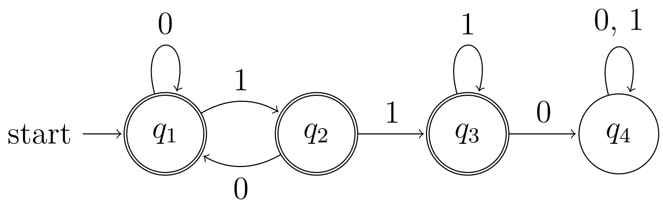


Problem 4.

a.



b.

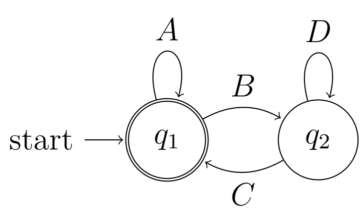


Problem 5.

Proof. Given $A_1 \cdot w \in A$ is regular. Then it's reverse $= w \cdot A_1^R$ is regular (the regular operation.) By induction, if A is regular, A^R is regular. $\square$

Problem 6.

Proof. Let $A = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$, $B = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$, $C = \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$, and $D = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$. The state diagram for B^R is as below.



Thus, B^R is regular, by Problem 5, B is regular. $\square$

Problem 7.

Proof. Let $\Sigma = \Sigma_1 \cup \Sigma_2$. Change $M_1 = (Q_1, \Sigma_1, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma_2, \delta_2, q_2, F_2)$ to $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$,then do the proof like before. $\square$